

Cosmology with galaxy clusters

III. Gravitationally lensed arc statistics as a cosmological probe

Asantha R. Cooray

Department of Astronomy and Astrophysics, University of Chicago, Chicago, IL 60637, USA (e-mail: asante@hyde.uchicago.edu)

Received 18 August 1998 / Accepted 24 September 1998

Abstract. We calculate the expected number of gravitationally lensed optical, radio and sub-mm lensed sources on the whole sky due to foreground galaxy clusters for different cosmological models. We improve previous calculations of lensed arc statistics by including redshift information for background sources and accounting for the redshift evolution of the foreground lensing clusters. The background sources are described based on the redshift and optical magnitude or flux distribution for sources in the Hubble Deep Field (HDF). Using the HDF luminosity function, we also account for the magnification bias in magnitude-limited observational programs to find lensed optical arcs. The foreground lensing clusters are modeled as singular isothermal spheres, and their number density and redshift distribution is calculated based on the Press-Schechter theory with normalizations based on the local cluster temperature function.

Based on the results from optical arc surveys, we find that the observed number of arcs can easily be explained in a flat universe ($\Omega_m + \Omega_\Lambda = 1$) with low values for cosmological mass density of the universe ($\Omega_m \lesssim 0.5$). However, given the large systematic and statistical uncertainties involved with both the observed and predicted number of lensed arcs, more reliable estimates of the cosmological parameters are not currently possible. We comment on the possibility of obtaining a much tighter constraint based on statistics from large area optical surveys. At radio wavelengths (1.4 GHz), we predict ~ 1500 lensed radio sources with flux densities greater than $10 \mu\text{Jy}$, and with amplifications due to lensing greater than 2, in a flat cosmology with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. Given the recent detection of a sub-mm selected lensed μJy radio source towards A370, it is suggested that deep radio observations of clusters should contain such lensed sources. At sub-mm wavelengths (850 μm), the number of lensed sources expected towards the same foreground lens population and cosmology is $\sim 3 \times 10^4$. We briefly consider the possibility of using the South Pole 10-m sub-mm telescope and the Planck surveyor to identify lensed sub-mm sources. A catalog of around 100 gravitationally lensed sources at 353 GHz may be a useful by-product of Planck.

Key words: radio continuum: galaxies – cosmology: large-scale structure of Universe – cosmology: gravitational lensing – cosmology: theory – cosmology: observations – galaxies: clusters: general

1. Introduction

It is now well known that gravitational lensing statistics is a useful probe of the geometry of the universe, especially for the determination of the cosmological constant. In a recent paper (Cooray et al. 1998a; hereafter CQM), we calculated the expected number of gravitationally lensed sources in the Hubble Deep Field (HDF; Williams et al. 1996) due to foreground galaxies as a function of the cosmological parameters, and estimated these parameters based on the observed lensing rate in the HDF. The expected lensing rate was calculated based on the redshift distribution of HDF galaxies as determined by the photometric redshift catalogs. Similar to multiple lensing events due to foreground galaxies, clusters of galaxies lens background sources. Such lensed sources with high magnification appear as arcs, and the number statistics of gravitationally lensed arcs can be used to determine the cosmological parameters (e.g., Wu & Mao 1996; Bartelmann et al. 1998) and study the galaxy evolution at high redshifts (e.g., Bézecourt et al. 1998).

The number statistics of lensed optical arcs have been studied by Wu & Mao (1996), where they considered the effect of Ω_Λ on the predicted lensing rate, and by Bartelmann et al. (1998), where simulations of galaxy clusters were used to calculate the number of lensed sources. The former study relied on the spherical singular isothermal potential to describe foreground lensing clusters, while the latter study used the cluster potentials observed with numerical simulations. In between these two studies, Hamana & Futamase (1997) showed that the evolution of background source population can affect the lensing rate, while Hattori et al. (1997) refined the observed lensing rate by including observational effects, such as seeing.

In the present paper, we extend our previous work on the HDF (CQM) to estimate the number of lensed optical, radio and sub-mm lensed sources on the sky due to foreground galaxy clusters. We describe the background galaxies by the redshift and magnitude or flux distribution of sources in the HDF. We also assume that the HDF luminosity function, as determined by Sawicki et al. (1997), is a valid description of the distant universe. Thus, one of the main differences between the present paper and previous studies involving arc statistics is that we use individual redshifts to calculate lensing probabilities, and use magnitude information to account for various systematic effects,

especially magnification bias present in magnitude-limited optical search programs to find lensed arcs towards galaxy clusters. A main difference between CQM and the present work is that we now describe the number density of foreground lensing objects, galaxy clusters, and their evolution using the Press-Schechter theory (PS; Press & Schechter 1974), normalized to the local cluster abundance.

Similar to optical arcs, galaxy clusters are also expected to lens background radio sources. Such lensed sources with high magnification should appear as arcs in radio surveys. The number statistics of lensed radio sources can be used to determine the cosmological parameters, to study the radio source evolution at high redshifts, and as discussed later, properties of star forming galaxies at moderate to high redshifts. The number statistics of lensed radio sources due to foreground clusters were first calculated by Wu & Hammer (1993). They predicted ~ 10 lensed radio sources on the sky down to a flux density limit of 0.1 mJy, and ~ 100 lensed radio sources down to 10 μ Jy at 2.7 GHz (Fig. 10 in Wu & Hammer 1993). At the source detection level of the VLA FIRST survey (~ 1 mJy; Becker et al. 1995), there are only ~ 2 to 3 lensed radio sources expected on the whole sky, and when compared to the area of the survey and its resolution, it is likely that there is no lensed source present. This prediction is compatible with observational attempts to find lensed radio sources; Andernach et al. (1997) searched the FIRST survey near Abell cluster cores and found no convincing candidates, and a statistical analysis of the radio positions towards clusters showed no preferential tangential orientation, as expected from gravitational lensing. Recently, a sub-mm selected source, SMM02399-0136, towards cluster A370 was found to be lensed with an amplification of 2.5 (Iverson et al. 1998). The source was detected at 1.4 GHz, with a flux density of $\sim 525 \mu$ Jy. This detection prompted us to calculate the expected number of lensed μ Jy sources present on the sky due to foreground clusters, and to refine the previous predictions in Wu & Hammer (1993). Since the predictions in Wu & Hammer (1993) for sources down to mJy level are still expected to be valid, we will only concentrate on the μ Jy sources here.

We also extend our calculation to estimate the number of expected sub-mm sources on the whole sky due to foreground clusters. Our calculation is prompted by recent observational results from the new Sub-millimeter Common-User Bolometer Array (SCUBA; see, e.g., Cunningham et al. 1994) on the James Clerk Maxwell Telescope, where a sample of gravitationally lensed sub-mm sources has now been observed by Smail et al. (1997, 1998). The gravitational lensing of background sub-mm sources due to foreground galaxy clusters was first studied by Blain (1997), using a model of a lensing cluster with predicted source counts for background sources. Blain (1997) showed that the surface and flux densities of lensed sources exceed those of galaxies within the lensing cluster, and their values. This behavior, primarily due to the slope of the source counts and the fact that the distance sources are intrinsically brighter at sub-mm wavelengths, has now allowed the observation of moderate to high redshift dusty star forming galaxies, which are amplified through the cluster potentials (e.g., Iverson et al. 1998). Even

though lensing of sub-mm sources has been studied in literature, no clear prediction has been made for the total number of sources lensed due to foreground clusters. Also, past calculations have relied mostly on models of background source counts that were based on different evolutionary scenarios for star forming galaxies.

In Sect. 2 we discuss our calculation and its inputs. In Sect. 3 we present the expected number of optical, radio and sub-mm lensed sources, and in Sect. 4 we outline possible systematic errors involved in our calculational method. In Sect. 5, we compare our predicted number of optical arcs in the whole sky to the observed number of arcs in the Le Fèvre et al. (1994) cluster sample. In the same section we discuss the possibility of large area optical arc survey, using the Sloan Digitized Sky Survey (SDSS; Loveday & Pier 1998) as an example. In Sect. 5 discuss the possibility of detecting lensed μ Jy sources and sub-mm sources. A summary is presented in Sect. 6. We follow the conventions that the Hubble constant, H_0 , is $100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$, the present mean density in the universe in units of the closure density is Ω_m , and the present normalized cosmological constant is Ω_Λ . In a flat universe, $\Omega_m + \Omega_\Lambda = 1$.

2. Galaxy clusters as lenses

In order to calculate the lensing rate for background galaxies due to foreground galaxy clusters, we model the lensing clusters as singular isothermal spheres (SIS) and use the analytical filled-beam approximation (see, e.g., Fukugita et al. 1992). In the case of the optical arcs, the lensed sources towards clusters have all been imaged in magnitude limited optical search programs. Such observational surveys are affected by the so-called ‘‘magnification bias’’ (see, e.g., Kochanek 1991), in which the number of lensed sources in the sample is larger than it would be in an unbiased sample, because lensing brightens sources that would otherwise not be detected. Thus, any calculation involving lensed source statistics should account for the magnification bias and associated systematic effects.

We refer the reader to CQM for full details of our lensing calculation involving foreground galaxies as lensing sources. Following CQM, if the probability for a source at redshift z to be strongly lensed is $p(z, \Omega_m, \Omega_\Lambda)$, as calculated based on the filled-beam formalism, we can write the number of lensed sources, $d\bar{N}$, with amplification greater than A_{\min} as:

$$\begin{aligned} \bar{N} &= \langle F(z_l) \rangle \times \sum_i p(z_i, \Omega_m, \Omega_\Lambda) \\ &\times \int_{A_{\min}}^{\infty} A^{-1-\alpha(z_i)} e^{L_i/L^*(z_i)} e^{-L_i/AL^*(z_i)} \\ &\times \Theta(m) \frac{2}{(A-1)^3} dA \\ &\equiv \langle F(z_l) \rangle \sum_i \tau(z_i), \end{aligned} \quad (1)$$

where the integral is over all values of amplification A greater than A_{\min} , and $\alpha(z)$ and $L^*(z)$ are parameters of the HDF luminosity function at various redshifts as determined by Sawicki et al. 1997. Here, the sum is over each of the galaxies in our sample. The index i represents each galaxy; hence, z_i and L_i are, respectively, the redshift, and the rest-frame luminosity of the i th galaxy. The step function, $\Theta(m)$, takes into account the lim-

iting magnitude, m_{lim} , of a given optical search to find lensed arcs in the sky, such that only galaxies with lensed magnitude brighter than the limiting magnitude is counted when determining the number of lensed arcs. Since rest-frame luminosities of individual galaxies are not known accurately due to uncertain K-corrections, as in CQM, we estimate the average amplification bias by summing the the expectation values of $\tau(z_i)$, which were computed by weighting the integral in above equation by a normalized distribution of luminosities L_i drawn from the Schechter function at redshift z_i .

The probability of strong lensing depends on the number density and typical mass of foreground objects, and is represented by the $\langle F(z_l) \rangle$ in above Eq. 1, where $\langle F(z_l) \rangle$ is the mean value over the lens redshift distribution, z_l . For SIS models, this factor can be written by the dimensionless parameter (CQM):

$$F \equiv 16\pi^3 n R_0^3 \left(\frac{\sigma_{\text{vel}}}{c} \right)^4, \quad (2)$$

where n is the number density of lensing objects, $R_0 \equiv c/H_0$, and σ_{vel} is the velocity dispersion in the SIS scenario. In general, the parameter F is independent of the Hubble constant, because the observationally inferred number density is proportional to h^3 . In the present calculation, the foreground objects are galaxy clusters, and thus n represent the number density of clusters and σ_{vel} represent their velocity dispersion. Since the number density of clusters in different cosmological models are expected to vary, we calculate the number density of galaxy clusters, $dn(M, z)$, between mass range $(M, M + dM)$ using a PS analysis (e.g., Lacey & Cole 1993):

$$\frac{dn(M, z)}{dM} = -\sqrt{\frac{2}{\pi}} \frac{\bar{\rho}(z)}{M} \frac{d\sigma(M)}{dM} \frac{\delta_{c0}(z)}{\sigma^2(M)} \times \exp \left[\frac{-\delta_{c0}^2(z)}{2\sigma^2(M)} \right] \quad (3)$$

where $\bar{\rho}(z)$ is the mean background density at redshift z , $\sigma(M)$ is the variance of the fluctuation spectrum filtered on mass scale M , and $\delta_{c0}(z)$ is the linear overdensity of a perturbation which has collapsed and virialized at redshift z . Following Mathiesen & Evrard (1998; Appendix A), we can write $\delta_{c0}(z)$ as:

$$\delta_{c0} = \frac{3}{20} (12\pi)^{\frac{2}{3}} (1+z) \quad (4)$$

when $\Omega_m = 1$, and

$$\delta_{c0}(z) = \frac{D(0)}{D(z)} \delta_c(z) \quad (5)$$

when $\Omega_m + \Omega_\Lambda = 1$ (flat) and $\Omega_\Lambda = 0$ (open). Here $D(z)$ is the linear growth factor and $\delta_c(z)$ is the critical overdensity.

For an open universe ($\Omega_\Lambda = 0$), $\delta_c(z)$ can be written as (Lacey & Cole 1993):

$$\delta_c(z) = \frac{3}{2} D(z) \left(1 + \frac{2\pi}{\sinh(\eta) - \eta} \right)^{2/3} \quad (6)$$

$$D(0) = 1 + \frac{3}{x_0} + \frac{3\sqrt{1+x_0}}{x_0^{3/2}} \ln(\sqrt{1+x_0} - \sqrt{x_0}) \quad (7)$$

where $x_0 \equiv \Omega_0^{-1} - 1$ and $\eta \equiv \cosh^{-1}(2/\Omega(z) - 1)$.

For a flat universe with $\Omega_0 + \Omega_\Lambda = 1$, $\delta_c(z)$ was parameterized in Mathiesen & Evrard (1998) as:

$$\delta_c(z) = 1.68660[1 + 0.01256 \log \Omega(z)], \quad (8)$$

which was derived by Kitayama & Suto (1997). The $\delta_{c0}(z)$ for a flat universe was calculated using the linear growth factor found in Peebles (1980),

$$D(x) = \frac{\sqrt{(x^3+2)}}{x^{3/2}} \int_0^x x_1^{3/2} (x_1^3+2)^{-3/2} dx_1 \quad (9)$$

where $x = a/a_e$, and $a_e = [(1-\Omega_\Lambda)/(2\Omega_\Lambda)]^{1/3}$, the inflection point in the scale factor. This function was integrated numerically to find the growth factor at redshifts z and 0.

In addition to growth factors suggested by Mathiesen & Evrard (1998), our calculation uses power-spectrum normalizations deduced by Viana & Liddle (1996) for $\sigma_8(\Omega_m)$ based on cluster temperature function:

$$\sigma_8 = 0.60 \Omega_m^{0.36+0.31\Omega_m-0.28\Omega_m^2}, \quad (10)$$

when $\Omega_\Lambda = 0$, and

$$\sigma_8 = 0.60 \Omega_m^{0.59-0.16\Omega_m+0.06\Omega_m^2}, \quad (11)$$

when $\Omega_m + \Omega_\Lambda = 1$. We have also assumed a scale-free power spectrum $P(k) \propto k^n$ with $n = -1.4$ ($\alpha = \frac{n+3}{6} \sim 0.27$), which corresponds to a power spectrum shape parameter Γ of ~ 0.25 in CDM models.

In order to calculate the parameter $\langle F(z_l) \rangle$, we also require knowledge of cluster velocity dispersion, σ_{vel} , which is the velocity dispersion of clusters in the SIS model. We assume that the σ_{vel} is same as the measured velocity dispersion for galaxy clusters based on observational data. To relate σ_{vel} with cluster mass distribution, we use the scaling relation between σ_{vel} and cluster temperature, T , of the form (Girardi et al. 1996):

$$\sigma_{\text{vel}}(T) = 10^{2.56 \pm 0.03} \times \left(\frac{T}{\text{keV}} \right)^{0.56 \pm 0.05}, \quad (12)$$

and the relation between T and cluster mass M (Bartlett 1997; see also Hjorth et al. 1998):

$$T(M, z) = 6.4 h^{2/3} \left(\frac{M}{10^{15} M_\odot} \right)^{\frac{2}{3}} (1+z) \text{ keV}, \quad (13)$$

to derive a relation between σ_{vel} and cluster mass M , $\sigma_{\text{vel}}(M)$. Finally, we can write the interested parameter F as a function of the lens redshift, z_l :

$$F(z_l) = \frac{16\pi^3}{cH_0^3} \int_{M_{\text{min}}}^{\infty} \sigma(M')^4 \frac{dn(M', z_l)}{dM'} dM', \quad (14)$$

and was numerically calculated, in addition to the above, by weighing over the redshift distribution of galaxy clusters derived based on the PS theory to obtain $\langle F(z_l) \rangle$, the mean value of $F(z_l)$, which is used in Eq. 1.

In order to compare the predicted number of bright arcs towards clusters with the observed number towards a X-ray luminosity, L , selected sample of galaxy clusters, we also need a relation between M and L . We obtain this relation based on the observed $L - T$ relation recently derived by Arnaud & Evrard (1998):

$$L = 10^{45.06 \pm 0.03} \times \left(\frac{T}{6 \text{ keV}} \right)^{2.88 \pm 0.15} 0.25 h^{-2} \text{ ergs s}^{-1}, \quad (15)$$

where L is the X-ray luminosity in the 2-10 keV band, and $M - T$ relation in Eq. 13. Since we will be comparing the predicted number of lensed arcs to the observed number, we will be setting the minimum mass scale, M_{\min} , which corresponds to the minimum luminosity, L_{\min} , of clusters in optical search programs to find lensed arcs. The luminosity cutoff of the EMSS cluster arc survey by Le Fèvre et al. (1994) is $8 \times 10^{44} h^{-2} \text{ ergs s}^{-1}$, which is measured in the EMSS band of 0.3 to 3.5 keV. By comparing the tabulated luminosities of EMSS clusters in Nichol et al. (1997), Mushotzky & Scharf (1997), and Le Fèvre et al. (1994), we evaluate that this luminosity, in general, corresponds to a luminosity of $\sim 12.8 \times 10^{44} h^{-2} \text{ ergs s}^{-1}$ in the 2 to 10 keV band. This luminosity is calculated under the assumption of $\Omega_m = 1$, and for different cosmological parameters, it is expected that the value will change as the luminosity distance relation is dependent on Ω_m and Ω_Λ . However, for clusters in the EMSS arc survey, with a mean redshift of 0.32, such variations are small compared to the statistical and systematic uncertainties in the scaling relations used in the calculation.

Ignoring various small changes due to the choice of cosmological model, we use a minimum mass M_{\min} of $8.8 \times 10^{14} h^{-1} M_\odot$, corresponding to above L_{\min} . Using the numerical values for $\sigma(M)$ and $dn(M, z)$, and performing numerical integrations we find $\langle F(z_l) \rangle$ to range from $\sim 3.7 \times 10^{-6}$ when $\Omega_m = 1$ to $\sim 2.8 \times 10^{-4}$ when $\Omega_m = 0.2$. The error associated with $\langle F(z_l) \rangle$ is rather uncertain. For example, the quoted random uncertainty in σ_8 from Viana & Liddle (1996) is $^{+37\%}_{-27\%}$. It is likely that $\langle F(z_l) \rangle$ has an overall statistical uncertainty of $\sim 50\%$, however, as we discuss later, there could also be systematic errors in our determination.

3. Predicted numbers

3.1. Optical arcs

In order to calculate the expected number of lensed arcs on the sky, we use the photometric redshift catalog by Sawicki et al. (1997) for HDF galaxies. This catalog contains redshift information for 848 galaxies and is complete down to a magnitude of 27 in I-band. However, HDF allows detection of sources down to a magnitude limit of 28.5 in I-band, and contains 1577 sources down to 28 in I-band, excluding 43 apparent stars (Sawicki et al. 1997). We use this extra information and complemented the photometric redshift catalog by equally distributing the additional number of optical sources between I-band magnitudes of 27 and 28, and between redshifts of 0 and 5. Since these sources are not expected to be at very low redshifts, where lensing probability is small, we do not expect to have created a systematic

bias in our study, other than perhaps underestimate the lensing rate, if all these sources were in fact at high redshifts. Also, since these additional sources have very low magnitudes, at the limit of HDF, we do not expect these sources to make a large contribution to the total number of lensed arcs when the limiting magnitude of lensed search programs are at the bright end. However, in order to calculate the true number of arcs at faint magnitudes, it is essential that these sources be accounted for. The HDF galaxies are within an area of 4.48 arcmin^2 . We extrapolate the predicted number of lenses in the HDF to the whole sky, by assuming that HDF is an accurate description of the distant universe everywhere on the sky. Since HDF was carefully selected to avoid bright sources, it is likely that we have missed a large number of low redshift galaxies, but, such galaxies are not expected to contribute to the lensing rate.

We have calculated the expected number of gravitationally lensed arcs in by using Eq. (1) as a function of Ω_m and Ω_Λ , and using A_{\min} of 10. Since we are using the SIS model, the amplification is simply equal to the ratio of length to width in observed lensing arcs (see, e.g., Wu & Mao 1996), allowing us an easy comparison between observed number of arcs with length to width greater than 10 in Le Fèvre et al. (1994) survey. In Table 1, we list the expected number of strongly lensed arcs in the sky for different Ω_m and Ω_Λ values, together with the number of lensed sources at radio and sub-mm wavelengths.

3.2. Lensed radio sources

In order to describe the background μJy sources, we describe the redshift and number distribution observed towards the HDF by Richards et al. (1998). The main advantage in using the HDF data is the availability of redshift information for μJy sources. Also, HDF is one of the few areas where a deep radio survey down to a flux limit of $\sim 2 \mu\text{Jy}$ at 1.4 GHz has been carried out. The HDF contains 14 sources with flux densities of the order ~ 6 to $500 \mu\text{Jy}$ at 8.5 GHz, and 11 of these sources have measured spectroscopic redshifts. We converted the 8.5 GHz flux densities to 1.4 GHz using individual spectral indices as presented by Richards et al. (1998). For sources with no measured spectral indices, we assumed an index of 0.4, the mean spectral index observed for μJy sources (Fomalont et al. 1991; Windhorst et al. 1993; Richards et al. 1998). For the 3 sources with no measured spectroscopic redshifts, we used photometric redshifts from the catalog of Fernández-Soto et al. (1998). We binned the redshift-number distribution in redshift steps of 0.25, and calculated the lensing probability using filled-beam formalism. The predicted number are tabulated in Table 1 for minimum amplifications of 2 and 10 respectively, and down to a flux limit of $10 \mu\text{Jy}$ at 1.4 GHz.

3.3. Lensed sub-mm sources

In order to describe the background sub-mm sources, we again use the redshift and number distribution observed towards the HDF sources by Hughes et al. (1998). The HDF contains 5 sources with flux densities of the order ~ 2 to 7 mJy . Hughes et

Table 1. Predicted number of lensed optical, radio, and sub-mm sources on the sky due to foreground clusters. N_{Planck} is the expected number of lensed sub-mm sources, with flux densities greater than 50 mJy at 850 μm , towards clusters that are expected to be detected with Planck Surveyor (see Sect. 5.3.1). The horizontal lines across the two optical arc columns contain the current range of observed arc statistics (see Sect. 5.1.1).

Ω_m	Ω_Λ	Optical ($A_{\text{min}} \geq 10$)		Radio ($f_{1.4 \text{ GHz}} \geq 10 \mu\text{Jy}$)		Sub-mm ($f_{850 \mu\text{m}} \geq 2 \text{ mJy}$)		
		$I_{\text{mag}} \leq 22$	$I_{\text{mag}} \leq 25$	$A_{\text{min}} \geq 2$	$A_{\text{min}} \geq 10$	$A_{\text{min}} \geq 2$	$A_{\text{min}} \geq 10$	N_{Planck}
0.1	0.0	2190	8329	1964	24	14815	183	115
0.2	0.0	935	1880	1188	15	9910	123	75
0.3	0.0	496	1880	631	8	6010	75	55
0.4	0.0	267	1010	343	4	3635	49	32
0.5	0.0	150	568	194	3	2270	28	21
0.6	0.0	87	329	113	1.5	1450	18	13
0.7	0.0	52	197	66	1	790	10	9
0.8	0.0	32	122	42	0.5	485	6	7
0.9	0.0	20	81	27	0.3	310	4	4
1.0	0.0	15	55	17	0.2	225	3	2
0.1	0.9	16462	60197	15012	180	270350	3340	2150
0.2	0.8	7017	25668	6105	74	101640	1255	785
0.3	0.7	2970	10870	1442	17	37835	468	290
0.4	0.6	1367	5010	569	7	15050	186	105
0.5	0.5	681	2488	298	4	6470	80	45
0.6	0.4	353	1294	163	2	2975	37	25
0.7	0.3	192	702	94	1	1440	18	15
0.8	0.2	111	496	58	1	735	9	7
0.9	0.1	67	245	37	0.5	390	5	4

al. (1998) studied the probable redshifts of the detected sources by considering the optical counterparts and assigning probabilities for likely associations. In Table 1, we list the expected number of lensed sources on the sky for $A_{\text{min}} = 2$ and 10. We have also tabulated the expected number of lensed sources towards clusters in Planck all sky survey data, as described in Sect. 5.3.1.

4. Systematic errors

Our lensing rate calculation relies on the assumption that the HDF is a reasonable sample of the distance universe and that it can be applied to the whole sky. In the case of optical arcs, we have included an additional number of faint sources to the photometric redshift catalog, and by doing so, may have introduced a systematic bias in our calculation. However, unless these sources are at either low or high redshift, we do not expect such sources to make a large change in the lensing rate. Also, there is a possibility that certain multiple sources, which we have counted as separate objects, may in fact represent star-forming regions within individual galaxies (e.g., Colley et al. 1997). If this is true, we may have overestimated the number of sources by as much as $\sim 40\%$, and may have caused a systematic increase in the lensing rate.

Another possible systematic error is involved with the determination of the $\langle F(z_l) \rangle$ parameter. We have used the PS theory normalized to local cluster abundance and relations between velocity dispersion, cluster temperature, mass and luminosity to

calculate $\langle F(z_l) \rangle$. The used scaling relations, as well as parameters in the PS function, have in some cases large uncertainties. It is likely that our predicted numbers may be accurate to within 40% to 50%. Other than statistical errors, there may also be systematic uncertainties. For example, the $M - T$ relation may have additional dependences on the cosmological parameters (see, e.g., Voit & Donahue 1998), which we have not fully considered. Since $\langle F(z_l) \rangle$ was inferred based on PS function normalized to observations, and since these observables depend on the assumed cosmology, $\langle F(z_l) \rangle$ will also depend on it. The dependence of the inferred $\langle F(z_l) \rangle$ on cosmology also depends on the scaling relations, as well as the lower limit of the luminosity used in the PS calculation, which varies with cosmology. As suggested earlier, for the most part, we can ignore such small changes due to the choice of cosmological model in our scaling relations and other observables; there are much larger statistical and systematic errors in our calculation involving the normalization of the PS function etc.

Even though we have used the PS theory to account for redshift evolution, it is possible that we have only partially accounted for evolutionary effects. For example, we have not taken into account the effects of Ω_Λ on cluster formation, where clusters are expected to be less compact in a universe with Ω_Λ than in a universe with $\Omega_\Lambda = 0$. Thus, our analytical calculation is different from the numerical study of Bartelmann et al. (1997), where effects of Ω_Λ on cluster structures are accounted based on numerical simulations. Our lens model is too simple to allow such effects, and by ignoring this important fact, we have

included an additional simplification in our present analysis. Bartelmann et al. (1997) found that the observed number of arcs can be explained in an open universe, while with a cosmological constant the number predicted is smaller than the observed statistics. However, we note that the results from Bartelmann et al. (1997) may be in conflict with present estimates of cosmological parameters based on other methods, which suggest a flat universe with non-zero Ω_Λ . In comparison, we find that the number predicted in an open universe is not enough to fully account for the observed statistics, unless $\Omega_m \sim 0.1$. Also, we find a considerably large number of arcs with a Ω_Λ dominated universe ($\Omega_\Lambda \sim 0.9$), which has been considered in the past to account for lensed arcs statistics (e.g., Wu & Mao 1996).

We have assumed that clusters can be described by singular isothermal spheres. However, for high amplification events such as arcs, substructures within clusters are important; substructures are also responsible for aspherical potentials. Such potentials have been considered important for lensing studies of individual clusters (e.g., Bézecourt 1998), where it has been shown that the true lensing rate can be as high as a factor of 2 from spherical potentials. However, it is likely that such biases may only exist for a small number of clusters, and thus, for overall statistics of lensed arcs, complex potentials can be ignored; a conclusion also supported by the numerical simulations of clusters and lensed arcs. However, for certain clusters, especially the ones that have been systematically studied in detail due to the large number of arcs, which includes A2218, A370 and A1689, complex potentials are important to model the individual arc distribution.

5. Discussion

5.1. Lensed optical arcs

5.1.1. Ω_m from observations and predictions

According to Wu & Mao (1996), there are 9 arcs towards 39 clusters with $L > 8 \times 10^{44} h^{-2} \text{ ergs s}^{-1}$ or roughly 0.2 to 0.3 arc per cluster in the bright EMSS arc surveys (Le Fèvre et al. 1994; Gioia & Luppino 1994). The current predictions for total number of clusters matching the criteria of EMSS arc survey clusters range from ~ 7500 to 8000 (see, e.g., Bartelmann et al. 1997). Thus we expect a total of ~ 1500 to 1900 such arcs. This estimate ignores the observational systematic effects in search programs, including observational constraints such as finite seeing (see, e.g., Hattori et al. 1997). Since, the result of such effects is to reduce the observed number, after making an additional correction, we estimate a total number of 1500 to 2500 arcs on the sky, which is slightly higher than the estimate made by Bartelmann et al. (1997). We find that our prediction is roughly in agreement with the observed number when $\Omega_m \lesssim 0.4$ in a flat universe or $\Omega_m \sim 0.1$ in an open universe. The range of Ω_m values when $\Omega_m + \Omega_\Lambda = 1$ is in agreement with our previous estimate based on the strong lensing rate in the HDF ($\Omega_m - \Omega_\Lambda > -0.39$ 95% C.I. CQM; also, Kochanek 1996), estimates of cosmological parameters based on the high redshift type Ia supernovae (Riess et al. 1998; $\Omega_m - \Omega_\Lambda \sim -0.5 \pm 0.4$), and galaxy cluster baryonic fraction (Evrard 1997).

However, in order to derive tighter constraints on the cosmological parameters, we need to consider both the statistical and systematic errors in the present calculation, as well as the observed number of lensed arcs. In general, we find that the predicted number in a Ω_m dominated universe ($\Omega_m > 0.6$) cannot be used to explain the observed number of lensed arcs, even when we consider the extreme errors in our calculation and the observed statistics.

5.1.2. Future outlook

We have predicted roughly 1500 to 3000 lensed arcs on the sky with I-band magnitudes greater than 22 towards foreground massive clusters. In order to use arc statistics as a probe of the cosmological parameters, it is necessary that reliable results from a large area survey be used. The current observed statistics on lensed arcs come from the optical observations towards X-ray selected clusters in the EMSS sample, which covers an area of ~ 750 sq. degrees. In the near future, the Sloan Digitized Sky Survey (SDSS) will take both imaging and spectroscopic data over π steradians of the sky. It is likely that the SDSS will image most of the foreground massive clusters, similar to the ones that we have considered here. The optical data from this survey are expected to allow detection of sources down to the I band magnitude of 22. The imaging data will be limited by seeing effects, which is expected to limit the image resolutions to between 1.1 and 1.4 arcsecs. For the purpose of finding lensed luminous arcs, the seeing effects would be of a minor concern; the spatial extent of lensed arcs with length-to-width ratios greater than 10 are not likely to be heavily affected by observational effects. Based on our predictions, it is expected that there will be roughly 375 to 750 arcs in the SDSS imaging data. However, there are various practical limitations which will affect the search for lensed arcs in SDSS data. Especially due to the large volume of data, it is unlikely that one would be able to select lensed arcs by just looking at the images; specific algorithms to find lensed arcs are needed. By testing such algorithms against simulated data, it is likely that selection effects involved in the arc search process can be properly studied. By considering such selection effects and the observed lensing rate of luminous arcs, it may be possible in the future to obtain reliable estimates on the cosmological parameters based on arc statistics.

5.2. Lensed radio sources

We have predicted ~ 1500 lensed μJy sources, with $A > 2$, for a cosmology with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. The number with $A_{\text{min}} > 4$ for the same cosmology is ~ 200 . When compared with the lensing rate for optical arcs down to I-band magnitude of 22 and amplifications greater than 10, we predict a similar, or slightly lower, rate for the μJy sources, down to a flux density limit of $10 \mu\text{Jy}$.

In comparison, Wu & Hammer (1993) predicted ~ 100 sources down to $10 \mu\text{Jy}$ towards clusters. They performed this calculation for a cosmological model of $\Omega_m = 1$, and using the X-ray luminosity function of Edge et al. (1990). For the same

cosmological model, we predict ~ 0.2 sources with amplifications greater than 10. The difference between two predictions is primarily due to the description of the background sources. We have used redshift information, while Wu & Hammer (1993) used the radio luminosity function with no evolution assumption, an assumption which may have overestimated the number of lensed sources. There are also other differences between the two methods. For example, we have accounted for the galaxy cluster evolution for different cosmological models using PS theory, where the number of available foreground lensing clusters strongly decreases with an increase in the cosmological mass density, Ω_m . Such changes have not been accounted in the previous calculation.

5.2.1. Possibility of detection

Unlike optical surveys, radio surveys with interferometers such as the VLA and the MERLIN are subjected to effects arising from instrumental limitations, primarily effects associated with resolution. For example, there is a minimum and a maximum size for sources that can be detected and resolved with an interferometer. The largest angular scale to which the interferometer is sensitive restricts the detection of high amplification sources, which are expected to appear as arcs, with length to width ratios equal to amplification factors. For the VLA A-array at 1.4 GHz, sources larger than $\sim 15''$ are not likely to be detected. Thus, observations of radio arcs with length to width ratios greater than 10 may not easily be possible. In SIS model for gravitational lensing, most of the lensed sources appear with amplification factors of 2 to 10. However, due to the convolution with synthesized beam, ranging from $\sim 1''$ to $5''$, such sources are not likely to appear as arcs. Therefore, detection of lensed sources with small amplifications are likely to be confused with foreground and cluster-member radio sources, requiring a selection process to remove such confusing sources. Most of the confusion is likely to come from cluster member sources, rather than the foreground sources, as there is an overabundance of radio sources in clusters relative to random areas of the sky. As discussed in Cooray et al. (1998b), based on cluster observations at 28.5 GHz, this overabundance is likely to be high as factors of 5 to 7. It is likely that this overabundance exists at low frequencies such as 1.4 GHz. However, certain cluster member sources may easily be identified through source properties and appearances; sources such as wide-angle tail sources are usually found in cluster environments with dense IGM. Such an analysis may be limited to few types of sources, and there is no direct radio property, such as the radio spectral index or luminosity, that can be used to separate cluster member sources from background ones. The identification process of candidate lensed sources needs to consider the optical counterparts of radio sources; a joint analysis between radio and optical data may be required to recover the background radio sources lensed through galaxy cluster potentials. Additional observations, especially redshifts may be required to establish the lensed nature of μJy sources selected towards clusters. This is contrary to optical searches, where

lensed galaxies can easily be established due to their arc-like appearances.

By considering the ratio between observed number of optical arcs and arclets and the ratio of surface density of optical to μJy sources, we expect to find \sim a total of 4 to 6 lensed μJy sources down to $10 \mu\text{Jy}$ at 1.4 GHz towards A2218 and A370. For A370, one such source has already been recovered (Ivison et al. 1998), through the sub-mm observations of Smail et al. (1997). The VLA A-array 1.4 GHz data (Owen & Dwarakanath, in prep.), in which the source was detected allows detection of sources down to a flux limit of $50 \mu\text{Jy beam}^{-1}$ (5σ). A quick analysis of the same archival data suggests that there is at least one more μJy lensed source towards A370 (Cooray et al., in prep.). It is likely that deep surveys of galaxy clusters with MERLIN and VLA will allow detection of μJy radio sources with amplifications of 2 to 10.

As discussed in Richards et al. (1998; see, also, Cram et al. 1998), μJy sources carry important information on the star formation rate and history. Thus, observational searches for lensed sources are expected to allow detection of moderate to high redshift star-forming galaxies. The search for such galaxies will be aided by the amplification due to gravitational lensing, allowing detections of faint sources, below the flux limits of regular surveys. It is likely that a careful analysis of lensed μJy sources will allow the study of star formation at moderate to high redshift galaxies. Also, the low redshift μJy sources, associated with spiral galaxies are not likely to be found through clusters, due to the low lensing rate. Based on our predictions and the detection of lensed sources towards A370, we strongly recommend that deep radio observations of lensing clusters be carried out to find lensed sources and that such detections be followed up at other wavelengths.

5.3. Lensed sub-mm sources

We have predicted $\sim 3 \times 10^4$ lensed sub-mm sources with flux densities greater than 2 mJy at $850 \mu\text{m}$, and with amplifications greater than 2, for a cosmology with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. The number with $A_{\text{min}} > 4$ for the same cosmology is ~ 3100 , while the number with $A_{\text{min}} > 10$ is ~ 500 . We predict a lensing rate of ~ 4 sources per cluster with amplifications greater than 2 down to a flux limit of 2 mJy.

We compare our predicted number of lensed sources to the observed number towards a sample of galaxy clusters imaged with the SCUBA by Smail et al. (1997, 1998). This sample contains 7 clusters with redshifts in the range ~ 0.2 to 0.4. All of these clusters are well known lensing clusters in the optical wavelengths. Unfortunately, this sample is incomplete either in terms of X-ray luminosity or total mass. This incompleteness doesn't allow us to perform a direct comparison between the predicted and observed numbers. Out of the 7 clusters, 3 clusters have X-ray luminosities greater than the lower limit imposed in our calculation. Towards these three clusters, A370, A2390 & A1835, there are 8 sub-mm sources, all of which may be gravitationally lensed. This implies a total of $\sim 2 \times 10^4$ lensed sub-mm sources on the whole sky. Based on our lensing rate, we

expect ~ 6 lensed sources towards 3 clusters; this exact number is strongly sensitive to the cosmological parameters. Here, we have assumed a spatially-flat cosmological model with $\Omega_m = 0.4$ and $\Omega_\Lambda = 0.6$. The predicted and observed numbers seem to be in agreement with each other for low Ω_m values in a flat universe ($\Omega_m + \Omega_\Lambda = 1$).

However, we cannot use the present observational data to derive cosmological parameters for several reasons. These reasons include source contamination in the lensed source sample and systematic biases in the foreground cluster sample. For example, it is likely that the lensed source sample presented by Smail et al. (1998) contain foreground and cluster-member sources. Since the foreground or cluster-member sources are less bright than the background lensed sources, this contamination is likely to be small (see, Blain 1997). An additional systematic bias comes from the selection effects associated with the foreground cluster sample. Since the observed clusters are well known lensing clusters with high lensing rates at optical wavelengths, it is likely that there may be more lensed sub-mm sources towards these clusters than generally expected. Therefore, it is likely that the Smail et al. (1998) sample is biased towards a higher number of lensed sub-mm sources.

In order to constrain cosmological parameters based on statistics of lensed sub-mm sources, results from a complete sample of galaxy clusters, preferably from a large area survey, are needed. Further SCUBA observations of galaxy clusters, perhaps the same cluster sample as the Le Fèvre et al. (1994) sample, would be helpful in this regard. However, such a survey will require a considerable amount of observing time, suggesting that current instruments may not be able to obtain the necessary statistics. However, in the near future there will be two opportunities to perform a large area sub-mm survey of galaxy clusters: the Planck Surveyor and the South Pole 10-m sub-mm telescope.

5.3.1. Survey opportunities

South Pole 10 m sub-mm telescope—The planned South Pole (SP) 10-m sub-mm telescope¹ is expected to begin observations around year 2003 (see, Stark et al. 1998). At $850 \mu\text{m}$, it is expected that within ~ 90 hours a square degree area will be surveyed down to a flux limit of 1 mJy. Given the resolution and flux sensitivity, it is likely that the SP telescope would be an ideal instrument to survey either a sample of clusters or random areas to obtain lensed source statistics down to few mJy. To obtain reliable values of the cosmological parameters based on the sub-mm lensed source statistics, a survey of several hundred square degrees down to few $\times 1$ mJy will be needed. A more direct approach within a reasonable amount of observing time would be to survey a carefully selected sample of galaxy clusters, either based on X-ray luminosity or total mass, from which lensed source statistics can easily be derived.

Planck Surveyor—Considering the amplification distribution for SIS lens model, and the number counts defined by Scott &

White (1998), we find that roughly 100 lensed sub-mm sources may be detected with the Planck Surveyor towards galaxy clusters². In Table 1, we list the number expected as a function of the cosmological parameters and assuming that the Planck data will allow detection of sources down to 50 mJy. However, given the limited observational data on source counts at $850 \mu\text{m}$, we note that the predicted numbers may have large errors. We also note that the Planck data will be highly confused, as the beam size of Planck is \sim few arcmins at $850 \mu\text{m}$; even with ~ 2 arcmin physical pixels for high signal-to-noise data, most of the sources down to 50 mJy would be separated by only one or two pixels. Assuming pixel sizes of the order beam size, the probability of finding two sources with flux densities greater than 50 mJy in one Planck pixel would be ~ 0.2 to 0.3. Thus, it is more likely that the Planck data will allow clear detection of sources down to ~ 100 mJy, but with additional information, such as from other frequency channels and filtering techniques (see, e.g., Tegmark & de Oliveira-Costa 1998), it may be possible to lower this flux limit.

Also, it is likely that the lensed background sources will contaminate the detection of Sunyaev-Zel'dovich (SZ) effect in galaxy clusters (see, Aghanim et al. 1997; Blain 1998). Given the source confusion and contamination, it is likely that that Planck data would not readily allow an adequate determination of lensed sub-mm source statistics to constrain cosmological parameters. It is more likely that the lensed sub-mm source catalog from Planck would be an important tool to study the star-formation history at high redshifts; since lensing brightens sources, such a lensed source catalog will contain sub-mm sources fainter than the current limit predicted to be observable with Planck for unlensed sources.

6. Summary

Using the redshift and magnitude, or flux, distribution observed towards the HDF to describe background sources and Press-Schechter theory and singular isothermal sphere models to describe the foreground lensing clusters, we have calculated the expected number of lensed arcs towards galaxy clusters. We have improved previous calculations on arc statistics by including redshift information for background galaxies and accounting for the redshift evolution of foreground lensing clusters in different cosmological models. We have also accounted for the magnification bias in magnitude-limited search programs based on the HDF luminosity function. Our predicted numbers are in agreement with an extrapolation of the observed number of arcs towards a sample of bright EMSS clusters to the whole sky when $\Omega_m \lesssim 0.5$ in a flat universe with $\Omega_m + \Omega_\Lambda = 1$. Given the large systematic effects involved with both the predicted and observed number of arcs, more reliable constraints on the cosmological parameters are not currently possible.

Using the redshift and flux information for HDF radio sources, and the same lens population, we have extended the calculation to predict the expected number of lensed μJy sources

¹ <http://cfa-www.harvard.edu/aas/tenmeter/tenmeter.html>

² <http://astro.estec.esa.nl/Planck/>; also, ESA document D/SCI(96)3.

towards galaxy clusters. In a cosmology with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$, we predict ~ 1500 lensed sources towards clusters with X-ray luminosities greater than $12.8 \times 10^{44} h^{-2} \text{ ergs s}^{-1}$, and with amplifications due to lensing greater than 2. We suggest that similar deep VLA observations may already contain lensed μJy sources and that a careful analysis may be required to establish the lensing nature of such sources.

At sub-mm wavelengths, we predict $\sim 3 \times 10^4$ lensed sources towards clusters for same cosmology as above. We have compared our predicted numbers to the observed number of lensed sub-mm sources towards a sample of galaxy clusters. However, various biases in this observed sample and possible source contamination, do not allow us to constrain cosmological parameters based on current statistics. We have briefly studied the possibility of using the Planck surveyor and the South Pole 10-m telescope data to perform this task. A catalog of ~ 100 lensed sources towards clusters is likely to be a useful by-product of Planck.

Acknowledgements. I would like to acknowledge useful discussions and correspondences with Jean Quashnock and Cole Miller on gravitational lensing, John Carlstrom on galaxy clusters, and Heinz Andernach, André Fletcher, Frazer Owen and Ian Smail on gravitational lensing of radio sources due to foreground clusters and the possibility of an observational search to find such sources. I would also like to thank the referees for their prompt refereeing of the three separate papers on optical, radio, sub-mm lensed sources, as well as the combined version. I have greatly benefitted from their detailed comments and valuable advice from Peter Schneider, which led to a significant improvement in this paper. This study was partially supported by the McCormick Fellowship at the University of Chicago, and a Grant-In-Aid of Research from the National Academy of Sciences, awarded through Sigma Xi, the Scientific Research Society.

References

- Aghanim N., De Luca A., Bouchet F.R., Gispert R., Puget J.L., 1997, *A&A* 325, 9
- Andernach H., Gubanov A.G., Slee O.B., 1997, In: Bremer M., Jackson N., Perez-Fournon I. (eds.), Kluwer Acad. Publ., p. 107-112 (astro-ph/9704136)
- Arnaud M., Evrard A.E., 1998, astro-ph/9806353
- Bartelmann M., Huss A., Colberg J.M., Jenkins A., Pearce F.A., 1998, *A&A* 330, 1
- Bartlett J.G., 1997, astro-ph/9703090
- Becker R.H., White R.L., Helfand D.J., 1995, *ApJ* 450, 559
- Bézécourt J., 1998, astro-ph/9802107
- Bézécourt J., Pelló R., Soucail G., 1998, *A&A* 330, 399
- Blain A.W., 1997, *MNRAS* 290, 553
- Blain A.W., 1998, *MNRAS* 297, 502
- Colley W.N., Gnedin O.Y., Ostriker J.P., Rhoads J.E., 1997, *ApJ* 488, 579
- Cooray A.R., Quashnock J.M., Miller M.C., 1998a, *ApJ*, in press (astro-ph/9806080) [CQM]
- Cooray A.R., Grego L., Holzapfel W.L., Marshall J., Carlstrom J.E., 1998b, *AJ* 115, 1532
- Cram L., Hopkins A., Mobasher B., Rowan-Robinson M., 1998, astro-ph/9805327
- Cunningham C.R., Gear W.K., Duncan W.D., Hastings P.R., Holland W.S., 1994, In: Crawford D.L., Craine E.R. (eds.) *Instrumentation in Astronomy VIII. Proc. SPIE* 2198, 638
- Edge A.C., Stewart G.C., Fabian A.C., Arnaud K.A., 1990, *MNRAS* 245, 559
- Evrard A.E., 1997, *MNRAS* 292, 289
- Fernández-Soto A., Lanzetta K.M., Yahil A. 1998, *ApJ*, in press
- Fomalont E.B., Windhorst R.A., Kristian J.A., Kellermann K.I., 1991, *AJ* 102, 1258
- Fukugita M., Futamase T., Kasai M., Turner E.L., 1992, *ApJ* 393, 3
- Gioia I.M., Luppino G.A., 1994, *ApJS* 94, 583
- Girardi M., Fadda D., Giuricin G., Mardirossian K., et al., 1996, *ApJ* 457, 61
- Hamana T., Futamase T., 1997, *MNRAS* 286, L7
- Hattori M., Watanabe K., Yamashita K., 1997, *A&A* 319, 764
- Hjorth J., Oukbir J., van Kampen E., 1998, *MNRAS* 298, L1
- Hughes D., Serjeant S., Dunlop J., et al., 1998, *Nat* 394, 241
- Iverson R.J., Smail I., Le Borgne J.-F., et al., 1998, *MNRAS* 298, 583
- Kitayama T., Suto Y., 1997, *ApJ* 490, 557
- Kochanek C.S., 1991, *ApJ* 379, 517
- Kochanek C.S., 1996, *ApJ* 466, 638
- Lacey C., Cole S., 1993, *MNRAS* 262, 627
- Le Fèvre O., Hammer F., Angonin M.C., et al., 1994, *ApJ* 425, L5
- Loveday J., Pier J., 1998, astro-ph/9809179
- Mathiesen B., Evrard A.E., 1998, *MNRAS* 295, 769
- Mushotzky R.F., Scharf C.A., 1997, *ApJ* 482, L13
- Nichol R.C., Holden B.P., Romer K.A., 1997, *ApJ* 481, 644
- Peebles P.J.E., 1980, *The Large Scale Structure of the Universe*. Princeton University Press, Princeton, 1980
- Press W.H., Schechter P., 1974, *ApJ* 187, 425
- Richards E.A., Kellermann K.I., Fomalont E.B., et al., 1988, *AJ*, in press (astro-ph/9803343)
- Riess A.G., Filippenko A.V., Challis P., et al., 1998, *AJ*, in press (astro-ph/9805201)
- Sawicki M.J., Lin H., Yee H.K.C., 1997, *AJ* 113, 1
- Scott D., White M., 1998, *A&A*, submitted (astro-ph/9808003)
- Smail I., Iverson R.J., Blain A.W., 1997, *ApJL* 490, 5
- Smail I., Iverson R.J., Blain A.W., Kneib J.-P., 1998, *ApJ*, in press (astro-ph/9806061)
- Stark A.A., Carlstrom J.E., Israel F.P., et al., 1998, astro-ph/9802326
- Tegmark M., de Oliveira-Costa A., 1998, *ApJ* 500, L83
- Vianna P.T.P., Liddle A.R., 1996, *MNRAS* 281, 323
- Voit G.M., Donahue M., 1998, *ApJ* 500, L111
- White D.A., Jones C., Forman W., 1997 *MNRAS*, 292, 419
- Williams R.E., Blacker B., Dickinson M., et al., 1996, *AJ* 112, 1335
- Windhorst R.A., Miley G.K., Owen F.N., Kron R.G., Koo D.C., 1985, *ApJ* 289, 494
- Windhorst R.A., Fomalont E.B., Partridge R.B., Lowenthal J.D., 1993, *ApJ* 405, 498
- Wu X.-P., Hammer F., 1993, *MNRAS* 262, 187
- Wu X.-P., Mao S., 1996, *ApJ* 463, 404