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Structural properties of s-Cepheid velocity curves*

Constraining the location of the $\omega_4 = 2\omega_1$ resonance

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Abstract. The light curves of the first overtone Pop. I Cepheids (s-Cepheids) show a discontinuity in their ϕ_{21} vs. P diagram, near $\mathrm{P} = 3.2$ day. This feature, commonly attributed to the 2:1 resonance between the first and the fourth overtones ($\omega_4 \approx 2\omega_1$), is not reproduced by the hydrodynamical models. With the goal of reexamining the resonance hypothesis, we have obtained new CORAVEL radial velocity curves for 14 overtone Cepheids. Together with 10 objects of Krzyt et al. (1999), the combined sample covers the whole range of overtone Cepheid periods. The velocity Fourier parameters display a strong characteristic resonant behavior. In striking contrast to photometric ones, they vary smoothly with the pulsation period and show no jump at 3.2 day. The existing radiative hydrodynamical models match the velocity parameters very well. The center of the $\omega_4 = 2\omega_1$ resonance is estimated to occur at $\mathrm{P_r} = 4.58 \pm 0.04$ day, i.e. at a period considerably longer than previously assumed (3.2 day). We identify two new members of the s-Cepheid group: MY Pup and V440 Per.

Key words: techniques: radial velocities – stars: oscillations – stars: variables: Cepheids

1. Introduction

The *sinusoidal* or s-Cepheids constitute about 30% of all Galactic Cepheids with periods below 5 days. Originally, they have been discriminated from other Cepheids by qualitative criteria – their small amplitudes and almost sinusoidal light, color and radial velocity curves. A more precise, quantitative definition, based on the Fourier decomposition of the light curves, has been introduced by Antonello et al.(1990). Armed with this new classification tool, the Italian group has identified 33 s-Cepheids, plus several likely suspects (Antonello & Poretti 1986; Antonello et al. 1990; Mantegazza & Poretti 1992; Poretti 1994).

The physical nature of the s-Cepheids has been a matter of debate. It has been suggested in the General Catalog of Variable Stars that these stars are either fundamental-mode pulsators during the first crossing of the instability strip or, alternatively, first overtone pulsators. The latter view has been adopted by Antonello et al. (1990). A different interpretation has been proposed by Gieren et al. (1990), who has argued that the short period s-Cepheids pulsate in the first overtone, but that the long period ones ($\mathrm{P} > 3.2$ day) are in fact fundamental-mode variables. The controversy has finally been settled with the massive photometry of the MACHO and EROS, which has unambiguously shown that *all* s-Cepheids pulsate in the overtone (Welch et al. 1995; Beaulieu et al. 1995). Independently, the first crossing hypothesis has been ruled out by the new spectroscopic abundance analysis (Kovtyukh et al. 1996).

The debate between Antonello et al. and Gieren et al. has been sparked off by the curious behavior of the s-Cepheid Fourier parameters. The light curve Fourier phase ϕ_{21} (cf. Simon & Lee 1981), when plotted vs. pulsation period, shows a very deep and very abrupt drop in the vicinity of $\mathrm{P} = 3.2$ day. At the same period the amplitude ratios R_{21} and R_{31} display a pronounced minimum. This behavior is reminiscent of what is observed for the fundamental-mode Cepheids at $\mathrm{P} \approx 10$ day (Simon & Moffett 1985). In the latter case the characteristic variation of the Fourier parameters has its origin in the 2:1 resonance between the fundamental mode and the second overtone (Simon & Schmidt 1976; Buchler et al. 1990; Kovács & Buchler 1989).

By analogy, Antonello & Poretti (1986) and Petersen (1989) have proposed that the variations observed for the s-Cepheid light curves are also caused by a resonance, namely the 2:1 coupling between the first and the fourth overtones. Also by analogy, it has been assumed that the resonance center coincides with the drop of the photometric ϕ_{21} and therefore occurs at $\mathrm{P} = 3.2$ day.

The resonance hypothesis, although very attractive, has encountered serious difficulties, when confronted with hydrodynamical calculations. Three sets of overtone cepheid models have been specifically computed to study the presumed resonance (Antonello & Aikawa 1993, 1995; Schaller & Buchler

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1994). To great disappointment, they have all failed to reproduce the properties of the s-Cepheid light curves. The theoretical ϕ_{21} and R_{21} display some features in the vicinity of $\mathrm{P} = 3.2$ day but they are *very far* from reproducing what is actually observed. The discrepancy is even more embarrassing, when compared with the very good agreement obtained with the same codes for the fundamental-mode pulsators (e.g. Moskalik et al. 1992).

Another potential problem has been pointed out by Buchler et al. (1996), who have considered the constraints imposed by resonances on the evolutionary Mass-Luminosity relation. Their linear calculations show that the proposed s-Cepheid resonance centered at 3.2 day and the well established f-mode Cepheid resonance at 10 day cannot be reconciled simultaneously with the same $\mathrm{M} - \mathrm{L}$ relation. For consistent picture, the s-Cepheid resonance has to occur at $\mathrm{P} = 4.3$ day.

So far, the analysis of the s-Cepheid pulsations has been performed almost exclusively in the photometry domain. This choice has been dictated by the lack of high quality V_r data for these low amplitude stars. The only attempt to compare the s-Cepheid velocity curves with the hydrodynamical models has been largely inconclusive (Antonello & Aikawa 1995). Analyzing overtone Cepheid velocity data is particularly desirable in light of the modeling difficulties discussed above. Velocity Fourier parameters can provide additional information on the overtone Cepheid pulsation dynamics and, thus, can shed new light on the resonance puzzle. The radial velocity, being a dynamical quantity, should display the effects of resonances in a more visible way. The models of BL Her-type stars give a good example of such a behavior (Moskalik & Buchler 1993, Buchler & Buchler 1994). The use of the velocity data is also preferred for comparison with the hydrodynamical computations, which are known to reproduce the observed velocity curves very well (e.g. Moskalik et al. 1992). As for the light curves, the models show small but persistent discrepancies of the Fourier phases, ϕ_{n1}, for every type of radial pulsators studied (Simon & Aikawa 1986, Simon 1990, Moskalik et al. 1992, Moskalik & Buchler 1993). The problem is most likely caused by an inadequate treatment of the radiative transfer in outer stellar layers, which however, has very little effect on the computed velocity curves (Feuchtinger & Dorfi 1996).

With the above reasoning in mind, we have collected new CORAVEL radial velocity data for several known overtone cepheids. Several others have recently been analyzed by Krzyt et al. (1999). The combined sample for the first time gives a complete and accurate description of the entire s-Cepheid velocity Fourier progression. The preliminary results of this project have been presented by Kienzle et al. (1998). In this paper, we present the final results and discuss their astrophysical implications. In particular, we discuss the constraint imposed by the velocity data on the location of the s-Cepheid resonance.

2. Observations

Krzyt et al. (1999) have made an extensive compilation of published radial velocity measurements for classical Cepheids and subsequently used these data to derive accurate Fourier parameters of their pulsation velocity curves. Their sample, over 100 objects in total, contains, however, only 10 overtone Cepheids. This number is not sufficient to adequately cover the Fourier progression for this group of stars. Therefore, we have taken new data for 13 other known overtone Cepheids, in order to enlarge the sample of Krzyt et al. The number of selected targets is particularly large close to the photometric ϕ_{21} drop (i.e. close to $\mathrm{P} = 3.2$ day), where the resonance has been expected, according to the previous results. In addition, 3 other Cepheids have been observed (AP Pup, MY Pup and IT Car), in an attempt to identify new long period overtone pulsators. Overtone Cepheids with $\mathrm{P} > 5$ day have recently been found in the LMC (Alcock et al. 1995, Fig. 5), their existence in the Galaxy has also been predicted on theoretical grounds (Buchler et al. 1997).

The V_r observations have been obtained with the northern and southern CORAVEL cross-correlation spectrophotometer (Baranne et al. 1979) at the 1-m Swiss telescope at the Haute Provence Observatory (France) and at the 1.54-m Danish telescope at the European Southern Observatory, La Silla (Chile). The cross-correlation function has been fitted with a gaussian profile in a standard way (Burki et al. 1982) in order to extract the radial velocities. Radial velocity standards have also been observed to check the instrumental drift. The majority of the data has been collected during four runs; in December 1996 (by FP), February 1997 (by FK), June 1997 (by DB) and during the last southern CORAVEL run at La Silla, in December 1997 (by FK). For all program Cepheids a very good phase coverage has been achieved, with more than 30 points per star (except V379 Cas – 27 points). The measurement errors range from 0.3 km/s to 0.8 km/s in most cases.

The data and Table 1 hereafter are available at the CDS[1].

3. Fourier decomposition

The radial velocity data are fitted with

$$V_r(t) = \mathrm{A}_0 + \sum_{k=1}^{N} \mathrm{A}_k \sin[k\omega(t - t_0) + \phi_k] \qquad (1)$$

where $\omega = 2\pi/\mathrm{P}$, P is the pulsation period of the star and N is the order of the fit. The parameters A_0, A_k, ϕ_k, P and their errors are estimated with a standard unweighted least-squares method. The variance of the residuals, σ^2, for M data points is estimated as:

$$\sigma^2 = \frac{\chi^2_{\min}}{M - 2N - 2} \qquad (2)$$

where $\chi^2_{\min}$ is the sum of squared residuals. The order of the fit, N, is increased until adding another harmonic does not decrease σ significantly. The points which are more than 2.5σ away from the fit (these are assumed to be poor quality data) are eliminated, and the fitting procedure is repeated. As the last step, the Fourier phases $\phi_{k1} \equiv \phi_k - k\phi_1$ and amplitude ratios $\mathrm{R}_{k1} \equiv \mathrm{A}_k/\mathrm{A}_1$ are calculated. Their errors are computed with the formulae of

[1] anonymous ftp to cdsarc.u.strasbg.fr (130.79.128.5); http://cdsweb.u-strasbg.fr/Abstract.html

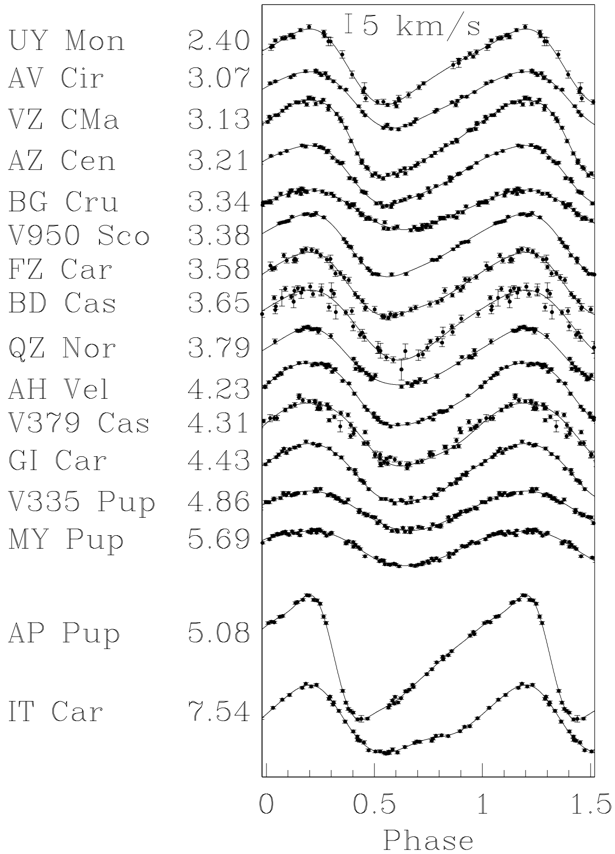

Fig. 1. Radial velocity curves for our program Cepheids. Overtone pulsators are sorted by increasing period, from *top* to *bottom*. Two fundamental-mode Cepheids, AP Pup and IT Car are shown at the bottom of the plot.

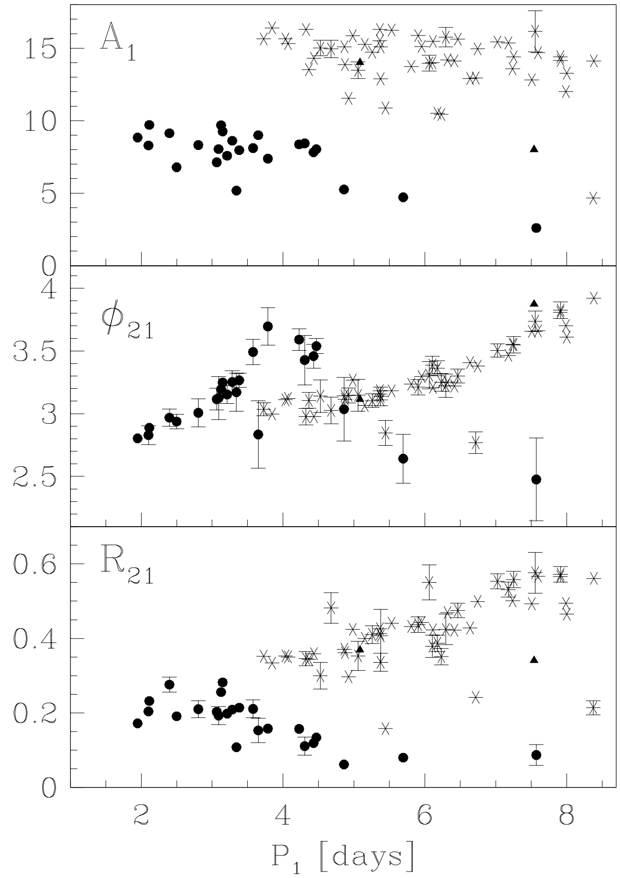

Fig. 2. A_1 (in km/s), ϕ_{21} and R_{21} vs. period for Cepheid radial velocity curves (*top*, *middle* and *bottom*, respectively). Fundamental-mode Cepheids are marked with asterisks and overtone Cepheids with filled circles. Filled triangles represent AP Pup and IT Car (see text for these two stars). Error bars are shown only when larger than the symbol size.

Petersen (1986). In Fig. 1 we display the phased velocity curves for the program stars, together with their Fourier fits. The Fourier parameters and their formal errors are given in Table 1.

3.1. *Comments on individual stars*

For 4 stars the phase coverage has been improved by supplementing our CORAVEL measurements with the published data:

BD Cas – 18 points from Gorynya et al. (1992, 1996)
FZ Car – 5 points from Pont et al. (1994)
UY Mon – 18 points from Imbert (1981). The archival data have been re-reduced. A vertical shift of -1.1 km/s has been applied, which reduces the variance of the fit, σ^2, by 48%.
QZ Nor – 5 points from Metzger et al. (1992). A vertical shift of -0.9 km/s has been applied, which reduced the variance of the fit by 25%.

4. Properties of Fourier parameters

The set of s-Cepheids observed for this paper has been supplemented with 10 overtone pulsators analyzed by Krzyt et al. (1999): SU Cas, EU Tau, IR Cep, DT Cyg, V351 Cep, EV Sct, SZ Tau, V532 Cyg, FF Aql and V440 Per. The lowest order Fourier parameters A_1, ϕ_{21} and R_{21} for the entire sample are plotted vs. period in Fig. 2 (filled circles). In the same plot we also display Krzyt's et al. fundamental-mode Cepheids (asterisks). In this case we have limited the sample to stars with the most secure Fourier solutions, namely those with 25 or more datapoints and with an error of $\sigma(\phi_{21}) < 0.15$. The third order parameters ϕ_{31} and R_{31} are displayed for completeness in Fig. 3.

Before we begin the general discussion of Figs. 2 and 3 preliminary clarifications for several stars are in order:

AP Pup and IT Car – These stars are displayed in the plots as filled triangles. All the Fourier parameters place AP Pup securely among the fundamental pulsators. The high values of A_1, R_{21} and R_{31} are incompatible with it being an overtone Cepheids. For IT Car, A_1 and R_{21} are somewhat low (and R_{31} very low), but ϕ_{21} and ϕ_{31} fall on the fundamental mode sequence. We classify both stars as fundamental-mode Cepheids and will not discuss them any further here.

MY Pup ($P = 5.69$ day) – The values of A_1, R_{21}, R_{31} and ϕ_{21} for this object are well below those derived for the fundamental-mode Cepheids. On these grounds, we classify this star as a new overtone Cepheid. This conclusion will be strengthened by comparison with the hydrodynamical models

Table 1. Fourier parameters for Cepheids measured with CORAVEL. M, N and σ are, respectively, the number of datapoints, the order of the fit and the standard deviation of residuals.

Star	Period [d]	M	N	σ	A_0	A_1	R_{21}	ϕ_{21}	R_{31}	ϕ_{31}	R_{41}	ϕ_{41}	R_{51}	ϕ_{51}
UY Mon	2.398240	36	3	0.611	33.750	9.136	0.276	2.968	0.043	6.088	–	–	–	–
	0.000005				0.116	0.147	0.020	0.069	0.017	0.415	–	–	–	–
AV Cir	3.065006	35	3	0.435	4.787	7.128	0.203	3.116	0.037	5.525	–	–	–	–
	0.000158				0.079	0.110	0.017	0.086	0.016	0.401	–	–	–	–
VZ CMa	3.126326	56	5	0.488	39.421	9.692	0.256	3.194	0.080	6.079	0.038	2.821	0.021	5.484
	0.000139				0.067	0.090	0.011	0.045	0.010	0.130	0.010	0.272	0.009	0.480
AZ Cen	3.211752	46	3	0.462	-10.343	7.585	0.198	3.153	0.061	5.967	–	–	–	–
	0.000126				0.073	0.111	0.015	0.072	0.014	0.207	–	–	–	–
BG Cru	3.342503	67	2	0.465	-19.708	5.178	0.108	3.171	–	–	–	–	–	–
	0.000056				0.057	0.084	0.017	0.152	–	–	–	–	–	–
V950 Sco	3.382519	34	3	0.340	13.687	7.962	0.214	3.266	0.051	6.352	–	–	–	–
	0.000012				0.071	0.101	0.012	0.057	0.013	0.216	–	–	–	–
FZ Car	3.577946	47	3	0.799	-0.960	8.110	0.211	3.492	0.056	6.417	–	–	–	–
	0.000023				0.121	0.175	0.024	0.101	0.020	0.394	–	–	–	–
BD Cas	3.650765	43	2	1.331	-49.084	8.996	0.153	2.834	–	–	–	–	–	–
	0.000063				0.221	0.306	0.033	0.268	–	–	–	–	–	–
QZ Nor	3.786553	43	3	0.536	-39.549	7.383	0.158	3.695	0.044	7.615	–	–	–	–
	0.000031				0.084	0.147	0.018	0.150	0.022	0.435	–	–	–	–
AH Vel	4.226777	50	4	0.410	25.801	8.368	0.157	3.590	0.042	5.873	0.023	3.304	–	–
	0.000053				0.070	0.085	0.009	0.086	0.011	0.233	0.010	0.468	–	–
V379 Cas	4.305816	27	3	0.669	-38.652	8.428	0.111	3.427	0.055	5.533	–	–	–	–
	0.000453				0.131	0.186	0.024	0.197	0.023	0.438	–	–	–	–
GI Car	4.431035	50	4	0.448	-20.204	7.819	0.119	3.458	0.037	6.644	0.019	2.222	–	–
	0.000287				0.066	0.093	0.013	0.095	0.012	0.318	0.012	0.644	–	–
V335 Pup	4.860555	70	2	0.466	40.041	5.249	0.062	3.036	–	–	–	–	–	–
	0.000354				0.056	0.082	0.015	0.254	–	–	–	–	–	–
MY Pup	5.694670	72	3	0.429	15.168	4.714	0.080	2.641	0.043	6.108	–	–	–	–
	0.000317				0.052	0.070	0.016	0.195	0.018	0.335	–	–	–	–
AP Pup	5.084534	45	5	0.521	15.393	14.016	0.369	3.115	0.183	6.384	0.090	3.325	0.030	0.327
	0.000090				0.098	0.125	0.009	0.031	0.009	0.055	0.010	0.089	0.008	0.360
IT Car	7.539680	40	3	0.402	-11.801	8.004	0.343	3.871	0.033	7.016	–	–	–	–
	0.002797				0.068	0.098	0.013	0.042	0.011	0.366	–	–	–	–

(Sect. 5). The light curve of MY Pup has been Fourier analyzed by Antonello & Poretti (1986). Antonello et al. (1990) have described it as a suspected s-Cepheid.

V440 Per ($\mathrm{P} = 7.57$ day) – This very low amplitude variable ($\mathrm{A}_1 = 2.7 \pm 0.1$ km/s) has been analyzed by Krzyt et al. (1999). It is similar to MY Pup and, for the same reasons, we classify this star as a new overtone Cepheid. Its light curve has been Fourier analyzed by Antonello & Poretti (1986). The classification scheme of Antonello et al. (1990), based on photometric ϕ_{21}, does not discriminate between the pulsation modes for $\mathrm{P} > 5.5$ day (see also Welch et al. 1995, Fig. 2). Therefore, the overtone nature of V440 Per has not been recognized in their paper.

BD Cas – This star has originally been classified as Pop. II Cepheid, according to its photometric and spectroscopic characteristics (Petit 1960), although its low galactic latitude ($b = -0.96$) is also compatible with Pop. I. With new CCD photometry, Schmidt (1991) and Poretti (1994) have reclassified BD Cas as a Pop. I s-Cepheid. However, their photometric data are scarce (16 points) and the Fourier parameters are plagued with very large errors ($\sigma(\phi_2) = 0.72$). The values of A_1 and R_{21} derived from the velocity curve place the star among the overtone Cepheids, but $\phi_{21} = 2.834 \pm 0.268$ does not, being instead similar to those of the fundamental pulsators. Clearly, more observations are needed to confirm the s-Cepheid status of this variable.

X Lac, V495 Cyg and V636 Cas – these 3 variables have been analyzed by Krzyt et al. (1999). For X Lac ($\mathrm{P} = 5.45$ day) and V495 Cyg ($\mathrm{P} = 6.72$ day) the values of ϕ_{21}, R_{21} and R_{31} fall well below the fundamental mode sequence and close to those of MY Pup and V440 Per. This behavior strongly suggests the s-Cepheid classification, but the amplitude of both stars is rather high and typical of fundamental pulsators (10.88 ± 0.16 km/s and 12.95 ± 0.24 km/s, respectively). In the case of V636 Cas ($\mathrm{P} = 8.38$ day), the values of $\mathrm{A}_1 = 4.66 \pm 0.08$ km/s and $\mathrm{R}_{21} = 0.214 \pm 0.019$ point towards the s-Cepheid classification, but the very high value of $\phi_{21} = 4.554 \pm 0.087$ is in conflict with such an interpretation. Although all 3 stars differ from the majority of the fundamental-mode Cepheids, at this point the evidence is not sufficient to consider them overtone pulsators.

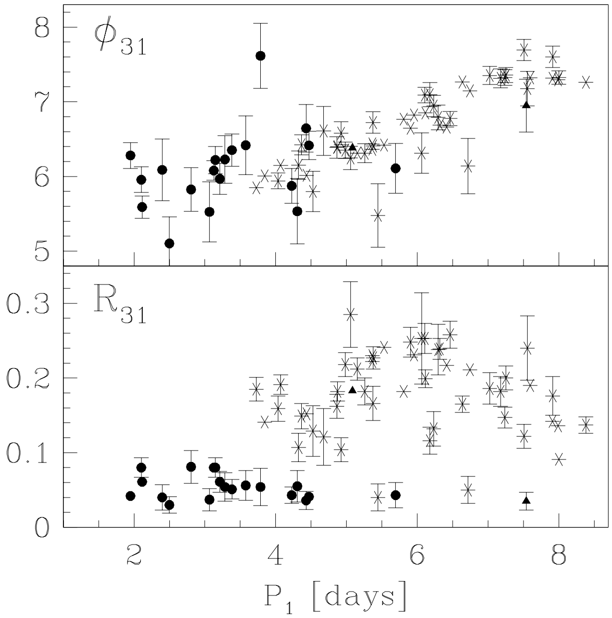

Fig. 3. ϕ_{31} and R_{31} vs. period for Cepheid radial velocity curves (*top* and *bottom*, respectively). Same symbols as in Fig. 2.

Our sample of 24 s-Cepheids covers the range of periods from 1.95 to 7.57 day. Except for BD Cas which deviates from the trend, the remaining stars display a remarkably tight progression of ϕ_{21}, R_{21} and to a lesser degree of A_1 with the pulsation period. As the period increases, ϕ_{21} rises, reaches a maximum at about $P = 4$ day and then falls down to ~ 2.5 rad. This variation is accompanied by a decrease of R_{21} at periods of $3.5 - 5.0$ day, followed by a very slow increase. The amplitude A_1 decreases uniformly throughout the whole range of periods. In contrast to the low order parameters, the behavior of the higher order terms ϕ_{31} and R_{31} shows essentially no features, perhaps because of their low accuracy.

The velocity Fourier parameters of the overtone Cepheids are distinctively different from those of the fundamental-mode pulsators. In particular, A_1, R_{21} and R_{31} are much lower, which is a testimony to the "sinusoidal" shape of the s-Cepheid velocity curves. The two groups are also clearly separated in the $\phi_{21} - P$ plane, except a narrow range of periods around 5 day. These properties allow to distinguish overtone from fundamental-mode Cepheids in the *entire* range of periods, including $P > 5.5$ day, where Antonello's et al. (1990) criterion based on the light curve ϕ_{21} no longer works.

In Fig. 4 we compare the s-Cepheid ϕ_{21} progression for light curves and for velocity curves. The light curve data are taken from Antonello & Poretti (1986), Antonello et. al. (1990), Mantegazza & Poretti (1992) and Poretti (1994). The two ϕ_{21} plots are remarkably different. For the light curves, ϕ_{21} undergoes a dramatic, essentially discontinuous drop at a period of ~ 3.2 day. It is this behavior, accompanied by a local minimum of R_{21}, which led to the hypothesis that the 2:1 resonance between the first overtone and the fourth overtone occurs at this place (Antonello & Poretti 1986; Petersen 1989). In case of the velocity curves, ϕ_{21} varies smoothly and displays *no jump at*

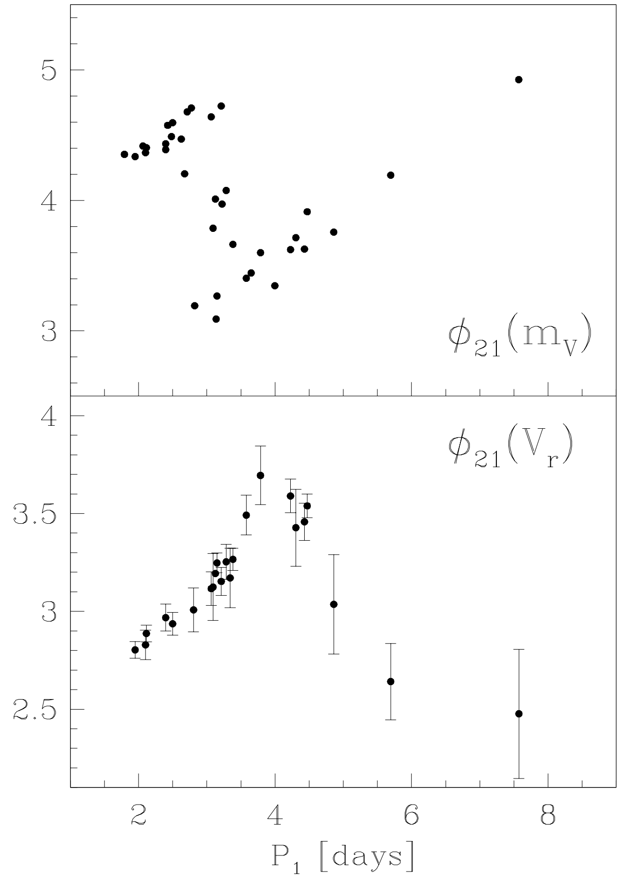

Fig. 4. s-Cepheid ϕ_{21} vs. period for light curves (*top*) and for radial velocity curves (*bottom*). For 2 stars photometric ϕ_{21} is not available: BG Cru (first harmonic not detectable) and V351 Cep (no published light curve Fourier decomposition). BD Cas is omitted in both plots.

3.2 day nor at any other period. The same is true for all the remaining velocity Fourier parameters. In other words, there is no spectacular change of velocity curve morphology at 3.2 day. This surprising and unexpected result contradicts the assumption of a resonance occurring at this particular period. The resonance does not have to cause discontinuous variations of the Fourier parameters. When it does, however, it happens for *both* the velocity curves and the light curves (e.g. Buchler & Kovács 1986). Because for the s-Cepheids this is not the case, we must conclude that the 3.2 day feature in their light curves is not related to the resonance. We will show in the next section that the resonance is nevertheless present in these stars, but its center is located at a very different period.

5. Comparison with hydrodynamical models

Two extended surveys of nonlinear first overtone Cepheid models have been performed in recent years, by Schaller & Buchler (1994) and by Antonello & Aikawa (1995). Those surveys have been aimed at investigating the effects of the $\omega_4 \approx 2\omega_1$ resonance. In order to study these effects in a systematic fashion, the models have been grouped in one parameter sequences, running either at constant luminosity (Antonello & Aikawa) or parallel to

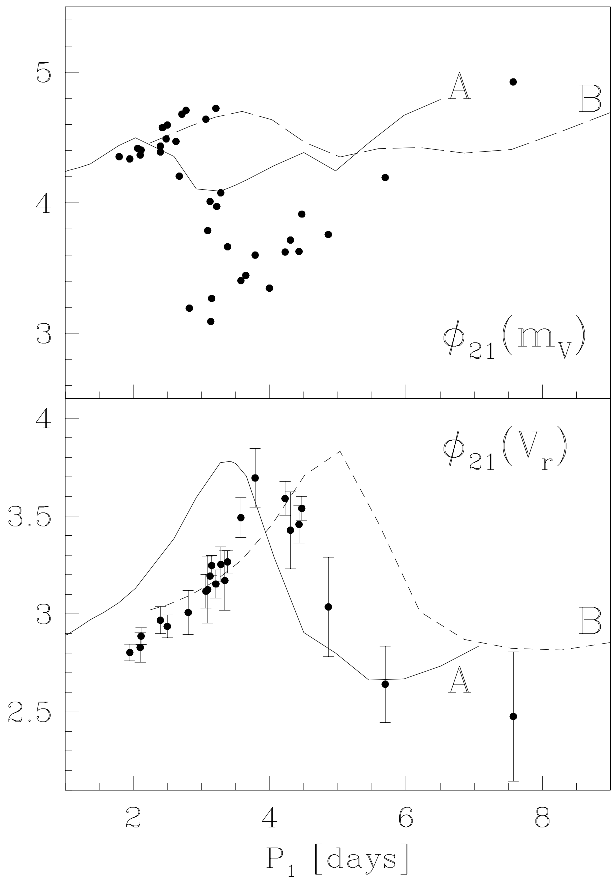

Fig. 5. Same as Fig. 4, but compared with two sequences of hydrodynamical models of Schaller & Buchler (1994). See text for details.

the theoretical first overtone Blue Edge (Schaller & Buchler). Both sets of calculations are performed with purely radiative hydrocodes and almost the same input physics (e.g. opacity tables). Consequently, they both give very similar results. In the following discussion we will use models of Schaller & Buchler, primarily because their sequences cover a wider range of period ratios, P_4/P_1.

In Fig. 5 we plot the observational data together with the theoretical values of ϕ_{21} for two sequences of models. Sequence A (solid line) obeys the Mass-Luminosity relation of Chiosi (1989) and runs on the H–R diagram parallel to, but 100K cooler of the first overtone Blue Edge. Sequence B (dashed line) follows the classical $M - L$ relation of Becker et al. (1977) and is 300K cooler than the Blue Edge. The resonance with the fourth overtone occurs in sequences A and B at, respectively, $P_1 = 4.02$ day and $P_1 = 5.40$ day. As already discussed, the calculations fail to reproduce the jump of the *photometric* ϕ_{21}. The variations displayed by the models are never as large or as sharp as actually observed. The situation is, however, entirely different in case of *velocity curves*. For both sequences shown, the progression of velocity ϕ_{21} has a shape remarkably similar to the observed one. The theoretical curves do not agree with the observations, though – they are displaced either to shorter or to longer periods in respect to the data. It is easy to notice that the displacement depends on the position of the $\omega_4 = 2\omega_1$ resonance within the sequence. Thus, a good match between the models and the radial velocity data should be possible, provided that the resonance period in the models is chosen properly.

5.1. Position of the resonance

We now use the radial velocity ϕ_{21} data to constrain the position of the s-Cepheid 2:1 resonance. Fig. 5 (bottom) already shows that it must be located somewhere between 4.02 and 5.40 day (i.e. between values for sequences A and B). We want to pinpoint the resonance center more precisely, though. To that aim we will try to “construct” the model sequence that matches the velocity data as closely as possible.

The hydrodynamical computations of Schaller & Buchler (1994) show that the velocity ϕ_{21} for the overtone Cepheid models is very tightly correlated with the resonant period ratio, P_4/P_1. In other words, to a very good approximation we can write:

$$\phi_{21} = f(P_4/P_1) \tag{3}$$

where function f is *the same for every model sequence*. Analogous property has also been found for the fundamental-mode Cepheids, where the $\omega_2 \approx 2\omega_0$ resonance plays an important role (Buchler et al. 1990). For *any* sequence of models, the relation between the period ratio and the period can be described by an approximate formulae

$$P_4/P_1 = 0.5 \left(\frac{P_1}{P_r}\right)^{-\alpha} \tag{4}$$

where P_r is the period at the resonance center. The parameters P_r and α are different for every sequence. Our goal is to find the values of these parameters, for which a sequence reproduces the observations best. Substituting Eq. (4) into Eq. (3) we get

$$\phi_{21}(P_1) = f\left[0.5 \left(\frac{P_1}{P_r}\right)^{-\alpha}\right] \tag{5}$$

P_r and α can now be determined by fitting the above expression to the observed values of velocity ϕ_{21}. This is done with a χ^2 method, minimizing

$$\chi^2(P_r, \alpha) = \sum \left[\frac{\phi_{21}^{\rm Obs}(P_1) - \phi_{21}(P_1)}{\sigma(\phi_{21})}\right]^2 \tag{6}$$

where $\phi_{21}^{\rm Obs}(P_1)$ and $\phi_{21}(P_1)$ are the observed and estimated values, respectively, and $\sigma(\phi_{21})$ is the error of the observed ϕ_{21}. For the function $f(P_4/P_1)$ we adopt the ϕ_{21} progression of sequence A. It is the longest and most densely sampled of all sequences of Schaller & Buchler (1994). BD Cas, which deviates from the trend is omitted from the fit. Another star, V440 Per, turns out to be outside the range of P_4/P_1 covered by the models. The minimization leads to an excellent fit with $\chi^2_{\rm min} = 18.97$, for χ^2 of 20 degrees of freedom (22 stars, 2 parameters). The low value of $\chi^2_{\rm min}$ shows that Eq. (5) indeed provides a good representation of physical reality. The values of

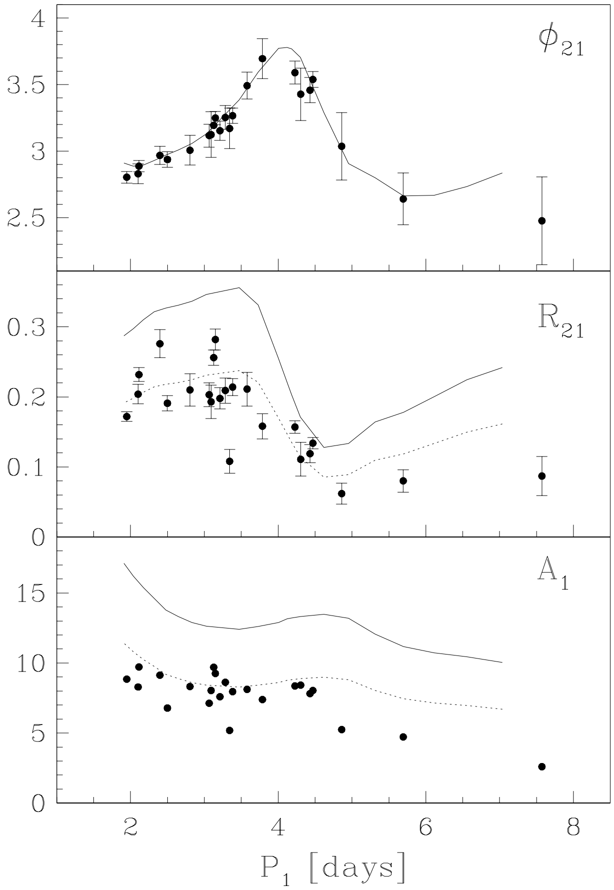

Fig. 6. Comparison of the s-Cepheid radial velocity observations with the models of sequence A (see text): ϕ_{21} (*top*), R_{21} (*middle*) and A_1 (*bottom*, in km/s). The theoretical pulsation amplitude is scaled to the observed amplitude with the projection factor of $p = 1.36$ (Burki et al. 1982). The abscissa for the models is transformed according to Eq. (4), with $\alpha = 0.141$ and $P_r = 4.58$ day. The dashed lines represent R_{21} and A_1 divided by constant factor of 1.5 (see text). BD Cas is omitted in all plots.

the parameters determined from the fitting procedure are $\alpha = 0.141 \pm 0.006$ and $P_r = 4.58 \pm 0.04$ day (1σ errors).

The resultant ϕ_{21} fit is shown in Fig. 6 (top). The plot confirms that a very good match between the models and the ϕ_{21} data has been achieved. In the middle and bottom panels of the figure we display the theoretical values of R_{21} and A_1 for the same sequence of models (solid lines). Aside from shifting and stretching of the abscissa (Eq. (4)) nothing else has been adjusted here. The computed amplitude ratio R_{21} is systematically too high as compared to the data (and so is A_1), but otherwise its progression with period bears a very strong resemblance to the observations. In particular, *both* the models and the data display a steep decrease of R_{21} between periods of 3.5 and 5.0 day, followed by a slow increase.

The excessively high values of theoretical A_1 and R_{21} should not be consider a serious problem, in fact they can be easily adjusted. Both quantities depend sensitively on the artificial viscosity, which is a numerical parameter in the hydrodynamical codes. By increasing the artificial viscosity, we can decrease R_{21} of all models by a certain constant factor. This will also decrease the amplitude A_1 by a similar (although somewhat larger) factor. The velocity Fourier phase ϕ_{21}, on the other hand, will remain almost *unchanged* and the good fit achieved for this parameter will not be spoiled. This is a very general property of the radiative hydrodynamical models (e.g. Kovács 1990; Kovács & Buchler 1993; Kovács & Kanbur 1998) and it holds for the s-Cepheid models as well (Schaller & Buchler 1994). The dashed lines in Fig. 6 show theoretical A_1 and R_{21} of sequence A, both divided by the same factor of 1.5. The scaled amplitude ratio R_{21} is in a very good *quantitative* agreement with observations. Not only the location, but also the size of the R_{21} drop is now reproduced. The scaled amplitude A_1 is also much closer to the measured values, although it is still somewhat too high.

Fig. 6 shows that the parameterization given by Eq. (4) can bring velocity ϕ_{21} and R_{21} into remarkably good, *simultaneous* agreement with the s-Cepheid observations. This proves that the radiative models are capable of describing the radial velocity variations in these stars. It also shows that the 2:1 resonance plays a dominant dynamical role in the s-Cepheid pulsations. In that respect, s-Cepheids are very similar to their fundamental mode counterparts. Indeed, even the shapes of the velocity ϕ_{21} and R_{21} progressions are in both groups of stars remarkably alike (Buchler et al. 1990; their Fig. 16).

The most important outcome of the fitting procedure is the determination of the s-Cepheid resonant period. We stress, that this result is based *solely* on matching the observed and computed ϕ_{21}. The derived value is somewhat dependent on the way we define the function $f(P_4/P_1)$. For example, if it is given by sequence B instead of sequence A, we obtain $P_r = 4.36 \pm 0.03$ day. This is an extreme case, however. For other sequences of models we *always* find higher values. As our final estimate of the resonant period we adopt the value resulting from fitting of sequence A. We consider it to be most trustworthy, because this particular sequence covers the widest range of P_4/P_1 and simultaneously gives the best match to the ϕ_{21} data (in terms of $\chi^2_{\rm min}$). We conclude, that the 2:1 resonance between the first overtone and the fourth overtone occurs in s-Cepheids at a period of $P_r = 4.58 \pm 0.04$ day.

5.2. Implications for the Mass-Luminosity relation

The evolutionary Mass-Luminosity relation for the classical Cepheids is still a matter of considerable debate (cf. Buchler et al. 1996). The newly derived position of the $\omega_4 = 2\omega_1$ resonance can be used to constrain this important relation. In the following, we adopt the slope of Becker et al. (1977) and assume that the $M - L$ relation has the form

$$\log(L/L_\odot) = 3.68 \log(M/M_\odot) + b \tag{7}$$

The zero point parameter b has to be adjusted, so as to place the resonance at the right period. In order to determine the value of b, we resort to the linear non-adiabatic (LNA) pulsation calculations. We use the linear pulsation code of Dziembowski (1977), with the latest version of the OPAL opacities (Iglesias & Rogers 1996) and assume the standard Pop. I metallicity of

$Z = 0.02$. The LNA calculations show that the first overtone Blue Edge and the fundamental Blue Edge are about 250 K apart. In the region between the two lines the models can pulsate in the first overtone only. We assume, somewhat arbitrarily, that the sample of Galactic s-Cepheids is represented best by the sequence of models running on the H–R diagram parallel the two Blue Edges, half-way between them. For a sequence constructed in such a way, the resonance condition ($\mathrm{P}_4/\mathrm{P}_1 = 0.5$ at $\mathrm{P}_1 = 4.58 \pm 0.04$ day) is satisfied for $b = 0.73 \pm 0.01$.

The derived value of b is not very far from the zero point of $b = 0.65$, inferred from the fundamental mode 2:1 resonance at $\mathrm{P}_0 = 10$ day (Moskalik & Krzyt 1998, 1999). The difference of $\Delta b = 0.08$ corresponds to 4.9% difference of mass at a given luminosity. Although not in full agreement, the two values of b are close enough that with better models it might be possible to match both resonances with the same $\mathrm{M} - \mathrm{L}$ relation. For the s-Cepheid resonance located at 3.2 day, as suggested in earlier literature, a simultaneous match is hardly possible. Such a low value of $\mathrm{P_r}$ would require a zero point of $b = 1.05$, which is incompatible with the fundamental mode constraint.

6. Conclusions

We have collected new CORAVEL radial velocity data and have then derived the Fourier parameters of the pulsation velocity curves for 14 overtone Cepheids. Our sample, combined with 10 variables of Krzyt et al. (1999), covers the entire range of s-Cepheid periods. As such, it is perfectly suited to discuss the group properties of this class of stars. The main results of our work can be summarized as follows:

1. The progressions of the s-Cepheid Fourier parameters for velocity curves and for light curves are very different. Velocity parameters vary smoothly with the period and do not undergo any rapid changes or jumps. The jump of the photometric ϕ_{21} at $\mathrm{P} = 3.2$ day is not related to the resonance, but must be caused by some other, as yet unidentified effect.
2. Velocity Fourier parameters ϕ_{21} and R_{21} can be reproduced remarkably well by the resonant (radiative) hydrodynamical models. This clearly shows that the 2:1 resonance with the fourth overtone is instrumental in shaping the s-Cepheid pulsations. The fit of the models to the velocity data yields a new estimate of the resonance period, which is $\mathrm{P_r} = 4.58 \pm 0.04$ day. This period implies the $\mathrm{M} - \mathrm{L}$ relation zero point of $b = 0.73 \pm 0.01$, not very different from value inferred from $\omega_2 = 2\omega_0$ resonance.
3. Velocity Fourier parameters can discriminate between the overtone and the fundamental-mode pulsators at all periods, including $\mathrm{P} > 5.5$ day. This property allows identification of two new overtone Cepheids: MY Pup ($\mathrm{P} = 5.69$ day) and V440 Per ($\mathrm{P} = 7.57$ day). The comparison with the hydrodynamical models (cf. Fig. 6) supports this identification.

The derived resonant period is based solely on the analysis of the s-Cepheid radial velocity curves, in particular of the velocity Fourier phase ϕ_{21}, while the light curve information has been entirely ignored. There are two arguments in favor of such an approach. First, the velocity ϕ_{21} is predicted by the existing hydrocodes more robustly than any Fourier parameter of the light curve. It shows little sensitivity to the the adopted artificial viscosity (as discussed above) or to the choice of particular numerical scheme (e.g. Buchler, Kollath & Marom 1997). In contrast to the light curve Fourier phases, it also shows little sensitivity to the treatment of radiative transfer in the optically thin outer layers of the star (Feuchtinger & Dorfi 1996). This last property is particularly important, since most hydrocodes do a rather poor job in this respect. When velocity data are available for comparison, the agreement between the models and observations is always very good, much better than ever achieved for the light curves. This is clearly the case for the fundamental-mode Cepheids (Moskalik et al. 1992). A very good match between computed and observed overtone Cepheid velocity curves strengthens our confidence in the above reasoning. The second argument comes from the fact that the resonant interaction of pulsation modes is a *dynamical* phenomenon. Since V_r is a *dynamical* quantity, studying its variations is, in our opinion, the most direct and perhaps more basic way of probing resonance effects in pulsating stars.

We want to end this paper with a word of caution. The hydrodynamical models (Schaller & Buchler 1994; Antonello & Aikawa 1995) which can match the velocity data so successfully, at the same time fail to reproduce the observed light curves. At this point we can offer no explanation for this discrepancy. A new modeling effort with better input physics is needed to address this problem and to confirm our conclusions. Such an effort, based on adaptive mesh hydrocode with time-dependent convection is already underway and its first results are very promising (Buchler, private communication).

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