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Towards understanding rapid line-profile and light variations of early-type stars

3. Some thoughts and reflections

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Abstract. The current situation in the research of rapid line-profile and light variations of early-type stars is critically reviewed. It is suggested that the ultimate understanding of the physical processes causing these variations can only come from an open-minded and complex approach to the problem and from systematic observational effort. It is argued that the results of the search for periodicities in the complicated variations of these objects depend critically on whether the method used is appropriate to the real physical situation. The danger of detection of a false multiperiodicity is pointed out for two particular situations: (i) a single-periodic signal which undergoes slow periodic change, e.g., due to the light time effect in a binary system, and (ii) a single-periodic signal with a complicated phase curve (a model of not exactly equidistant corotating spokes). It is argued that the observed rapid variations need not be due to classical non-radial pulsations but may arise from more complicated velocity fields in the stellar atmospheres and/or mantles.

Two early-type stars, ε Per and ζ Oph, are discussed in detail. It is argued that both may be the cases where the variations are caused by corotating structures slightly above the stellar photosphere. For ε Per, the pattern of the variations can also be affected by the motion of the star in a binary orbit. For ζ Oph, a double-wave light curve with the corotation period of $0\overset{d}{.}64$ (suggested by the author earlier for the line-profile variations) was found from Hipparcos V photometry and its presence can also be suspected in other existing photometric data and in the recurrence times of the narrow features seen in the UV spectra.

Key words: stars: early-type – stars: emission-line, Be – stars: individual: ε Per – stars: individual: ζ Oph – stars: oscillations – stars: rotation

1. Introduction

By "early-type" or "hot" stars I shall understand here the stars of spectral classes O and B, i.e. stars with effective temperatures $\geq$ 10 000 K. From the point of view of their internal structure, early-type stars are objects for which nuclear burning via the CNO cycle plays a dominant role.

The terms "rapid" or "slow" are always only relative. For early-type stars I conclude that what most of their students interpret as *rapid changes* are *variations comparable to or shorter than the rotational periods of the stars in question.* That is what I shall mean by rapid changes in this paper.

The history of the subject has been reviewed by several investigators who addressed different aspects of the problem. Interested readers are referred to studies by Baade (1987, 1992, 1998), Balona (1991, 1995a,c), Gies (1991, 1994a), Percy (1986, 1987), Smith (1985b, 1986b), Sterken & Jerzykiewicz (1993), Waelkens(1991), Waelkens & Rufener (1985) and Waelkens et al. (1998), among many others. My own recent review on the subject has recently been published and may serve as an introduction to the discussion presented below – see Harmanec (1998c).

I shall mention only a few relevant things here. Smith (1977) and Baade (1979) in their discovery studies suggested that the variations are due to non-radial pulsations (NRP hereafter). Walker, Yang & Fahlman (1979) suggested a model of corotating surface structures which they later abandoned in favour of two possibilities: either corotating structures in circumstellar matter above the photosphere or NRP – see Walker, Yang & Fahlman (1981). These two models were developed quantitatively by Vogt & Penrod (1983) who – after some discusion – preferred the NRP model in spite of the fact that the model of the circumstellar corotating structures fitted their observations equally well if not better. In support of the NRP model, Baade (1983) pointed out that both, the overall line asymmetry and travelling sub-features are simultaneously present in the line profiles of some hot stars. Soon thereafter, the NRP model gained a widespead support.

When I, and soon thereafter also Luis Balona, started to criticize the NRP interpretation and argued in favour of corotating structures (Harmanec 1983, 1984a, 1987b,c, 1989: TOW1 hereafter, 1991 a,b, Balona and Engelbrecht 1986, 1987, Balona 1990, 1991, Harmanec and Tarasov 1990a: TOW2 hereafter), our arguments were not widely accepted. Since then, the situation has changed a bit. Analyses of new systematic observations

Send offprint requests to: P. Harmanec demonstrated that the physical processes behind the observed rapid changes are quite complicated and my feeling is that even the most convinced defenders of each of the existing models have now realized the shortcomings of their favourite scenario. On one side, it seems hard to deny the presence of some velocity fields in the stellar atmospheres. On the other hand, it is equally clear that these velocity fields are more complicated than the classical NRP introduced in the seminal paper by Ledoux (1951). For some support of this statement, readers are referred to several contributions in Balona et al. (1994) (especially to Gies 1994a and Baade & Balona 1994), to Balona (1995a,c) and also to systematic studies of η Cen (Štefl at al. 1995, Štefl, Baade & Cuypers 1995) and μ Cen (Rivinius et al. 1998a,b). It appears as if the real stars combine properties of several of the simple models with, perhaps, something not as yet disclosed.

One can note that the continuing dispute begins to repeat already presented arguments. Here, I shall show particular examples of how seemingly strong arguments in the literature can have alternative explanations. New facts are gradually emerging and an attempt is made here to put them into a new perspective.

2. Illustrative discussion of two particular stars

2.1. ε Per

ε Per (45 Per, HR 1220, HD 24760, ADS 2888A) was used for years as a spectrophotometric and MKK classification standard for spectral subclass B0.5III (Morgan et al. 1943). Nowadays, it is known as an archetype of early-type line-profile variables. Current knowledge about this star is summarized in TOW 1 & 2 and in Tarasov et al. (1995) who also confirmed that the star is the primary component of a 14.076-d binary. Therefore, only investigations of its rapid variability are summarized here.

Smith (1985a) obtained the first high-S/N electronic spectra and concluded that ε Per is a non-radial pulsator and that the line-profile variations arose from a superposition of 3 prograde modes with l = 4, 6 and 1(2). Using additional spectral observations and UBV photometry, Smith (1986a) and Smith et al. (1987) concluded that two prograde modes of non-radial pulsations, with l=4 and 6 and with periods of 3.85 $\pm$ 0.02 and 2.25 $\pm$ 0.03 hours are excited in ε Per. Gies and Kullavanijaya (1988) identified as many as four periods of 3.837, 2.264, 4.466, and 3.036 hours. They also attributed them to pulsations of modes l= 4, 6, 3, and 5, respectively. Their interpretation was challenged by Harmanec (1987b) who pointed out that all four reported periods are integer submultiples of a single period of 1.12 days (26.9 hours) which he tentatively attributed to the stellar rotational period of ε Per. TOW1 measured positions of individual travelling subfeatures in the selection of line profiles published by Smith and Gies and Kullavanijaya and reconstructed their radial-velocity (RV hereafter) curves. Fitting these curves to a sinusoid he derived a revolution period of the subfeatures equal to 0.567 days. TOW2 analyzed a new series of CCD spectrograms and confronted their data with various possible models of line-profile changes (pulsations, surface spots, corotating circumstellar structures). They concluded that the model of circumstellar structures seems to be the most plausible one but pointed out that none of the models could be definitively confirmed or ruled out. Harmanec and Tarasov (1990b) noted that the period analyses of their observations indicated in fact the presence of two similar families of frequencies. They found that the difference of corresponding frequencies was equal to the mean orbital frequency. They speculated that one family of the frequencies is related to the rotational period of the primary and the other to the period with which a given meridian "sees" the secondary. As pointed out to me by the referee, this cannot be the correct explanation. Since ε Per is a binary with a highly eccentric orbit, the secondary moves with very different speeds in various parts of the orbit. Gies (1991) refuted Harmanec's (1987b, 1989) interpretation that the line-profile variations are governed by only one principal period, arguing that the superperiods of the four modes identified by Gies and Kullavanijaya (1988), and by Gies (1991) in Smith's 1984 spectral observations, differ significantly from each other. Moreover, when Gies (1991) modelled the light variations associated with the putative strongest pulsational mode, he found that the light maxima should coincide with the instant when the spectral line is deepest and claimed an agreement with Smith et al. (1987) simultaneous spectral and photometric observations.

There are, therefore, two different views of the character of line-profile variations of ε Per: Smith (1985a, 1986a), Smith et al. (1987), Gies and Kullavanijaya (1988) and Gies (1991) maintain that the variations are multiperiodic, with as much as four distinct periods according to Gies, while Harmanec (1987b) suggested that they follow only one physical period with a complicated phase curve.

Let us have a look at the original arguments once more. Table 1 compares the periods of rapid variations of ε Per reported by various authors with the submultiples of a single period of 1.11878 d. The agreement between the observed and predicted values is generally a very good one, the difference between the mean observed and predicted values is not larger than the difference between two values of the same period derived by two different authors and/or period searching techniques – with the only exception of the Crimean data (see the last column of the table), which were analyzed in such a way as to find the frequencies in the neighbourhood of the four frequencies reported earlier. The very fact that each of the investigators recovered *different submultiples of the 1.12-d period* from each separate data set also seems to speak strongly in favour of my interpretation.

It appears that the existing analyses *do not* provide any compelling evidence for the multifrequency description of the line-profile changes which would rule out my one-frequency model. Note in particular that my interpretation *was not* challenged by the analysis of Gies (1991). What Gies and Kullavanijaya (1988) and Gies (1991) did in fact was the following: They used a sophisticated period-analysis technique and derived four distinct periods of line-profile variations of the Si III lines of ε Per. As the next step, they discussed and adopted certain physical elements of ε Per (mass, luminosity, radius) and using them, they excluded rotation and corotation models from consideration. Then, they *a priori* identified the four observed periods with sectorial modes of free non-radial pulsations with modal

Table 1. Comparison of the periods (in days) of line-profile variations of ε Per reported by various authors, including my analysis of the central line depths of the Crimean He I 6678 line profiles, with integer submultiples of the 1.11878-d period.

N	P/N	PBF84	S85	SFP87	GK88	G91	TS97	TOW2+SFP87
1	1.11878							
2	0.55939		0.550					
3	0.37293							
4	0.27969							
5	0.22375	0.216						
6	0.18646				0.18608	0.18958	0.1862	0.1868194
					± .00096		± .0031	± .0000016
7	0.15983	0.160		0.1604	0.15987	0.16004	0.1597	0.15982218
				± .0008	± .00021		± .0023	± .00000070
8	0.13985		0.138					
9	0.12431				0.12650	0.12804	0.1264	0.12683831
					± .00042		± .0014	± .00000094
10	0.11188							
11	0.10171							
12	0.09323		0.0913	0.0937	0.09433	0.09475	0.0943	0.09452605
			± .0008	± .0013	± .00050		± .0008	± .00000049
13	0.08606							
14	0.07991						0.0798	
							± .0006	

The first column contains the number of the submultiple, second the value of the predicted submultiple and the remaining columns contain the periods observed by various authors indicated in column headings as follows: PBF84: Percy et al. (1984), S85: Smith (1985a), SFP87: Smith et al. (1987), GK88: Gies & Kullavanijaya (1988), G91: Gies (1991): re-analysis of Smith's observations, TS91: Telting & Schrijvers (1997): re-analysis of Gies and Kullavanijaya (1988) observations. Note that they were unable to decide between the $0\overset{d}{.}1264$ period and its 1-d alias of $0\overset{d}{.}1441$. The errors of the periods are quoted from the original papers if available. In the last column (TOW2+SPF87) which contains my analysis of the combined Crimea data and a selection of KPNO line profiles, the errors are the rms errors calculated from a covariance matrix using a sinusoidal fit to the data subsequently prewhitened for individual frequencies

numbers $m = -3$ to -6. Gies (1991) argues that the superperiods mP of these four modes are significantly different and thus the pattern speeds are not fully commensurable. They are not, of course, *for this particular mode decomposition* but this does not exclude the 1.119-d period (or its integer multiple) as the true physical period or "superperiod" of the variations observed.

In Table 2 the superperiods mP corresponding to individual detected periods and their NRP interpretation as sectorial modes $m = -3$ to -6 are listed. It is immediately seen that the superperiods of modes -3 and -6, and of modes -4 and -5 are *clearly identical* within the limits of accuracy of their detemination. The frequencies 1.566 c d^{-1} and 1.771 c d^{-1} corresponding to these "superperiods" differ by 0.205 c d^{-1}. Notably, the period corresponding to this difference frequency, $4\overset{d}{.}88$, could perhaps be identified with the period of the spin-orbit pseudosynchronization near periastron (where the attractive effect of the secondary is largest), at least within the limits of current uncertainties of the orbital elements – cf. Hut (1981). Similarly, an investigation of the observed accelerations of the four detected signals in the Gies and Kullavanijaya (1988) study (see their Table 4B) reveals that the signals S3 and S4 (in their notation) have very nearly the same acceleration (of about 1171 km s^{-1}d^{-1}), while the signals S5 and S6 both have an acceleration of 1292 km s^{-1}d^{-1}. Assuming that the line-profile variations are associated with the equatorial region on the stellar surface and adopting v sin i = 134 km s^{-1} as did Gies and Kullavanijaya, one obtains the recurrence periods of the two couples of signals of $0\overset{d}{.}719$, and $0\overset{d}{.}651$, respectively. These "superperiods" differ from those found above and could be made identical to them only by decreasing v sin i below 120 km s^{-1}. It is important to note in this

Table 2. Comparison of the superperiods of line-profile variations of ε Per corresponding to various periods reported by several authors on the explicit assumption of the NRP interpretation.

m	SFP87	GK88	G91	Mean
-3		0.5583	0.5687	0.5635
		± .0029		± .0074
-4	0.6417	0.6395	0.6402	0.6404
	± .0033	± .0008		± .0011
-5		0.6325	0.6402	0.6364
		± .0021		± .0055
-6	0.5625	0.5660	0.5685	0.5657
	± .0075	± .0030		± .0030

The first column contains the modal number m while the remaining columns contain the superperiods mP for the periods reported by various authors indicated in the headings of columns as follows: SFP87: Smith et al. (1987), GK88: Gies and Kullavanijaya (1988), G91: Gies (1991) re-analysis of Smith's observations. The errors of the periods are quoted from the original papers whenever available. The last column contains the arithmetic mean of the corresponding values and its r.m.s. error. All quoted superperiods are in days.

connection that the series of spectra used by Smith et al. (1987) was obtained on the rapidly descending part of the orbital RV curve near the periastron while the data analyzed by Gies & Kullavanijaya were obtained in orbital phases far away from the periastron passage. All this shows that the future analyses of the line-profile variations of this star will have to take the binary nature of the object into account quite seriously.

Conceivably, only *two, not four* different physical periods may be controlling the line profile changes: the orbital period and one dominant period of line-profile changes (which itself may perhaps vary slightly with the phase of the orbital period). Because of the complicated character of the variations, different investigators simply described the observed changes by different Fourier decompositions.

Since I now have at my disposal the longest homogeneous set of the line profiles from the 555 Crimean CCD data analyzed in TOW2, I repeated period analyses for the line depths of the He I 6678 line measured in the line centre of the difference profiles which resulted from the subtraction of the overall mean line profile from individual observed ones. The line centre was derived taking the orbital RV changes into account. Negative values correspond to deeper line profiles. I also used the central line depths in the selection of the Si III 4552 line profiles published by Smith et al. (1987). I found that if one subtracts the mean value of the central line depth from these Si III strengths, their range of variations is very nearly identical to that for the He I 6678 line profiles. It is, therefore, possible to refine the value(s) of the period(s) from an analysis of the combined data set of Si III and He I central line depths. Stellingwerf (1978) PDM θ statistics in the neighbourhood of the "mean bump separation" period of $0^{\rm d}\!.16$ for the individual as well as combined data sets is shown in Fig. 1. It is immediately seen that no matter how good the Kitt Peak Si III series of observations is, the relatively short time span covered by the data allows only moderate accuracy in the period determination. Only the combination of both data sets leads to a unique determination of an accurate value of the period.

A subsequent non-linear least-squares fit by program FOTEL (Hadrava 1990, 1995) led to the following value of the period:

$0^{\rm d}\!.15982591 \pm 0^{\rm d}\!.00000033$.

This is a period identical to the most dominant period found by the previous investigators – see Table 1. A phase plot of the central line depths for this period is shown in Fig. 2. It is important to realize that this period characterizes the *mean time separation* of the travelling sub-features passing across the line centre. Also plotted in Fig. 2 are the existing photoelectric observations of ε Per. In spite of their good quality, the S/N is low due to the fact that the light changes have only a very small amplitude. However, one can note that there is indeed some indication that the light of the star may vary with the $0^{\rm d}\!.1598$ period. If so, then the light maxima do indeed occur at phases when the spectral lines are deepest, as predicted by Gies (1991) on the assumption of NRP. Regrettably, none of the existing photometric data sets is suitable for an independent period analysis. In particular, 54 good Hipparcos observations span an interval over 1000 days. A

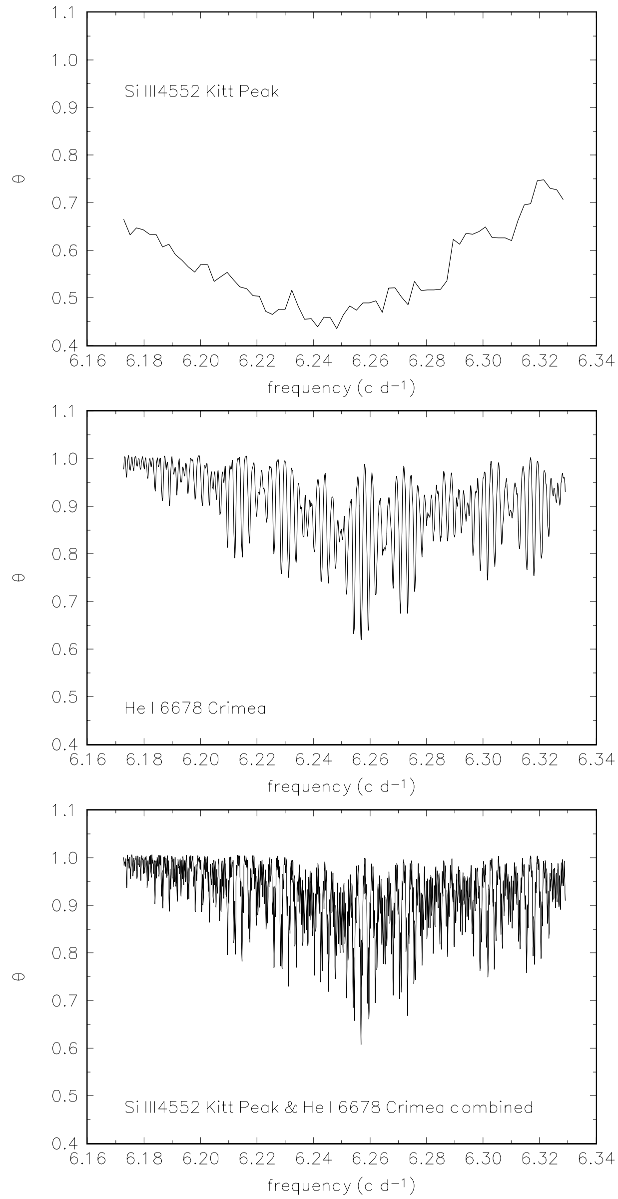

Fig. 1. Stellingwerf's (1978) PDM period search in the central line strengths of ε Per in the neighbourhood of the 'bump separation' period of $0^{\rm d}\!.1598$

formal period search on these data for roughly sinusoidal variation indicates a period of $0^{\rm d}\!.1911079$ as the best one but the result can be quite accidental.

As another test, I carried out a period search in line depths using the PDM method and looking for very complicated curves (i.e. using the coverage with 70 bins and 5 'covers'). One of the deepest minima found occurs indeed for a period of $1^{\rm d}\!.11878265$, i.e. the (super)period advocated by Harmanec (1987b). Fig. 3 shows the phase curve for this period. It is seen that the line depths define a remarkably stable curve with 7 minima and maxima of slightly different amplitudes and time separations. The agreement of Si III and He I strengths is quite notable. The data span an interval of 8 years. Available photometric observations of ε Per are also plotted vs. phase of the

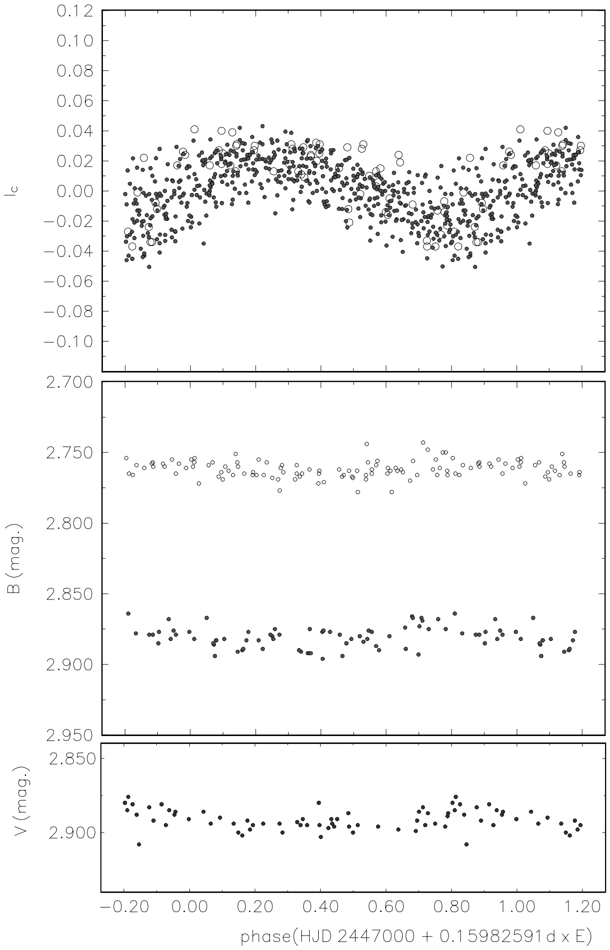

Fig. 2. Phase plots for the best-fit period of $0\stackrel{\rm d}{.}15982591$: *Upper panel:* Combined Si III (open circles) and He I (dots) residual central line depths. Note that the more negative values of I_c, the deeper the absorption-line core is. *Central panel:* Blue photoelectric observations from Percy et al. (1984) (dots) and Smith et al. (1987) (open circles) in the instrumental b and B magnitudes, respectively. *Bottom panel:* Hipparcos H_p magnitudes transformed into Johnson V after Harmanec (1998b)

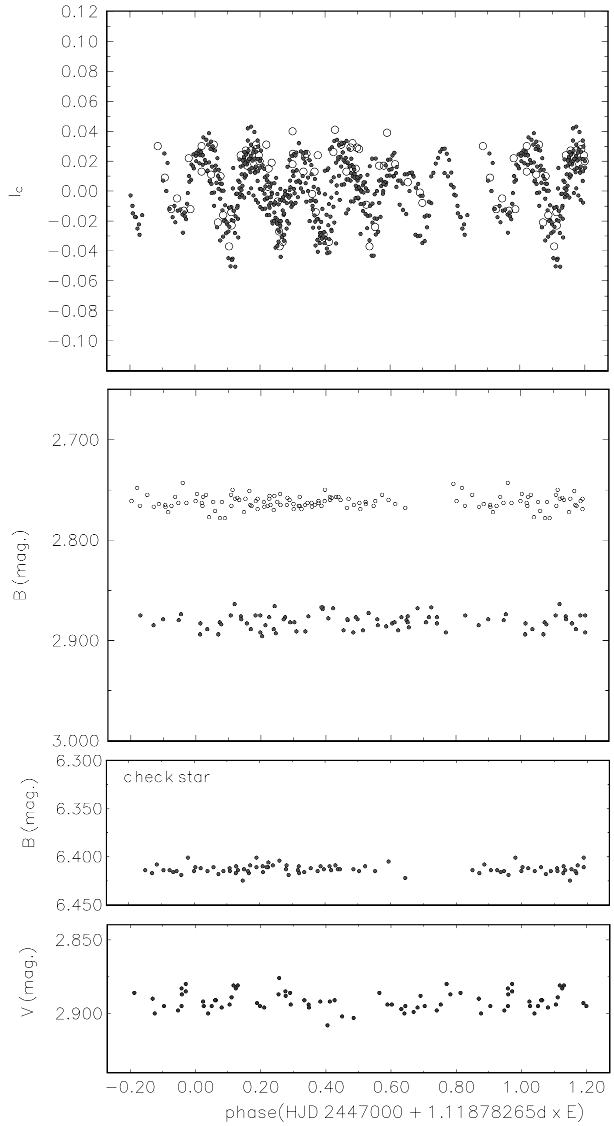

Fig. 3. Phase plots for the best-fit non-sinusiodal period of $1\stackrel{\rm d}{.}11878265$: *Upper panel:* Combined Si III (open circles) and He I (dots) residual central line depths. Note that the more negative values of I_c, the deeper the absorption-line core is. *Central and bottom panels:* Blue photometry of ε Per and the check star HR 1234 from Percy et al. (1984) (dots) and Smith et al. (1987) (open circles) and H_p Hipparcos photometry coverted to Johnson V magnitude after Harmanec (1998b)

$1\stackrel{\rm d}{.}119$ period in Fig. 3. It seems that in cases when the S/N and phase distribution of the photometric data make the detection of light changes above the noise level possible, the light maxima coincide with the instants of deepest line profiles. It is clear, however, that yet much more systematic observations would be needed to confirm the reality of the complicated phase curve beyond doubt. Note that the phase coverage in Fig. 3 is quite uneven which leaves room for some doubts on the reality of the differences between individual maxima and minima.

Although the variations displayed in Fig. 3 show a remarkable degree of repeatability (especially since there may be some percentage of observations with higher-than-usual errors in such a large data set), I admit that the description of the rapid variations observed in terms of only the 1.119-d period may not be complete. Besides, the PDM search indicated slightly better fits for periods of 2.078547 d, 2.237565 d, and 2.379386 d. However, the number of phase overlaps for these periods is still so small that the superiority of some of them over the $1\stackrel{\rm d}{.}119$ period cannot be demonstrated conclusively. The only other detected period of a high significance is a period of 0.959304 d. Note that the basic short period of 0.159826 d detected by the Fourier analysis is, to a high degree of accuracy, equal to 1/7 of the best-fit 1.119-d period found by the PDM method while the period of 0.959 d is six times longer than the 0.159-d period.

Perryman et al. (1997) published the Hipparcos observations of ε Per. It is a bit disappointing that the improved parallax of ε Per they obtained, $\pi = 0\farcs00606 \pm 0\farcs00082$, still leaves room for various alternatives open. The range of distances, from 145 to 191 pc, leaves uncertain whether ε Per is the member of the α Per cluster, although the nominal value of the parallax would now place the star in the cluster. This value, together with the observed V magnitude of the star ($2\overset{m}{.}892$ according to H_p Hipparcos photometry transformed to Johnson V after Harmanec 1998b), effective temperature derived by Tarasov et al. (1995) and interpolated bolometric correction from Code et al. (1976), leads to a radius of 6.0 $R_\odot$ (5.3 to 6.9 $R_\odot$ within the parallax error) for the ε Per primary. This is a quite normal radius for a slightly evolved (B0.5IV) star – see, e.g., Harmanec (1988). It also basically agrees with the values adopted in TOW1 and Tarasov et al. (1995): 6.89 $R_\odot$ and 6.9 $\pm$ 0.2 $R_\odot$, respectively. Note, however, that the value advocated by Gies (1991), 8.6 $R_\odot$, seems to be too large.

How can we interpret the observed variability? Here is my reflection of the current situation:

1. The analysis of several independent sets of data covering many years shows that the pattern of variations of ε Per appears to be persistent over 8 years.
2. The discussion of the advantages and shortcomings of the competing models which was presented in TOW2 remains sound. In particular, the discussion of accelerations of individual sub-features moving accross the line profiles and the confrontation with *the dynamical and/or geometrical* predictions of the models seem to exclude the model of corotating *surface* structures, raise certain problems for the model of non-radial oscillations and seem to agree best with the model of corotating circumstellar structures which *are not very high above the stellar photosphere*. No quantitative model of theoretical line-profile and light changes of such corotating structures is currently available. One can argue, however, in favour of the following qualitative scenario: It seems that for the known Be stars, the formation of a new Be envelope begins with formation of a “pseudophotosphere”, an optically thick region which mimics a normal stellar photosphere and manifest itself only by its secular variability (absorption line profiles correspond to slightly different T_{eff} and log g values at each epoch of observation) – see, e.g., Hirata (1978), Harmanec (1983) or Koubský et al. (1997). The presence of some amount of circumstellar matter around ε Per can at least be suspected: Gry et al. (1984) reported “transient shell components” in the Lyman hydrogen lines while Tarasov et al. (1995) suspected secular variation of the equivalent width of the Hα line. It is conceivable that the putative pseudophotosphere is more concentrated towards the stellar equator. If such a structure consists of regions of higher and lower density and temperature, it must produce observable line-profile and light changes as it corotates. At the same time, the mutual phase relation of these two kinds of changes may depend on the inclination under which the rotating star is observed, similarly as for the disks of Be stars (Harmanec 1983): The same region of higher density may produce a light decrease when seen equator-on (“shielding” the stellar disk) or a light increase when seen under an intermediate inclination, i.e. projected outside the stellar disk (therefore increasing the total luminous area). No doubt, only a realistic quantitative model can confirm or disprove such a scenario but the same is also true for the NRP model (note, e.g., Townsend’s 1997 warning that the effects of rotation can change the character of associated light changes).
3. It is important to realize that for the model of corotating *surface* features, one would have to assume that the period of rotation is $1\overset{d}{.}12$. Since the mean separation of the sub-features is about $0\overset{d}{.}16$, one should observe 3 to 4 sub-features in the line profiles at any time. In reality, however, only 2 sub-features are usually seen in the line profiles. While this argument eliminates the surface corotating structures, it is not so restrictive for the structures above the stellar photosphere since only a part of them is projected against the stellar disk, depending on their distance from the photophere and inclination of the axis of rotation. Note that a rotational period of $2\overset{d}{.}24$ was preferred in TOW2 and it was found that the corotating structures responsible for the observed sub-features have to be at distances of about 3 stellar equatorial radii. For such a configuration, only 2 of the overall number of 14 sub-features would indeed be projected against the stellar disk, in accordance with the observations.
4. The observed commensurability of the periods detected by Fourier decompositions of the line-profile changes of ε Per must be taken into account in further development of the model of *free* NRP. It will be important to demonstrate that it can be explained consistently as the rotational splitting of the pulsational frequencies within the current uncertainties of the basic stellar properties of ε Per. Note that the observed v sin i and the probable mass and radius now define a rather narrow interval for the rotational frequency of ε Per between about 0.45 and 0.65 c d^{-1}.
5. An important question is what is the cause for such corotating structures. Some investigators believe they are produced by magnetic fields. I must repeat what I suggested earlier (Harmanec 1987c): These can be resonant phenomena in the circumstellar disk caused by the gravitational force of the companion. There are some indications that there may be some interesting numerical coincidences present. If $2\overset{d}{.}24$ is indeed the true rotational period of the primary, it is conceivable that the primary is rotating twice as far as the spin-orbit synchronization rate at periastron.
6. It may also be rewarding to investigate a related alternative that the observed line-profile changes are in fact *forced* non-radial oscillations (cf., e.g., De Mey et al. 1997).
7. To complicate the story even more, I also note that the recurrence period(s) of the signals detected in the line profiles are numerically comparable to the expected period of *Keplerian rotation* near the stellar surface of the primary.

2.2. ζ Oph

ζ Oph (HD 149757, HR 6175) is an O9.5Ve star. This is the very first star for which sub-features of apparent absorption and/or emission, moving from the blue to red across the line profiles, were discovered (Walker, Yang & Fahlman 1979). They first believed they detected corotating surface ‘spots’. Later, Walker, Yang & Fahlman (1981) argued that the travelling sub-features are either due to corotating structures in the circumstellar disk or due to NRP. Vogt & Penrod (1983) tested all three models quantitatively. They found excellent fits to the line profiles on the assumption that the corotating structures in the disk are modelled as spokes of higher density, slightly tilted in the direction of the stellar rotation. Yet, they preferred the NRP model since they estimated that the transit of each spoke across the stellar disk should cause a $0\overset{\mathrm{m}}{.}05$ light decrease which they did not find during one night of photometry. I pointed out, however, that two to three spokes had remained projected against the stellar disk at any time in their spoke model of ζ Oph which inevitably decreases the amplitude and changes the phase of any observable light changes (see Harmanec 1987a, TOW1). In TOW1, I also re-analysed all line profiles of ζ Oph available at that time. Measuring accelerations of individual sub-features I showed that all observed changes could be reconciled on the assumption that there were 7 different features at the stellar equator which corotated with the star with a period of $0\overset{\mathrm{d}}{.}64308$. I also suggested that the equatorial features could be roots of circumstellar spokes, responsible for the formation of the narrow absorption components in the UV resonance lines.

Proponents of the NRP interpretation have continued their effort to decompose the observed line-profile changes into a sum of several sinusoidal variations, however. Kambe, Ando & Hirata (1993) concluded that the line-profile changes arise from a superposition of two sinusoidal variations with periods of $0\overset{\mathrm{d}}{.}139$ and $0\overset{\mathrm{d}}{.}101$. In some seasons, they also detected another period of either $0\overset{\mathrm{d}}{.}0775$ or $0\overset{\mathrm{d}}{.}0587$. They warned that 1-d aliases of the two main periods, $0\overset{\mathrm{d}}{.}0921$ and $0\overset{\mathrm{d}}{.}162$ cannot be excluded and pointed out that these are then nearly exact submultiples of the period of $0\overset{\mathrm{d}}{.}643$ period which I found. Their conclusion was that NRP is the “simplest explanation” of the observed changes but admitted that they cannot exclude corotating models completely. Reid et al. (1993) concluded that the line-profile variations could be adequately represented by a sum of four sinusoids with periods of $0\overset{\mathrm{d}}{.}1391$, $0\overset{\mathrm{d}}{.}1015$, $0\overset{\mathrm{d}}{.}07746$ and either $0\overset{\mathrm{d}}{.}05692$ or $0\overset{\mathrm{d}}{.}05383$. They pointed out approximate but *not exact* commensurability of these periods. Reid et al. (1993) also challenged my rough estimate of the equatorial radius of ζ Oph (see formula (13) in TOW1) claiming that one should equate

$$\pi\, R_p\, R_e \;=\; 1.5\pi\, R^2,$$

where R_p, R_e and R are the polar, equatorial and observed apparent radius of the star, respectively. They recalled that the polar radius of a critically rotating star remains virtually unchanged with respect to than of a non-rotating star. Well, that is exactly what I assumed. I adopted indeed $R_p = 2/3\, R_e$ but equated the apparent projected area of an ellipse to the projected area of a circle with radius R. Both, the interferometric and the BCD estimates of the radius of ζ Oph, a mean of which I adopted, assume, of course, a spherical star with an effective radius R. I insist, therefore, that one has to use the formula

$$\pi\, R_p\, R_e \;=\; \pi\, R^2$$

to the estimate of the equatorial radius of a critically rotating star. For $R = 6.5\,\mathrm{R}_\odot$ this gives $R_e = 8\,\mathrm{R}_\odot$. I, therefore, believe that the criticism of models of corotating structures, presented by Reid et al. (1993) in their Sect. 4.5, is not as sound as they claim. Incidentally, the Hipparcos parallax of $0\overset{\prime\prime}{.}00712$ also supports a normal main-sequence value of the mean stellar radius: the range is 6.9 – 8.5 $\mathrm{R}_\odot$ if the measured angular diameter and its error are assumed, and 5.1 $\mathrm{R}_\odot$ if one uses the distance modulus and the effective temperature corresponding to spectral class O9.5. In passing, I remind that a more accurate estimate would require some modelling which would also take into account the effects of the limb and gravity darkening.

Howarth et al. (1993) analyzed IUE and HST UV spectra of ζ Oph and concluded that the time variable ‘discrete absorption components’ in the stellar-wind lines reappear on a time scale of about $0\overset{\mathrm{d}}{.}8$. They also concluded that there is no obvious evidence that the UV discrete components would be directly related to the photospheric travelling sub-features in the optical line profiles. They, therefore, favoured the line-driven instability as the most promising model of the origin of the discrete components of UV lines.

Unfortunately, their results are only published in the form of a time plot (Fig. 6 of their paper). From that it appears that the three occurrences of discrete components starting at lower velocities were observed – very roughly – on JDs 2447642.2,...43.5 and 44.3. It is possible that these three episodes are separated by integer multiples of the $0\overset{\mathrm{d}}{.}643$ period derived in TOW1.

So far the last word has been said by Kambe et al. (1997) (see Kambe et al. 1995 for a preliminary report) who organized a successful international multisite campaign. They concluded that two periods, $0\overset{\mathrm{d}}{.}084083$ and $0\overset{\mathrm{d}}{.}13904$, dominate the line profile changes. They also suspected the presence of two other periods, $0\overset{\mathrm{d}}{.}10133$ and $0\overset{\mathrm{d}}{.}087792$, but admitted these may be in fact aliases of the period of $0\overset{\mathrm{d}}{.}084083$. Kambe et al. also noted that the two main periods have a nearly identical superperiod of $0\overset{\mathrm{d}}{.}41875$.

It is a pity that the unique body of quantitative data obtained and analyzed by Kambe et al. (1997) has not been made available to others. Using the tables and diagrams provided in their study, I can only make the following remarks:

1. Within the accuracy with which I was able to measure the accelerations of the $0\overset{\mathrm{d}}{.}084$ and $0\overset{\mathrm{d}}{.}139$ signals near the centre of the He I line profile (from Figs. 8a, 8b of their paper), I found *identical* values near some 3600 km s^{-1} per day for both of them. Such an acceleration can be reconciled with the $0\overset{\mathrm{d}}{.}643$ period (TOW1) and the v sin i observed.
2. I also noted that the following relation holds

   $$0.084^{-1} - 0.139^{-1} = 3 \times 0.638^{-1}.$$

   This shows that even their analysis reflects somehow the time scale found in TOW1. To investigate the problem more closely, I carried out the following numerical experiment: I assumed my model of rapid variations of ζ Oph (cf. TOW1), i.e. that the line-profile changes are caused by seven spokes around the stellar equator corotating with a period

of $0^{\rm d}.64308$. I adopted the time separations of the consecutive spokes which I derived from an analysis of Vogt & Penrod's (1983) series of spectra (see Table 3 in TOW1), namely $0^{\rm d}.1051$, $0^{\rm d}.0710$, $0^{\rm d}.1042$, $0^{\rm d}.0929$, $0^{\rm d}.0957$, $0^{\rm d}.0889$, and $0^{\rm d}.0853$. I modelled the line-strength variations in the line centre by a function which consisted from a sequence of sinusoids with periods equal to the above-mentioned spoke time separations and semiamplitudes which somewhat arbitrarily ranged from 0.85 to 1.0. The model function was so constructed that the whole pattern repeats regularly with the period of $0^{\rm d}.64308$. Then I estimated the times of observations from Fig. 4 of Kambe et al. (1997) and calculated the values of the model function for these times. Using Breger's (1990) program PERIOD I searched the artificial time series for periodicity over the range of frequencies from 1 to 30 c d^{-1}. I found the following periods:
$0^{\rm d}.0919$, $0^{\rm d}.1607$, $0^{\rm d}.1286$, $0^{\rm d}.0643$, $0^{\rm d}.0804$, $0^{\rm d}.1072$, and $0^{\rm d}.0585$.
This is to be compared to the results of Kambe et al. (1997) who reported the following periods:
$0^{\rm d}.0841$, $0^{\rm d}.1390$, $0^{\rm d}.1013$, $0^{\rm d}.0524$, $0^{\rm d}.0420$, $0^{\rm d}.0878$, $0^{\rm d}.0804$, and $0^{\rm d}.0695$.
Since I created the model function somewhat arbitrarily, without having the chance to analyze the real spectra, it is not surprising that I could not model the real situation. It is notable, however, that the strongest two periods are similar. The first one corresponds to the mean bump separation and the second one is 1.75 times and 1.65 times longer than the first one in the simulated and real time series, respectively. More generally, the structural similarity of both series of periods is notable.
I am not, therefore, fully convinced that the presence of a real multiperiodicity of the line-profile variations of ζ Oph has been proven beyond doubt.

Investigation of light variability of ζ Oph led to rather inconclusive results. Vogt and Penrod (1983) reported one 5 hours long series of photometric observations relative to HR 6278 (HD 152585) and HR 6179 (HD 149911) which seemed to indicate a slight monotonic change within about $0^{\rm m}.02$. HR 6179 is a magnetic Ap star and a light variable (Maitzen 1976). Cuypers et al. (1989) published a systematic series of photometric observations of ζ Oph relative to ρ Oph AB and 16 Oph obtained in Sutherland and in 1987 also at La Silla. For the 1985 data, they found periodic light variation with a period of $0^{\rm d}.193$ but they noted that the same data can also be reconciled with a longer period of $1^{\rm d}.075$ and a complicated light curve, the full amplitude being $0^{\rm m}.03$. No significant variations were found during 1987, however. Some rapid variations were detected in 1988, when also a steep light decrease for about $0^{\rm m}.03$, taking place during a few days, was recorded. Balona (1992) continued the b observations at Sutherland in 1989 and 1990. He detected a period of either $0^{\rm d}.376$ (or its alias of $0^{\rm d}.273$) or $1^{\rm d}.14$ in 1989 data, and $0^{\rm d}.435$ and $0^{\rm d}.376$ in the 1990 series. He concluded that the short-periodic variations had no stable period. Both comparison stars seem to be constant over the period of Hipparcos observations (Perryman et al. 1997). Also Kambe et al. (1997) obtained photometry of ζ Oph, relative to HR 6096 (HD 147550) and HR 6278 (which are about three magnitudes fainter than ζ Oph), covering a period of about 12 days. They found variability within $0^{\rm m}.03$, with some indication of a $0^{\rm d}.380$ period or its alias of $0^{\rm d}.6146$ in u observations, and $0^{\rm d}.367$ in other passbands.

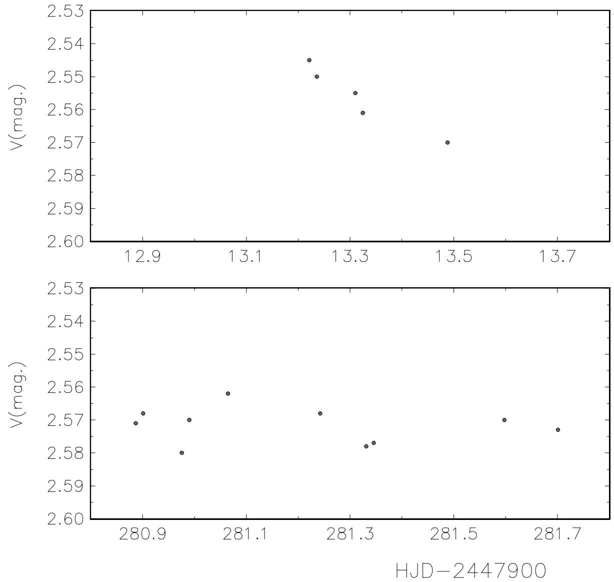

Fig. 4. The V magnitude of ζ Oph (derived from the Hipparcos H_p magnitude) plotted vs. time for the two longest series of observations. It is seen that the variations occur on a time scale longer than $0^{\rm d}.1$

I inspected the Hipparcos photometry of ζ Oph (Perryman et al. 1997) and found that the star is undoubtedly a small-amplitude light variable. Since Dr. L.A. Balona kindly put at my disposal all published and unpublished 1985-90 SAAO and La Silla photometry, I decided to carry out independent period analyses of the existing photometry.

Since the light variations of ζ Oph have a very low amplitude, there is always some danger that even a small error in the nightly reductions (extinction, zero point drifts) can affect the observed variations non-negligibly, especially the observations at larger air masses. Therefore, to set some limit on the shortest timescale of variations seen in the data, I first inspected the Hipparcos photometry (transformed to Johnson V after Harmanec 1998b). Fig. 4 shows the time plots of the two longest series of Hipparcos observations. It is seen that there is no clear evidence of variations on a time scale of about $0^{\rm d}.1$, comparable to periods derived by Kambe et al. (1997). If the variations are periodic, the period must be longer than about $0^{\rm d}.3$. Time plots of the SAAO and La Silla observations (not shown here) also indicate that light decreases for $0^{\rm m}.02$-$0^{\rm m}.03$ occur on a time scale of several weeks. This represents an extra problem for any search for periodic changes of ζ Oph.

When I analyzed all good Hipparcos observations for periodicity in the neighbourhood of the $0^{\rm d}.64$ period derived in TOW1, I indeed found that the data can be well reconciled with a period of $0^{\rm d}.6447663$, close to the value derived in TOW1. I must emphasize again that the Hipparcos photometry is not ideal

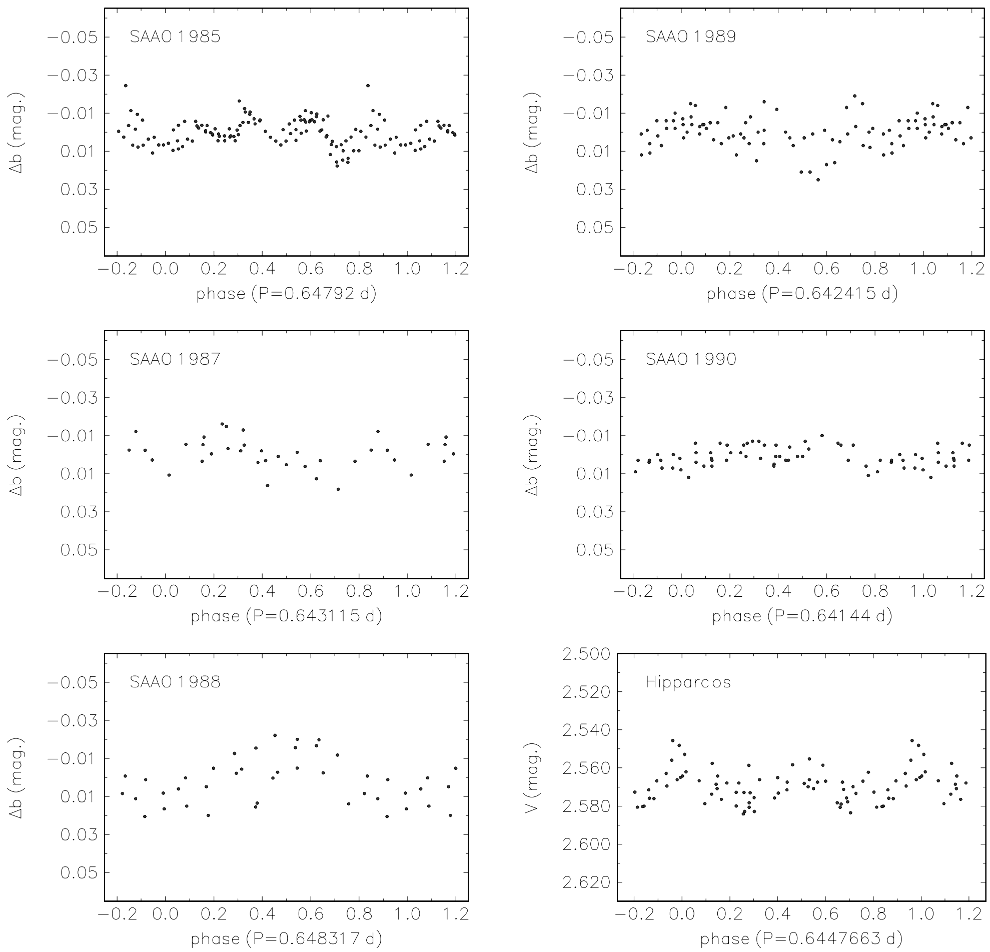

Fig. 5. Several optical (b-band) light curves and the Hipparcos V light curve of ζ Oph for the locally best-fit periods near $0.^{\rm d}64$; see the text for details

for an independent period analysis because of a limited number of observations distributed over several years. However, the $0.^{\rm d}645$ period is among the best fits for the periods longer than $0.^{\rm d}3$ and certainly gives the best light curve. It is true, however, that a $0.^{\rm d}32$ period with a roughly sinusoidal light curve cannot be excluded at the S/N of the photometric data.

An inspection of all b observations used by Balona (1992) shows that the variations were below the detection treshold during the first part of 1987 observations and also at the end of the 1989 season. In most of the other seasons (besides 1989 and 1990), there were overlapping variations on a time scale of a few weeks mentioned above. Using the smoothing technique by Vondrák (1969, 1977), I locally removed these trends from the data and subjected the prewhitened observations to period analyses from $0.^{\rm d}3$ to $2.^{\rm d}0$. Although it was possible to detect several formally better periods in each of the data set studied (for instance about $0.^{\rm d}7$ in the second half of the 1987 and in 1988 data and about $1.^{\rm d}2$ in 1985 and 1989), only periods near $0.^{\rm d}64$ could be detected in all investigated data sets. Photometric observations used by Kambe et al. (1997) are not available but the authors remarked that they detected a period of $0.^{\rm d}615$ as one of the two possibilities.

Fig. 5 shows the corresponding light curves for the locally derived best periods near $0.^{\rm d}64$. It is seen that the shape of the light curve varies from nearly sinusoidal to rather complicated curve with several maxima and minima. Such a behaviour is known also for other Be stars (cf., e.g., Harmanec et al. 1987). All this shows that the variations of ζ Oph are certainly quite complex. It is hard to decide whether the photometric periods found in different seasons differ significantly. Since the shape of the curve varies even during one observing season, it is not clear how to define the phase coherence between the curves. For a rapidly rotating star like ζ Oph, the corotation period could vary with time, depending on the varying extent of the circumstellar envelope.

I certainly cannot claim that the results presented here provide a firm proof that the $0.^{\rm d}64$ period is the principal physical period of the variations observed. Actually, the referee pointed out to me that the grey scale plots of line-profile changes published by Kambe et al. (1997) do not seem to offer support for a pattern repeating every $0.^{\rm d}64$. However, even if the $0.^{\rm d}64$ period will not ultimately be confirmed, the analyses presented above show that real photometric changes of ζ Oph occur on a time scale which is several times longer than the periods derived from Fourier decompositions of line-profile changes. Further systematic simultaneous photometric and spectral observations of the star are very desirable. As pointed out to me by Dr. A.E. Tarasov, a period analysis of the V/R variations during the next emission-line episode could also help in testing the reality of the $0.^{\rm d}64$ period.

3. My observations, suggestions and speculations

It is interesting to realize that some two-three decades ago, the variations of hot stars, especially of emission-line objects, were generally considered to be of stochastic, irregular nature, with the only exception of β Cep stars. Several phenomenological groups were distinguished. Nowadays, evidence is being accumulated that these phenomenological groups need not be mutually exclusive. Thus, for instance, the prototype of β Cep stars, β Cep itself has long been known to be a Be star (cf. Hadrava & Harmanec 1996 for a recent summary) and was recently reported to be a magnetic oblique rotator (Henrichs et al. 1993). There are certain indications of the presence of circumstellar matter around the archetype of 'non-emission' line-profile variables, ε Per (Gry et al. 1984, Tarasov et al. 1995), and its southern analog, δ Sco, has been found to be a Be star (Coté & van Kerkwijk 1993). Also, some Be stars were found to be chemically peculiar or vice versa (cf., e.g., Walborn 1974, Smith, Robinson & Corbet 1998).

A recent tendency is to look for periodic components of the variations observed. This is certainly a very good trend but one should keep in mind that especially the Fourier decomposition of the variations is a "blind" method which will *always* find some frequencies in a limited string of data. In particular, cyclic or stochastic variations will be described by a number of frequencies and if there is some regularity or a characteristic time scale in the pattern of such changes, it is even possible that two or more series of observations would be formally decomposed into similar families of frequencies. Rivinius et al. (1998c) suggested that each strong positive interference of several close $0\overset{\mathrm{d}}{.}5$ periods which they detected in line-profiles changes of μ Cen led to the formation of a new Be envelope around the star. This is certainly an exciting finding which could bring us closer to understanding the problem of the Be phenomenon. In the light of what I mentioned above, I feel, however, that it will be extremely important to ask first what is the cause and what are the consequences. If I am right in my belief that the line-profile changes are somehow related to the circumstellar matter not very high above the stellar photosphere, then it is conceivable that the whole scenario should be reversed. What if some (still unknown) physical process is causing formation of new Be envelopes from time to time, irregularly but with a certain characteristic time scale? These changes could, of course, affect the effective radius of the (pseudo)photosphere and, therefore, the principal corotation period of μ Cen. This in turn could lead to the detection of several close periods in period analyses by the standard techniques.

Below, I offer some modified views of various aspects of the whole problem. To make the following text shorter, I shall introduce two terms to denote two different types of variations:

1. BCL (β Cep -like) variation: spectral and/or light variation which can – at least locally – be modelled by a single-wave curve with a period between about $0\overset{\mathrm{d}}{.}05$ and $0\overset{\mathrm{d}}{.}25$, i.e. a typical period of β Cep stars.
2. RL (rotation-like) variation: spectral and/or light variation which can – at least locally – be modelled by a generally non-sinusoidal curve with a period comparable to the expected period of stellar rotation for the star in question.

i. *BCL vs. RL variations* There were various suggestions concerning the occurrence and mutual relation of these two types of changes. Some investigators argued that the BCL changes, especially in the form of line-profile disturbances, represent a more general type of variability, found among many OB stars, and claimed that the RL changes are only typical for Be stars (cf., e.g., Balona in Baade & Balona 1994). I would like to offer a slightly different view of the problem:

Let us assume that one observes a local disturbance which travels across a line profile. (It may be either a 'spot' of locally different contrast with respect to the photosphere or a wavecrest carried out across the profile by combined effects of phase velocity and rotation.) If the star in question is a rapid rotator, a spectroscopist will record the disturbance as the so-called 'moving bump' or 'travelling sub-feature', first observed by Walker et al. (1979) for ζ Oph. If, however, one observes the same phenomenon for a slow rotator, he or she will detect it only through RV, line strength, FHWM and perhaps also EW changes.

It may be highly significant that the time separation of the individual sub-features travelling across line profiles of all so far found rapidly rotating LPV, is in the range from $0\overset{\mathrm{d}}{.}05$ to $0\overset{\mathrm{d}}{.}2$, i.e. in the range of the periods found from period analyses of the spectra and photometry of the β Cep stars. It would be important to look carefully for the photometric counterparts of the line-profile variations due to passing sub-features.

There is also another aspect of this problem. Štefl et al. (1995) found for the Be star η Cen that the strong travelling sub-features reappear in its line profiles always *at the same interval of phases* of its $0\overset{\mathrm{d}}{.}6424$ photometric and RV period. It is, therefore, conceivable that the light variability is dominated by the structural difference between strong and weak sub-features, integrated over the visible part of the stellar disk at each phase rather than by individual passing sub-features. This may explain the simultaneous presence of BCL and RL variations in the same star and that is why I must oppose Balona's (1991) classification of rapid variables into λ Eri variables (those which exhibit also RL type changes) and ζ Oph stars (those characterized by only BCL changes). Obviously, these two phenomena are not, or at least need not be independent of each other.

Statement 1 *BCL and RL variations may be simultaneously present in any OB variable, though with largely different relative amplitudes, and should be looked for carefully in each particular case. It should also be established beyond doubt whether or not the period of reappearance of a particular sub-feature in the line centre is, or is not, identical to the photometric and/or RV period observed for the given star.*

ii. *Do RL variations reflect true rotational periods?* The discussion on the nature of the RL changes is complicated by the following fact: There is not a single RL Be variable the radius of which has been derived accurately enough. Available systematic studies of long-term spectral and light variations of various Be stars (cf., e.g., Koubský et al. 1997) show that a 'pseudophotosphere' i.e. an optically thick region, can be formed early in

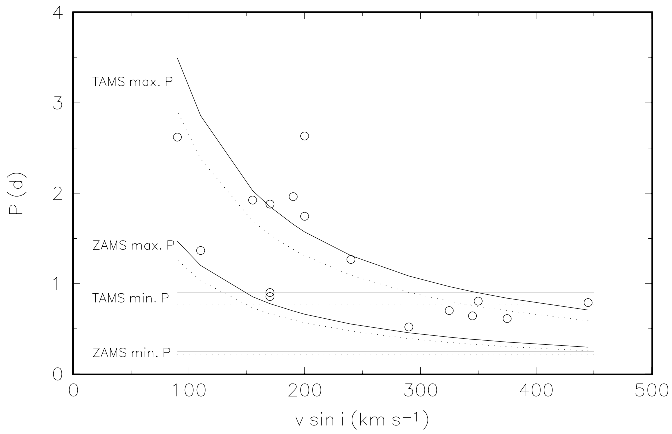

Fig. 6. Observed photometric periods of B2e stars vs. v sin i (circles). Also shown are the minimum and maximum rotational periods for a 7 $M_\odot$ star (dotted lines) and a 9 $M_\odot$ star (solid lines) for the ZAMS and TAMS stellar interior models of Schaller et al. (1992)

Table 3. Periods and v sin i values of known periodic B2e light variables

Star	HD	P (d)	v sin i (km s^{-1})	Ref.
DX Eri	30076	1.2671	240	1
λ Eri	33328	0.7017	325	2
V1046 Ori	37017	0.9012	170	9
ω Ori	37490	1.9616	190	2
HR 2142	41335	0.79	445	3
FT CMa	48917	2.632	200	2
FV CMa	54309	0.52	290	3
ω CMa	56139	1.365	110	4
HR 3857	65875	0.855	170	2
IU Vel	77320	0.612	375	2
δ Cen	105435	1.923	155	2
ν Cen	120307	2.621	90	5
η Cen	127972	0.6424	345	6
HD 160886	160886	0.806	350	7
Ahmed 01		1.745	200	8
Ahmed 15		1.88	170	8

References: 1: Štefl & Balona (1996); 2: Balona, Cuypers & Marang (1992); 3: Barrera, Mennickent & Vogt (1991); 4: Baade (1982); 5: Cuypers, Balona & Marang (1989); 6: Štefl et al. (1995); 7: Kilkenny & Lynas-Gray (1987); 8: van Vuuren, Balona & Marang (1988); 9: Bolton et al. (1998).

the process of formation of a new Be envelope. This in turn means that without knowing the pattern of the long-term spectral changes of the star, one *cannot rely* on the spectroscopic determination of its spectral and luminosity class. It is virtually impossible to distinguish the secularly changing pseudophotosphere from the true stellar photosphere corresponding to the *evolutionary stage* of the star in question. Therefore, even a very careful determination of the luminosity class, based on the detailed comparison of the observed and synthetic line profiles, may give an incorrect answer about the true photospheric radius of the star in question. This warning may also apply to at least some B stars which *are not known* to have the Balmer emission. Systematic monitoring of their Hα profiles with electronic detectors may reveal many of them as marginal Be stars.

For a long time, the dispute between the advocates of NRP model and advocates of models assuming corotating structures has concentrated on the issue of whether or not the periods of the RL changes observed for Be stars are indeed comparable to the rotational periods of stars in question. Their mutual similarity was first suggested by Harmanec (1983, 1984a, 1991a) and the *identity* of the two was advocated by Balona (1990, 1991, 1995a,b). There was a dispute between Luis Balona, Doug Gies and Dietrich Baade (Gies 1994b, 1996, Balona 1995b, Baade 1996) on this topic. As I pointed out earlier (Harmanec 1991a), the only possibility how to cope with the existing uncertainties in the stellar radii is to assume that any classical Be star has a radius somewhere between the zero-age (ZAMS) and terminal-age (TAMS) main sequence and investigate the minimum and maximum rotational periods for each observed star individually. To eliminate at least the dependence of radii on the spectral type, which affects Balona's statistics, I selected a subset of rapid Be variables of spectral class B2 only – see Table 3. Fig. 6 is a plot of observed RL periods vs. v sin i for all more reliable determinations. There is indeed some correlation between the two quantities. To take the uncertainty in masses and radii into account, I adopted 7 $M_\odot$ and 9 $M_\odot$ evolutionary models from Schaller et al. (1992) to bracket the minimum and maximum rotation periods. The minimum period (shown in each case by a straight line) is defined as the break-up period at the equator (Roche-model approximation) while the maximum period is based on equating the equatorial rotational velocity to the v sin i observed (see Harmanec 1991a for details). If the photometric periods measure the rotational periods of the stars in question, then the observed points should fall between some pair of the limiting lines. Fig. 6 shows that most of the observed periods could indeed be identified with the rotational periods of the respective stars. The only star which really strongly violates the upper-limit test is FT CMa (at v sin i of 200 km s^{-1}) for which Balona et al. (1992) found a double-wave curve with a period of $2\overset{\mathrm{d}}{.}632$ and a rather high scatter. For the same star, Baade (1984) suggested a period of $1\overset{\mathrm{d}}{.}334$ from his RV and photometric observations. Also Hubert & Floquet (1998) derived a period of $1\overset{\mathrm{d}}{.}339$ from their analysis of Hipparcos photometry of FT CMa. This shorter period would pass the test. How serious are the other, milder discrepancies needs to be investigated further. Some other periods may not be identified properly as well (note, for instance, that Štefl et al. 1995 found a period three times shorter than that reported earlier for η Cen while Štefl and Balona 1996 revised the period of DX Eri from $1\overset{\mathrm{d}}{.}113$ to $1\overset{\mathrm{d}}{.}267$) and there may also be large errors in v sin i values (due to unrecognized effects of the pseudophotosphere, for example). Another uncertainty comes from the fact that non-rotating stellar evolutionary models were used. The effect of rotation on models depends on the (unknown) internal angular-velocity distribution (cf., e.g., Cotton & Connon Smith 1983). On the other hand, Stalio et al. (1987) found no evidence for critical rotation in Be stars from Voyager UV data. This probably more

Table 4. Comparison of the observed periods of rapid variations of several brightest B and Be stars with the probable periods of Keplerian orbits close to the stellar photosphere

Star	HD	Sp.class	d (pc)	R ($R_\odot$)	$P_{\rm Kepl.}$ (d)	$P_{\rm sp.}$ (d)	$P_{\rm phot.}$ (d)	Source
γ Cas	5394	B0.5V	188± 20	6.5	0.52	0.8:	1.16	1,2
ε Per	24760	B0.5IV	165± 23	6.0	0.47	0.56	?	1
μ Cen	120324	B2IV	162± 18	5.2	0.46	0.50	1.1	3
η Cen	127972	B2IV	95± 07	5.5	0.54	0.64	0.64	4
ζ Oph	149757	O9.5V	140± 14	5.2	0.60	0.64	0.64	1
o And	217675	B6III	212± 31	10.	1.90	?	1.57	5

References: 1: this paper; 2: Ninkov et al. (1983) [from their best-fit sinusoidal fit], Smith et al. (1998); 3: Rivinius et al. (1998b); 4: Štefl et al. (1995); 5: Harmanec et al. (1987);

than compensates for the possible increase of the polar radii of rotating stars since only very near to break-up the equatorial radius differs significantly from the polar one (1.5 times for the break-up and the Roche model). Therefore, almost twice shorter minimum periods than those shown in Fig. 6 could still be possible (cf. Harmanec 1988).

Statement 2 *Whether the periods of RL variations, derived from the periodic light variations, RV variations or line-width changes, are identical to the true rotational periods for all rapidly rotating Be stars, independently of the true nature of the RL changes, still remains to be seen. The issue is important since in case of an affirmative answer, one would have a very useful method of determination of the rotational periods of Be stars.*

iii. *RL variations and Keplerian rotation?* Thanks to accurate parallaxes measured by Hipparcos (Perryman et al. 1997), it is possible to estimate the radii of the brightest B or Be stars using the visual magnitude (from phases without emission for Be stars), bolometric corrections and effective temperatures derived from spectral types or line-profile modelling. I also adopted masses corresponding to spectral types using the calibration by Harmanec (1988) and estimated the periods of Keplerian rotation just above the photosphere for several bright B stars. In Table 4, I compare these periods with the periods derived either from RV variation or acceleration of sub-features ($P_{\rm sp.}$) and also with the periods of light changes ($P_{\rm phot.}$).

For γ Cas, Smith et al. (1998) give $1\overset{\rm d}{.}125$. Marchenko et al. (1998) derived periods of either $1\overset{\rm d}{.}634$ or $1\overset{\rm d}{.}045$ from Hipparcos photometry but they did not remove the secular trend. I, therefore, repeated their analysis, using only data with error flag zero and with rms of individual observations better than $0\overset{\rm m}{.}01$ and removing first a small linear trend from the data. Searching for periods between $1\overset{\rm d}{.}1$ and $1\overset{\rm d}{.}2$, I found a period of $1\overset{\rm d}{.}15655 \pm 0\overset{\rm d}{.}00012$ with a semiamplitude of $0\overset{\rm m}{.}0064 \pm 0\overset{\rm m}{.}0012$ as the best one. The corresponding light curve is shown in Fig. 7.

No doubt, the estimates presented in Table 4 suffer from errors which are hard to evaluate properly. For instance, varying the distance derived for η Cen within the errors of the Hipparcos parallax, one arrives at Keplerian periods between $0\overset{\rm d}{.}39$ and $0\overset{\rm d}{.}67$. Yet, it is interesting to note that the estimated periods of Keplerian rotation are usually comparable to the observed

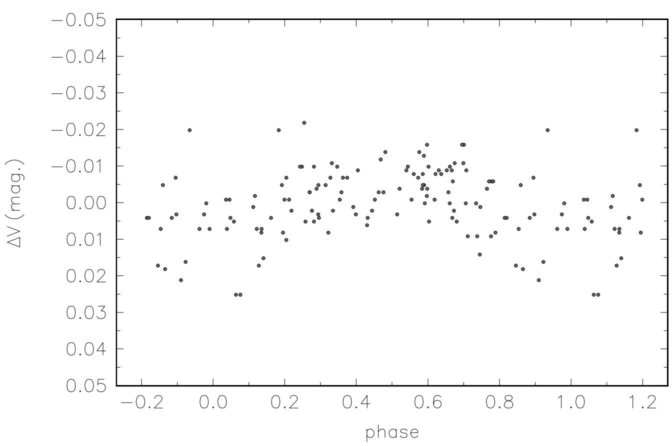

Fig. 7. Residual V magnitude after removal of a linear trend from Hipparcos observations plotted vs. phase of the $1\overset{\rm d}{.}15655$ period. The epoch of minimum brightness, HJD 2448500.806 ± 0.030, was adopted as phase zero

spectroscopic periods. In some cases, the photometric periods are (twice??) longer. If this speculation could be confirmed by more accurate determinations, this would constitute another argument in favour of the hypothesis that the observed variations originate somehow in the circumstellar matter near the stellar photospheres.

iv. *Problems of harmonic analyses of line profiles* Note that the success of the period analyses based on Fourier decomposition into a sum of several sinusoidal variations depends strongly on the real physical situation. If – for instance – the line-profile variations are governed solely by NRP, such an analysis is sound and should return indeed the frequencies of NRP modes in observer's frame. If, however, the variations are due to several corotating density enhancements ("spokes") as I suggested in TOW1 and as students of UV spectra of O stars are now suggesting (Owocki, Cranmer & Fullerton 1995), such an analysis may fail, partly or completely, depending on how regularly or irregularly such structures are spaced around the stellar equator. It is obvious that in this case, the Fourier analysis will detect "periods" comparable to the *mean* time separation between the consecutive sub-features as they are crossing the line centre. The more irregular spacing the structures will have, the more "periods" will be detected in the periodograms. This, however, *will not be a real multiperiodicity*, although it may appear so since there need not be *any obvious relation* between the frequencies found (cf. the discussion of ζ Oph).

Special attention should be paid to OB variables which are the components of close binaries. It is conceivable — as the star rotates with respect to an observer on Earth — that, for example, the observed line strengths would vary with the *sidereal* rotational period while the line width (affected by the changing shape of the star due to varying gravitational pull of the companion) would vary with the *synodic* period of rotation (which itself can vary in case of eccentric orbits).

Statement 3 *Sinusoidal fits of the RV curves of the travelling sub-features and a reliable determination of their accelerations with which they cross the line centre should remain a natural part of period analyses of line-profile variations. Only conclusive evidence of significantly different accelerations of different detected periods should be adopted as a proof of real multiperiodicity of the variations observed. In the case of binaries, one should also verify whether the two seemingly different accelerations found do not have any obvious relation to the sidereal and synodic period of the binary component in question.*

As already mentioned, another potential problem of period analyses of line strengths at certain wavelengths across the line profile is that they often implicitly assume a constant RV of the profile.

Statement 4 *A particularly suitable quantity to be analyzed for periodicity is the RV at various depths across the line profile. This quantity correctly measures the variations of the profile on all time scales involved without any a priori assumptions to be made, as my – as yet unpublished – analyses of real data seem to confirm.*

The Fourier analysis of the time variations may also fail in cases when the variations are governed by a single period which is very slowly and periodically changing with time, for instance because of the light-time effect in a wide binary system. This was recently demonstrated for ω CMa (HD 56139) by Harmanec (1998a) and it also applies to β Cep (Pigulski & Boratyn 1992, Hadrava & Harmanec 1996).

Statement 5 *It would be useful to test objects like β Cep stars in binaries, periodic Be stars or "slowly pulsating stars" for the possible presence of a slow cyclic variation of the principal period using the O-C diagrams for locally derived epochs (a technique routinely used in the binary star research).*

4. Concluding remarks

1. I tried to demonstrate that the RL and BCL variations may not only be simultaneously present in any early-type variable but that they may also be *causally* related. For this reason, I suggest that at least for the known emission-line stars, the phenomenologically defined groups of λ Eri and ζ Oph stars should be abandoned since they do not describe the physically different types of variability: both show the Balmer emission which comes and goes, both have travelling sub-features in their line profiles and both show RL light variations. All these aspects have already been known for the historical prototype of the early-type rapid variables o And (cf., e.g., Guthnick 1941, Archer 1958, 1959, Horn et al. 1982, Harmanec 1983, 1984b, Harmanec et al. 1987, Stagg et al. 1988, Hill et al. 1989 and references therein).
2. The main purpose of this paper, however, is to convey the message that the rapid line-profile and light variations of early-type stars represent a part of more complex variations of these objects on various time scales. An analysis of only one aspect of the problem or a limited set of data may often lead to incorrect conclusions. I also tried to reiterate my earlier warning that the phenomenologically defined groups of early-type stars should not be taken too literally. I find it hard to accept the picture which seems — let me exagerrate a bit – to be emerging from the recent studies: O stars and B and A supergiants vary because of their non-homogeneous wind streams corotating with the stars, early-B stars pulsate and late-B and many A stars are chemically peculiar oblique rotators. Do we really know enough to interpret the same kind of variations, let us say a change of the EW of a spectral line on a time scale comparable to stellar rotation, once as a consequence of a varying velocity field, once as a temperature change and once as the change of chemical abundance? I think that all conceivable physical processes should be kept in mind and their possible effects investigated in each particular case.

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