Astron. Astrophys. 342, 353–362 (1999)

ASTRONOMY AND ASTROPHYSICS

Cosmological parameters from statistics of strongly lensed radio sources

Asantha R. Cooray

Department of Astronomy and Astrophysics, University of Chicago, Chicago, IL 60637, USA (e-mail: asante@hyde.uchicago.edu)

Received 26 October 1998 / Accepted 25 November 1998

Abstract. We calculate the expected number of strongly lensed radio sources in a sample of ~ 6500 sources observed with the Very Large Array as part of the Cosmic Lens All Sky Survey (CLASS) during the first two sessions of its observations. A comparison between the predicted and the observed number of lensed radio sources allows a determination of the current value of $\Omega_m - \Omega_\Lambda$, where Ω_m is the cosmological mass density of the universe and Ω_Λ is the normalized cosmological constant. If there are six strongly lensed sources in this sample, our 95% confidence lower limit on $\Omega_m - \Omega_\Lambda$ is -0.58. For a flat universe with $\Omega_m + \Omega_\Lambda = 1$, then, $\Omega_\Lambda < 0.79$ (95% C.L.). If there are ten strongly lensed sources, the 95% confidence lower limit on $\Omega_m - \Omega_\Lambda$ is -0.90. These lower limits are consistent with estimates based on high redshift supernovae and with previous limits based on gravitational lensing. Instead of considering a simple cosmological constant, we also consider the possibility of a quintessence scalar field responsible for the additional energy density of the universe, with an equation of state of the form $w = P_x/\rho_x$, where P_x and ρ_x are the pressure and energy density of the field. We present our constraints on the $\Omega_x - w$ plane, where Ω_x is the present day normalized energy density of the scalar-field component, assuming a flat universe such that $\Omega_m + \Omega_x = 1$. If there are 6 strongly lensed sources in the present CLASS sample, gravitational lensing statistics allow us to rule out the region with $\Omega_x \gtrsim (1.2\text{–}0.5w^2) \pm 0.05$ (95% C.L.). We discuss the region allowed by combined gravitational lensing statistics, high redshift Type Ia supernovae distances, and globular cluster ages.

Instead of a cosmological model, we can constrain the redshift distribution of faint radio sources based on the observed gravitational lensing rate and an assumed cosmological model. If there are six strongly lensed sources, the 68% confidence upper limit on the average redshift $\langle z \rangle$ of radio sources with flux densities less than 150 mJy at 8.4 GHz is $\langle z \rangle < 1.4 + (\Omega_m - \Omega_\Lambda) \pm 0.1$. In order to obtain a much tighter estimate on the cosmological parameters, it is essential that the redshift distribution for radio sources at the faint flux density levels be observationally determined. We strongly recommend that statistically complete optical spectroscopic programs be carried out to obtain redshifts for a representative subsample of faint background radio sources. Until such redshifts are obtained, it is unlikely that a major improvement could be made with respect to lensed radio source constraints on cosmological parameters.

Key words: galaxies: luminosity function, mass function – cosmology: observations – cosmology: gravitational lensing – radio continuum: galaxies

1. Introduction

The number of strongly lensed sources found in individual observational programs can be used to constrain the cosmological constant (e.g., Turner 1990). Using statistics of optical lensing programs, various studies have now derived upper limits on the present day value of the cosmological constant (Kochanek 1996a; Chiba & Yoshi 1997; Cooray et al. 1998a). As discussed in the literature, however, lensed source search programs at optical wavelengths can be affected by systematics arising from effects due to extinction and reddening (e.g., Malhotra et al. 1996), and issues related to completeness (Kochanek 1991). Such effects, especially extinction for which there is no clear consensus, cannot easily be quantified in cosmological studies with gravitational lensing statistics. At radio wavelengths, neither of these effects are likely to affect the statistics (see, Falco et al. 1998), including completeness, when flux-limited systematic surveys are carried out. Also, high resolution interferometers such as the VLA and MERLIN allow detection of lensed sources with image separations much smaller than that allowed by ground-based optical telescopes. Thus, statistics from lensed radio source surveys are generally preferred over the optical ones to obtain cosmological parameters.

Recently, Falco et al. (1998) studied the statistics of lensed radio sources in the Jodrell Bank-VLA Astrometric Survey (JVAS; Browne et al. 1997; Patnaik et al. 1992) sample of 2500 flat-spectrum sources brighter than 200 mJy at 5 GHz. They were able to derive a 2σ lower-limit on the mass density of the universe of $\Omega_m > 0.27$ ($0.47 < \Omega_m < 1.38$ at 1σ). Recently, the number of strongly lensed radio sources has steadily increased with results from the Cosmic Lens All Sky Survey (CLASS), which has observed ~ 6500 flat-spectrum sources during the first two sessions of its observations using the VLA. The total number of currently confirmed lensed radio sources from these observations amount to six: B1608+656 (Myers et al. 1995), B2045+265 (Fassnacht et al. 1998), B1933+503 (Sykes et al. 1998), B0712+472 (Jackson et al. 1998a), B1600+434 (Jackson et al. 1995), B1127+385 (Koopmans et al. 1998). The CLASS is a collaboration between groups at Jodrell Bank, Caltech, Dwingeloo and Leiden, and is an extension to the previous JVAS by observing flat-spectrum sources ($\alpha \geq -0.5; S_\nu \propto \nu^\alpha$) down to a flux density limit of 30 mJy at 5 GHz.

Here, we calculate the expected number of lensed radio sources in the CLASS sample of $\sim$ 6500 sources for different cosmological parameters, and constrain these parameters by comparing the predictions with the observations. In Sect. 2, we discuss our calculation and its inputs, including the redshift distribution for background radio sources and the luminosity distribution of foreground lenses. In Sect. 3, we present our resulting constraints on cosmological parameters, and in particular on $\Omega_m - \Omega_\Lambda$, as well as on the redshift distribution of the faint radio sources. In Sect. 4, we discuss the potential effect of systematic errors in the predicted and observed rate of strong lensing. Finally, in Sect. 5, we summarize and discuss future prospects for tighter constraints. We follow the conventions that the Hubble constant, H_0, is $100\,h$ km s^{-1} Mpc^{-1}, the present normalized mean density of the universe Ω_m is $8\pi G\rho_m/3H_0^2$, and the present normalized cosmological constant Ω_Λ is $\Lambda/3H_0^2$. In a flat universe, $\Omega_m + \Omega_\Lambda = 1$. We have simplified the notation by not including the subscript ‘0’ usually taken to denote the present day values of these quantities.

2. Expected number of strongly lensed sources

2.1. Calculational method

Our present calculational method is quite similar to that of Cooray et al. (1998a; hereafter CQM), in which we calculated the expected lensing rate in the Hubble Deep Field (HDF; Williams et al. 1996), as implied by various photometric redshift catalogs for HDF sources. A comparison between the observed number of lensed sources in the HDF and the predicted number, allowed us to calculate the present value of $\Omega_m - \Omega_\Lambda$. We follow a procedure similar to CQM by first establishing the redshift distribution of the observed CLASS sources, and then by using the standard lensing calculations to estimate the expected number of strongly lensed radio sources present in this sample.

In Fig. 1, we show the flux density distribution of CLASS sources. Except for a handful of sources with flux densities greater than 200 mJy, most of the observed sources follow a distribution that sharply rises around 30 mJy. This is only an observational artifact due to the 30 mJy cutoff imposed on the initial source selection. Since the source sample was selected based on previous Green Bank 5 GHz surveys (e.g., Gregory et al. 1996) and that we are now using the measured flux densities from CLASS VLA snapshot maps at 8.4 GHz, the presence of sources with flux densities less than 30 mJy at 8.4 GHz suggest that the current CLASS sample includes a small number of non-flat spectrum radio sources; The CLASS definition with respect to flat spectrum sources is based on an upper frequency point at 5 GHz rather than 8.4 GHz. Sources with flux densities

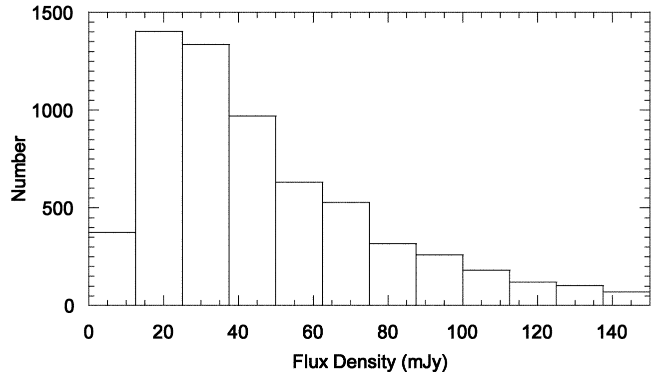

Fig. 1. The flux density distribution for CLASS sources. We have only shown the distribution of sources with flux densities less than 150 mJy, where most of the sources are found

less than 30 mJy are also expected given uncertainties in the Green Bank survey flux density measurements and the possibility for intrinsic flux density variation between Green Bank and CLASS survey epochs. The total number of radio sources in the publicly available catalog of CLASS sources[1] during the first two sessions is 6528.

As discussed in Falco et al. (1998), one of the main problems associated with lensed radio source statistics is the non-availability of redshifts for radio sources. However, it has also been shown by Falco et al. (1998), and previously by Kochanek (1996b), that the radio sources exhibit a strong correlation between their redshift and flux density distributions. Thus, even though complete redshift surveys may not exist, it is still possible to construct with existing data a statistically accurate redshift distribution for a given radio flux density. This assumes that the correlation between redshift and flux density distribution remains to be valid at flux levels lower than observationally studied. Since redshifts cannot be determined exactly for individual sources, each of the assigned redshifts at a given flux density will have an uncertainty which can be accounted when calculating cosmological parameters. Also, since the CLASS radio source sample is substantially large, on average, the modeled redshift distribution can be expected to represent the true redshift distribution. Since no new radio source redshift surveys have been published recently, we use the parameterization of the flux$-$redshift relation in Falco et al. (1998) to calculate the probable redshifts of CLASS sources. For each redshift, we also assign an uncertainty based on the error in the parameterization presented by Falco et al. (1998; see, their Fig. 4). Since no redshift surveys exist for radio sources with flux densities less than $\sim$ 200 mJy, we use the extrapolation of the flux$-$redshift distribution to estimate the redshifts of these sources. However, we assign a slightly higher uncertainty for these redshifts than expected from the simple extrapolation presented in Falco et al. (1998). We assume that the model A in Falco et al. (1998) is the mean redshift distribution, while models B and C, with an increase in uncertainty, represents our higher and lower redshift distributions, respectively. With redshift and flux density

[1] http://www.jb.man.ac.uk/$\sim$njj/glens/class.html

for each source, we calculate the radio luminosity, L, and its uncertainty. We do not consider any errors in the flux density measurement as these are likely to be small compared to the ones associated with the lensing calculation.

In order to calculate the lensing rate for radio sources, we model the lensing galaxies as singular isothermal spheres (SIS) and use the analytical filled-beam approximation (e.g., Fukugita et al. 1992). At redshifts $z \leq 4$, the analytical filled-beam calculation in Fukugita et al. (1992) agree to better than 2% with numerical calculations (e.g., Fig. 1 in Holz et al. 1998).

Gravitational lensing statistics are affected by the so-called "magnification bias" (Kochanek 1991) in which the number of lensed sources in the sample can be different from an unlensed sample down to the same flux density level. Thus, any calculation involving lensed source statistics should account for the magnification bias and associated systematic effects. The effect of this bias, whether to increase or decrease the number of lensed sources, depends on the slope of the number counts. The magnification bias is particularly pronounced in quasar lensing surveys (Maoz & Rix 1993), because the faint end of the quasar luminosity function rises steeply, increasing the number of lensed sources. In Sect. 2.2, we discuss this effect for radio sources based on the flat-spectrum radio luminosity function.

Following CQM, if the probability for a source at redshift z to be strongly lensed is $p(z, \Omega_m, \Omega_\Lambda)$, as calculated based on the filled-beam equation, we can write the number of lensed sources, $\bar{N}$, as (see, also, Maoz et al. 1992):

$$\bar{N} = \sum_i p(z_i, \Omega_m, \Omega_\Lambda) B(L_i, z_i) g(\Delta\theta, \Delta f) \equiv \sum_i \tau(z_i) \,, \tag{1}$$

where $B(L_i, z_i)$ is the magnification bias for a radio source at redshift z with luminosity L, and $g(\Delta\theta, \Delta f)$ is the selection function defined based on image separation, $\Delta\theta$, and flux ratio, Δf, between components of the strongly lensed sources. Here, the sum is over each of the radio sources in our sample. The index i represents each object; hence, z_i and L_i are, respectively, the redshift, and the rest-frame luminosity of the ith source.

For simplicity, following Kochanek (1996b), we assume that the selection function is simply a uniform function in the range $0\farcs3 \leq \Delta\theta \leq 6\farcs0$, such that all lensed sources in this range are recovered. For the SIS model, the fraction of lensed sources in this range of image separations is 0.917 (e.g., Kochanek 1996b). We have assumed that the image separation, $\Delta\theta$, under the SIS assumption for foreground galaxies is given by $\Delta\theta = 8\pi(\sigma/c)^2$, where σ is the velocity dispersion of the foreground lensing galaxies, and that the characteristic velocity dispersion for a L^* galaxy, σ^*, is 220 km s^{-1} (see, Eq. 22 in Kochanek 1996b). Given that the observations are carried out to a much lower flux density level than the flux density of the observed source, and that the candidate lensed sources are selected when flux ratios between strong components are ≤ 10 (Browne et al. 1997), it is likely that the selection function is independent of the flux ratio between lensed images. For SIS models, the fraction of lensed sources with image flux ratios ≤ 10 is 0.9997, and thus, for the purpose of this calculation we take $g(\Delta\theta, \Delta f)$ to be a constant with numerical value of 0.916.

Table 1. The adopted luminosity function for radio sources assuming a pure luminosity evolution model based on Dunlop & Peacock (1990)

Parameter	Value
Log(Φ_0/Mpc$^{-3}\Delta$ Log L^{-1})	-8.15
α_1^*	0.83
β_1^*	1.96
γ_1	1.83
γ_2	2.96
a_0	25.26
a_1	1.18
a_2	-0.28

2.2. Magnification bias for radio sources

The bias, $B(L, z)$, can be calculated based on the radio luminosity function for background sources at redshift z, $\Phi(L, z)$:

$$B(L, z) = \frac{\int_{A_{\rm min}}^{\infty} \Phi(L/A, z)\, P(A)\, dA\, d(L/A)}{\Phi(L, z)\, dL}, \tag{2}$$

where $P(A)dA$ is the probability distribution of amplifications A. The GHz radio luminosity function for flat-spectrum sources, and its evolution, is still not well determined observationally. For the purpose of the present paper, we follow the work of Dunlop & Peacock (1990), where luminosity functions are calculated for sources at 2.7 GHz. Since we are dealing with a sample of flat spectrum sources, we can safely assume that the 1 to 10 GHz luminosity function is equivalent to that at 2.7 GHz. We use the pure luminosity-evolution model (PLE) for flat-spectrum sources, which is motivated by optical quasar and X-ray AGN luminosity functions (e.g., Boyle et al. 1987):

$$\Phi(L, z) = \Phi_0 \left[\left(\frac{L}{L_c(z)} \right)^{\alpha_1^*} + \left(\frac{L}{L_c(z)} \right)^{\beta_1^*} \right]^{-1}, \tag{3}$$

where α_1^* and β_1^* are the faint and bright end slopes of the luminosity function with respect to the critical, or break, luminosity L_c. For the purpose of this paper, the quantity L is the radio luminosity (in W Hz^{-1} sr^{-1}). Since this luminosity function is defined in terms of an element $d\log L$ instead of the dL required in Eq. 2, following King & Browne (1996), we rewrite the luminosity function in terms of a linear element, with new slopes γ_1 and γ_2 below and above the critical luminosity:

$$\Phi(L, z) = \frac{\Phi_0}{L_c(z) \ln 10} \left[\left(\frac{L}{L_c(z)} \right)^{\gamma_1} + \left(\frac{L}{L_c(z)} \right)^{\gamma_2} \right]^{-1}. \tag{4}$$

The critical luminosity, L_c, has been found to evolve with the form:

$$\log L_c(z) = a_0 + a_1 z + a_2 z^2. \tag{5}$$

In Table 1, we list each of the coefficients.

Since we are using the SIS model, the minimum amplification, $A_{\rm min}$, is 2 and the probability distribution for amplifications, $P(A)$, is $8A^{-3}$ (see, e.g., Maoz & Rix 1993). The

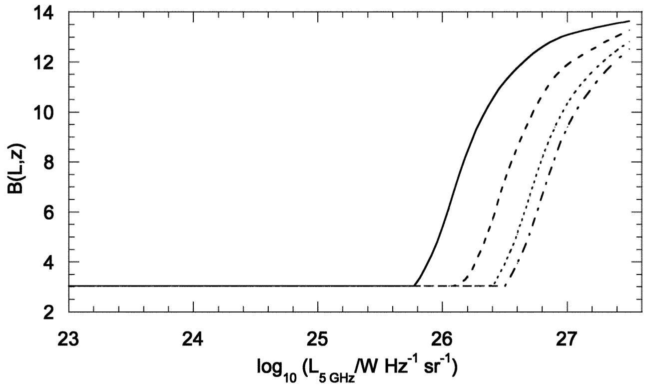

Fig. 2. The calculated $B(L,z)$ vs. L for sources at redshifts, z, of 0.5 (solid), 1 (dashed) and 1.5 (dotted) and 2 (dot-dashed). When $L(z) \leq L_c(z)$, the magnification bias is a constant determined by the slope of the lower end of the luminosity function γ_1

magnification bias can be written as (Maoz & Rix 1993):

$$B(L,z) = \tag{6}$$
$$\begin{cases} 8\left(\frac{L}{L_c}\right)^{\gamma_2-3}\left(\frac{1}{3-\gamma_1}-\frac{1}{3-\gamma_2}\right)+\frac{2^{\gamma_2}}{3-\gamma_2}, & L > 2L_c; \\ \frac{2^{\gamma_1}\left(\frac{L}{L_c}\right)^{(\gamma_2-\gamma_1)}}{3-\gamma_1}, & L_c < L < 2L_c; \\ \frac{2^{\gamma_1}}{3-\gamma_1}, & L_c > L. \end{cases}$$

In Fig. 2, we show the magnification bias as a function of radio luminosity using the luminosity range for most of the sources in the CLASS Survey. For sources with luminosities such that $L(z) \leq L_c(z)$, the magnification bias is simply determined by the slope γ_1 and is numerically equivalent to $\sim$ 3.04.

2.3. *Properties of lensing galaxies*

The probability of strong lensing depends on the number density and typical mass of lensing galaxies. For singular isothermal spheres, this factor is conveniently represented by the dimensionless parameter (Turner, Ostriker & Gott 1984):

$$F \equiv 16\pi^3 n_0 R_0^3 \left(\frac{\sigma}{c}\right)^4 . \tag{7}$$

Here n_0 is the number density of galaxies, $R_0 \equiv c/H_0$, and σ is the velocity dispersion. The parameter F is independent of the Hubble constant, because the observationally inferred number density is proportional to h^3. We can estimate F at a given redshift from the galaxy luminosity function, which we describe based on the Schechter function (Schechter 1976), in which the comoving density of galaxies at redshift z and with luminosity between L and $L + dL$ is:

$$\phi(L,z)\,dL = \phi^*(z)\left[\frac{L}{L^*(z)}\right]^{\alpha(z)} e^{-L/L^*(z)}\,dL . \tag{8}$$

In order to relate velocity dispersion, σ, with luminosity, we assume a dependence between absolute magnitude, M, and σ of the form:

$$-M = a + b\log\sigma, \tag{9}$$

which is known as the Faber-Jackson relation for early-type galaxies (Faber & Jackson 1976) and the Tully-Fisher relation for spiral galaxies (Tully & Fisher 1977).

Thus, using Eqs. 8 and 9, Eq. 7 can be written as (Fukugita & Turner 1991):

$$F = \frac{16\pi^3}{cH_0^3}\phi^*\sigma_*^4\Gamma\left(\alpha+\frac{10}{b}+1\right) , \tag{10}$$

where Γ is the normal gamma function and σ^* is given by the characteristic magnitude M^*: $-M^* = a + b\log\sigma^*$.

To estimate F, we use a recent determination of the foreground galaxy luminosity function by Zucca et al. (1997) based on observations of 3342 galaxies in the ESO Slice Project (ESP). A comparison between the ESP luminosity function and the ones previously used in lensing statistics calculations (e.g., Kochanek 1996, Falco et al. 1998) suggest no large variations. Thus, we expect our F parameter to be within the uncertainty of previous F parameter estimations. However, we note one change in our estimate for F from a recent study of lensed source statistics: We consider lensing galaxies to be composed of both early-type as well as spiral galaxies. This is contrary to the recent suggestion by Chiba & Yoshi (1998) that spiral and dwarf-type galaxies do not contribute to the observed lensing rate. We argue against this possibility simply based on the observational data, where spiral galaxy lenses have been found (e.g., B1600+434 in the present CLASS sample, Jaunsen & Hjorth 1997; Koopmans et al. 1998). Zucca et al. (1997) find that the luminosity function for galaxies is represented by a Schechter function with parameters:

$$M_B^* = -19.61^{+0.06}_{-0.08}\,, \qquad \alpha = -1.22^{+0.06}_{-0.07}\,,$$
$$\phi^* = 0.020 \pm 0.004\, h^3\mathrm{Mpc}^{-3} . \tag{11}$$

These parameters agree with the recent estimation of the luminosity function of known lenses (Kochanek et al. 1998; see their Fig. 4).

In order to derive the characteristic velocity dispersion σ^* for the observed M^*, we use the following Faber-Jackson and Tully-Fisher relations:

$$-M_B^* + 5\log h$$
$$= (19.37 \pm 0.07) + 10(\log\sigma_E^* - 2.3) \quad \text{for E}$$
$$-M_B^* + 5\log h$$
$$= (19.75 \pm 0.07) + 10(\log\sigma_{S0}^* - 2.3) \quad \text{for S0}$$
$$-M_B^* + 5\log h$$
$$= (19.18 \pm 0.10) + (6.56 \pm 0.48)(\log\sigma_{Sp}^* - 2.05)$$
$$\text{for Sp}, \tag{12}$$

as presented by de Vaucouleurs & Olson (1982) for early-type galaxies (E/S0) and Fukugita et al. (1991) for spiral galaxies (Sp). To estimate the overall F from individual contributions, we use the galaxy fractions presented by Postman & Geller (1984) with ratios E:S0:Sp=12$\pm$2:19$\pm$4:69$\pm$4. These ratios also agree with the relative numbers of spiral and early type galaxies in the Southern Sky Redshift Survey (Marzke et al. 1998). In Table 2, we list these parameters and estimate for F using the luminosity function from the ESP survey.

Table 2. F value based on the ESP luminosity function

Type	Fraction	σ^*	F
E	0.12 ± 0.02	210^{+10}_{-11}	0.009 ± 0.004
S0	0.19 ± 0.04	194^{+12}_{-10}	0.010 ± 0.003
Sp	0.69 ± 0.04	131^{+15}_{-14}	0.006 ± 0.004
Total			0.026 ± 0.006

The best estimate for F is 0.026 $\pm$ 0.006, with a relative contribution of 31% from spiral galaxies. Such a spiral galaxy contribution agrees with the estimate by Helbig (1998) that 2 of the 6 known lensed CLASS sources are due to foreground spirals. This observation also justifies our inclusion of spiral galaxies in the F parameter estimation.

Our estimate for F is a factor of 2.5 higher than the value used by Chiba & Yoshi (1998). According to Kochanek et al. (1998), the two luminosity functions used by Chiba & Yoshi (1998) are the two models most discrepant with the luminosity function from known lenses. Using a low F value can decrease the number of expected lensed sources enhancing the significance of a cosmological constant in producing strongly lensed sources. In order to obtain reliable estimates of the cosmological parameters based on statistics of lensed sources, as well as for comparative purposes, a consistent value of F is needed among different studies. We note that our estimate for F is consistent with previous estimates by Kochanek (1996a,1996b) and Falco et al. (1998). Since there could be additional errors, we allow for a slightly higher uncertainty (30%) and take F to be 0.026 $\pm$ 0.008. This estimate for F is factor of four less than the value we recently estimated for gravitational lensing in the HDF (CQM). This was both due to the factor of 2 higher number density of sources in the HDF as suggested by its luminosity function (Sawicki et al. 1997) and our partly incorrect assumption that all such sources are elliptical galaxies. The revised constraints on cosmological parameters from HDF lensing rate based on a more appropriate F value could be found in Cooray et al. (1998b).

We calculate the expected number of lensed sources by first estimating the redshift, luminosity and lensing rate for individual CLASS sources as a function of cosmology, with luminosity used to account for the magnification bias. We also vary the redshift according to flux$-$redshift correlation and recalculate the expected number of strongly lensed sources. In Fig. 3, we show the expected number of strongly lensed radio sources in the CLASS survey as a function of Ω_m and Ω_Λ when the mean redshift is considered: A universe dominated with Ω_Λ has a higher number of multiply-imaged sources than a universe dominated with a large Ω_m. As shown in Fig. 3, $\bar{N}$ is essentially a function of the combined quantity $\Omega_m - \Omega_\Lambda$, which is mainly due to a coincidence; the exact dependence is a function of background source redshifts. For background source redshifts in the range 1 to 3, the curves are essentially $\Omega_m - \Omega_\Lambda$. We can use this degeneracy in the lensing rate (e.g., Carroll, Press & Turner 1992; Kochanek 1993) to constrain $\Omega_m - \Omega_\Lambda$ rather than Ω_m or Ω_Λ individually. In Table 3, we list the expected number of strongly

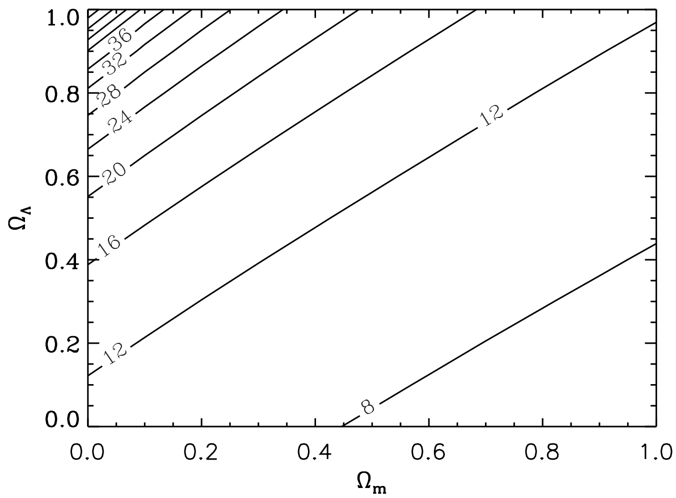

Fig. 3. Expected number of lensed radio sources, $\bar{N}$ as a function of Ω_m and Ω_Λ. $\bar{N}$ is constant along lines of constant $\Omega_m - \Omega_\Lambda$, allowing for direct constraints on this quantity. Shown here is the expected number of lensed sources for model A in Falco et al. 1997

Table 3. Predicted number of lensed radio sources in the CLASS survey

$\Omega_m - \Omega_\Lambda$	$\bar{N}$
-1.0	$55.1^{+8.0}_{-22.5}$
-0.8	$30.7^{+2.4}_{-10.8}$
-0.6	$21.5^{+1.3}_{-7.0}$
-0.4	$16.6^{+0.8}_{-5.2}$
-0.2	$13.5^{+0.5}_{-4.1}$
0.0	$11.3^{+0.4}_{-3.8}$
0.2	$9.7^{+0.3}_{-2.8}$
0.4	$8.5^{+0.2}_{-2.4}$
0.6	$7.6^{+0.1}_{-2.2}$
0.8	$6.8^{+0.1}_{-1.9}$
1.0	$6.2^{+0.1}_{-1.8}$

lensed radio sources along the $\Omega_m + \Omega_\Lambda = 1$ line as a function of $\Omega_m - \Omega_\Lambda$. The error bar accounts only for the uncertainty in redshift. We account for the error in F when cosmological parameters are estimated, which is facilitated by the fact that the number of expected lensed sources is directly proportional to F.

3. Constraints on cosmological parameters

3.1. Observed number of lensed radio sources

As listed in Sect. 1, the CLASS survey has produced six new lensed sources during the first two sessions of its observations from a sample of 6500 sources. We assume this number to be our canonical case. However, to account for the possibility that a subsample of lensed sources has been missed, possibly due to unaccountable selection effects and other biases in the lens search strategy, we consider the possibility that there are total of ten strongly lensed sources within this sample and study the variation in cosmological parameters from the canonical case. We also study the case when there are four lensed sources.

3.2. Constraints on $\Omega_m - \Omega_\Lambda$

We follow CQM to constrain the quantity $\Omega_m - \Omega_\Lambda$ by comparing the observed and predicted number of lensed radio sources. We adopt a Bayesian approach, and take a uniform prior for $\Omega_m - \Omega_\Lambda$ between -1 and +1, since we do not yet have a precise determination of this quantity, and because we do not wish to consider cosmologies in which either Ω_m or Ω_Λ lie outside the interval [0,1]. For values outside the interval of -1 to +1 in $\Omega_m - \Omega_\Lambda$, we assume an a priori likelihood of zero. We do not constrain Ω_m or Ω_Λ separately; thus, no prior is required for these quantities. Since the prior for $\Omega_m - \Omega_\Lambda$ is uniform, the posterior probability density is simply proportional to the likelihood.

As derived in CQM, the likelihood $\mathcal{L}$ — a function of $\Omega_m - \Omega_\Lambda$ — is the probability of the data, given $\Omega_m - \Omega_\Lambda$. The likelihood for n observed sources (at redshifts z_j) when $\bar{N}$ is expected is given by:

$$\langle \mathcal{L}(n) \rangle = \prod_{j=0}^{n} \tau(z_j) \times e^{-\bar{N}} \times \left(1 + \sigma_{\mathcal{F}}^2 \left[\frac{\bar{N}^2}{2} - n\bar{N} + \frac{n(n-1)}{2}\right]\right) . \quad (13)$$

Here, n is the observed number of strongly lensed radio sources, while $\bar{N}$ is the expected number of lensed sources for a given cosmology. We have also taken into account the uncertainty in F by defining $\mathcal{F} \equiv F/0.026$ and taking $\mathcal{F}$ to have a mean of unity and standard deviation $\sigma_{\mathcal{F}} = 0.3$ allowing for a 30% uncertainty in F. The factor $\mathcal{F}$ is then an overall correction to the expected lensing rate, due to a systematic uncertainty in F.

In order to constrain $\Omega_m - \Omega_\Lambda$, we also need the redshifts z_j of the observed lensed sources. Using Table 1 of Jackson et al. (1998b), we obtained redshifts for 4 sources. For the 2 sources with no known redshifts, we assume a source redshift of 1.5, consistent with other four redshifts as well as the predicted redshift distribution for radio sources in the CLASS sample. Changing these redshifts to reasonably different values do not change our constraints on the cosmological parameters greatly. We assume that there are 4, 6 and 10 lensed sources in the CLASS sample. This is primarily to study the variation in cosmological parameters with the observed number of lensed sources.

In Table 4, we summarize our 95% confidence lower limits on $\Omega_m - \Omega_\Lambda$ for the three cases, with $z_{\rm mid}$, $z_{\rm low}$, and $z_{\rm high}$ representing the mid, low and high values for the redshift distribution of radio sources. The cumulative probabilities for observing six lensed sources with the three different estimates of the redshift distribution is shown in Fig. 4. As shown and tabulated, our lowest 95% confidence on $\Omega_m - \Omega_\Lambda$ is -0.58. Accordingly, in a flat universe with $\Omega_m + \Omega_\Lambda = 1$, $\Omega_\Lambda < 0.79$ (95% C.L.). If the true redshift distribution for faint radio sources is represented by the high end allowed by the flux$-$redshift parameterization, then $\Omega_m - \Omega_\Lambda > -0.23$ and $\Omega_\Lambda < 0.61$ (95% C.L.). If there are ten lensed sources in the present sample of $\sim$ 6500 sources in the CLASS survey, assuming that four lensed sources have been missed by current searches, then $\Omega_\Lambda < 0.95$ (95% C.L.). These results are consistent with previous lensing statistics with a 95% confidence upper limit on Ω_Λ of 0.57 (Kochanek 1996a) and cosmological parameters based on high redshift type Ia Supernovae (Riess et al. 1998). However, these limits, especially when $n = 6$, are marginally inconsistent with recent estimates of the cosmological parameters based on the combined cosmic microwave background (CMB) power spectrum analysis and the high redshift supernovae (Lineweaver 1998; however, see, Tegmark 1998), with the current best estimate for $\Omega_m - \Omega_\Lambda$ of -0.38 ± 0.18 (1σ). To be consistent, we need more than the currently known six lensed sources within this CLASS sample or a lower redshift distribution for faint radio sources than allowed by current studies ($z \leq 1$, see below) if there are six or less lensed sources. The case for a low redshift distribution can be rejected based on the observed distribution of redshifts for known lensed sources.

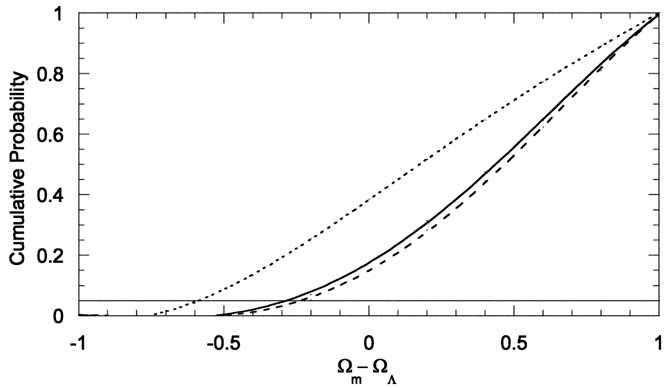

Fig. 4. Cumulative probability distribution for $\Omega_m - \Omega_\Lambda$, if there are six strongly lensed sources assuming that the redshifts are distributed according to model A of Falco et al. (1998) (*solid*), higher (*dashed*), and lower (*dotted*) from this mean (see, Sect. 3). The intercepts with the horizontal line show the 95% confidence lower limits on $\Omega_m - \Omega_\Lambda$ (see, also, Table 4)

Table 4. 95% confidence lower limits on $\Omega_m - \Omega_\Lambda$

Case	$\Omega_m - \Omega_\Lambda$
n=6, $z_{\rm mid}$	-0.29
n=6, $z_{\rm low}$	-0.58
n=6, $z_{\rm high}$	-0.23
n=10, $z_{\rm mid}$	-0.58
n=10, $z_{\rm low}$	-0.90
n=10, $z_{\rm high}$	-0.52
n=4, $z_{\rm mid}$	-0.08
n=4, $z_{\rm low}$	-0.28
n=4, $z_{\rm high}$	-0.04

As presented in Table 4, our lower limit on $\Omega_m - \Omega_\Lambda$ varies widely with the used parameterization of flux$-$redshift relation for background radio sources. Since the number of sources at the lower flux density end are quite large for the CLASS sample and that the redshift distribution at this low flux density end is not known, it is necessary that the redshift distribution be accurately determined to obtain firm limits on cosmological parameters. This can be carried out in two ways: observational determination based on spectroscopic observations of optical counterparts of radio sources with flux densities less than 150 mJy or theoretical determinations based on the number counts and other information (e.g., Jackson & Wall 1998).

A comparison of Fig. 3 and Table 3, suggest that an upper limit to $\Omega_m - \Omega_\Lambda$ can also be calculated, in addition to the lower limit. Since the number of lensed sources does not vary greatly when $\Omega_m - \Omega_\Lambda > 0$, such an upper limit is weak compared to constraints from other cosmological probes. Also, this upper limit from lensing lies outside the current range of interest in $\Omega_m - \Omega_\Lambda$ of -1 to +1, and will depend on what one assumes for a prior likelihood. Also, flat cosmologies where $\Omega_m - \Omega_\Lambda$ is greater than +1 are incompatible with current constraints on cosmological parameters (e.g., White 1998). Therefore, we do not attempt to calculate an upper limit to $\Omega_m - \Omega_\Lambda$, hence a lower limit to Ω_Λ in a flat universe, using current lensing statistics.

Since the CLASS survey is expected to observe a total of $\sim$ 10000 sources, it is possibile that the current constraints on cosmological parameters may be improved. In order to investigate this possibility, we used the full catalog of $\sim$ 10500 sources publicly available from the CLASS collaboration, which includes sources that are planned to be observed, as well as the $\sim$ 6500 sources in the current sample. The present constraints on $\Omega_m - \Omega_\Lambda$ do not vary greatly when the true number of lensed sources for the whole sample remains to be what is obtained based on the current observed lensing rate. If for some reason, the additional sample of $\sim$ 3500 sources were to contain twice as more lensed sources than currently expected, then cosmological parameter estimates change to be consistent with a high Ω_Λ. However, since this is not likely to be the case, unless the source selection process is heavily affected systematics, an increase in the source sample is not likely to improve the radio lensing constraints on cosmological parameters greatly.

3.3. *Constraints on* $\Omega_x - w$

Recent studies have suggested the existence of a scalar field as viable alternatives to the cosmological constant (see, e.g., Caldwell, Dave & Steinhardt 1998) with an equation of state of the form $w = P_x/\rho_x$, where P_x and ρ_x are the pressure and density of the scalar-field, respectively. Then, as the Universe expands $\rho_x \propto a^{-3(1+w)}$, where a is the scale factor with $a = (1+z)^{-1}$. The vacuum energy or the cosmological constant corresponds to $w = -1$, while texture or tangled strings correspond to $w = -1/3$.

We modified our lensing probability calculation to account for the scalar field, and recalculated the likelihoods using the same technique as above. We assume a flat universe, and take $\Omega_m + \Omega_x = 1$, where Ω_x is the present day normalized energy density of th scalar-field component. We assume a prior in which Ω_x is uniform between 0 and +1. Our new constraints are summarized in Fig. 5 on the $\Omega_x - w$ plane. The 95% and 68% upper limits (*thick solid lines*) from gravitational lensing are labeled 95% GL and 68% GL respectively. The 95% confidence. upper limit on Ω_x can be written as $\Omega_x \lesssim (1.2 - 0.5w^2) \pm 0.05$.

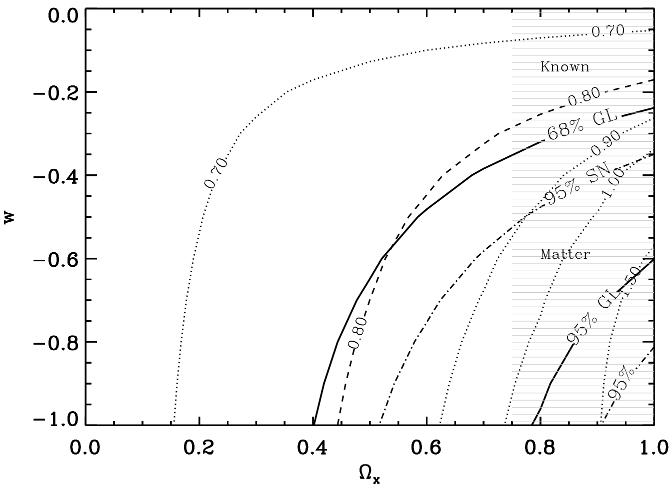

Fig. 5. Constraints on the $\Omega_x - w$ plane based on gravitational lensing, high redshift Type Ia supernovae, and globular cluster ages. We have assumed a spatially flat universe such that $\Omega_m + \Omega_x = 1$. The dotted lines are the constant H_0t_0 values, with $H_0t_0 \sim 0.8$ (dashed-line) for the minimum age of the universe. The solid lines are the upper limits based on gravitational lensing statistics at the 68% and 95% confidence. the 95% confidence contours from high redshift type Ia supernovae are shown as dot-dashed lines. If the universe is flat, we can put an upper limit on Ω_x based on the known matter content of the universe as outlined and discussed in the text

In addition to lensing constraints, we also show the resulting constraints on Ω_x and w based on type Ia supernovae at high redshifts (95% SN) as recently studied by Garnavich et al. (1998; *dot-dashed lines*) and constant age lines of the Universe as a function of H_0t_0 (*dotted lines*). For a Hubble constant, H_0, of $65 \pm 10\,\mathrm{km\,s^{-1}}$ Mpc^{-1} (Riess et al. 1998) and a minimum age of the Universe, t_0, from globular clusters of 12.5 ± 1 Gyr (Chaboyer et al. 1996), a conservative lower limit on H_0t_0 is ~ 0.8 (*dashed line*). For an age of the universe of $\sim 15 \pm 2$ Gyr, $H_0t_0 \sim$ 0.90 to 1.15. These three cosmological probes, Type Ia supernovae, age and lensed sources, qualitatively agree with each other when $0.5 \lesssim \Omega_x \lesssim 0.8$ and $w \lesssim -0.4$. These combined results rule out tangled strings with an equation of state of the form $w = -1/3$ as the scalar-field component.

As studied by Turner & White (1997), the combined large scale structure, age and supernovae data are consistent with $w \sim -0.6$ when $\Omega_m \sim 0.3$ to 0.4. The constraints on Ω_m primarily comes from the clustered mass-density in galaxy clusters based on observed baryonic mass combined with a nucleosynthesis determined baryonic mass density of the universe, $\Omega_b = (0.019 \pm 0.001)h^{-2}$ (e.g., Burles & Tytler 1997), resulting in $\Omega_m = (0.28 \pm 0.07)h^{-2/3}$ (Evrard 1997). In Fig. 5, We have outlined a conservative lower limit on the known matter content considering these and similar measurements. Even with known matter considered, there is still a large region left on the $\Omega_x - w$ plane to form a flat universe allowing various possibilities for the physical nature of the scalar-field component. In future, constraints on the equation of state of the unknown scalar-field component is likely to be improved based on CMB anisotropy and large scale structure data combined with standard cosmological probes such as high redshift supernovae and gravitational lensing statistics (see, Hu et al. 1998).

3.4. Constraints on $\langle z \rangle$

Since redshifts for radio sources with flux densities less than 150 mJy are not accurately known, we can use the observed lensing rate with an assumed cosmological model to constrain the redshift distribution of background sources. This approach is very similar to the one used by Holz et al. (1998) to constrain the redshift distribution of gamma-ray burst sources detected by BATSE based on lensing statistics. For the purpose of this calculation, we vary the redshifts of the background sources with flux densities less than 150 mJy, while keeping the rest fixed at the appropriate redshifts as determined earlier. Since we are only interested in an upper limit to the average redshift of faint radio sources, we calculate lensing probabilities as a function of $\langle z \rangle$, the *effective* average redshift, under the assumption that all sources are at this redshift. This methodology parameterizes our ignorance of the faint radio source redshift distribution (see, Holz et al. 1998 for a discussion). However, this assumption is incorrect given that at a given flux density level, radio sources have a rather broad distribution in redshift. Nevertheless, our approach is to suggest a possible upper limit to the mean redshift of this distribution, assuming that the distribution is not shaped such that more sources are at higher redshifts than suggested by the mean. Since the lensing rate is highly nonlinear in redshift, a slowly decreasing distribution with increasing redshift can increase the lensing rate faster than the assumed average value. In general, our upper limit underestimates the true average redshift.

In Table 5, we tabulate the 68% confidence upper limit on the redshift of the subsample of sources with flux densities less than 150 mJy and assuming that the total number of lensed sources in the current CLASS sample is either 6 or 10. This upper limit on the redshift is again a function of $\Omega_m - \Omega_\Lambda$. If there are 6 lensed sources in the present CLASS sample, then $\langle z \rangle < 1.4 + (\Omega_m - \Omega_\Lambda) \pm 0.1$ (68% C.L.). Since the observed redshift distribution of known lensed sources in the CLASS sample are between 1.3 and 1.6 (see, Table 1 in Jackson et al. 1998b), cosmological models where Ω_Λ is greater than 0.8 in a flat universe are disfavored.

Table 5. 68% upper limit on $f_{8.4\,\mathrm{GHz}} \leq 150\,\mathrm{mJy}$ source redshift distribution as a function of $\Omega_m - \Omega_\Lambda$

$\Omega_m - \Omega_\Lambda$	$n = 6$	$n = 10$
−1.0	0.57	0.84
−0.4	1.05	1.39
0.0	1.44	1.78
0.4	1.83	2.24
1.0	2.37	2.88

4. Systematic errors

As presented in Table 4, a major uncertainty comes from the unknown redshift distribution of faint radio sources with flux densities less than 150 mJy. Thus, it is unlikely that the current observational data on radio sources would be useful to constrain the cosmological parameters based on CLASS results, unless the redshift distribution for its sources is properly established. Since the redshift and flux density distributions for radio sources are strongly correlated, it is not necessary that redshifts for all ~ 6500 sources be known. However, redshifts for a representative subsample of CLASS sources are much needed. The determination of redshifts would involve the process of optical identifications and spectroscopic observations; a task that would take a considerable amount of telescope time and man power to complete. However, as a result of this study, where we have shown the strong variation in cosmological parameters with a change in redshift distribution, it is necessary that the true redshift distribution be used to constrain the cosmological parameters. We strongly recommend that a subsample of sources be defined from the CLASS catalog and be followed up optically to obtain redshifts. The determination of redshifts would also aid in the estimation of a reliable luminosity function for faint radio sources, which can be used to improve the calculation associated with the magnification bias.

In addition to redshift distribution, the selection function for lensed radio source discovery process is badly needed. For the present calculation, we have assumed that all lensed sources with image separations between 0.3 and 6 arcsecs are recovered. However, the selection function is likely to be complicated; it is much easier to recognize a four-image lensed source than a doubly-imaged lensed source. An initial study of such biases as applied to the JVAS survey could be found in King & Browne (1996). However, as discussed earlier, since radio lens statistics are much ‘cleaner’ than the optical ones, we do not expect biases and selection effects to make a big difference in present constraints on the cosmological parameters, unless the selection function is heavily affected by such effects. Nevertheless, King & Browne (1996) found that the JVAS survey contains at least a factor of 2 less doubly imaged sources than expected from SIS and external shear models. Such a result, if also confirmed for CLASS survey, suggests the possibility that a large number of doubly-imaged sources has been missed by the search process or that a bias not yet understood affects the observations.

Apart from background sources, an understanding of the foreground lenses is clearly needed. Use of different F values by neglecting certain galaxy populations can lead to rather diverse results, as was recently demonstrated by Chiba & Yoshi (1998). Current deep and wide-area redshift surveys are likely to increase our knowledge on the foreground lenses and their composition over the next few years. A consistent description of foreground lenses are also needed for the comparative purposes between optical and radio lensed source sample studies. Currently, however, this task is complicated by individual studies suggesting different values for the F parameter, resulting from different luminosity functions used to describe foreground lenses. With increasing knowledge on the foreground lenses, an ideal approach would be to use the luminosity function determined from known lensing galaxies (e.g., Kochanek et al. 1998), including the variation of their number density with redshift.

5. Summary and conclusions

Using flux density distribution for a sample of $\sim$ 6500 radio sources observed during the first two sessions of the Cosmic Lens All Sky Survey, and the observed number of strongly lensed sources within this sample, we have constrained the cosmological parameters. We have considered the possibility that there are total of six (as observed), four or ten lensed sources within this sample to investigate the changes in our constraints on the cosmological parameters.

We find that the expected number of lensed sources is primarily a function of $\Omega_m - \Omega_\Lambda$. A comparison of the predicted number of lensed sources with the observed number allows us to constrain its current value. Based on the current determinations of the redshift distribution for radio sources, our 95% confidence lower limit on $\Omega_m - \Omega_\Lambda$ is between -0.58 to -0.23 depending on the assumed model for the redshift distribution of faint radio sources. For a flat universe with $\Omega_m + \Omega_\Lambda = 1$, then, $\Omega_\Lambda < 0.61$ to 0.79 (95% C.L.). If there are ten strongly lensed sources, the 95% confidence lower limit on Ω_Λ is between -0.90 and -0.52. These limits are in agreement with previous limits based on gravitational lensing (Kochanek 1996a, 1996b; Falco et al. 1998), and are not in conflict with estimates based on high redshift supernovae (viz., $\Omega_m - \Omega_\Lambda \sim -0.5 \pm 0.4$ [Riess et al. 1998]).

As has been performed recently in various papers, combining $\Omega_m - \Omega_\Lambda$ results from high redshift supernovae measurements with $\Omega_m + \Omega_\Lambda$ results from CMB power spectrum analysis constrains Ω_m and Ω_Λ separately, with much higher accuracy than the individual experiments alone. We note that gravitational lensing constraints on $\Omega_m - \Omega_\Lambda$ should also be considered in such an analysis, to obtain a consistent picture on the present cosmological parameters.

In addition to the cosmological constant, we have also studied the possibility of a scalar field responsible for the additional energy density of the universe. We have shown our constraints on the $\Omega_x - w$ plane. If there are six strongly lensed sources in the present CLASS sample, gravitational lensing statistics allow us to rule out the region with $\Omega_x \gtrsim (1.2–0.5w^2) \pm 0.05$ (95% C.L.). We study the region allowed using combined constraints from gravitational lensing statistics, high redshift Type Ia supernovae distances and limits on the age of the universe from globular clusters. We have also considered the current estimates on the known matter content of the universe.

Since we have assumed a redshift distribution for radio sources to constrain cosmological models, alternatively, we can constrain the redshift distribution of faint radio sources based on the observed gravitational lensing rate and an assumed cosmological model. If there are six strongly lensed sources, the 68% confidence upper limit on the redshift distribution of flat-spectrum radio sources with flux densities less than 150 mJy at 8.4 GHz is $\langle z \rangle \, < \, 1.4 + (\Omega_m - \Omega_\Lambda) \pm 0.1$.

In order to obtain a much tighter estimate on $\Omega_m - \Omega_\Lambda$, it is essential that the redshift distribution for radio sources at the faint flux density levels be observationally determined. Also, estimates of the flat-spectrum radio luminosity function and the selection function in lensed source discovery process are needed to complete the story. We strongly recommend that statistically complete optical spectroscopic programs be carried out to obtain redshifts of faint background radio sources. Until such redshifts are obtained, it is unlikely that a major improvement could be made with respect to radio lensing constraints on cosmological parameters.

Acknowledgements. I thank members of the CLASS collaboration for making their source catalog publicly available, Phillip Helbig for useful correspondences, and Jean Quashnock, Cole Miller, André Fletcher, and John Carlstrom for useful discussions. I would also like to thank the anonymous referee for constructive comments on the paper. This work was supported in part by the McCormick Fellowship at the University of Chicago, and a Grant-In-Aid of Research from the National Academy of Sciences, awarded through Sigma Xi, the Scientific Research Society.

References

Boyle B.J., Fong R., Shanks T., Peterson B.A., 1987, MNRAS 227, 717

Browne I.W.A., Jackson N.J., Augusto P., et al., 1997, In: Bremer M., Jackson N.J.F; Perez-Fournon I. (eds.) Observational Cosmology with the Radio Surveys. Kluwer, Dordrecht, 305

Burles S., Tytler D., 1997, AJ 114, 1330

Caldwell R.R., Dave R., Steinhardt P.J., 1997, PRL 80, 1582

Carroll S.M., Press W.H., Turner E.L., 1992, ARA&A 30, 499

Chaboyer B., Kernan P.J., Krauss L.M., Demarque P., 1996, Sci 271, 957

Chiba M., Yoshi Y., 1997, ApJ 489, 485

Chiba M., Yoshi Y., 1998, ApJ in press (astro-ph/9808321)

Cooray A.R., Quashnock J.M., Miller M.C., 1998a, ApJ in press (astro-ph/9806080) [CQM]

Cooray A.R., Quashnock J.M., Miller M.C., 1998b, In: Holt S.S., Smith E.P. (eds.) After the Dark Ages: When Galaxies were Young. (astro-ph/9811115)

de Vaucouleurs G., Olson D.W., 1982, ApJ 256, 346

Dunlop J.S., Peacock J., 1990, MNRAS 247, 19

Evrard A.E., 1997, MNRAS 292, 289

Faber S.M., Jackson R.E., 1976, ApJ 204, 668

Falco E.E., Kochanek C.S., Munoz J.A., 1998, ApJ 494, 47

Fassnacht C.D., Blandford R.D., Cohen J.G., et al. 1998, AJ in press (astro-ph/9811167)

Fukugita M., Turner E.L., 1991, MNRAS 253, 99

Fukugita M., Okamura S., Tarusawa K., Rood H.J., Williams B.A., 1991, ApJ 376, 8

Fukugita M., Futamase T., Kasai M., Turner E.L., 1992, ApJ 393, 3

Garnavich P.M., Jha S., Challis P., et al., 1998, ApJ in press (astro-ph/9806369)

Gregory P.C., Scott W.K., Douglas K., Condon J.J., 1996, ApJS 103, 427

Helbig P., 1998, MNRAS 298, 395

Holz D.E., Miller M.C., Quashnock J.M., 1998, ApJ in press (astro-ph/9804271)

Hu W., Eisenstein D.J., Tegmark M., White M., 1998, PRD submitted (astro-ph/9806362).

Jackson C.A., Wall J.V., 1998, In: Mellier Y., Colombi S. (eds.) Wide Field Surveys in Cosmology. (astro-ph/9808294)

Jackson N., et al., 1998a, MNRAS, submitted

Jackson N., Helbig P., Browne I., et al., 1998b, A&A 334, L33

Jackson N., de Bruyn A.G., Myers S., et al., 1995, MNRAS 274, L25
Jaunsen A.O., Hjorth J., 1997, A&A 317, L39
King L.J., Browne I.W.A. 1996, MNRAS 282, 67
Kochanek C.S., 1991, ApJ 379, 517
Kochanek C.S., 1993, MNRAS 261, 453
Kochanek C.S., 1996a, ApJ 466, 638
Kochanek C.S., 1996b, ApJ 473, 595
Kochanek C.S., Falco E.E., Impey C.D., et al., 1998, In: Holt S.S., Smith E.P. (eds.) After the Dark Ages: When Galaxies were Young. (astro-ph/981111)
Koopmans L., et al., 1998, MNRAS submitted
Koopmans L., de Bruyn A.G., Jackson N., 1998, MNRAS 295, 534
Lineweaver C., 1998, ApJ 505, L69
Malhotra S., Rhoads J.E., Turner E.L., 1997, MNRAS 288, 138
Maoz D., Bahcall J.N., Doxsey R., et al., 1992, ApJ 394, 51
Maoz D., Rix H.-W., 1993, ApJ 416, 425
Marzke R.O., da Costa L.N., Pellegrini P.S., et al., 1998, ApJ 503, 617
Myers S.T., Fassnacht C.D., Djorgovski S.G., et al., 1995, ApJ 447, L5
Patnaik A.R., Browne I.W.A., King L.J., et al., 1992, In: Kayser R., Schramm T., Nieser L. (eds.) Gravitational Lenses. Springer, Berlin, 140
Postman M., Geller M.J., 1984, ApJ 281, 95
Riess A.G., Fillippenko A.V., Challis P., et al. 1998, AJ 116, 1009
Sawicki M.J., Lin H., Yee H.K.C., 1997, AJ 113, 1
Schechter P., 1976, ApJ 203, 297
Sykes C.M., Browne I.W.A., Jackson N.J., et al., 1998, MNRAS in press (astro-ph/9710358)
Tegmark M., 1998, ApJL submitted (astro-ph/9809201)
Tully R.B., Fisher J.R., 1977, A&A 54, 661
Turner E.L., Ostriker J.P., Gott J.R., 1984, ApJ 284, 1
Turner E.L., 1990, ApJ 365, L43
Turner M., White M., 1997, PRD 56, 2419
White M., 1998, ApJ 506, 495
Williams R.E., Blacker B., Dickison M., et al., 1996, AJ 112, 1335
Zucca E., Zamorani G., Vettolani G., et al., 1997, A&A 326, 477