

Helioseismic tests of diffusion theory

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Abstract. We present a quantitative estimate of the accuracy of the calculated diffusion coefficients, by comparing predictions of solar models with observational data provided by helioseismology. By taking into account the major uncertainties in building solar models we conclude that helioseismology confirms the diffusion efficiency adopted in SSM calculations, to the 10% level.

Key words: diffusion – Sun: fundamental parameters – Sun: oscillations

1. Introduction

In recent years inclusion of elemental diffusion has been an essential ingredient of stellar evolutionary codes for achieving agreement between predicted and helioseismic values of properties of the convective envelope. Bahcall & Pinsonneault (1992) were the first to show quantitatively that diffusion significantly improves the agreement with observation, see also Cox et al. (1989), Bahcall & Pinsonneault (1995), Bahcall et al. (1997), Ciacio et al. (1996), Degl'Innocenti et al. (1997), Richard et al. (1996), Turck-Chièze et al. (1998). The success of solar models with diffusion, and the corresponding failures of models that neglect diffusion, see Fig. 1, suggest that the diffusion process has been properly treated. The goal of this paper is to present a quantitative estimate of the accuracy of the calculated diffusion coefficients, by comparing predictions of solar models with observational data provided by helioseismology.

Until recently, few stellar evolution codes included diffusion. Apart from practical difficulties, the omission of diffusion was justified since elemental diffusion occurs on a large time scale (larger than 10^{13} yr. to diffuse a solar radius under solar condition), which should imply that the effect on stellar structure are small. Nevertheless, the precision required for calculating solar neutrino fluxes and for matching the accuracy of helioseismic observations is so great that diffusion has to be taken into account, see e.g. Bahcall & Loeb (1990), Proffitt (1994).

Most importantly, diffusion together with gravitational settling hide below the convective envelope a significant fraction

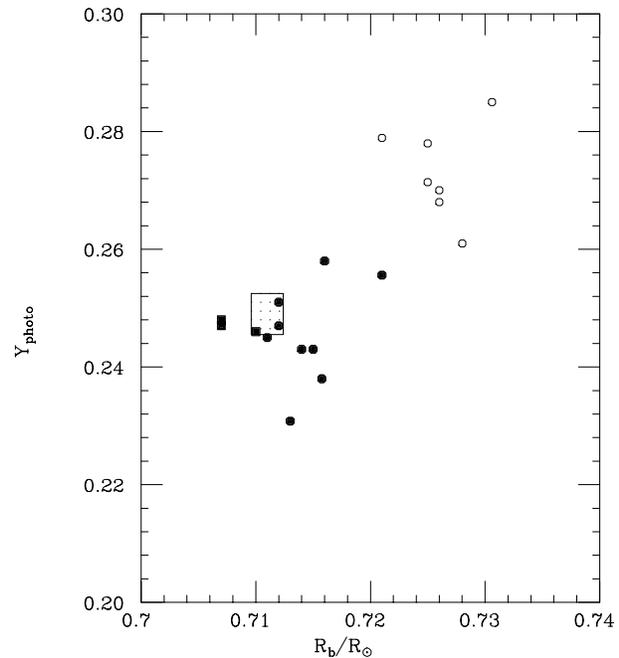


Fig. 1. The photospheric helium mass fraction Y_{ph} and the depth of the convective zone (R_b/R_\odot). The dotted rectangle is the 1σ interval allowed by helioseismology from Degl'Innocenti et al. (1997); the open circles are the predicted values by solar models without diffusion, from top to bottom correspond to Dar & Shaviv (1996), Christensen-Dalsgaard et al. (1993), Richard et al. (1996), Turck-Chièze & Lopes (1993), Proffitt (1994), Bahcall & Pinsonneault (1995), Ciacio et al. (1996). The full squares are the predicted values by solar model with helium diffusion, from top to bottom correspond to Christensen-Dalsgaard et al. (1993), Bahcall & Pinsonneault (1992), Proffitt (1994). The full circles are the predicted values by solar model with helium and heavy elements diffusion, from top to bottom correspond to Richard et al. (1996), Cox et al. (1989), Proffitt (1994), Bahcall & Pinsonneault (1995), Christensen-Dalsgaard et al. (1996), Turck-Chièze et al. (1998), Bahcall et al. (1998), Ciacio et al. (1996), Dar & Shaviv (1996).

(about 10%) of the initial helium and heavy metals, and this effect is crucial when comparing solar models with the observed properties of the convective envelope (e.g. the present heavy el-

element abundance and helium fraction, this latter inferred by means of helioseismology).

One of the most detailed calculations of diffusion coefficients has been presented by Thoul et al. (1994), hereafter TBL. It is based upon exact numerical solutions of the fundamental equations for element diffusion and heat transfer discussed in Burgers (1969). Good agreement exists between the diffusion rates computed by Thoul et al. and the results obtained with a very different treatment of Michaud & Proffitt (1993). The accuracy of the TBL coefficients is estimated to be about 15% by comparison with the results of other authors, see Bhacall and Pinsonneault (1995) and refs. therein.

However, some warning is needed: the separation of elements by diffusion might be inhibited by the presence of mixing in the solar interior, see Schatzman (1969), Lebreton & Maeder (1987), Pinsonneault et al. (1989), Pinsonneault et al. (1990), Chaboyer et al. (1992), Chaboyer et al. (1995). We also remind the anomalous lithium depletion of the sun with respect to meteorites, signalling some deficiency of present standard solar models.

Clearly quantitative observational constraints on the calculated diffusion coefficients would be welcome. The purpose of this paper is to discuss the constraints posed by helioseismology.

As previously remarked recent Standard Solar Models (SSMs) which include diffusion, as well as accurate equations of state and updated opacity tables, are in good agreement with helioseismology. In essence we address the following question: *which variation of the diffusion efficiency is allowed without spoiling the agreement with helioseismic data?*

In the next section we study how the predictions of solar models are affected when the diffusion efficiency is changed and we summarize the relevant helioseismic information.

In Sect. 3 we discuss the comparison of helioseismic data with the predictions of solar models, when the diffusion efficiency is varied.

2. The diffusion efficiency and solar models predictions

For our standard solar model calculations we use the diffusion coefficients calculated following TBL. For each element one has to consider three different coefficients (A_P , A_T , and A_H following the notation of TBL), each depending on the value of several chemical and physical variables (ρ , T , X , Z_i ...) in the solar interior, so that in principle one has really many parameters which could be varied for numerical studies. All the diffusion coefficients, however, are approximately constant along the solar profile apart from the innermost core, see Fig. 11 in TBL.

For simplicity, we shall introduce just one *diffusion efficiency parameter* d and we shall study solar models obtained by rescaling the TBL coefficients by the factor d , assumed to be the same for all elements at any point in the solar interior.

By numerical experiments with our evolutionary code FRANEC (Cieffi & Straniero (1988)) we have determined solar models (i.e. stellar structures with luminosity $L_\odot = 3.844 \cdot 10^{33}$ erg/s, radius $R_\odot = 6.9599 \cdot 10^{10}$ cm and photospheric

Table 1. The solar observables Q_i used in our analysis, their predicted values Q_{SSMi} from BP95, the helioseismic determination $Q_{\odot i}$ with their 1σ error ΔQ_i , from Degl'Innocenti et al. (1997) and the calculated coefficient α , corresponding to Eq. 1.

Q_i	Q_{SSMi}	$Q_{\odot i}$	ΔQ_i	α_Q
Y_{ph}	0.247	0.249	0.0035	-0.091
R_b/R_\odot	0.712	0.711	0.0014	-0.016
ρ_b [g/cm ³]	0.187	0.192	0.0018	0.14

abundance $Z/X_{ph} = 0.0245$ at age $t_\odot = 4.57$ Gy for a mass $M_\odot = 1.989 \cdot 10^{33}$ g) obtained for different values of d .

By means of helioseismology one can reconstruct accurately three independent properties of the convective envelope: its depth R_b , its density at the bottom ρ_b and the photospheric helium abundance Y_{ph} . The resulting values together with estimated 1σ uncertainties are shown in Table 1 together with SSM predictions from the model with helium and heavy element diffusion of Bahcall and Pinsonneault (1995) hereafter BP95.

By calculating these observables for our solar models, we have studied their dependence on d . For each observables Q we introduce a parametrization of the form:

$$Q(d) = Q_{SSM} d^{\alpha_Q} \quad (1)$$

The coefficients α are collected in Table 1.

The numerical results can be qualitatively explained by simple considerations. Consider as an example the case of increased diffusion efficiency ($d > 1$), with respect to the SSM ($d = 1$):

- i) As a larger fraction of helium is hidden below the convective envelope, whereas the initial helium abundance is essentially fixed by the present luminosity, this implies a lower photospheric helium than in the SSM;
- ii) the metal abundance below the convective envelope (for a fixed photospheric abundance) is increased so that opacity increases and convection starts deeper and consequently at higher density. In addition from helioseismology one can reconstruct the sound speed profile of the solar interior with a 1σ accuracy of about 0.15% in the intermediate region ($0.2 < R/R_\odot < 0.6$).

This piece of information, however, is not particularly illuminating for studying of diffusion. As already remarked in Degl'Innocenti et al. (1997), solar models without diffusion can have sound speed profile quite consistent with the helioseismic one.

3. Diffusion efficiency and the properties of the convective envelope

We now determine the acceptable range of d by requiring that the three independent observables of the convective envelope, R_b , ρ_b and Y_{ph} , are predicted within their helioseismic ranges, by using Eqs. (1) to determine the dependence of these properties on d .

We remind that there are at least three major uncertainties in building standard solar models that also have the potentiality of

Table 2. Deviations from standard diffusion allowed by helioseismic measurements. The first five columns show whether the parameter is kept fixed (F) at its SSM value or it is allowed to vary (V) as a free parameter. The sixth column shows the resulting χ^2 per degree of freedom. The last two columns show the best fit value for d and its 1σ error.

κ	ζ	S_{pp}	d	χ^2/dof	d_{Best}	Δd
F	F	F	F	8.61 / 3		
F	F	F	V	2.92 / 2	1.14	0.06
F	F	V	V	1.81 / 2	1.07	0.09
F	V	V	V	0.98 / 2	1.05	0.09
V	V	V	V	0.75 / 2	1.08	0.11

affecting the three properties under investigation, and, therefore, that could interfere with/hinder the effect of d : the astrophysical factor S_{pp} , the solar opacity κ , the heavy element abundance $\zeta = Z/X$. We shall add all these effects one after the other, and determine a range of d that takes into account these uncertainties.

We start by defining a χ^2 as:

$$\chi^2(d) = \sum_{i=1,3} \left(\frac{Q_i(d) - Q_{\odot i}}{\Delta Q_i} \right)^2, \quad (2)$$

where the sum include the three independent observables of the convective envelope, $Q_i(d)$ are computed by using Eqs. (1), and $Q_{\odot i}$ are the helioseismic values reported in Table 1 together with the 1σ errors ΔQ_i . The value $\chi^2(d = 1)$ indicates how well the SSM reproduces these helioseismic properties. The first row of Table 2 shows the good agreement between BP95 and helioseismology ($\chi^2/\text{dof} = 8.61/3$).

If we use d as free parameter (second row of Table 2), we find the following best fit value ($\chi^2/\text{dof} = 2.92/2$) and 1σ range:

$$d = (1.14 \pm 0.06), \quad (3)$$

i.e. helioseismology suggests a diffusion efficiency slightly larger than the SSM estimate.

3.1. Uncertainties on S_{pp}

A conservative estimate of the uncertainty is provided by the range of the published results (NACRE coll. (1998)), whereas a 1σ estimate has been provided in Kamionkowski & Bahcall (1994); we shall use $\Delta S_{pp}/S_{pp}^{\text{SSM}} = 0.05/3$ at 1σ (5% is the “3 σ error” estimate). The dependence of Q_i on S_{pp} has been determined numerically in Degl’Innocenti et al. (1998). By re-defining a suitable $\chi^2(d, S_{pp})$:

$$\chi^2(d, S_{pp}) = \sum_i \left(\frac{Q_i(d, S_{pp}) - Q_{\odot i}}{\Delta Q_i} \right)^2 + \left(\frac{S_{pp} - S_{pp, \text{SSM}}}{\Delta S_{pp}} \right)^2, \quad (4)$$

we find now:

$$d = (1.07 \pm 0.09), \quad (5)$$

3.2. Uncertainties on κ and ζ

The heavy element abundance ζ and the solar opacity κ are known with a conservative accuracy of about 10%, see Bahcall & Pinsonneault (1992), V. Castellani et al. (1997). Therefore, our 1σ relative error estimate will be 0.1/3. The dependence We remark that we are considering a constant rescaling of opacity along the solar profile. of Q_i on κ and ζ has been determined numerically in Degl’Innocenti et al. (1998). In this case, the relevant χ^2 is:

$$\chi^2(d, S_{pp}, \zeta, \kappa) = \sum_i \left(\frac{Q_i(d, S_{pp}, \zeta, \kappa) - Q_{\odot i}}{\Delta Q_i} \right)^2 + \left(\frac{S_{pp} - S_{pp, \text{SSM}}}{\Delta S_{pp}} \right)^2 + \left(\frac{\kappa - \kappa_{\text{SSM}}}{\Delta \kappa} \right)^2 + \left(\frac{\zeta - \zeta_{\text{SSM}}}{\Delta \zeta} \right)^2. \quad (6)$$

We find a small change of the best fit value and, of course, an increase of the 1σ range of d :

$$d = (1.08 \pm 0.11). \quad (7)$$

3.3. Solar model “theoretical uncertainties”

At last we try to estimate how much our results could depend on having used BP95 as reference standard model. To this end, we consider one of the standard solar models (models that include all the state-of-the-art solar physics), whose helioseismic properties differ the most from BP95 and, consequently, fit less well the experimental data. We repeated the above-described analysis by using FR97 (Ciaccio et al. (1996)) as standard solar model. When all parameters are varied, the 1σ range and best fit value, cf. Eq. (7), become:

$$d = (1.13 \pm 0.12), \quad (8)$$

in substantial agreement with Eq. (7).

In conclusion, helioseismology confirms the diffusion efficiency adopted in SSM calculations, to the 10% level (1σ C.L.).

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References

- Bahcall J.N., Loeb A., 1990, ApJ 360, 267
- Bahcall J.N., Pinsonneault M.H., 1992, Rev. Mod. Phys. 64, 885
- Bahcall J.N., Pinsonneault M.H., 1995, Rev. Mod. Phys. 67, 781
- Bahcall J.N., Pinsonneault M.H., Basu S., Christensen-Dalsgaard J., 1997, Phys. Rev. Lett. 78, 171
- Bahcall J.N., Basu S., Pinsonneault M.H., 1998, astro-ph/9805135, to appear on Phys. Lett. B.
- Burgers J.M., 1969, Flow Equations for Composite Gases. Academic Press, New York
- Castellani V., Degl’Innocenti S., Fiorentini G., Lissia M., Ricci B., 1997, Phys. Rep. 281, 309
- Chaboyer B., Demarque P., Pinsonneault M.H., 1985, ApJ 441, 865

- Chaboyer B., Vauclair S., Zahn J.P., 1992, *A&A* 255, 191
- CD Christensen-Dalsgaard J., Proffitt C.R., Thompson M.J., 1993, *ApJ* 403, L75
- Christensen-Dalsgaard J., et al., 1996, *Sci* 272, 1286
- Ciacio F., Degl'Innocenti S., Ricci B., 1996, *A&AS* 123, 44
- Cieffi M., Straniero O., 1989, *ApJS* 71, 47
- Cox A.N., Guzik J.A., Kidman R.B., 1989, *ApJ* 342, 1187
- Dar A., Shaviv G., 1996, *ApJ* 468, 933
- Degl'Innocenti S., Dziembowski W.A., Fiorentini G., Ricci B., 1997, *Astroparticle. Phys.* 7, 77
- Degl'Innocenti S., Fiorentini G., Ricci B., 1998, *Phys. Lett. B* 416, 365
- Kamionkowski M., Bahcall J.N., 1994, *ApJ* 420, 884
- Lebreton Y., Maeder A., 1987, *A&A* 175, 99
- Michaud G., Proffitt C.R., 1993, In: Baglin A., Weiss W.W. (eds.) *Inside the Stars*. IAU Col. 137, PASP, San Francisco
- NACRE collaboration, 1998, private communication, to appear within the publication plane of the collaboration
- Pinsonneault M.H., et al., 1989, *ApJ* 338, 424
- Pinsonneault M.H., Kawaler S.D., Demarque P., 1990, *ApJS* 74, 501
- Proffitt C.R., 1994, *ApJ* 425, 849
- Richard O., Vauclair S., Charbonnel C., Dziembowski W.A., 1996, *A&A* 312, 1000
- Schatzman E., 1969, *A&A* 3, 331
- Thoul A.A., Bahcall J.N., Loeb A., 1994, *ApJ* 421, 828
- Turck-Chièze S., Lopes I., 1993, *ApJ* 408, 347
- Turck-Chièze S., et al., 1998, *astro-ph/9806272*