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Upper limit at 1.8 kHz for a gravitational-wave stochastic background with the *ALTAIR* resonant-mass detector

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Abstract. The sensitivity to a cosmic stochastic background of gravitational waves of the resonant cryogenic antenna *ALTAIR*, operating at 1.8 kHz, is reported. An experimental upper limit of *ALTAIR* is compared with the upper limits given by the other detectors at lower frequencies.

Key words: gravitation

1. Introduction

All resonant bar detectors currently in operation, as well as the more advanced bar and interferometric detectors presently in the planning and proposal stage, are sensitive in frequency bands whitin the range $10\ Hz$ - 3 kHz. These detectors were developed or designed with the main purpose to detect impulsive radiation, as that coming from a collapsing star or from the fall of large masses into a black hole, or other types of gravitational waves, like those due to pulsars or to coalescence of binary systems (Thorne 1987). Recently, it has been pointed out (Ferrari 1996) that a different source of gravitational waves (g.w.), the stochastic background, could be one of the most interesting as it might give information on the very early stages of the Universe and its formation. In particular a new source for stochastic background, based on the string theory of matter, has been more deeply investigated (Brustein et al. 1995). On the other hand several sources of stochastic background of g.w. have been considered in the past years (Flanagan, 1993), but all the models tend to predict g.w. in the frequency range below $1\ Hz$, lower than the operating frequency of the present detectors already in operation (resonant bars) or entering in operation in the next years (long-arm interferometers).

The interesting feature of this model based on string cosmology, from the observer's point of view, is that it predicts relic g.w. whose energy spectral density increases, in a certain range, with the frequency ν to the third power. This energy density remains below the limit imposed by nucleosynthesis considerations that put an upper limit to the stochastic g.w.'s background (Brustein et al. 1995).

The limit is usually expressed in terms of the fraction of the cosmological closure density ρ_c that is in gravitational wave energy ρ_{gw} per unit logarithmic frequency interval,

$$\Omega_{gw}(\nu) = \frac{1}{\rho_c}\frac{d\rho_{gw}}{dln\nu} \tag{1}$$

We recall that for a g.w. with a dimensionless amplitude (strain) h, the general relationship between the g.w. density $\Omega_{gw}(\nu)$ and the power spectrum $S_h(\nu)$ of h (in units of $1/Hz$), is given by:

$$\Omega_{gw}(\nu) = \frac{4\pi^2}{3}\frac{\nu^3}{H^2}S_h(\nu)\,, \tag{2}$$

where H is the Hubble's constant. As the predictions of the new string cosmology models depend on a number of parameters, then the results obtained with a measurement, even an upper limit, would help very much in delineating the exact model.

A recent paper (Astone et al. 1996) reported the experimental upper limits at about $900\ Hz$ for the g.w. stochastic background, obtained with the *Explorer* and *Nautilus* resonant detectors. Here we report the corresponding upper limit at about 1.8 kHz, using the data collected with the cryogenic resonant detector *ALTAIR*.

2. The *ALTAIR* detector

The *ALTAIR* detector (Bonifazi et al. 1993) consists of a cylindrical bar equipped with a capacitive resonant transducer and a d.c.SQUID amplifier. *ALTAIR* is located at IFSI-CNR laboratories in Frascati (Rome). The antenna is an Al-6061 alloy cylinder, $150\ cm$ long, $35\ cm$ in diameter, with mass $M = 389\ kg$, resonant frequency (first longitudinal mode) $\nu_a = 1763\ Hz$. The antenna is equipped with a capacitive resonant transducer, consisting of a small disk, which vibrates at a frequency

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$\nu_t = 1782\ Hz$. Because of the resonant transducer there are two resonant modes at frequencies about $\nu_- = 1752\ Hz$ and $\nu_+ = 1785\ Hz$. The read-out system, very similar to that used with the *Explorer* (Astone et al. 1993) and *Nautilus* (Astone et al. 1994a, 1997a) detectors, has been described elsewhere (Bassan et al. 1990).

The signal from the SQUID amplifier is sent to lock-in amplifiers (with integration time of $100\ ms$) that extract the signal components at the frequencies of interest: at the two mode frequencies ν_- and ν_+ and at the frequency of a reference signal ($\nu_c = 1716\ Hz$), used to monitor the gain of the SQUID. The data, sampled at $10\ Hz$, are processed and recorded on a Digital MicroVax 3300, using the DAGA2 acquisition system (Frasca 1993).

We report the results of our analysis only for the mode at the frequency of $1785\ Hz$, at $4.2\ K$, where sensitivity was best. Extra-noise, probably due to mechanical resonances in the suspensions, driven by the Helium boil-off, affected the measurement at the other mode at frequency ν_- in normal operation. Results at the ν_- mode are instead reported for the antenna operating at $2.0\ K$, in superfluid helium regime.

3. Sensitivity for stochastic waves

By equating to unity the ratio of the noise spectrum (due to the thermal noise of the detector and electronic noise contributed by the readout system), and the spectrum of the bar end displacement due to a g.w. excitation with power spectrum $S_h(\nu)$, we obtain (Astone et al. 1993) the *g.w. spectrum detectable with* $SNR = 1$, that is the detector noise spectrum referred to the input

$$S_h(\nu) = \frac{\pi}{8}\frac{kT_e}{MQL^2}\frac{\nu_0}{\nu^4} \times \left(1 + \Gamma\left[Q^2\left(1-(\nu/\nu_0)^2\right)^2 + (\nu/\nu_0)^2\right]\right) \quad (3)$$

where T_e is the equivalent temperature of the detector that includes the heating effect (back-action) due to the electronic amplifier, k is the Boltzmann constant, Γ is the spectral ratio between electronic and brownian noise (Pizzella 1975), Q is the overall quality factor, L is the length of the bar, and ν_0 is the resonance frequency of the bar.

Being $\Gamma \ll 1$, at a resonance ν_0 we have

$$S_h(\nu_0) = \frac{\pi}{2}\frac{kT_e}{MQv_s^2}\frac{1}{\nu_0} \quad (4)$$

(neglecting the transducer constant and the gain of the amplifiers), where v_s is the velocity of sound in the bar. We notice that, for a bar of given material and resonance frequency ν_0, the best spectral sensitivity, obtained on resonance, only depends, according to Eq. (4), on the temperature T, the mass M and the quality factor Q of the detector, provided $T \simeq T_e$, that is the coupling between bar and read-out system is sufficiently small.

The bandwidth, with $SNR = 1$, estimated at half-height of the power spectrum, is given by (Pallottino et al. 1984)

$$\Delta\nu = \frac{\nu_0}{Q}\frac{1}{\sqrt{\Gamma}} \quad (5)$$

The previous formulas are valid for a bar equipped with a non-resonant transducer. In the case of a detector with a resonant transducer, we have to take into account the stochastic force acting on the transducer oscillator. If the transducer is well tuned to the bar, the effect of this additional force is equivalent to double the force spectrum. This means that the final spectral sensitivity is reduced by a factor of 2: $S_h(\nu_-)$ and $S_h(\nu_+)$, given by Eq. (4) are twice than before. For any arbitrary tuning of the transducer formulas (3) and (4) can be also used, but the equivalent force spectra for the two modes are different

$$\tilde{S}_h(\nu_-) = \frac{S_h(\nu_-)}{a_-} \ \ and\ \ \tilde{S}_h(\nu_+) = \frac{S_h(\nu_+)}{a_+} \quad (6)$$

$$\text{with } 0 \le a_\pm \le 1 \text{ and } a_- + a_+ = 1 \quad (7)$$

This means that at one mode we can obtain a better spectral sensitivity at the expense of a reduced sensitivity at the other mode (Astone et al. 1997a).

The g.w. spectrum for the *ALTAIR* detector is estimated by analyzing the outputs x(t) and y(t) of the lock-in amplifiers, operating at the two resonant modes. For each mode, the two outputs of the instruments provide two independent noise processes with spectra (Astone et al. 1994b)

$$S^n_{xx}(\omega^*) = S^n_{yy}(\omega^*) = S_{uu}(|W_1|^2 + \Gamma)|W_2|^2 \quad (8)$$

where S_{uu} is the spectrum of the noise forces driving the mechanical oscillator, $\omega^* = |\omega - \omega_i|$, and ω_i is the angular resonance frequency of the mode considered.

$W_1 = (1 + j\omega^*\tau_i)^{-1}$ represents the transfer function of the antenna as seen through the lock-in amplifier, and $W_2 = (1 + 2j\omega^* t_0)^{-1}$ represents the filtering action of the lock-in (neglecting the gain of the amplifiers). τ_i is the amplitude decay time of the mode and t_0 is the integration time of the lock-in amplifier and it is set equal to the sampling time Δt.

The corresponding spectra for a flat g.w. spectrum S_h, will be

$$S_{xx}(\omega^*) = S_{yy}(\omega^*) = S_h|W_1 W_2|^2 \quad (9)$$

In order to estimate $S_h(\nu)$ at each mode we must divide the power spectrum obtained from the two discrete sequences $x(\Delta t)$ and $y(\Delta t)$ by the square modulus of W_1W_2 ($1/W_1W_2$ is the inverse filter that cancels the dynamic of the antenna and of the lock-in integrator).

In Fig. 1 we report the square modulus of the FFT (periodogram) relative to the data of the mode at $\nu_+ = 1784.853 Hz$ for a time period of one hour starting on 17 UT, 7 January 1992, and in Fig. 2 the corresponding estimation of the square root of the g.w. spectrum detectable with SNR=1 (amplitude strain $\tilde{h}$), obtained from the square root of the product between each data of the periodogram and the corresponding value of the inverse transfer function $1/|W_1W_2|$ computed with $\tau_+ = 780\ s$ and $t_0 = 0.1\ s$. We note that the minimum value of $\tilde{h}$ is on resonance and corresponds to the value of $1.6\ 10^{-21}\ Hz^{-1/2}$, that it is only a factor two greater than the *Explorer* one, at $900\ Hz$. On the other hand, the bandwidth of *ALTAIR* is a factor two greater than that of the *Explorer*.

Table 1. Sensitivity and bandwidth of cryogenic gravitational wave detectors of the Roma group.

	Frequency	Bandwith with $SNR = 1$	exp. strain sensitivity	theor. pulse sensitivity h
	[Hz]	[Hz]	$1/\sqrt{Hz}$	(0.5 ms)
EXPLORER, Cern	900	1	$7\ 10^{-22}$	$3\ 10^{-19}$
ALTAIR, Rome	1800	2	$2\ 10^{-21}$	$1\ 10^{-18}$
NAUTILUS, Rome	900	1	$7\ 10^{-22}$	$3\ 10^{-20}$

* experimental strain sensitivity is dimensionless h

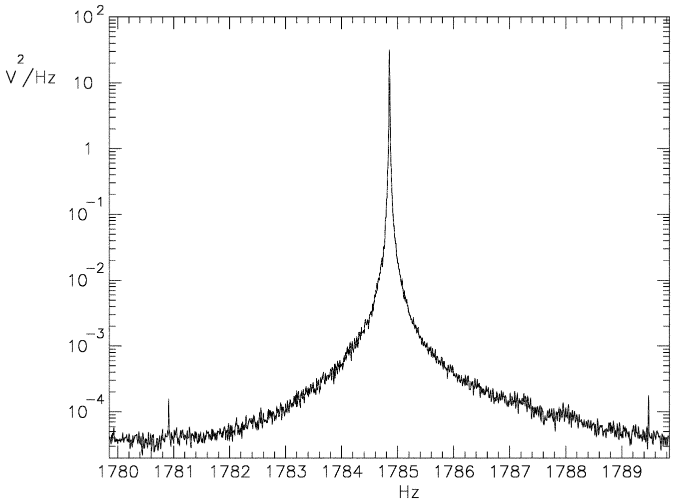

Fig. 1. The square modulus of the FFT (periodogram), for one hour of data of ALTAIR, at the mode+ ($1784.853 Hz$).

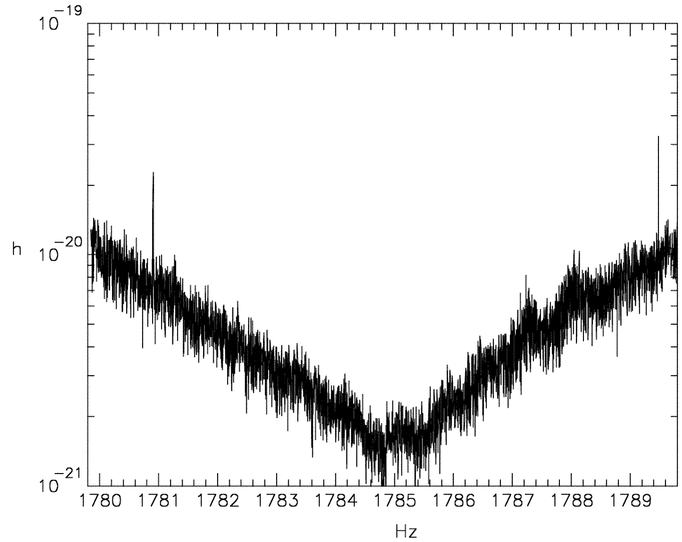

Fig. 2. Sensitivity to stochastic g.w. background with SNR=1 for ALTAIR at $1785 Hz$. $T = 4.2\ K$, $Q = 4.4\ 10^6$, spectrum averaged over one hour. The normalization in terms of h units is obtained computing from the integral value of the periodogram the corrisponding value of T_e in kelvin, and imposing that the value of $\tilde{h}(\nu_+) = \sqrt{\tilde{S}_h(\nu_+)}$ is given by Eq. (6) using the value of $a_+ = 0.478$.

The bandwidth is an important parameter to measure the g.w. stochastic background, by crosscorrelating the output of two identical antennas close to each other, within a distance much smaller than the g.w. wavelength (Astone et al. 1996).

In Fig. 3 we show the amplitude (strain) $\tilde{h}$ at $1784.853\ Hz$ computed by the hourly periodogram of the mode+ for a period of 84 hours starting on 19 UT, 5 January 1992. We note that we have a duty cycle of 90% of useful data.

In Fig. 4 we report the value of $h(\tilde{\nu_-}) = \sqrt{S_h(\nu_-)}$ at $\nu_- = 1751.945\ Hz$ with $T = 2.0\ K$, $a_- = 0.466$, $\tau_- = 167\ s$, and $t_0 = 0.1\ s$ relative to one hour of data starting on 22 UT, 8 April 1992.

The sensitivities and bandwidth for the gravitational wave resonant detectors of the Roma group are shown in Table 1. *ALTAIR* operates at about $1800\ Hz$ and all the others operate at about $900\ Hz$. The other operating antennas: Allegro (Mauceli et al. 1996), Niobe (Blair et al. 1995) and Auriga (Vitale et al.1997) have parameters very similar to those of Explorer and Nautilus.

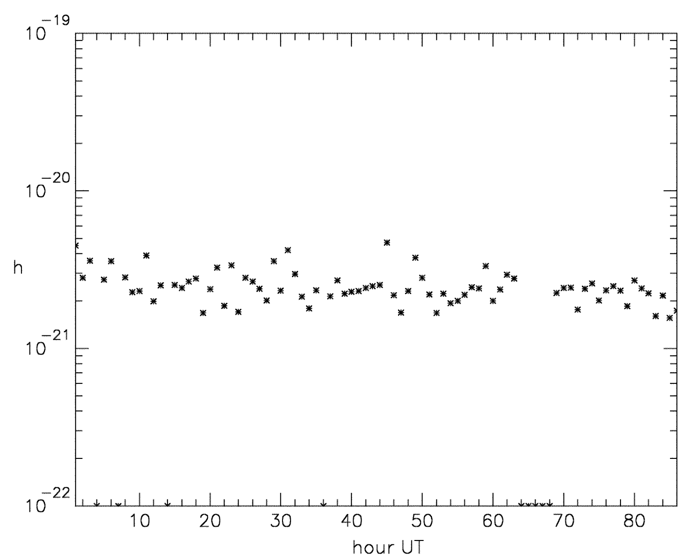

Fig. 3. Sensitivity to stochastic g.w. background with SNR=1 for ALTAIR at 1784.85 Hz, computed by hourly periodograms starting on 19 UT, 5 January 1992, $T = 4.2\ K$, $Q = 4.4\ 10^6$.

4. Conclusions

Upper limits on $\Omega_{gw}(\nu)$ by direct experimental measurements on ground, in the kHz range, have been performed in the past. The first limit by ground-based gravitational wave detectors was $\Omega_{gw}(900\ Hz) < 3\ 10^3$, set by a coincidence

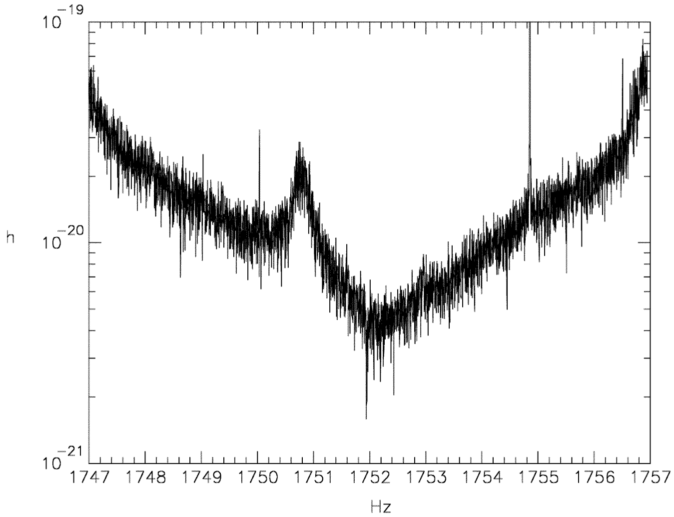

Fig. 4. Sensitivity to stochastic g.w. background with SNR=1 for ALTAIR at $1752 Hz$. $T = 2.0\ K$, $Q = 9.2\ 10^5$, spectrum averaged over one hour.

observation between bar detectors in Glasgow (Hough et al. 1975). More recent estimates from the background noise of cryogenic resonant bar detectors of the Rome group (Astone et al. 1996) give $\Omega_{gw}(920\ Hz) < 3\ 10^2$. The value obtained with the ALTAIR detector is $\Omega_{gw}(1785\ Hz) < 5\ 10^3$. With only one detector it is possible to give an *upper limit* only, and in order to improve the sensitivity we need at least two detectors. The problem of crosscorrelating gravitational wave detectors to search for a stochastic background has been discussed by a number of authors. We recall (Vitale et al. 1997) that the background appears as a noise in each detector, but if the detectors are in the same location and orientation then the effect of the background will be perfectly correlated between them. Detectors that are separated by more than, roughly, a wave-length will be only partly correlated, since waves from different directions excite them with different time delays. The analysis of the data of the 100-hour experiment, obtained with the Glasgow and Garching interferometers, for a preliminary 6 minutes of data (Compton et al. 1996) gives a value for $\Omega_{gw} = 3\ 10^5$. Obviously all these values of Ω_{gw} do not set interesting limits on the stochastic background, but they give us the opportunity to develop adeguate tecniques of data analysis. In fact these upper limits have to be compared with the nucleosynthesis limit (Brustein et al. 1995) of $\Omega_{gw} < 10^{-5}$ at all scales of frequency.

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