

Asynchronous rotation in the polars

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Abstract. Four AM Herculis systems are known to have slightly asynchronous white dwarf primary stars, and several others show features which could be consistent with some form of asynchronism.

We address the problems of the attainment and maintenance of corotation, and show why it is possible that some systems may have asynchronous motions, while others can be locked. Differences in system parameters such as the white dwarf mass and magnetic field are related to the existence and attainment of locked states. Oscillations about a locked state are also possible with typical periods of a few decades. Given the constraints necessary to attain a stable synchronous state, we note that there are probably more asynchronous systems than those known. Theory suggests that there may be systems with degrees of asynchronism two orders of magnitude lower than those presently observed. We emphasise the need for observations of AM Her binaries over longer time bases in order to search for such systems.

Key words: accretion, accretion disks – stars: magnetic fields – stars: novae, cataclysmic variables

1. Introduction

The AM Herculis binaries are now well established as a major class of magnetic cataclysmic variables. They have been characterized by their unique property of having accreting white dwarfs spinning in synchronous rotation with the orbital motion. The large magnetic moment of the white dwarf primary leads to strong fields which prevent the formation of an accretion disc and channel material transferred from the L_1 region of the red dwarf secondary. The resulting hot, localized accretion column that forms close to the primary's surface emits radiations with intensities modulated by its rotation. The periodic variations seen in the optical and X-ray light curves, and the optical polarization curves, yield the spin rate of the white dwarf. Accurate measurements of orbital periods are generally more difficult, principally due to the relatively low luminosity of the secondary. However, orbital periods have been measured in eclipsing systems (e.g. Biermann et al., 1985), from ellip-

soidal variations in the infra-red (e.g. Bailey et al., 1985), from photospheric absorption lines in the near infra-red (Mukai & Charles, 1987) and from narrow emission lines originating on the X-ray heated side of the secondary star (e.g. Schwope et al., 1991).

The primary and orbital angular velocities are usually found to be the same, to the accuracy limits of the methods employed. However, there are four systems in which the white dwarf rotates asynchronously: BY Cam (Silber et al., 1997; Mason et al., 1998), RXJ 1940-10 (e.g. Geckeler & Staubert, 1997), V1500 Cyg (e.g. Katz, 1991) and RXJ 2115-58 (Schwope et al., 1997). AM Her, WW Hor and DP Leo show secular shifts in their accretion regions which may be due to small asynchronous motions.

The present paper considers possible reasons why some AM Her systems may have small degrees of asynchronism, while others may not. In Sects. 2, 3 and 4 various aspects relating to the attainment and maintenance of the synchronous state are presented, and useful formulae are derived and discussed. In Sect. 5 the observational data are analysed using this theory, and the nature of the asynchronism is assessed. The conclusions are given in Sect. 6, regarding the observed systems and the possibility of other asynchronous AM Her binaries.

2. The approach to synchronism

The removal of asynchronous spin motions, at least from an over-synchronous state, requires some form of dissipative coupling to the orbital motion. The lobe-filling red dwarf secondary star is expected to be corotating with the orbit, as a result of the action of tidal torques. An asynchronously rotating magnetic primary leads to induced electric fields and current flows in the conducting secondary star. If there is sufficient ionized material surrounding the stars, currents will also flow between them. A highly diffusive, turbulent secondary allows the dissipation of currents in material of large thermal capacity. A surrounding tenuous magnetosphere would have relatively low thermal capacity and high conductivity, and hence support little dissipation. However, such currents may enhance the dissipation in the secondary star. The dissipation of the induced magnetic field in the secondary leads to a torque on the primary which spins it towards synchronism. This torque was calculated in Campbell (1983) and is given by

$$\mathbf{T}_D = -\frac{5\mu_0 m_p^2 R_s^3 \sin^2 \alpha}{4\pi D^6} f(|\omega|/\eta) \mathbf{k}, \quad (1)$$

where m_p is the primary's magnetic moment, α the angle between \mathbf{m}_p and the orbital angular velocity vector $\boldsymbol{\Omega} = \Omega \mathbf{k}$, R_s and D are the mean radius of the secondary and the orbital separation, η the diffusivity of the secondary and ω the synodic angular velocity of the primary. The dissipation torque is an average over the synodic period $2\pi/\omega$. The function $f(|\omega|/\eta)$ has the form of a resonance curve, with a single maximum. We note that the associated dissipation in the secondary is $\dot{E} = \omega T_D$ and typically $|\dot{E}| \lesssim 10^{-3} L_s$, where L_s is the stellar luminosity. Hence the perturbation by magnetic dissipation to the thermal balance in the secondary is small.

A synchronization time-scale can be defined as

$$t_s = \frac{I|\omega|}{|T_D|}, \quad (2)$$

where I is the moment of inertia of the primary. For small degrees of asynchronism the dissipation torque given by (1) has the asymptotic form

$$T_D = \mp \frac{\pi}{12\mu_0} \frac{(B_p)_0^2 R_p^2 R_s^5 \sin^2 \alpha}{\eta D^6} \left[1 + \frac{48}{35} \left(\frac{R_s}{D} \right)^2 \right] |\omega|, \quad (3)$$

where $(B_p)_0$ is the polar field strength on the primary and the negative and positive signs apply for $\omega > 0$ and $\omega < 0$ respectively. We note that higher multipole additions to the primary's magnetic field would only make a small difference to the torque, since they fall faster with distance than the dipole field and the torque depends on B^2 .

The magnetic field resulting from the electric currents induced in the secondary leads to a non-central force on the primary (see Campbell, 1998). This force results in an orbital torque which enables the white dwarf to exchange angular momentum and energy with the orbit. In particular, this allows an under-synchronous primary to be spun up to corotation by gaining the required energy from the large orbital source, via dissipative spin-orbit coupling.

A lobe-filling secondary satisfies the formula

$$\frac{M}{M_s} \left(\frac{R_s}{D} \right)^2 = 0.1, \quad (4)$$

(Paczynski, 1967), where $M = M_s + M_p$. This holds to good accuracy for $0.1 < M_s/M_p < 0.8$. Lower main sequence stars obey the approximate mass-radius relation

$$\frac{R_s}{R_\odot} \simeq n \frac{M_s}{M_\odot}, \quad (5)$$

with $n \simeq 1.1$ (Kippenhahn & Weigert, 1990). Using this, together with the orbital period relation

$$\frac{4\pi^2}{P^2} = \frac{GM}{D^3}, \quad (6)$$

yields the mass-period relation

$$\frac{M_s}{M_\odot} = \frac{1}{9n^{\frac{3}{2}}} \left(\frac{P}{\text{hr}} \right). \quad (7)$$

Eqs. (2) and (3) give a synchronization time independent of $|\omega|$, corresponding to an exponential approach to corotation for small degrees of asynchronism ($|\omega|/\Omega < 10^{-3}$). The resulting expression for t_s can be simplified using (4), (5) and (7) to give

$$t_s = \frac{\left(\frac{k_p^2}{0.18} \right) \left(\frac{M_p}{0.5 M_\odot} \right) \left(\frac{M}{M_s} \right)^2 \left(\frac{\eta}{10^9 \text{ m}^2 \text{ s}^{-1}} \right) \left(\frac{P}{3 \text{ hr}} \right)}{n^{\frac{1}{2}} \left(\frac{(B_p)_0}{40 \text{ MG}} \right)^2 \left(\frac{R_p}{9.7 \times 10^6 \text{ m}} \right)^4 N_1 \sin^2 \alpha} \text{ yr}, \quad (8)$$

where

$$N_1 = 4.6 \left[1 + 0.3 \left(\frac{M_s}{M} \right)^{\frac{2}{3}} \right], \quad (9)$$

and $k_p R_p$ is the radius of gyration of the primary, being related to its moment of inertia by $I = k_p^2 R_p^2 M_p$. White dwarf models give $0.2 < k_p < 0.1$. The radius of a helium white dwarf is related to its mass by

$$R_p = 7.8 \times 10^6 \left[\left(\frac{M_p}{M_c} \right)^{-\frac{2}{3}} - \left(\frac{M_p}{M_c} \right)^{\frac{2}{3}} \right]^{\frac{1}{2}} \text{ m}, \quad (10)$$

(Nauenberg, 1972), where the Chandrasekhar mass is $M_c = 1.44 M_\odot$.

The accretion torque on the primary, time averaged over a synodic period, is

$$\mathbf{T}_A = A^2 \Omega \dot{M}_p \mathbf{k}, \quad (11)$$

(Campbell, 1986), where \dot{M}_p is the accretion rate and A is the distance of the primary from the L_1 point. This distance is related to the orbital separation by

$$\frac{A}{D} = 0.500 - 0.227 \log \left(\frac{M_s}{M_p} \right), \quad (12)$$

(Plavec & Kratchovil, 1964). The accretion torque has a horizontal component, even for late field channelling of the accretion stream (Campbell, 1997). However, this essentially averages to zero over a synodic rotation period.

The function $f(|\omega|/\eta)$ in (1) has a single maximum. It follows that, for \dot{M}_p below a maximum rate, there will be two values of ω for an over-synchronous primary at which the dissipation torque T_D cancels the accretion torque T_A . The torque balance at the higher value of ω is unstable, but the lower value is stable and corresponds to a slightly asynchronous state. The synodic period of this state follows by equating (3) and (11) and using (4), (5) and (7). The result is

$$P_a = \frac{n^{\frac{1}{2}} N_2 \left(\frac{(B_p)_0}{40 \text{ MG}} \right)^2 \left(\frac{R_p}{9.7 \times 10^6 \text{ m}} \right)^6 \left(\frac{M_s}{M} \right)^2 \sin^2 \alpha}{\left(\frac{\eta}{10^9 \text{ m}^2 \text{ s}^{-1}} \right) \left(\frac{\dot{M}_p}{10^{-10} M_\odot \text{ yr}^{-1}} \right) \left(\frac{A}{3.8 \times 10^8 \text{ m}} \right)^2} \text{ yr}, \quad (13)$$

where

$$N_2 = 53 \left[1 + 0.3 \left(\frac{M_s}{M} \right)^{\frac{2}{3}} \right]. \quad (14)$$

The critical maximum value of \dot{M}_p , denoted \dot{M}_c , corresponds to T_A equalling the maximum value of T_D . This leads to

$$\dot{M}_c = \frac{n^{\frac{3}{2}} N_3 \left(\frac{(B_p)_0}{40 \text{ MG}} \right)^2 \left(\frac{R_p}{9.7 \times 10^6 \text{ m}} \right)^6 \left(\frac{M_s}{M} \right)^2}{\left(\frac{A}{3.8 \times 10^8 \text{ m}} \right)^2 \left(\frac{P}{3 \text{ hr}} \right)^2} M_\odot \text{ yr}^{-1}, \quad (15)$$

where

$$N_3 = 2.5 \times 10^{-9} \left[0.86 + 0.52 \left(\frac{M_s}{M} \right)^{\frac{3}{2}} \right] \sin^2 \alpha. \quad (16)$$

Provided $\dot{M}_p < \dot{M}_c$, the dissipation torque exceeds the accretion torque and the primary is spun towards synchronism.

Allowing for a conducting magnetosphere, corotating with the white dwarf, is likely to enhance the dissipation torque. However, the qualitative form of the torque dependence on $|\omega|$ should be similar to that given by (1). There will be a shear between the magnetosphere and the secondary star, related to the difference in their rotation rates. This will induce currents and act like an effective frequency $|\omega|$, with lower field penetration at higher asynchronism. Hence a resonance curve will again result and the foregoing picture will still apply. The dissipation torque will depend on α , but remain finite for $\alpha = 0$. Motions induced in the surface layers of the secondary by the time varying magnetic force are also likely to enhance the dissipation.

3. The locked state

The dissipation torque which spins the primary towards synchronism cannot maintain corotation, since it vanishes with $|\omega|$ while the accretion torque remains finite. Hence, in the absence of other torques, a slightly asynchronous state is reached with a synodic period given by (13). A non-dissipative torque is required to cancel the accretion torque and create a locked state. Possible sources of such torques are the interaction of the primary with a dynamo-generated magnetic field extending from the secondary, or a tidal torque on a magnetically distorted primary (Campbell, 1985, 1989, 1990; King et al., 1990).

The physical principles relating to such torques are illustrated by considering the simplest case in which the primary and secondary have dipole moments lying in the orbital plane. The sum of the accretion and magnetic torques acting on the white dwarf is then

$$\mathbf{T} = [A^2 \Omega \dot{M}_p - m_p B_{sp} \sin(\beta - \delta)] \mathbf{k}, \quad (17)$$

where

$$B_{sp} = \frac{\mu_0 m_s}{2\pi D^3} \frac{1}{(3 \cos^2 \delta + 1)^{\frac{1}{2}}}, \quad (18)$$

The angle β gives the orientation of \mathbf{m}_p , while δ is related to the fixed orientation of the secondary's dipole moment \mathbf{m}_s (Campbell, 1985). The locked state $\beta = \beta_s$ corresponds to $T = 0$ and

can exist provided the accretion torque does not exceed the maximum value of the magnetic torque, occurring at $\beta = \delta + \pi/2$. Eq. (17) yields this torque ratio as

$$Q = \frac{N_4 \left(\frac{A}{3.8 \times 10^8 \text{ m}} \right)^2 \left(\frac{\dot{M}_p}{10^{-10} M_\odot \text{ yr}^{-1}} \right) \left(\frac{M}{M_s} \right)}{\left(\frac{R_p}{9.7 \times 10^6 \text{ m}} \right)^3 \left(\frac{(B_p)_0}{40 \text{ MG}} \right) \left(\frac{(B_s)_0}{10^2 \text{ G}} \right) \left(\frac{P}{3 \text{ hr}} \right)}, \quad (19)$$

where

$$N_4 = 2.9 \times 10^{-2} (3 \cos^2 \delta + 1)^{\frac{1}{2}}. \quad (20)$$

The condition for a magnetically locked state to be possible is then $Q < 1$. A similar limit arises for the more general case with the dipole moments lying out of the orbital plane, and for the case of locking due to gravitational torques. However, the three-dimensional torque balances are more restrictive since certain combinations of the secondary's surface field strength and orientation do not allow synchronous states. There are also more unstable states. These restrictions are essentially lifted when $Q \ll 1$ (Campbell, 1997). It is noted that such torques, being non-dissipative, do not play a part in the approach to synchronism.

The period of small oscillations about the two-dimensional locked state can be expressed as

$$P_o = \frac{36 \left(\frac{k_p^2}{0.18} \right)^2 \left(\frac{M_p}{0.5 M_\odot} \right)^{\frac{1}{2}} \left(\frac{R_p}{9.7 \times 10^6 \text{ m}} \right) \left(\frac{P}{3 \text{ hr}} \right)^{\frac{1}{2}}}{\left(\frac{A}{3.8 \times 10^8 \text{ m}} \right) \left(\frac{\dot{M}_p}{10^{-10} M_\odot \text{ yr}^{-1}} \right)^{\frac{1}{2}} F(\beta_s, \delta)} \text{ yr}, \quad (21)$$

where $F(\beta_s, \delta) \sim 1$. A similar oscillation period arises in the three-dimensional cases.

Wickramasinghe & Wu (1991) considered the torque generated by a quadrupolar component in the white dwarf's magnetic field. Generally, this would be small relative to the dipole torque, but they considered the special case with the primary and secondary dipole moments aligned. The dipole-dipole torque then vanishes and the quadrupole-dipole torque is left to balance the accretion torque. Detailed balances were not found, and hence the stability was not tested. However, if such balances are possible, they lead to oscillation periods about the synchronous state with similar values to those given by (21).

4. The attainment of synchronism

The existence of a locking mechanism does not ensure that synchronism will be attained. An over-synchronous primary must have its synodic rotational energy plus the work done by the accretion torque dissipated in a synodic period $2\pi/\omega$, close to corotation. This is essentially equivalent to the condition $t_s \lesssim P_a$, where P_a is the slightly asynchronous period given by (13). This condition places a maximum limit on the magnetic diffusivity of the secondary of

$$\eta_{\max}/\text{m}^2 \text{ s}^{-1} = \quad (22)$$

$$\frac{n^{\frac{1}{2}} N_5 \left(\frac{k_p^2}{0.18} \right)^{-\frac{1}{2}} \left(\frac{(B_p)_0}{40 \text{ MG}} \right)^2 \left(\frac{R_p}{9.7 \times 10^6 \text{ m}} \right)^5 \left(\frac{M_s}{M} \right)^2}{\left(\frac{M_p}{0.5 M_\odot} \right)^{\frac{1}{2}} \left(\frac{\dot{M}_p}{10^{-10} M_\odot \text{ yr}^{-1}} \right)^{\frac{1}{2}} \left(\frac{A}{3.8 \times 10^8 \text{ m}} \right) \left(\frac{P}{3 \text{ hr}} \right)^{\frac{1}{2}}},$$

where

$$N_5 = 3.7 \times 10^9 \left[1 + 0.3 \left(\frac{M_s}{M} \right)^{\frac{2}{3}} \right] \sin^2 \alpha. \quad (23)$$

If $\eta > \eta_{\max}$ then synchronism cannot be attained in the presence of accretion, since the dissipation will not be great enough to prevent the primary from over-shooting the locked state. Then, despite the fact that a stable synchronous state exists, the lowest value attained by ω will be $2\pi/P_a$ with P_a given by (13).

5. Comparison with observations

The foregoing theory can be applied to assess the likely origin of the asynchronous motions observed in four AM Herculis systems. Table 1 shows these systems with their orbital periods, degrees of asynchronism and observed synchronization time-scales. The latest results for RXJ 2115-58 are due to Ramsay et al. (1998).

The asymptotic expression given by (8) can be used to estimate the synchronization time-scale. It is noted that (8) is valid for $\omega/\Omega < 10^{-3}$, while the observed values of asynchronism in Table 1 are somewhat larger than this. However, the asymptotic form gives a lower limit in such cases. Taking V1500 Cyg as an example, use of $P = 3.35$ hr in (7) gives $M_s = 0.32 M_\odot$. Taking the parameters to have the normalized values shown in (8), corresponding to $M_p = 0.5 M_\odot$, with $n = 1.1$ and $\alpha = 45^\circ$, yields $t_s = 55$ yr. The general formula, using the full form of $f(|\omega|/\eta)$ in (1) and a slightly higher value of M_p , gives the observed value of 150 yr. The other values of t_s shown in Table 1 can be found for reasonable parameters.

Using the foregoing parameters in (13) gives the slightly asynchronous synodic period as $P_a = 4.9$ yr, corresponding to an asynchronism of $|\omega|/\Omega = 7.7 \times 10^{-5}$. The fact that the observed values of $|\omega|/\Omega$ are significantly higher than this is consistent with the non-asymptotic form of t_s giving the best agreement with the observed synchronization time-scales. It indicates that for these systems the asymptotic limit of low asynchronism has not yet been reached.

Without an accurate knowledge of quantities such as the secondary's surface magnetic field and the mass of the primary, it is impossible to predict whether these systems will become phase locked or not. The analysis of Sect. 3 shows that $Q < 1$ is a necessary condition for magnetic locking, and this requires a sufficiently strong secondary magnetic field.

It is possible that the maximum limit for the secondary's diffusivity consistent with attaining corotation, discussed in Sect. 4, could be different even in systems with similar orbital periods. This possibility arises since, although the actual value

Table 1. Asynchronous Polars

System	P/hr	ω/Ω	$(t_s)_{\text{obs}}/\text{yr}$
RXJ 1940-10	3.3655	-2.85×10^{-3}	110
BY Cam	3.3544	9.54×10^{-3}	1107
V1500 Cyg	3.3507	1.76×10^{-2}	150
RXJ 2115-58	1.8467	1.21×10^{-2}	

of η in stars of similar mass will not differ much, the limiting value given by (22) depends significantly on parameters such as $(B_p)_0$ and R_p which can differ even in systems of similar periods.

An AM Herculis system having $Q > 1$, and/or $\eta > \eta_{\max}$, would reach the slightly asynchronous state with synodic period P_a . Such small degrees of asynchronism may be difficult to detect, especially since the value of $|\omega|$ will vary over the spin period in response to the time-varying torques. This effect would be largest for η slightly exceeding η_{\max} , corresponding to $t_s \simeq P_a$, so the primary has maximum response to the torque variations.

Even if the locked state has been attained, oscillations of ω about zero may be induced by some perturbation. The period of such oscillations is given by (21). There is some observational evidence for secular shifts of the accretion regions in AM Her, WW Hor and DP Leo, consistent with such periods (Bailey et al., 1993).

6. Conclusions

It seems unlikely that the systems shown in Table 1 are approaching orbital corotation for the first time. They are also not in a steady asynchronous state, since the observed values of ω/Ω are too high for this and the white dwarf period is significantly time-dependent. It is known that V1500 Cyg underwent a nova explosion in 1975, and this is probably the cause of its asynchronism. Such events may be the cause of the asynchronism observed in the other systems. It is noteworthy that RXJ 1940-10 is in an under-synchronous state.

Given the range of conditions needed to attain a locked state, it is plausible that a significant fraction of AM Her systems may have small degrees of asynchronism. Careful monitoring over sufficiently long time bases would be needed to detect these. This would be especially true in cases in which $|\omega|$ varies over a synodic rotation period, making detection harder when $|\omega|$ is near a minimum value.

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