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Modelling of magnetic fields of CP stars

II. Analysis of longitudinal field, crossover, and quadratic field observations

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Abstract. In recent years, the introduction and systematic application of new diagnostic techniques has enormously increased the opportunities to investigate magnetic fields of chemically peculiar (CP) stars. To approach the problem of modelling these fields, in previous papers we set up a theory aimed at describing the magnetic configuration due to the superposition of a dipole with an arbitrary quadrupole. The present work is a first application of this theory to spectro-polarimetric observations of Stokes I and V. We have attempted to model nine magnetic CP stars by analysing their curves of longitudinal field, crossover and quadratic field. We found that the classical dipolar model is adequate in only one case, while in six cases it should definitely be ruled out. For two stars a specific dipole plus quadrupole model has been recovered.

Key words: polarization – stars: chemically peculiar – stars: magnetic fields

1. Introduction

Most of our knowledge of magnetic fields of chemically peculiar (CP) stars is based on measurements of mean longitudinal magnetic field, which has been detected in about 150 stars so far (Romanyuk 1997). Many observations were carried out by Babcock (1958), Borra & Landstreet (1980), and, more recently, by Mathys (1991, 1994). Other techniques have been applied to a more limited sample of CP stars, although the number of new observations is rapidly increasing. Mathys (1994, 1995a, 1995b) and Mathys & Hubrig (1997) have started a campaign of observations of magnetic CP stars at ESO. By means of the so-called "moment technique" (Mathys 1993), they observe – in addition to the mean longitudinal field, $\langle B_z \rangle$ – the "crossover", $v_{\rm e} \sin i \, \langle dB_z \rangle$, and the "mean quadratic field", $\langle B^2 + B_z^2 \rangle^{1/2}$. These quantities have been measured in about 50 stars so far, although only a fraction of them has been thoroughly monitored. The mean field modulus, $\langle |B| \rangle$, has been detected in about 40 stars, mostly by Mathys et al. (1997). Finally, Leroy (1995) has carried out observations of broad band linear polarization (BBLP) in CP stars, detecting a signal certainly to be ascribed to the magnetic field in about 15 of them.

The modelling of the magnetic structure of CP stars was initially based on the oblique rotator model (ORM) with a dipolar field (Stibbs 1950). This simplified assumption has been proved sufficient to explain most of the observations of mean longitudinal field. However, an analysis based on the observations of $\langle B_z \rangle$ alone *cannot* demonstrate that the magnetic structure of CP stars is a dipolar one. Even severe departures from the dipolar configuration leave the curve of mean longitudinal field almost a purely sinusoidal one, like that expected from a dipole field. Deviations of the curve of $\langle B_z \rangle$ from the sinusoidal behaviour were indeed observed (see, e.g., Preston 1971), but in most cases they were explained as due to systematic effects in the interpretation of photographic plates (Borra 1974). Only few stars, e.g., HD 37776 (Thompson & Landstreet 1985) and HD 137509 (Mathys 1994, Mathys & Hubrig 1997), exhibit a curve of mean longitudinal field that is definitely not sinusoidal. On the other hand, several attempts to reproduce the curve of $\langle B_z \rangle$ together with other observable quantities within the framework of the ORM with a dipolar field did not succeed (see, e.g., Huchra (1972) and Landstreet (1992) for the interpretation of $\langle B_z \rangle$ plus $\langle |B| \rangle$ observations, and Bagnulo et al. (1995) for the interpretation of $\langle B_z \rangle$ plus BBLP observations).

If the magnetic structure of CP stars cannot be described in terms of a dipolar field, on the other side it must be roughly homogeneous, otherwise the observed polarization, after integration over the stellar disk, would be null. Several authors proposed more sophisticated magnetic structures, like the decentred dipole model (e.g., Stift 1975, Hensberge et al. 1977) or the equatorially symmetric rotator model (Oetken 1977). Recently, Bagnulo et al. (1996) developed a more general formalism based on a (non-axisymmetric) multipolar expansion of the magnetic field. This approach, which has also the advantage of providing *analytical* formulas for some of the observable quantities, was then revisited by Landolfi et al. (1998, hereafter referred to as Paper I) in order to make it more directly related to the problem of interpreting the observations.

More precisely, in Paper I we described a method for recovering the magnetic field of CP stars from mean longitudinal

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field, crossover, and mean quadratic field observations, under the assumption of either a pure dipole or a dipole plus quadrupole magnetic structure. This second paper is a direct application of that method to the observations obtained by Mathys (1994, 1995a, 1995b) and Mathys & Hubrig (1997) by means of the moment technique. In Sect. 2 the main points of the method for the analysis are recalled, in view of a direct analysis of the observations which is described in Sect. 3. In Sect. 4 the main results are summarised and commented upon.

2. Method of the analysis

The method described in Paper I consists in the minimization of a suitable χ^2, which takes into account all three kinds of observational data: longitudinal field, crossover, and quadratic field. We developed two distinct codes, based on the assumption of a pure dipole and of a dipole plus quadrupole field, respectively. The χ^2 corresponding to the dipole case depends on 5 parameters,

$$i \ , \ \beta \ , \ f_0 \ , \ B_{\rm d} \ , \ v_{\rm e} \ ,$$

while the χ^2 corresponding to the dipole plus quadrupole case depends on 10 parameters,

$$i \ , \ \beta \ , \ \beta_1 \ , \ \beta_2 \ , \ f_0 \ , \ \gamma_1 \ , \ \gamma_2 \ , \ B_{\rm d} \ , \ B_{\rm q} \ , \ v_{\rm e} \ .$$

The quantities $B_{\rm d}$ and $B_{\rm q}$ are the dipole and quadrupole field strength respectively, $v_{\rm e}$ is the equatorial velocity of the star, i is the inclination angle between the rotation axis and the line of sight; the remaining parameters are angles that specify the magnetic configuration (see Paper I). The stellar rotation period P is considered a fixed data.

The main results obtained in Paper I can be summarized as follows.

2.1. Uniqueness of the fit

Owing to the large observational errors, it is *not* possible, in general, to recover a unique magnetic configuration from a given set of data. In some cases it is not even possible to ascertain whether the stellar configuration is a pure dipole or a dipole plus quadrupole configuration; in other cases the dipole configuration can be ruled out, but several dipole plus quadrupole configurations are possible; only in the most favourable cases (which usually correspond to a largely dominant quadrupolar component and a large i value) the magnetic configuration can be unambiguously established. Defining as "good" a fit with a reduced χ^2 less than 2.2, the different situations that we expect to encounter are classified as follows:

Class $(++)$: a good fit with the dipole model, and two or more good fits with the dipole plus quadrupole model (corresponding to different sets of parameters) are found. In this case, both the pure dipole and the dipole plus quadrupole configuration are possible; furthermore:

a) if the star has a dipole field, the parameters derived assuming the dipole model *are* correct;
b) if the star has a dipole plus quadrupole field, one of the parameters sets derived assuming the dipole plus quadrupole model *may* be correct.

Class $(-+;{\rm B2})$: no good fit with the dipole model, two or more good fits with the dipole plus quadrupole model. In this case:

a) the star does *not* have a dipole configuration;
b) one of the parameters sets derived assuming the dipole plus quadrupole model *is likely* to be correct.

Class $(-+;{\rm B1})$: no good fit with the dipole model, one good fit with the dipole plus quadrupole model. In this case:

a) the star does *not* have a dipole configuration;
b) the parameters set derived assuming the dipole plus quadrupole model *is* correct.

We recall that the above classification is based on extensive numerical simulations with data sets of 20 points for each quantity (longitudinal field, crossover, and quadratic field) affected by errors of $0.2\,{\rm kG}$, $3\,{\rm kG}\,{\rm km}\,{\rm s}^{-1}$, and $1\,{\rm kG}$, respectively. This represents a typical situation, but in some cases the errors of real observations may be much larger than these values. Furthermore, the observed magnetic curves of some stars that we study in this work have less than 20 points (e.g., only nine in the case of HD 96446).

2.2. Degenerate magnetic configurations

Apart from observational errors, certain magnetic configurations cannot be distinguished from each other because of intrinsic symmetry properties of the expressions for the longitudinal field, crossover, and quadratic field. There are four such configurations for the dipole field, characterized by the values

$$\begin{array}{l}(i,\, \beta,\, f_0,\, B_{\rm d},\, v_{\rm e}) \\ (\pi - i,\, \pi - \beta,\, f_0,\, B_{\rm d},\, v_{\rm e}) \\ (\beta,\, i,\, f_0,\, B_{\rm d},\, v_{\rm e}) \\ (\pi - \beta,\, \pi - i,\, f_0,\, B_{\rm d},\, v_{\rm e})\,,\end{array} \qquad (1)$$

and two for the dipole plus quadrupole field, characterized by the values

$$\begin{array}{l}(i,\, \beta,\, \beta_1,\, \beta_2,\, f_0,\, \gamma_1,\, \gamma_2,\, B_{\rm d},\, B_{\rm q},\, v_{\rm e}) \\ (\pi - i,\, \pi - \beta,\, \pi - \beta_1,\, \pi - \beta_2,\, f_0,\, \gamma_1,\, \gamma_2,\, B_{\rm d},\, B_{\rm q},\, v_{\rm e})\,;\end{array} \qquad (2)$$

their physical meaning is illustrated in Paper I.

2.3. Constraints to the fits

Suppose that the magnetic observations of a given star are consistent with the pure dipole model; this means that four different configurations, related as in Eqs. (1), are possible. However, the value of $v_{\rm e} \sin i$ corresponding to the first (and second) configuration is in general different from the value corresponding to the third (and fourth) one. In other words, an independent measurement of $v_{\rm e} \sin i$ can be used to favour, or to rule out, certain dipole configurations.

In general, an independent determination of $v_\mathrm{e}\sin i$ sets a constraint to the possible values of v_e. A similar constraint comes from the relation between the stellar radius R_*, the rotation period P, and v_e,

$$\frac{R_*}{R_\odot} = 0.0198\,P\,v_\mathrm{e}\,, \tag{3}$$

where P is in days and v_e in $\mathrm{km\,s^{-1}}$: since P is usually known, an estimate of R_* is equivalent to an estimate of v_e. In the analysis presented in this paper, such constraints on the values of v_e are not taken into account explicitly. v_e is considered as a free parameter, and the values of $v_\mathrm{e}\sin i$ and R_* deduced from the fits are, eventually, compared to the existing estimates of these quantities.

Such procedure, where both R_* and $v_\mathrm{e}\sin i$ are considered as secondary outputs of the inversion algorithm, is intrinsic to the method described in Paper I. It has the advantage of being a general procedure, since it leaves aside the estimates (sometimes uncertain or conflicting) of these quantities derived from other methods.

It is probably naïve to expect that the "measurements" of R_* and $v_\mathrm{e}\sin i$ obtained by a so indirect method agree automatically with the existing estimates. As a matter of fact, such agreement is lacking in several cases, as shown by the following analysis. In order to achieve a more realistic model for a specific star, one could set in the inversion algorithm explicit constraints to the values of v_e (or R_*) and $v_\mathrm{e}\sin i$. This would be equivalent to fitting two additional kinds of data. However, the present investigation already implies the *combined analysis of three different kinds of measurement*, and we believe interesting to check their intrinsic diagnostic potential and consistency with the dipole or dipole plus quadrupole magnetic model, independently of other constraints. Furthermore, the following "unconstrained" analysis has some advantages. On the one hand, a magnetic configuration corresponding to a best-fit with an exceedingly large χ^2-value can be definitely ruled out, since the inclusion of additional constraints can only worsen the quality of the fit. On the other hand, a magnetic configuration which "automatically" yields R_* and $v_\mathrm{e}\sin i$ values consistent with independent estimates should be regarded as strongly reliable.

3. Analysis of the observations

3.1. Selection of targets

We selected from the literature all the stars for which it was possible to define the curves for $\langle B_z\rangle$, $v_\mathrm{e}\sin i\,\langle dB_z\rangle$, and $\langle B^2+B_z^2\rangle^{1/2}$. Observations of $\langle B_z\rangle$, $v_\mathrm{e}\sin i\,\langle dB_z\rangle$, and $\langle B^2+B_z^2\rangle^{1/2}$ were performed by Mathys (1994, 1995a, 1995b) and by Mathys & Hubrig (1997). Up to now, they published observations for about 50 stars, and monitored ten of them, but for one of these stars (HD 116458), the rotation period is not known with a sufficient precision to permit us a secure analysis (147.9 ± 0.6 d according to Hensberge 1993). The list of the targets is given in Table 1, and all the measurements considered in this analysis are taken from the above references.

Table 1. List of selected targets. Spectral type is from Mathys (1991). References for the periods are given in Column 4

HD	Sp. Type	Period (d)	
83368	A8 Sr Cr Eu	2.851982	Kurtz et al. (1992)
96446	B2 IIIp He-strong	0.85137	Matthews & Bohlender (1991)
119419	A0 Si Cr Eu	2.60090	Mathys & Hubrig (1997)
125248	A1 Eu Cr	9.2954	Babcock (1960)
137509	B8 Si Cr Fe	4.4916	Mathys & Lanz (1997)
137909	A9 Sr Eu Cr	18.4877	This work
147010	B9 Si Cr Sr	3.920676	Catalano & Leone (1993)
153882	A1 Cr Eu	6.00890	Mathys (1991)
175362	B6 He-weak Si	3.67375	Mathys (1991)

Actually, more observations of $\langle B_z\rangle$ were available in the literature for some of the stars in Table 1 (see Mathys 1991). However, we found it difficult to mix up observations obtained by different authors, finding that in many cases they were not consistent among themselves. Mathys (1991) has widely discussed the reasons for such discrepancies, and following his arguments, we decided to include in our analysis only the observations of $\langle B_z\rangle$ obtained by means of the so-called "photographic technique" where the actual detector is a CCD. At present, for the selected targets, this restricts the analysis to the observations published by Mathys (1994) and Mathys & Hubrig (1997).

3.2. Modelling of nine magnetic CP stars

We have applied the two inversion codes mentioned in Sect. 2 to each star of our sample. The limb-darkening coefficient u was set to 0.5. However, the overall picture is unaffected by the value of u; by setting $u=1$ we always obtained results consistent, within the errors, with those for $u=0.5$.

In all cases, the χ^2 hypersurface corresponding to the dipole model has one minimum; by contrast, the χ^2 hypersurface corresponding to the dipole plus quadrupole model has one minimum in some cases, two or three minima in other cases. The values of the reduced χ^2 at these minima are shown in Table 2, together with the "class" of the fit defined according to Sect. 2.1.

3.2.1. Fits of class $(++)$

Four of the stars in our sample belong to this class. This means that a pure dipole configuration cannot be excluded on the basis of the existing longitudinal field, crossover and quadratic field observations, and that the parameters values derived from the dipole fit are to be considered correct provided the stellar configuration *is in fact* dipolar.

For HD 83368, a possible magnetic model is a pure dipole specified by the parameters

Table 2. Overall number of observations (n), reduced χ^2 for the pure dipole ($\chi^2_{\rm d}$) and the dipole plus quadrupole ($\chi^2_{\rm dq}$) model, and class of the fit

HD	n	$\chi^2_{\rm d}$	$\chi^2_{\rm dq}$	Class
83368	38	1.77	1.76 ; 1.79 ; 1.85	(++)
96446	27	2.06	2.00 ; 2.10	(++)
119419	66	1.24	0.95 ; 0.98	(++)
125248	63	2.40	1.31 ; 1.41 ; 1.41	(−+; B2)
137509	42	4.45	1.05 ; 1.46	(−+; B2)
137909	45	1.02	0.70 ; 0.76	(++)
147010	57	3.51	1.52 ; 1.66	(−+; B2)
153882	51	2.36	2.04	(−+; B1)
175362	89	4.59	2.18	(−+; B1)

$$\begin{aligned} i &= 90^\circ \pm 1^\circ \\ \beta &= \ \ 8^\circ \pm 1^\circ \quad \text{or} \quad 172^\circ \pm 1^\circ \\ f_0 &= \ \ 0^\circ \pm 5^\circ \end{aligned}$$

$$\begin{aligned} B_{\rm d} &= 14.1 \pm 0.5\,{\rm kG} \\ v_{\rm e} &= 30.6 \pm 5.6\,{\rm km\,s^{-1}}\,. \end{aligned}$$

The zero-phase point corresponds to JD $= 2450002.11$, and the corresponding fit is shown in Fig. 1. It should be pointed out that the values of R_* and $v_{\rm e}\sin i$ associated with this set of parameters *are consistent with the independent estimates of these quantities*. In fact, we get $R_* = 1.7 \pm 0.3\ R_\odot$, which is quite reasonable for a (non peculiar) star of spectral type A8 ($1.6\,R_\odot$), and $v_{\rm e}\sin i = 30.6 \pm 5.6\ {\rm km\,s^{-1}}$, which is in excellent agreement with the value $32.6 \pm 2.6\ {\rm km\,s^{-1}}$ obtained by Mathys (1995b). Note that the alternative models derived from the third and fourth line of Eqs. (1) can be ruled out because of the small value predicted for $v_{\rm e}\sin i$.

On the other hand, the presence of a quadrupolar component *cannot* be excluded; but, according to Sect. 2.1, we cannot be certain that any of the recovered models (two of which are as well consistent with the estimates of R_* and $v_{\rm e}\sin i$) is correct.

Particularly interesting is the case of HD 137909 (β CrB), where excellent fits are found both with the pure dipole and with the dipole plus quadrupole model (cf. Table 2).

For the rotation period, Mathys & Hubrig (1997) propose the value of 18.4868 d. We looked for the best period by including the observations of $\langle B_z\rangle$ by Mathys (1994) and Mathys & Hubrig (1997), and the observations of BBLP by Leroy (1995). The best-fit to the $\langle B_z\rangle$ curve was obtained by means of a second-order Fourier expansion, and to the BBLP data by means of a fourth-order Fourier expansion: this yielded the value 18.4877 ± 0.0015 d, which has been adopted in this analysis.

The pure dipole model predicts $v_{\rm e} = 4.4 \pm 0.8\,{\rm km\,s^{-1}}$, hence a stellar radius of $1.6 \pm 0.3\,R_\odot$, which is consistent with the expected radius of a (non peculiar) A9 star ($1.6\,R_\odot$). The derived value for the projected equatorial velocity is $v_{\rm e}\sin i = 1.6 \pm 0.4\,{\rm km\,s^{-1}}$ or $4.4 \pm 0.8\,{\rm km\,s^{-1}}$ depending on which of the configurations in Eqs. (1) is considered. Such small values are both compatible with the estimate by Wade (1996) of $3.5 \pm 1.5\,{\rm km\,s^{-1}}$; therefore, the direct Doppler broadening measurement does not help, in this case, to distinguish between the degenerate configurations of Eqs. (1).

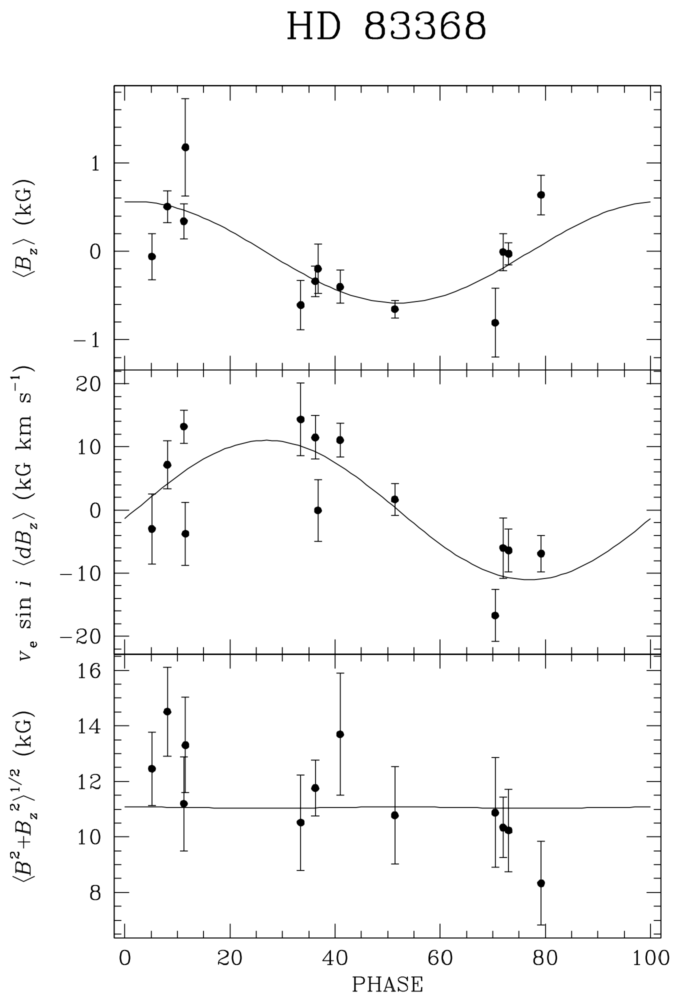

Fig. 1. Pure dipole fit to the longitudinal field, crossover, and quadratic field observations of HD 83368

However, for β CrB it is possible to make use of a further independent constraint given by the observations of BBLP. Although a comprehensive analysis of all the observable quantities is outside the scope of this work, it is straightforward to identify certain typical features of the diagrams of BBLP which, together with certain features of the $\langle B_z\rangle$ curve, set definite constraints to the magnetic configuration. The observed BBLP diagram of β CrB exhibits a clockwise double loop (cf. Leroy 1995), and the curve of longitudinal field has

$$\langle B_z\rangle^{\rm max} > 0 > \langle B_z\rangle^{\rm min} \quad \text{and} \quad |\langle B_z\rangle^{\rm max}| < |\langle B_z\rangle^{\rm min}|\,.$$

According to Landolfi et al. (1997), in the case of pure dipole field, these features locate i and β in the h' region of the (i,β) domain shown in their Fig. 2. We can thus identify the *unique* dipole model specified by

$$\begin{aligned} i &= 159^\circ \pm 2^\circ \\ \beta &= \ \ 84^\circ \pm 1^\circ \\ f_0 &= \ \ \ 0^\circ \pm 3^\circ \end{aligned}$$

$$B_{\rm d} = 7.2 \pm 0.3\,{\rm kG}$$
$$v_{\rm e} = 4.4 \pm 0.8\,{\rm km\,s^{-1}}\ ,$$

where the zero-phase point corresponds to JD = 2450010.80.

Although this model is consistent both with the $\langle B_z \rangle$, $v_{\rm e} \sin i\, \langle dB_z \rangle$, $\langle B^2 + B_z^2 \rangle^{1/2}$ measurements, and with the independent estimates of R_* and $v_{\rm e} \sin i$, the observations of mean field modulus $\langle |B| \rangle$ (Wolff & Wolff 1970; Mathys et al. 1997) *cannot* be explained in terms of a dipolar configuration – which is further inconsistent with a combined, detailed interpretation of $\langle B_z \rangle$ and BBLP observations (Leroy et al. 1996).

This difficulty is not removed by assuming a dipole plus quadrupole configuration. Both the recovered models (which are characterized by field strengths $B_{\rm d} \simeq 8\,$kG, $B_{\rm q} \simeq 11\,$kG and $B_{\rm d} \simeq 7\,$kG, $B_{\rm q} \simeq 5\,$kG, respectively) yield R_* and $v_{\rm e} \sin i$ values compatible with the above estimates; however, the corresponding curves of $\langle |B| \rangle$ (which we have calculated by performing numerical integrations over the stellar disk) *fail* to reproduce the observations.

The preceding analysis suggests that higher-order multipoles might be necessary to describe the magnetic configuration of β CrB. However, the possibility of a dipole plus quadrupole configuration cannot definitely be ruled out. In fact, since the fit of this star belongs to Class $(++)$, we cannot exclude the possibility of a dipole plus quadrupole model *different* from the two models recovered (and compatible with $\langle |B| \rangle$ and BBLP measurements). Such possibility is strengthened by the small values found for the reduced χ^2 at the two minima.

For the remaining stars of Class $(++)$ – HD 96446 and HD 119419 – no definite conclusion can be drawn, since the predicted values for R_* and/or $v_{\rm e} \sin i$ are not consistent with the estimated or measured values. We will come back to this point in Sect. 4.

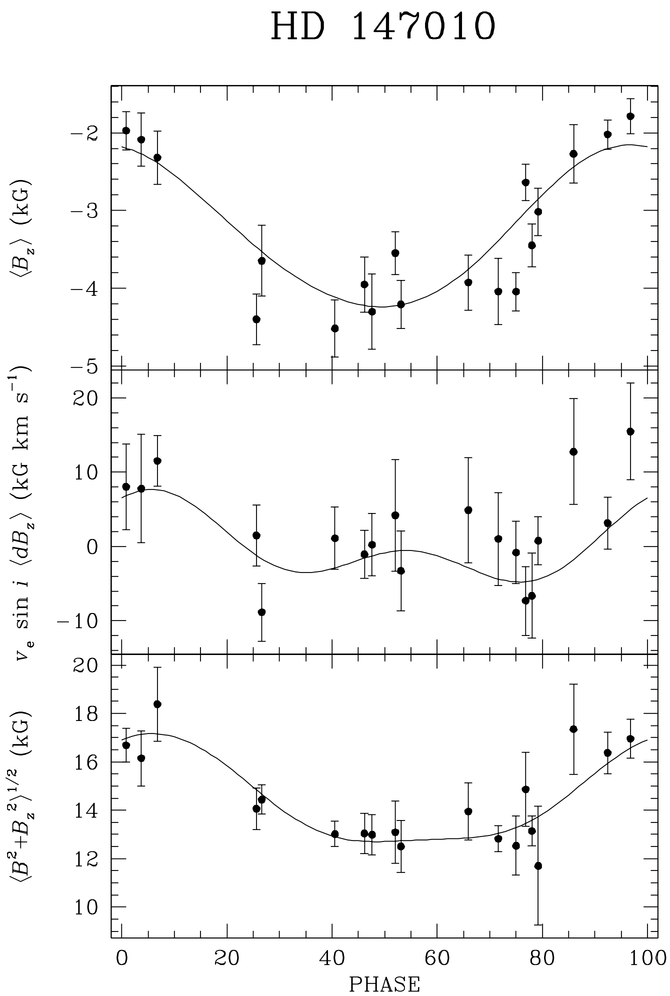

Fig. 2. Dipole plus quadrupole fit to the longitudinal field, crossover, and quadratic field observations of HD 147010

3.2.2. Fits of class $(-+;{\rm B2})$

Three of the stars in our sample belong to this class: for all of them, the pure dipole model should be ruled out.

For HD 147010, the dipole plus quadrupole model corresponding to the lowest χ^2 value (1.52) is specified by the parameters

$$i = 34° \pm 26°$$
$$\beta = 142° \pm 27°$$
$$\beta_1 = 14° \pm 12°$$
$$\beta_2 = 59° \pm 35°$$
$$f_0 = 0° \pm 7° \qquad B_{\rm d} = 17.9 \pm 1.8\,{\rm kG}$$
$$\gamma_1 = 251° \pm 33° \qquad B_{\rm q} = 29.5 \pm 4.9\,{\rm kG}$$
$$\gamma_2 = 162° \pm 7° \qquad v_{\rm e} = 20.1 \pm 9.2\,{\rm km\,s^{-1}}\ ,$$

where the zero-phase point corresponds to JD $= 2450002.88$ (according to Eqs. (2), the alternative model is specified by the supplementary values for i, β, β_1, β_2 and the same values for the remaining parameters). This model, characterized by a quadrupole strength almost twice the dipole strength, predicts $R_* = 1.6 \pm 0.7\ R_\odot$, to be compared with the value $2.1\,R_\odot$ expected for a (non peculiar) B9 star, and $v_{\rm e} \sin i\ = 11.2 \pm 12.6\ {\rm km\,s^{-1}}$, which is compatible with the Doppler broadening observations of Wolff (1981) ($v_{\rm e} \sin i < 20\,{\rm km\,s^{-1}}$) and of Mathys (1995b) ($v_{\rm e} \sin i\ = 22.1 \pm 4.0\,{\rm km\,s^{-1}}$). By contrast, the model corresponding to the other χ^2 minimum (1.66) predicts unrealistically small values for R_* and is not compatible with the observations of Doppler broadening.

Although, according to Sect. 2.1, the reliability of the fits of this class cannot be assured, the above features make the recovered model strongly plausible. The fit, shown in Fig. 2, well accounts for a characteristic pointed out by Mathys (1995b): the curve of $\langle B^2 + B_z^2 \rangle^{1/2}$ has a maximum where the absolute value of $\langle B_z \rangle$ has a minimum.

For the remaining stars of Class $(-+;{\rm B2})$ – HD 125248 and HD 137509 – the dipole plus quadrupole fits predict R_* and $v_{\rm e} \sin i$ values smaller (approximately by a factor 2) than the expected or measured values.

3.2.3. Fits of class $(-+;{\rm B1})$

The last two stars of the sample belong to this class: the dipole model should be ruled out, and the recovered dipole plus qua-

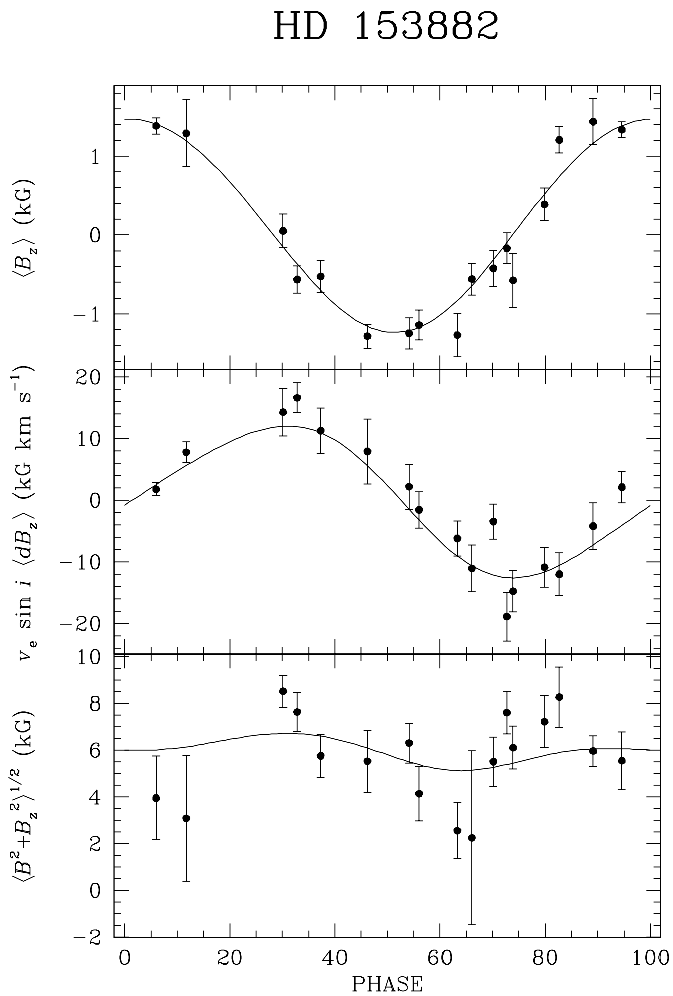

Fig. 3. Dipole plus quadrupole fit to the longitudinal field, crossover, and quadratic field observations of HD 153882

drupole model (which is unique, apart from the degeneracy of Eqs. (2)) should be considered reliable.

For HD 153882, the best-fit parameters are

$$\begin{array}{rcrcl} i &=& 80^\circ &\pm& 8^\circ \\ \beta &=& 70^\circ &\pm& 8^\circ \\ \beta_1 &=& 8^\circ &\pm& 5^\circ \\ \beta_2 &=& 108^\circ &\pm& 14^\circ \\ f_0 &=& 0^\circ &\pm& 5^\circ \qquad B_\mathrm{d} = 4.8 \pm 0.3\,\mathrm{kG} \\ \gamma_1 &=& 264^\circ &\pm& 42^\circ \qquad B_\mathrm{q} = 8.2 \pm 0.7\,\mathrm{kG} \\ \gamma_2 &=& 246^\circ &\pm& 24^\circ \qquad v_\mathrm{e} = 14.4 \pm 1.6\,\mathrm{km\,s^{-1}}\,, \end{array}$$

and the zero-phase point corresponds to JD $= 2450005.56$ (again, the alternative model is specified by the supplementary values for i, β, β_1, β_2 and the same values for the remaining parameters). This model, characterized by a quadrupole twice as strong as the dipole, predicts a stellar radius $R_* = 1.7 \pm 0.2\,R_\odot$, fully consistent with the typical value for a (non peculiar) star of spectral type A1 ($1.8\,R_\odot$). The derived projected equatorial velocity is $v_\mathrm{e} \sin i = 14.2 \pm 1.9\,\mathrm{km\,s^{-1}}$, which is fully consistent with the value measured by Abt & Morrel (1995) of $15\,\mathrm{km\,s^{-1}}$, but not compatible with the values given by Preston (1970) ($v_\mathrm{e} \sin i = 26\,\mathrm{km\,s^{-1}}$) and Mathys (1995b) ($v_\mathrm{e} \sin i = 23.2 \pm 0.9\,\mathrm{km\,s^{-1}}$). The fit is shown in Fig. 3.

By contrast, the model recovered for HD 175362 predicts too small values both for the stellar radius and for the projected equatorial velocity: $R_* = 1.0 \pm 0.1\,R_\odot$, to be compared with $2.6\,R_\odot$ expected for a (non peculiar) star of spectral type B6, and $v_\mathrm{e} \sin i = 13.2 \pm 0.9\,\mathrm{km\,s^{-1}}$, to be compared with the (conflicting) estimates of $28\,\mathrm{km\,s^{-1}}$ (Wolff & Wolff 1976) and $45.4 \pm 5.1\,\mathrm{km\,s^{-1}}$ (Mathys 1995b). Since, in addition, the reduced χ^2 has a rather large value, the presence of higher-order multipoles looks very likely.

4. Discussion and conclusions

We have analysed the observations of magnetic curves (mean longitudinal field, crossover and mean quadratic field) of nine CP stars, to test their consistency with the dipole or with the dipole plus quadrupole magnetic model.

For one star (HD 83368) the pure dipole configuration was found sufficient to reproduce the observations, and a magnetic model has been recovered which is fully consistent with all the available data.

For six stars the dipole configuration should be ruled out, and for two of them (HD 147010 and HD 153882) a plausible dipole plus quadrupole model was worked out: in both cases, the quadrupole field predominates over the dipole field. In the case of HD 175362 there are indications that the magnetic configuration is even more complex than the dipole plus quadrupole one.

The fitting technique adopted in this paper is a direct application of a method recently proposed (Landolfi et al. 1998), which is the first attempt to interpret simultaneously longitudinal field, crossover, and quadratic field observations. The above results show clearly the diagnostic potential of this kind of analysis.

However, a magnetic model could not be unambiguously recovered for several stars of our sample. In some cases (particularly HD 137509) this can be partly ascribed to the small number of observations and/or insufficient phase coverage. But the present analysis also shows that, in general, two additional kinds of data should be included in the fitting technique. In this technique, the stellar radius and the projected equatorial velocity (which are strictly related to the parameters of the fit) are *not* taken into account explicitly, rather they are considered as by-products of the analysis. Within the framework of the oblique rotator model, such quantities could indeed be recovered by analysing the magnetic curves alone. However, the observational errors make this possibility rather unrealistic: thus it is not surprising that only in a few cases the derived values of these quantities were "automatically" consistent with the expected values.

A first improvement of the analysis presented in this paper can be obtained by a slight modification of the fitting technique, that is, by setting explicit constraints to the values of the stellar radius and the projected equatorial velocity; work on this subject is in progress. At the same time, a detailed magnetic modelling requires more and more accurate observations of longitudinal field, crossover and quadratic field. Obviously, the ultimate step toward a comprehensive diagnostic method for magnetic fields of CP stars is the combined analysis of *all* kinds of measurement obtained via spectropolarimetric and broad band observations.

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