

# Oblate spheroid model of nucleus of comet 46P/Wirtanen

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**Abstract.** An improved forced-precession model of Comet 46P/Wirtanen is presented. The nongravitational motion of the comet has been investigated based on the forced precession model of the rotating cometary nucleus. The least squares method applied to observational equations allows us to determine six basic nongravitational parameters:  $A$ ,  $\eta$ ,  $I$ ,  $\phi$ ,  $f_p$ , and  $s$  together with six orbital elements. The solutions were obtained with additional assumptions: 1. The cometary activity with respect to the perihelion is asymmetric and, 2a. The time shift parameter which describes the displacement of maximum activity with respect to the perihelion could change its value during the investigated apparitions or 2b. The real activity of the comet changed during the time interval considered. All eight recorded apparitions of the comet were linked using all astrometric observations covering the period 1948–1997 with a mean RMS residual of  $1''.6$ . According to the best solution, the nucleus of Comet Wirtanen is oblate along the spin-axis with a ratio of equatorial to polar radius of about  $R_a/R_b = 1.1$ . This precession model yields a value of 4.9 hrs/km for the ratio of  $P_{\text{rot}}/R_a$ . Assuming that the nucleus radius of 46P/Wirtanen most likely is in the range of 0.5–2.0 km we obtain a range 2.5–10.0 hours for the rotational period.

**Key words:** comets: individual: 46P/Wirtanen

## 1. Introduction

The short-period comet 46P/Wirtanen will be investigated for two years from 2011 during the ROSETTA cometary rendezvous mission. Therefore, precise predictions of the future perihelion passages and space positions of the comet are crucial for successful planning of the mission.

Comet 46P/Wirtanen has been observed at eight returns during the almost fifty-year interval since its discovery in 1948 by C.A. Wirtanen. Two close approaches of the comet to Jupiter in 1972 and 1984 are responsible for significant changes of orbital elements. Apart from the planetary perturbations the comet is subject to nongravitational forces (Marsden & Williams 1997). The method of determining the nongravitational effects in the orbital motion of the comet was proposed by Marsden et al.

(1973). In Marsden's formalism the three orbital components of a nongravitational force acting on the comet have the form:

$$F_i = A_i g(r), \quad A_i = \text{const for } i = 1, 2, 3, \quad (1)$$

where  $F_1, F_2, F_3$  represent the radial, transverse and normal component of the nongravitational force, respectively. The analytical function  $g(r)$  simulates the ice sublimation rate as a function of the heliocentric distance  $r$ .

In a previous paper (Królikowska & Sitarski 1996, hereafter Paper I) we concluded that the forced precession model of the rotating nonspherical cometary nucleus adequately explains the variations of the nongravitational effects observed in the periodic comet Wirtanen and is suitable for making predictions of the future returns. In the present paper, we continue the study of nongravitational motion of the 46P/Wirtanen including the astrometric observations from the last apparition.

A forced precession of the spin axis is caused by the torque which arises when the vector of the jet force (exerted by the outgassing on the nucleus) does not pass through the mass center of the nonspherical nucleus. The precession rate is a function of the nucleus orientation, the lag angle  $\eta$  of the maximum outgassing behind the subsolar meridian, the modulus of the reactive force, the nucleus oblateness  $s$  and the precession factor  $f_p$  which depends on rotation period and nucleus size (Sekanina 1984, 1988). The orientation of the nucleus is defined by two angles: the obliquity  $I$  of the orbital plane to the cometary equator and the longitude  $\phi$  of the Sun at perihelion. The nongravitational force components (which vary along the orbit) are related to the angular parameters  $\eta, I, \phi$  of the rotating nucleus by:

$$F_i(t) = A \cdot C_i(\eta, I(t), v(t) + \phi(t)) \cdot g(r), \quad \text{for } i = 1, 2, 3,$$

where  $A = (F_1^2 + F_2^2 + F_3^2)^{1/2} / g(r)$ ,  $v$  is the true anomaly of the comet and  $C_i$  denote the direction cosines which are time-dependent due the orbital and precessional motion of the comet. We adopt Marsden's water production curve,  $g(r)$ , because of its analytical form which is useful for orbital computations. Expressions for time variations of the angles  $I$  and  $\phi$ , the precession rate, and the direction cosines of the nongravitational force are given by Królikowska et al. (1998).

The values of five precessional parameters:  $A, \eta, I_0, \phi_0, f_p$  were derived in Paper I; the sixth – oblateness of the comet nucleus – could not be determined because of poor observational material which then consisted of 67 positional observations covering the period 1948–1991. Since that analysis, the new, high-

**Table 1.** Orbital elements and physical parameters of the nucleus for forced precession models linking all apparitions of 46P/Wirtanen. Angular elements  $\omega$ ,  $\Omega$ ,  $i$  refer to Equinox J2000.0. Parameters  $A$ ,  $A^I$  and  $A^{II}$  are in units of  $10^{-8}$  AU/day<sup>2</sup>, the precession factor  $f_p$  is in units of  $10^6$  day/AU, and time shifts  $\tau$ ,  $\tau_1$  and  $\tau_2$  are in days.

	Model 1	Model 2a	Model 2b
	<i>Orbital elements</i>		
Equinox:	J2000.0		
Epoch:	1998 01 27		
	ET = JD 2450840.5		
T	1997 03 14.15654	1997 03 14.15568	1997 03 14.15611
$q$	1.06342371	1.06342736	1.06343483
$e$	0.65681460	0.65680686	0.65680623
$\omega$	356° 32749	356° 33091	356° 33058
$\Omega$	82° 20129	82° 19688	82° 19803
$i$	11° 72345	11° 72330	11° 72321
	<i>Nongravitational and nuclear parameters</i>		
$A$	+0.62615 ± 0.00523	+0.76153 ± 0.00986	–
$A^I$	–	–	+0.80188 ± 0.00873
$A^{II}$	–	–	+0.67797 ± 0.00722
$\eta$	21° 02 ± 0° 42	15° 90 ± 0° 58	15° 48 ± 0° 66
$I_0$	136° 00 ± 1° 13	154° 64 ± 0° 52	140° 83 ± 1° 28
$\phi_0$	259° 60 ± 3° 82	23° 00 ± 8° 98	318° 32 ± 1° 82
$f_p$	0.62528 ± 0.02536	+1.7461 ± 0.1242	+1.2337 ± 0.0694
$s$	0.33419 ± 0.01318	+0.12148 ± 0.04720	+0.10190 ± 0.03421
$\tau$	–	–	–23.476 ± 1.247
$\tau_1$	–	–31.623 ± 1.691	–
$\tau_2$	–	–11.236 ± 0.529	–
Res	1'' 82	1'' 68	1'' 59

quality data have become available from the latest apparition 1995–1997. This allows us to refine the previous forced precession model. The aim of this paper is to obtain the best model of the comet by linking all the astrometric observations made during the eight recorded appearances of the comet.

## 2. Observational material

The distribution of the 67 observations of Comet 46P/Wirtanen carried out before 1995 is given in Table 1 of the Paper I. At present, 247 positional observations (494 residuals) are available from the recent apparition covering the time interval of 26 June 1995–30 December 1997. Since the observations are distributed highly nonuniformly over the apparitions, we replaced more than two observations of the same day by one average comet position (so-called normal place), which yields 300 residuals for the last return of the comet. Combining mean residuals 1''96, 1''67, 1''52, 2''29, 2''44, 1''97 and 1''01 calculated separately for apparitions in 1948, 1954, 1961, 1967–1975 (two apparitions), 1986, 1991 and 1996, respectively, we obtained an *a priori* mean residual (Bielicki & Sitarski 1991) of 1''38.

## 3. Forced precession model of the rotating nonspherical cometary nucleus

The equations of the comet motion have been integrated numerically by the recurrent power series method (Sitarski 1979, 1984),

taking into account all the planetary perturbations. A detailed description of the forced precession model is given in Paper I. The first set of equations contained six unknown parameters  $A$ ,  $\eta$ ,  $I_0$ ,  $\phi_0$ ,  $f_p$  and  $s$  which were determined simultaneously with six orbital elements from the observational equations by an iterative least squares process. This solution is given in the first column of Table 1 (hereafter Model I). The positive value of  $s$  implies that – according to our model – 46P/Wirtanen rotates on its shorter axis.

However, a comparison of the mean residual of this solution with the *a priori* mean residual shows that a better solution is required for the motion of the comet. Therefore, we no longer make the assumption that the activity reached a maximum exactly at perihelion. We have found (from our orbital calculations) that between the apparitions of 1986 and of 1991 (one revolution after the second closest approach of the comet to Jupiter) something changed in the cometary activity and/or in the geometry of the sublimation area on the nucleus surface. These changes can be modelled by setting the discontinuity of parameters  $A$  and/or  $\tau$  close to the moment of aphelion passage between 1986 and 1991:

$$A = \begin{cases} A^I \text{ for } t < 1989.0 \\ A^{II} \text{ for } t \geq 1989.0 \end{cases} \text{ and/or } \tau = \begin{cases} \tau_1 \text{ for } t < 1989.0 \\ \tau_2 \text{ for } t \geq 1989.0 \end{cases}$$

where parameter  $\tau$  represents a time shift of the maximum value of  $g(r)$  with respect to the perihelion time. Thus, the function

$g(r)$  is then replaced by  $g(r')$ , where  $r' = r(t - \tau)$  (Sekanina 1988, Sitarski 1994).

However, it turned out that due to the relatively poor observational material (especially that collected before 1995), only two additional parameters could be numerically determined. Thus, we were limited to introduce the discontinuity of  $\tau$  and  $A$  in two separate models, denoted as Model 2a and Model 2b respectively in Table 1.

From this reason, the final two solutions are described by eight parameters:

$$A, \eta, I_0, \phi_0, f_p, s, \tau_1, \tau_2 \quad (\text{Model 2a}),$$

and

$$A^I, A^{II}, \eta, I_0, \phi_0, f_p, s, \tau \quad (\text{Model 2b}),$$

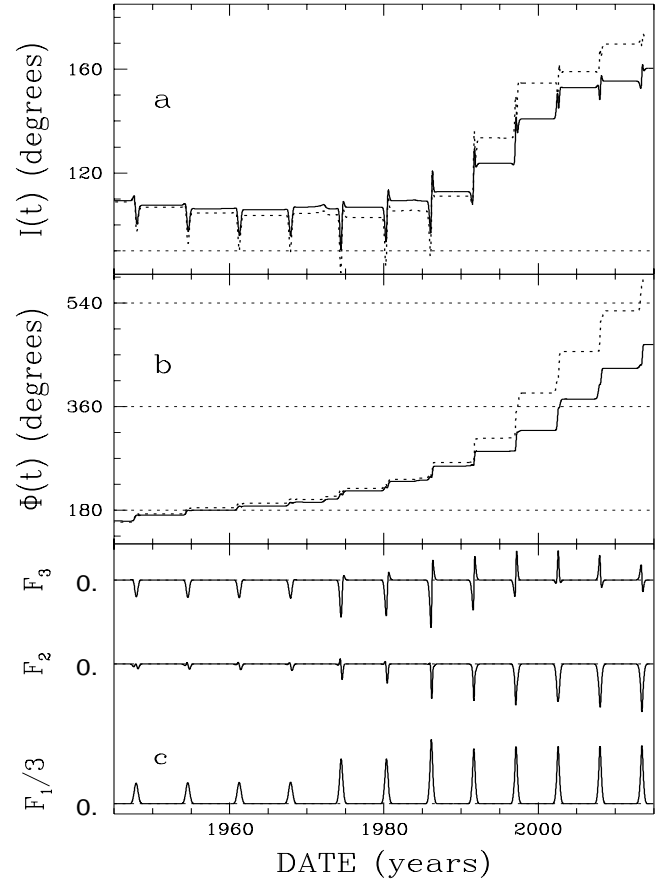
which were determined from the observational equations along with the orbital elements. These solutions, characterized by mean residuals of  $1''.68$  and  $1''.59$ , are given in Table 1. Both models give a prediction of perihelion passage in February 2008 (the last before the ROSETTA rendez-vous with the comet) with a time dispersion of 0.038 day.

The motion of the nucleus rotation axis represented by angles  $I$  and  $\phi$  and the qualitative variations of the nongravitational force components  $F_1, F_2, F_3$ , acting on the comet during its successive returns to the Sun are presented in Fig. 1 for Model 2b (solid curves). Dashed curves in the same figure represent the variation of  $I$  and  $\phi$  for Model 2a; the variations of  $F_1, F_2, F_3$  are undistinguishable from those for Model 2b. This means that the same character of the nongravitational accelerations can be interpreted by two different models of cometary activity. The time variations of  $I, \phi$  and  $F_1, F_2, F_3$  are a consequence of the forced precession and orbital changes caused by two close approaches to Jupiter in 1972 and 1984. These close encounters with Jupiter reduced the perihelion distance from 1.63 AU in 1947 to 1.08 AU in 1986, which caused an increase of the nongravitational force during this period (Fig. 1c). Both solutions imply a retrograde rotation of the cometary nucleus ( $I > 90^\circ$ ). This is in agreement with the Jorda & Rickman (1995) conclusion from the positive lightcurve asymmetry combined with the negative rate of comet orbital period change.

#### 4. Discussion

The present precession models impose some constraints on the physical properties of the nucleus of Comet 46P/Wirtanen. Using these solutions it is possible to calculate values of Sekanina's (1984) torque factor  $f_{tor} = f_p/s$ . Using this parameter we are able to calculate the value of the  $P_{rot}/R_a$  ratio from the formula:  $P_{rot}/R_a = 4\pi \cdot f_p/(5s)$ , where the equatorial radius,  $R_a$ , is related to the oblateness and the polar radius,  $R_b$ :  $R_b/R_a = 1 - s$ . Our models give:

$$P_{rot}/R_a = \begin{cases} 0.75 \pm 0.01 & \text{for Model 1} \\ 5.80 \pm 1.85 & \text{for Model 2a} \\ 4.88 \pm 1.36 & \text{for Model 2b} \end{cases} \quad (2)$$



**Fig. 1a–c.** Temporal variation of the angle  $I$  – **a**,  $\phi$  – **b**, and components  $F_1, F_2, F_3$  of the nongravitational force  $\mathbf{F}$  – **c**, for Comet 46P/Wirtanen due to the spin-axis forced precession of the comet's nucleus. Solid curves represent Model 2b and dashed curves – Model 2a. Dotted horizontal lines in **b** indicate moments when the sunlit pole of nucleus in perihelion changes from 'northern' to 'southern' and vice versa.

where  $P_{rot}$  is in hours and  $R_a$  in kilometers. Taking into account observations of the nucleus radius of 46P/Wirtanen it is possible to verify our models.

From several estimates of the nucleus radius of Comet Wirtanen it appears that this comet possesses one of the smallest nuclei detected so far. From photographic magnitudes Jorda & Rickman (1995) obtained a radius of the nucleus "of less than 2 km, probably 1.5–1.8 km". This is in agreement with Almeida et al. (1997), who derived a lower limit of the nucleus radius of 1.0 km from visual magnitudes of the comet. However, systematically smaller sizes of the nucleus are reported from CCD photometry. CCD observations done by Boehnhardt et al. (1997) give an upper limit of 0.8 km and Lamy et al. (1998) obtained a mean effective radius of  $0.60 \pm 0.02$  km. Such small radii imply that a large fraction of the surface of the nucleus is active. The active area of  $1.80 \text{ km}^2$  obtained by Farnham & Schleicher (1998) covers about 7% of the sunlit surface area for a radius of 2 km, 45% for a radius of 0.8 km, and rises to more than 80% for a radius of 0.60 km.

Assuming that the nuclear radius of 46P/Wirtanen most likely is in the range 0.5–2.0 km we obtain from Eq. (2):

**Table 2.** Evolution of orbits of 46P/Wirtanen according to Model 2b given in Table 1; perihelion distance  $q$  is in AU, period  $P$  is in years, angular parameters  $\omega$ ,  $\Omega$ ,  $i$  refer to Equinox J2000.0.

T	$q$	$e$	$P$	$\omega$	$\Omega$	$i$	$I$	$\phi$	Epoch
1997 03 14.14929	1.06376188	0.65673107	5.46	356°34091	82°20593	11°72249	140°18	309°42	1997 03 13
2002 08 26.63663	1.05876883	0.65780736	5.44	356°39605	82°17493	11°73837	145°09	357°62	2002 09 03
2008 02 02.13771	1.05750021	0.65807737	5.44	356°34611	82°17490	11°73953	149°17	38°25	2008 01 15
2013 07 08.64672	1.05209806	0.65923863	5.42	356°33829	82°16315	11°75731	161°23	85°46	2013 07 07

$$R_{\text{eff}}[\text{km}] : 0.5 \quad 1.0 \quad 2.0$$

$$P_{\text{rot}}[\text{hrs}] = \begin{cases} .42 \pm .01 \quad .83 \pm .01 \quad 1.66 \pm .02 & \text{for Mod 1} \\ 3.0 \pm 0.9 \quad 6.0 \pm 1.8 \quad 12.0 \pm 3.7 & \text{for Mod 2a} \\ 2.5 \pm 0.7 \quad 5.0 \pm 1.4 \quad 10.0 \pm 2.8 & \text{for Mod 2b} \end{cases}$$

where  $R_{\text{eff}}$  is the effective radius of the nucleus. Thus, Model 1 gives very small rotational periods (fast rotation). Assuming that the mean density of the cometary material is equal to  $1 \text{ g/cm}^3$  and the comet radius is equal to 1 km, one finds that the gravitational force at the nucleus surface is almost 20 times smaller than the centrifugal force that should cause rotational splitting of the nucleus. Alternatively, Model 1 requires the comet radius to be greater than 4 km.

Based on observations made at intermediate heliocentric distances and assuming a double-peaked light curve, Meech et al. (1997) found a possible rotational period of about 7.6 hours. Lamy et al. (1998) derived a double peaked light curve with a rotational period of  $6.0 \pm 0.3$  hr. Models 2a and 2b are in agreement with these results. Assuming 6 hr for the rotational period our Models 2a and 2b give a radius in the range of 0.7–1.5 km and of 0.9–1.7 km, respectively.

From the range of variation of the light curve Lamy et al. (1998) found that the ratio of the axes must be greater than 1.20 for the assumed ellipsoidal body. Our values of  $1.14 \pm 0.05$  (Model 2a) and  $1.11 \pm 0.05$  (Model 2b) for the  $R_a/R_b$  ratio are only slightly below their lower limit. Thus, we concluded that Models 2a and 2b are to be preferred over Model 1 because of the smaller mean residual, and the reasonable prediction of the  $P_{\text{rot}}/R_a$  ratio.

However, not all observational facts are explained by our models. The negative values of time shifts  $\tau$  (Model 2b) and  $\tau_1$ ,  $\tau_2$  (Model 2a) suggest that the gas production rate peaks before the perihelion passage. This is in contradiction with the positive light curve asymmetry given in Jorda & Rickman (1995) assuming that the jet force is produced only by sublimation directly from the nuclear surface. This question was also pointed out by Rickman & Jorda (1998). To explain this problem, they discussed a model of 46P/Wirtanen in which the nucleus is subject to fragmentation along with sublimation. The icy chunks which break from the surface are expelled by the drag of the outflowing gas from the pores and propelled by a jet force due to sublimation. They argued that if the outgassing from icy circumnuclear material is predominant then it gives rise to the postperihelion brightness. Thus, it is possible that the gas production curve related to the nongravitational effects has no correspondence to the postperihelion shift of lightcurve maximum observed for

46P/Wirtanen. Furthermore, it is possible that the discontinuity in global activity obtained from orbital calculations would not be detected in the visible brightness of the comet. More studies are needed to judge whether the fragmentation hypothesis is true.

For all of these reasons, we conclude that our forced precession model for the rotating comet nucleus (Model 2b or 2a) reproduces – with very good precision – the nongravitational motion of 46P/Wirtanen during an almost fifty-year interval and gives nucleus parameters (e.g. size and rotational period) consistent with the available photometric observations as well. Future evolution of the orbital elements and orientation of the comet spin-axis is presented in Table 2 for Model 2b since this model represents the orbital motion with the smallest mean residual. One should note that the nongravitational forces have a significant influence on the cometary position even during a single apparition of the comet. Thus, these effects – even within a single apparition – must be included into any accurate orbit calculations.

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