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Gravitational lensing by rotating stars

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Abstract. The equations giving the position of the images of a point source by a rotating point lens are derived by a new, elementary method. It is shown that only two of the three images are visible. It is argued that the projection of the angular momentum of the lens star on the lens plane can be measured if the lens is a rapidly rotating early type star. This is achieved by performing a series of astrometric measurements of the position of the images.

Key words: gravitation – stars: rotation – cosmology: gravitational lensing

1. Introduction

Gravitational lensing has become an important tool for observational astrophysics. A formalism based on the Fermat principle, first advocated by P.Schneider (Schneider 1985) gives physical insight on a range of problems related to lensing, such as the classification of lensing topologies (Blandford & Narayan 1986) or evaluations of time delays (Krauss & Small 1991). The calculations of gravitational lensing do not in general take into account stellar rotation, which is expected to be a very small second order effect. However, gravity induced by stellar rotation is important from the point of view of principles, since it is the most straightforward example of gravity induced by a current of mass.

The problem of the deviation of light by rotating stars has been studied in the general case of a Kerr metric (Dymnikova 1986). Other calculations have been performed based on the PPN expansion (Epstein & Shapiro 1980) or on a power series in gravitational constant G (Ibañez 1983). We show here that a simple modification of Fermat's principle can take the lowest order effects of stellar rotation into account. Possible physical effects induced by stellar rotation include a small additional deflection of the light ray in the plane which contains the observer, the lens and the source star ("planar deviation"), a small deflection away from this plane ("aplanar deviation"), and a time delay between the images of the source. Gravitational lensing provides a natural framework to test for these effects, which affect differently the multiple images of the source. In

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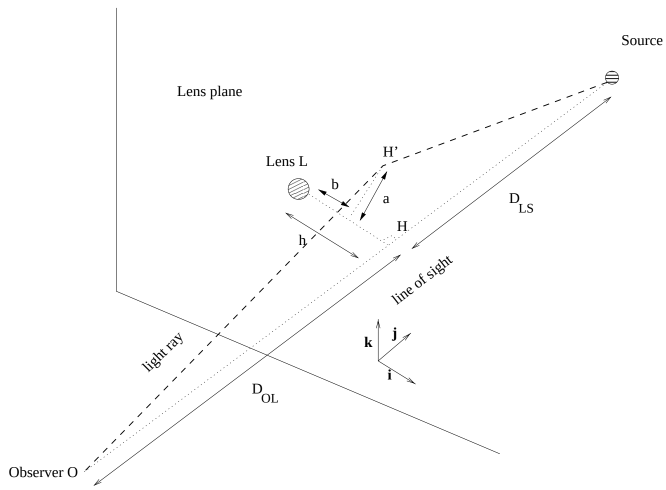


Fig. 1. An observer located at O receives photons from a source star at S. The source star is lensed by a lens star at L. The distance between the observer and the lens is D_{OL}, the distance between the lens and the source is D_{LS}. The lens plane is normal to the line of sight OS. H is the intersection of the lens plane with OS.

the case of the lensing of a point source by a rotating point lens, we show that only 2 images are visible. Orders of magnitude of the various rotation-induced effects are evaluated. A method for measuring the aplanar deviation with accurate astrometry is given. The angular momentum of a rapidly rotating early type star at distance of a few tens of parsec can be evaluated with astrometric measurements in the μas (microarcsec) range.

2. Lensing by a non-rotating point-like star

This section is devoted to establishing notations and recalling well known results on the theory of lensing by point masses. The lensing geometry is shown on Fig. 1. A source star located at S is lensed by a massive lens star (mass M) at L and observed at O.

The Schwarzschild radius r_S and the Einstein radius r_E are defined by:

$$D_{OS} = D_{OL} + D_{LS} \tag{1}$$

$$r_S = \frac{2GM}{c^2} \tag{2}$$

$$r_E = \sqrt{\frac{2r_S D_{LS} D_{OL}}{D_{OS}}} \tag{3}$$

The generic point P on the light ray has spherical coordinates r_1, θ_1, ϕ_1. The gravitational time delay is:

$$\Delta T_s = (1/c) \int ds + (1/c) \int \frac{r_S}{r_1} ds \tag{4}$$

The first term on the right hand side of the equation is the geometric time delay, while the second is the "Shapiro" effect (Weinberg 1972). The right hand side of Eq. (4) can be evaluated to first order by replacing the true light ray by 2 straight lines OH' and H'S. Defining a,b and h by

$$\boldsymbol{LH'} = a\boldsymbol{k} + b\boldsymbol{i}, \ \boldsymbol{LH} = h\boldsymbol{i} \tag{5}$$

where $\boldsymbol{j}$ is along the line of sight to the actual position of the source, $\boldsymbol{k}$ is the normal to the SLO plane and $\boldsymbol{i} = \boldsymbol{j} \wedge \boldsymbol{k}$, one has:

$$\Delta T_s = \frac{1}{c}(D_{OS} + \frac{1}{2}(\frac{D_{OS}}{D_{OL} D_{LS}})((b-h)^2 + a^2) \\ -2r_S \ln \sqrt{b^2 + a^2}) \tag{6}$$

By differentiating (6) with regard to a, one finds that:

$$a = 0$$

so the photon trajectory is in the SLO plane. The equation obtained by differentiating with regard to b is

$$b - h - r_E^2/b = 0 \tag{7}$$

The source has 2 images located at:

$$b_+ = 1/2(h + \sqrt{h^2 + 4{r_E}^2}) \tag{8}$$

$$b_- = 1/2(h - \sqrt{h^2 + 4{r_E}^2}) \tag{9}$$

Note that:

$$b_+ b_- = -r_E^2 \tag{10}$$

$$b_+ + b_- = h \tag{11}$$

$$b_+ - b_- = \sqrt{h^2 + 4{r_E}^2} \tag{12}$$

It is somewhat puzzling that this equation has only 2 solutions, since one can show (Blandford & Narayan 1986) that lensing systems have odd numbers of images. The rotating lens star case, to which we now turn will allow us to solve this paradox.

3. Time delay induced by stellar rotation

The lens star has a radius R and is assumed to rotate rigidly with period T. The star spin is (see Fig. 2)

$$\boldsymbol{J} = J(\cos\theta_0 \boldsymbol{k} + \sin\theta_0(\cos\phi_0 \boldsymbol{i} + \sin\phi_0 \boldsymbol{j})) \tag{13}$$

The equatorial velocity of the lens is $v = 2\pi R/T$. It can be related to the angular momentum J by:

$$J \sim CMRv \tag{14}$$

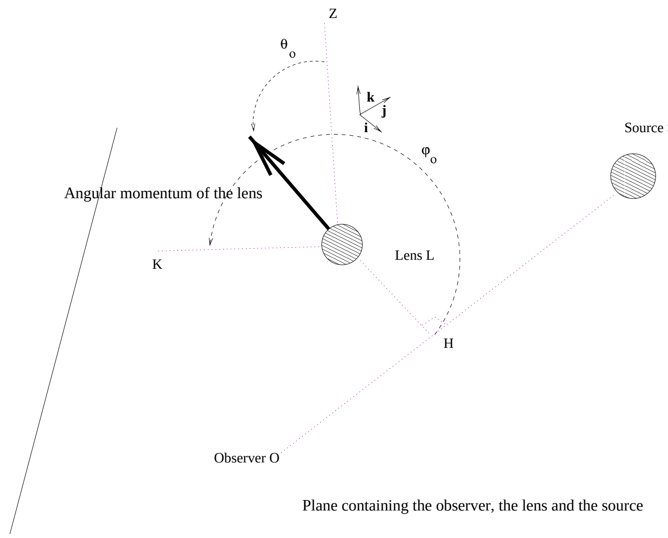


Fig. 2. Angular momentum of the lens star. Lines LH and LZ are in the lens plane. Line LK lies in the direction of the projection of $\boldsymbol{J}$ on the SLO plane.

where $C = 2/5$ for a uniform mass distribution or $C \sim 0.2$ for a more realistic mass distribution. The impact parameter in the lens plane of light rays from the source is always large in comparison of the Schwarzschild radius of the lens. In this limit, the metric around the rotating lens can be described to first order in r_S/r_1 and GJ/r_1c^3 by the Lense-Thirring (Lense & Thirring 1918) metric

$$ds^2 = (1 - r_S/r_1)dt^2 - (1 + r_S/r_1)d\sigma^2 \\ +4GJ/r_1c^3 \sin^2\theta_1 dt d\phi_1 \tag{15}$$

where $d\sigma = \sqrt{{dr_1}^2 + r_1^2 {d\theta_1}^2 + \sin^2\theta_1 d\phi^2}$ is the euclidian spatial line element at the generic point P on the light ray. Since rotation effects are in general very small, it is not obvious that the Lense-Thirring metric applies to a real life star. This will be true only if the metric around the star is such that second order terms in r_S/r_1 are negligible compared to the first order term in $GJ/{r_1}^2c^3$. This in turn is true provided

$$\frac{r_S}{R} \leq \frac{v}{c} \tag{16}$$

For a main sequence star, $r_S/R \simeq r_{S\odot}/R_\odot \sim 10^{-5}$. The condition (16) holds for an early type fast rotating star, such as F or B stars, since for these stars $v/c \sim 10^{-3}$. For the Sun, one has $v/c \sim 0.6\ 10^{-5} \sim r_{S\odot}/R_\odot$ so that the condition (16) is still marginally valid. The condition (16) is also close to be fulfilled for a fast rotating white dwarf with $r_S/R \sim 3\ 10^{-4}$, but is invalid for a neutron star or a black hole.

With this choice of the metric, the extra delay due to rotation (Sagnac effect) is (Landau & Lifschitz 1970)

$$\Delta T_r = -\int (\frac{2GJ}{r_1c^3}\frac{1}{(1 - r_S/r_1)}\sin^2\theta_1 d\phi_1) \\ \simeq -\int (\frac{2GJ}{r_1c^3}\sin^2\theta_1 d\phi_1) \tag{17}$$

The integral is again evaluated on the path shown on Fig. 1, which should be accurate to first order.

After a straightforward, but lenghty calculation, one finds

$$\Delta T_r = -4GJ/c^3 \frac{(-b\cos\theta_0 + a\sin\theta_0\cos\phi_0)}{(a^2+b^2)} \quad (18)$$

The total time delay is

$$\begin{aligned}\Delta T &= \Delta T_s + \Delta T_r \\ &= \frac{1}{c}(D_{OS} + \frac{1}{2}(\frac{D_{OS}}{D_{OL}D_{LS}})((b-h)^2 + a^2) \\ &\quad -2r_S \ln\sqrt{b^2+a^2} \\ &\quad -4GJ/c^3 \frac{(-b\cos\theta_0 + a\sin\theta_0\cos\phi_0)}{(b^2+a^2)}) \quad (19)\end{aligned}$$

The equations for b and a are found by differentiating the total time delay with regard to b and a. The equation for a is

$$(\frac{D_{OS}}{D_{OL}D_{LS}})a = -(4GJ/b^2c^3)\sin\theta_0\cos\phi_0 + o(a^2) \quad (20)$$

or equivalently:

$$a = -\frac{J\sin\theta_0\cos\phi_0}{Mc}(\frac{r_E}{b})^2 + o(a^2) \quad (21)$$

This equation describes the motion outside the plane containing the observer, the source and the lens, a “non-planar” deviation caused by rotation. Note that a scales like $1/b^2$. The equation for b is, anticipating that $\frac{a}{b} = \theta \ll 1$,

$$(\frac{D_{OS}}{D_{OL}D_{LS}})(b-h) = 2\frac{r_S}{b} + 4GJ/b^2c^3\cos\theta_0 \quad (22)$$

These equations agree with those of Ibañez (Ibañez 1983) which were obtained by more involved calculations based on the gravitational scattering of spinning particles. Eq. (22) describes the usual (or “planar”) deviation of light rays in the plane containing the lens, the source and the observer. The right hand side is the deviation angle of a light ray in the gravitational field of a rotating mass. This formula for the deviation angle was first found by Cohen and Brill (Cohen & Brill 1958).

For a rigidly rotating star, the size of the last term in the right hand side of the “planar” equation is of the order of $(r_S R/r^2)(v/c)$. It is always much smaller than the 1/r term by a factor $(R/r)(v/c)$. Typically $v/c \simeq 3\ 10^{-3}$ for a fast rotating star and $r \geq R$ so the effect of the lens rotation on the planar deviation is expected to be very small. The effect on planar deviation is largest for a light ray travelling in the equatorial plane of the spin ($\theta_0 = 0$).

4. Number of images

Eq. (22) has 3 solutions, so in principle one should see 3 images unless one is eclipsed by the lens. The distance of the third image to the lens is maximal in the equatorial plane of the lens spin, so the best chance to see the third image is when $\theta_0 = 0$. The equation for b becomes (with the extra assumption that $\frac{a}{b} \ll 1$):

$$b - h = \frac{r_E^2}{b} \pm \frac{J}{Mc}\frac{r_E^2}{b^2} \quad (23)$$

where the $\pm$ sign depends on the orientation of the photon trajectory relative to the star spin axis. One has a third order equation in b, with 3 real solutions (on physical grounds). This means that we have now explicitely the 3 lensing images. Two of the solutions of the equation in b are close to b_+ and b_- and the third solution is

$$b_0 = \frac{J}{Mcr_S} \sim CR(\frac{v}{c}) \sim 0.2R(\frac{v}{c}) \ll R \quad (24)$$

Hence, the image at b_0 is always eclipsed by the lens star. The number of visible images is 2, like in the non-rotating case.

Possible signatures of the lens rotation are now investigated by comparing the properties of the 2 images with and without rotation. Two possible signatures for rotation are an extra time delay between the 2 images (Sagnac effect) and a misalignement between the 2 images and the source (gravitomagnetic effect).

5. Time delay

The time delay between the 2 images at b_+ and b_- caused by the rotation of the lens is easily calculated from Eq. (18). It is largest when

$$\cos\theta_0 = \pm 1$$

and given in that case by:

$$\begin{aligned}\delta T_\pm &= \Delta T_r(b_+) - \Delta T_r(b_-) \quad (25)\\ &= \pm\frac{4GJ}{c^4}(\frac{1}{b_+} - \frac{1}{b_-}) \quad (26)\\ &= \pm\frac{4GJ}{c^4}\frac{\sqrt{(h^2+4r_E^2)}}{r_E^2} \quad (27)\\ &= \pm\frac{4GJ}{r_E c^4}\sqrt{(x^2+4)} \quad (28)\end{aligned}$$

In the following, x is defined by $x = h/r_E$. For early type stars with $M > 1.12M_\odot$, J given by the empirical formula (Kraft 1967)

$$J \simeq 100J_\odot(\frac{M}{M_\odot})^{5/3} \quad (29)$$

If the distance to the lens is much closer than the distance to the source, then

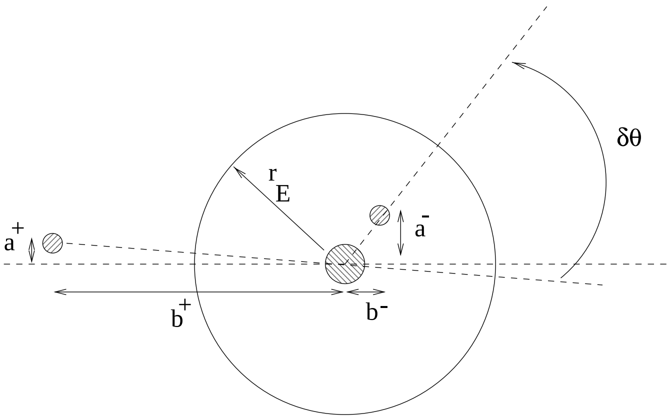
$$\delta T_\pm \sim \pm 0.5\ 10^{-4}\ \mu\mathrm{s}\ (\frac{100\ \mathrm{pc}}{D_{OL}})^{1/2}(\frac{M}{M_\odot})^{7/6}\sqrt{(x^2+4)} \quad (30)$$

Even for very massive star, this time is in the 10 ns range at best, much smaller than the time delay between the two images of a non-rotating star which is in the 10 μs range (Krauss & Small 1991).

6. Misalignement of the images

The rotation of the lens induces a non-planar deviation of the light rays. In particular, Eq. 21 shows that $a_\pm \propto J(r_E/b_\pm)^2$. The order of magnitude of this deviation is

$$CR(\frac{v}{c}) \sim 0.6\ 10^{-5}(\frac{v}{300\ \mathrm{km\ s^{-1}}})(\frac{R}{10\ R_\odot})AU \quad (31)$$



Lens plane

Fig. 3. Misalignement between the lens (in the center) and the projected images of the source. $\delta\theta$ is the deviation to a straight line. Angles are grossly exagerated

for $b = r_E$. The deviation depends only on v, R and the parallax of the lens. It is in the 0.1 μas range or above if the lens is closer than 50 pc. a_+ scales with $1/x^2$ while a_- scales with x^2.

The aplanar deviation is different for the images at b_+ and b_-, which gives a very small misalignement between the images and the lens (see Fig. 3). The non-planar deviation is largest when

$$\cos\phi_0 = \sin\theta_0 = 1$$

Under this condition, the differential deviation between the images at b_+ and b_- is

$$\delta\theta = \theta(b_+) - \theta(b_-) \tag{32}$$

$$= (\frac{J}{Mc}) r_E^2 (\frac{1}{b_+^3} - \frac{1}{b_-^3}) \tag{33}$$

The latter expression can be simplified with the help of Eqs. (12) to yield

$$\delta\theta = (\frac{J}{Mc}) \frac{(h^2 + r_E^2)\sqrt{h^2 + 4{r_E}^2}}{r_E^4} \tag{34}$$

$$= (\frac{J}{Mcr_E})(x^2+1)\sqrt{x^2+4} \tag{35}$$

Hence

$$\delta\theta \sim (\frac{v}{c})(\frac{R}{r_E}) \tag{36}$$

It is maximal for a large fast rotating star. For a early type star at a few tens of parsec, $\delta\theta$ is in the arcsec range.

The deviations $a_\pm$ and $\delta\theta$ are not directly measurable by astrometry. We now examine how these quantities can be extracted in practice.

7. Measurement of the angular momentum of the rotating lens

Before considering the astrometry of the 2 images, we need to know if they are resolved. We assume an angular resolution of 10 mas and a positional accuracy of 1 μas. This is within the capabilities of the planned "Spatial Interferometry Mission" (Allen et al. 1997). In traditional photometric surveys such as EROS (EROS collaboration 1993) or MACHO (MACHO collaboration 1993), the lensed images are not resolved. If the source is very far from the lens, the Einstein radius is:

$$r_E \simeq \sqrt{2r_S D_{OL}} \sim .66(\frac{D_{OL}}{50\,\mathrm{pc}})^{1/2}(\frac{M}{M_\odot})^{1/2}\,\mathrm{AU} \tag{37}$$

This gives $r_E \sim 40$mas for a star with $M = 10M_\odot$ located at 50 pc. Since the separation between the closest image and the lens is roughly r_E/x for $x \gg 1$, the image at b_- can be tracked down to x=4. This assumes of course that the photometry is sensitive enough to detect this faint object.

The lens velocity is the source of another potential problem. Gravitational lensing by stars is usually detected when lenses move in front of distant sources. In the previous sections, the position of the images of the source was calculated in a coordinate system where the lens was at rest. The actual positions seen by the observer can be obtained by performing a Lorentz transformation from the lens rest frame to the observer rest frame. It turns out that the relative positions of the images and the lens are modified by corrections of order W^2/c^2 and $(W/c)(r_E/D_{OL})$, where $\boldsymbol{W}$ is the velocity of the lens in the observer frame. Eq. (37) shows that $r_E/D_{OL} \ll 10^{-6}$. Hence the only terms which can affect seriously the relative position of the images are of order W^2/c^2. These terms are comparable to the deviation caused by the rotation of the lens (Eq. (31)) for velocities larger than 300 km s^{-1}. The velocity of nearby bright stars is typically less than 50 km s^{-1}. The effect of the motion of the lens on the relative positions of the lens and the images of the source is thus expected to be small.

Next, arbitrary coordinate axis are chosen in the lens plane and the positions of the far image $M_+ : (x_+ y_+)$ and the near image $M_- : (x_- y_-)$ are measured relatively to the lens position. Since the actual position of the source is not known, the axis taken are not those defined in Sect. 2. The line forming the intersection of the plane containing the observer, the lens and the source with the lens plane has to be found from the measured positions of the images and the lens. The equation of this line is $y = \alpha x$, where α has to be determined.

Eq. (21) gives the constraint

$$\frac{y_- - \alpha x_-}{y_+ - \alpha x_+} = \frac{x_+^2 + y_+^2}{x_-^2 + y_-^2} \tag{38}$$

Solving for α gives an estimate for a_+:

$$a_+ = \frac{(x_- y_+ - x_+ y_-)(x_+^2 + y_+^2)}{\sqrt{(x_+^2+y_+^2)^3 + (x_-^2+y_-^2)^3 - 2(x_+x_- + y_+y_-)(x_+^2+y_+^2)(x_-^2+y_-^2)}}$$

$$\simeq \frac{(x_- y_+ - x_+ y_-)(x_+^2 + y_+^2)}{x_-(x_-^2 + y_-^2) - x_+(x_+^2 + y_+^2)} \tag{39}$$

A similar expression holds for a_-. Taking the relation (12) into account, one obtains:

$$a_+ = \frac{(x_- y_+ - x_+ y_-)}{x_-(x_-^2 + y_-^2) - x_+(x_+^2 + y_+^2)} \frac{r_E^4}{(x_+^2 + y_+^2)} \tag{40}$$

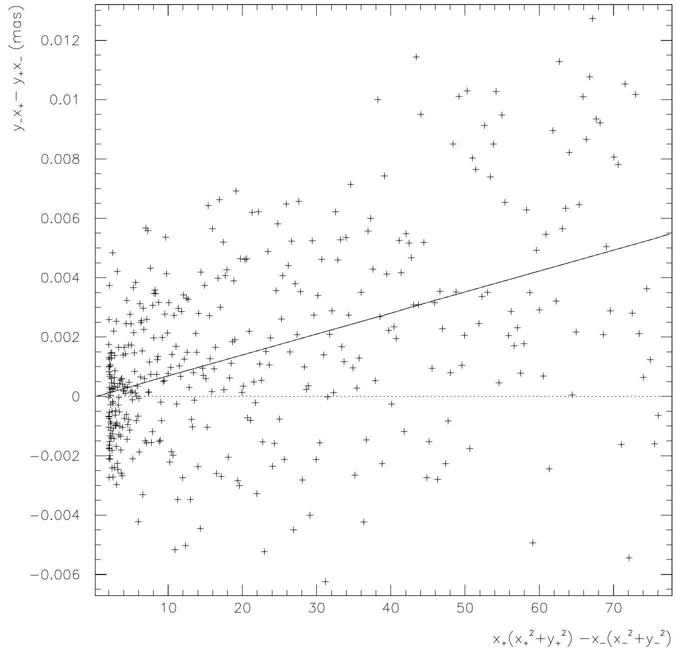


Fig. 4. Simulated measurements of $(x_+y_- - x_-y_+)$ versus $x_+(x_+^2 + y_+^2) - x_-(x_-^2 + y_-^2)$. The positions of the images (x_+y_+) and (x_-y_-) are obtained with a 1 μas accuracy. The $x_\pm$ are normalized to r_E. $\boldsymbol{J}$ is assumed to be along the x axis and $u_0 = 0$. The values $M = 30M_\odot$ and $J/Mcr_E = 7.9\,10^{-5}$ were used in the generation. The lens moves with a velocity $V \ll 300\,\mathrm{km\,s^{-1}}$. The time interval between measurements is $10^{-2}r_E/V$, which is a few hours for typical values of V. The fit is shown by the solid line. The fitted value with 400 measurement points is $J_{fit}/Mcr_E = 7.1\ 10^{-5} \pm 0.7\ 10^{-5}$. The dotted line is $J = 0$.

Comparing with Eq. (21), it is clear that:

$$\frac{J \sin\theta_0 \cos\phi_0}{Mc} = \frac{(x_+y_- - x_-y_+)r_E^2}{x_-(x_-^2 + y_-^2) - x_+(x_+^2 + y_+^2)} \quad (41)$$

The term $\sin\theta_0 \cos\phi_0$ changes as the lens moves in front of the source. In a coordinate system where the lens direction is taken as fixed and the source moves along the x axis with a projected velocity V, one has:

$$\boldsymbol{OS} = \begin{pmatrix} V(t-t_o) \\ D_{OS} \\ u_0 r_E \end{pmatrix}, \quad \boldsymbol{J} = \begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}$$

where u_o is the so-called "impact parameter" and t is the time. Eq. (41) can be rewritten as

$$\frac{(J_x V(t-t_0)/r_E + J_z u_0)}{Mc\sqrt{u_0^2 + V^2\frac{(t-t_0)^2}{r_E^2}}} = \frac{(x_+y_- - x_-y_+)r_E^2}{x_-(x_-^2 + y_-^2) - x_+(x_+^2 + y_+^2)} \quad (42)$$

Eq. (42) can be used to obtain J_x/M and sometimes J_z/M. First u_0 and V/r_E are easily found, for instance by the usual "microlensing" photometric method. Eq. 42 shows that:

$$\frac{\delta(J_x V(t-t_0) + J_z u_0 r_E)}{(J_x V(t-t_0) + J_z u_0 r_E)} \simeq \frac{\Delta}{y^-} \quad (43)$$

where $\Delta \sim 1\,\mu$as is the accuracy of a single position measurement. For a $30M_\odot$ lens at 50 pc, y_- is expected to be in the $0.1\ \mu$as range and scales as x^2, while positions are measured with a 1 μas accuracy. For a $\boldsymbol{J}$ measurement, one has

$$\frac{\delta(J_x V(t-t_0) + J_z u_0 r_E)}{(J_x V(t-t_0) + J_z u_0 r_E)} \simeq \frac{10}{x^2} \quad (44)$$

For N measurements up to an x of x_{max}, one has (with the simplifying assumption that the J_x term dominates):

$$\frac{\delta J_x}{J_x} = \frac{10\sqrt{5}}{x_{max}^2\sqrt{N}} \quad (45)$$

Hence J_x can be measured with a reasonable accuracy with a few hundred measurements as long the M_- image is tracked close enough to the lens. This is illustrated on Fig. 4. In the general case, both J_x and J_z contribute and can be evaluated by a 2-parameter fit. The J_x term dominates when x_{max}/u_o is large. This is presumably the case for an event triggered by an Earth-based photometric survey ($u_o < 1$) and tracked by an astrometry satellite close to the lens ($x \gg 1$).

8. Conclusion

Gravitational lensing by rotating stars has been investigated. It has been shown that the best prospect for observing a rotation effect is a precise astrometric measurement of the position of the images of the source. The lens has to be a luminous nearby located fast rotating star, which unfortunately are rare.

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