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The effect of non-radial motions on the X-ray temperature distribution function and the two-point correlation function of clusters

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Abstract. We show how non-radial motions, originating in the outskirts of clusters of galaxies, may reduce the discrepancy between the Cold Dark Matter (CDM) predicted X-ray temperature distribution function of clusters of galaxies and the observed one and also the discrepancy between the CDM predicted two-point correlation function of clusters of galaxies and that observed. We compare Edge et al. (1990) and Henry & Arnaud (1991) data with the distribution function of X-ray temperature, calculated using Press & Schechter's (1974 - hereafter PS) theory and Evrard's (1990) prescriptions for the mass-temperature relation and taking account of the non-radial motions originating from the gravitational interaction of the quadrupole moment of the protocluster with the tidal field of the matter of the neighboring protostructures. We find that the model produces a reasonable clusters temperature distribution. We compare the two-point cluster correlation function which takes account of the non-radial motions both with that obtained by Sutherland & Efstathiou (1991), from the analysis by Huchra et al. (1990) deep redshift survey, and with the data points for the Automatic Plate Measuring (APM) clusters, computed by Efstathiou et al. (1992a), showing that non-radial motions reduce the discrepancy between the theoretical and the obsservational correlation function.

Key words: cosmology: large-scale structure of Universe – cosmology: theory – galaxies: formation

1. Introduction

Although at the begining the standard form of CDM was very successful in describing the structures observed in the Universe (galaxy clustering statistics, structure formation epochs, peculiar velocity flows) (Peebles, 1982; Blumenthal et al. 1984; Bardeen et al. 1986 - hereafter BBKS; White et al. 1987a,b; Frenk et al. 1988; Efstathiou 1990) recent measurements have shown several deficiencies in the model, at least, when any bias of the distribution of galaxies relative to the mass is constant with scale (see Babul & White 1991; Bower et al. 1993; Del Popolo & Gambera 1998a,b,c). Some of the most difficult problems that must be reconciled with the theory are:

– the magnitude of the dipole of the angular distribution of optically selected galaxies (Lahav et al. 1988; Kaiser & Lahav 1989);
– the possible observations of clusters of galaxies with high velocity dispersion at $z \geq 0.5$ (Evrard 1989);
– the strong clustering of rich clusters of galaxies, $\xi_{cc}(r) \simeq (r/25h^{-1}Mpc)^{-2}$, far in excess of CDM predictions (Bahcall & Soneira 1983);
– the X-ray temperature distribution function of clusters, overproducing the observed clusters abundances (Bartlett & Silk 1993);
– the conflict between the normalization of the spectrum of the perturbation which is required by different types of observations;
– the incorrect scale dependence of the galaxy correlation function, $\xi(r)$, on scales 10 to 100 $h^{-1}Mpc$, having $\xi(r)$ too little power on the large scales compared to the power on smaller scales (Lahav et al. 1989; Maddox et al. 1990a; Saunders et al. 1991; Peacock 1991; Peacock & Nicholson 1991).

These discrepancies between the theoretical predictions of the CDM model and the observations led many authors to conclude that the shape of the CDM spectrum is incorrect and to search alternative models (Peebles 1984; Shafi & Stecker 1984; Valdarnini & Bonometto 1985; Holtzman 1989; Efstathiou et al. 1990a; Turner 1991; Schaefer 1991; Cen et al. 1992; White et al. 1993a,b; Schaefer & Shafi 1993; Holtzman & Primack 1993; Bower et al. 1993).

In this paper we address two of the quoted and most serious problems of the CDM model, namely that of the discrepancy between the predicted and observed X-ray temperature distribution function of clusters and that of the discrepancy between the CDM predicted two-point correlation function of clusters of galaxies and that observed, showing how them may be reduced when non-radial motions, that develop during the collapse process, are taken into account.

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X-ray studies of clusters of galaxies have provided a great number of quantitative data for the study of cosmology. The mass of a rich cluster is approximately $10^{15}h^{-1}M_{\odot}$, where $h = H_0/(100\,kms^{-1}Mpc^{-1})$, being H_0 the Hubble constant at current epoch (in the paper we adopt $h = 1/2$). This mass comes from a region of diameter $\simeq 20h^{-1}Mpc$ and consequently the observations of clusters can provide information on the mass distribution of the Universe on these scales. Furthermore, since rich clusters are rare objects, their properties are expected to be sensitive to the underlying mass density field from which they arose. On scales of $\simeq 20h^{-1}Mpc$ the density field is still described by the linear perturbation theory so the measurement at the present epoch can be related to the initial conditions. While the gravitational mass in bound objects is easily computed using analytic approaches such as the PS formalism or N-body simulations, it is not so easily measured. The most robust measurements of the clusters abundance currently realy on the clusters temperature function, that is the number density of clusters above a certain temperature $N(> kT)$ expressed in units of h^3 Mpc^{-3}. The reasons of this choice are several. The integrated temperature does not suffer from projection effects, and complete samples of clusters may be obtained from all sky X-ray surveys. Clusters are close to isothermal, both observationally and in simulations, which makes their temperature determination robust and insensitive to numerical resolution or telescope angular resolution. The temperature of a cluster depends primarily on the depth of the potential well of the dark matter, and the state of equilibrium of the gas. This is in contrast to the clusters luminosity, which depends strongly on small scale parameters like clumping and core radius.

The clusters abundance is one of the best observables available for determining the density field. In fact, according to the gravitational instability scenario, galaxies and clusters form where the density contrast, δ, is large enough so that the surrounding matter can separate from the general expansion and collapse. Consequently the abundance of collapsed objects depends on the amplitude of the density perturbations. In the CDM model these latter follow a gaussian probability distribution and their amplitude on a scale R is defined by $\sigma(R)$, the r.m.s. value of δ, which is related to the power spectrum, $P(k)$. In hierarchical models of structure formation, like CDM, $\sigma(R)$ decreases with increasing scale, R, and consequently the density contrast required to form large objects, like clusters of galaxies, rarely occurs. The present abundance of clusters is then extremely sensitive to small change in the spectrum, $P(k)$. Moreover the rate of clusters evolution is strictly connected to the density parameter, Ω_0. Then clusters abundance and its evolution are a probe of Ω_0 and $P(k)$ and can be used to put some constraints on them.

Henry & Arnaud (1991) and Edge et al. (1990) have analysed the HEAO-1 A2 all sky X-ray survey to obtain temperatures for clusters at a flux limit of $F_{2\div 10\mathrm{keV}} > 3 \times 10^{-11}$ erg/(cm^2 sec). The sample contains the 25 brightest X-ray clusters and is complete for more than 90 % at galactic latitude $|b| > 20^o$. By virtue of their brightness the temperatures are easily and well determined. They also constructed temperature distributions that has been compared with the CDM predicted X-ray temperature distribution function (Bartlett & Silk 1993). Using this data it was shown that galaxy clusters are too numerous in the CDM scenario (Bartlett & Silk 1993).

The other problem of the CDM model that we are addressing is that of the discrepancy between the two-point correlation function of clusters of galaxies and the observed one.

Measurements of galaxy clustering on large scales (Maddox et al. 1990a; Efstathiou et al. 1990b; Saunders et al. 1991) revealed rms fluctuations of the order of 50% within spheres of radius $20h^{-1}Mpc$. These amplitudes, relative to non-linear clustering on scales $5h^{-1}Mpc$, are $2 \div 3$ times larger than that predicted by the CDM model. At first the suggestion of excess power arose from the estimates of the autocorrelation of Abell clusters (Hauser & Peebles 1973; Bahcall & Soneira 1983; Klypin & Kopylov 1983). As shown by White et al. (1987a), the cluster correlations in *Standard* Cold Dark Matter (SCDM) should have an amplitude $2 \div 3$ times smaller than these estimates. Although the integrity of Abell cluster has been repeatedly called into question (Sutherland 1988; Dekel et al. 1989; Olivier et al. 1990; Sutherland & Efstathiou 1991) and a new sample of rich clusters identified from the APM survey exhibits weaker correlations, marginally consistent with the CDM predictions (Dalton et al. 1992), X-ray clusters samples (Lahav et al. 1989) have a large correlation length and observations of bright radio galaxies (Peacock 1991; Peacock & Nicholson 1991) are also strongly clustered on large scales.

Finally, as shown in some studies of galaxy clustering on large scales (Maddox et al. 1990a; Efstathiou et al. 1990b; Saunders et al. 1991), the measured rms fluctuations within spheres of radius $20h^{-1}Mpc$ have values $2 \div 3$ times larger than that predicted by the CDM model. In order to solve the quoted discrepancies between CDM model previsions and observations, alternative models have been introduced. Several authors (Peebles 1984; Efstathiou et al. 1990a; Turner 1991) have lowered the matter density under the critical value ($\Omega_m < 1$) and have added a cosmological constant in order to retain a flat Universe ($\Omega_m + \Omega_\Lambda = 1$). The spectrum of the matter density is specified by the transfer function, but its shape is affected because the epoch of matter-radiation equality (characterized by the redshift z_{eq}) is earlier, $1 + z_{eq}$ being increased by a factor $1/\Omega_m$. Around the epoch of redshift z_Λ, where $z_\Lambda = (\Omega_\Lambda/\Omega_m)^{1/3} - 1$, the effect of the cosmological constant becomes important and the growth of the density contrast slows down and ceases after z_Λ. As a consequence, the normalisation of the transfer function begins to fall, even if its shape is retained. In particular Bartlett & Silk (1993) showed that a model with $\Omega_0 = 0.2$ and $\Omega_\Lambda = 0.8$ produces a reasonable temperature distribution of clusters, but a higher normalization with $\sigma_8 \simeq 2$, (where σ_8 is the rms value of $\frac{\delta M}{M}$ in a sphere of $8h^{-1}$Mpc), is needed to explain the peculiar velocity field (Efstathiou et al. 1992b).

Mixed dark matter models (MDM) (Shafi & Stecker 1984; Valdarnini & Bonometto 1985; Schaefer et al. 1989; Holtzman 1989; Schaefer 1991; Schaefer & Shafi 1993; Holtzman & Primack 1993) increase the large-scale power because neutrinos free-streaming damps the power on small scales. In particular the model with $\Omega_{HDM} = 0.3$ simulated by Davis et al.

(1992), gives a good reproduction of the clusters abundance with a normalization within the 1 σ errors of the COBE. These last models have some difficulty in reproducing peculiar velocities (Efstathiou et al. 1992b).

Alternatively, changing the primeval spectrum, several problems of CDM are solved (Cen et al. 1992). Finally it is possible to assume that the threshold for galaxy formation is not spatially invariant but weakly modulated ($2\% \div 3\%$ on scales $r > 10h^{-1}Mpc$) by large scale density fluctuations, with the result that the clustering on large-scale is significantly increased (Bower et al. 1993). This model follows the spirit of the well known high-peak model but differs from it because non-local physical processes produce different shapes of the mass and galaxy correlation function.

In any case the solution to the quoted problems till now proposed is related to alternative models with more large-scale power than CDM.

Here, we propose a solution to the problem using the CDM model and taking account of the non-radial motions originating from the gravitational interaction of the quadrupole moment of the protocluster with the tidal field of the matter of the neighboring protostructures.

The plan of this work is the following: in Sect. 2 we introduce the model used to find the effects of non-radial motions on the X-ray temperature distribution function and the two-point correlation function. In Sect. 3 we compare the results of this model with the X-ray temperature distributions given by Henry & Arnaud (1991) and Edge et al. (1990) while in Sect. 4 we compare our model with the two-point correlation function obtained by Sutherland & Efstathiou (1991), from the analysis by Huchra et al. (1990) deep redshift survey, and with the data points for the Automatic Plate Measuring (APM) clusters, computed by Efstathiou et al. (1992a). Sect. 5 is devoted to the conclusions.

2. The X-ray temperature function

The PS theory provides an analytical description of the evolution of structures in a hierarchical Universe. In this model the linear density field, $\rho(\mathbf{x},t)$, is an isotropic random Gaussian field, the non-linear clumps are identified as over-densities (having a density contrast $\delta_c \sim 1.68$ - Gunn & Gott 1972) in the linear density field, while a mass element is incorporated into a non-linear object of mass M when the density field smoothed with a top-hat filter of radius R_f, exceeds a threshold δ_c ($M \propto R_f^3$). The comoving number density of non-linear objects of mass M to $M+dM$ is given simply by:

$$N(M,t)dM = -\rho_b\sqrt{\frac{2}{\pi}}\nu\exp\left(-\nu^2/2\right)\frac{1}{\sigma}\left(\frac{d\sigma}{dM}\right)\frac{dM}{M} \quad (1)$$

where ρ_b is the mean mass density, $\sigma(M)$ is the rms linear mass overdensity evaluated at the epoch when the mass function is desidered and $\nu = \frac{\delta_c}{\sigma(M)}$. The redshift dependence of Eq. (1) can be obtained remembering that

$$\nu = \frac{\delta_c(z)D(0)}{\sigma_o(M)D(z)} \quad (2)$$

being $D(z)$ the growth factor of the density perturbation and $\sigma_o(M)$ the current value of $\sigma(M)$. In Eq. (1) PS introduced arbitrarily a factor of two because $\int_0^\infty dF(M) = 1/2$, so that only half of the mass in the Universe is accounted for. Bond et al. (1991) showed that the "fudge factor" 2 is naturally obtained using the excursion set formalism in the sharp k-space while for general filters (e.g., Gaussian or "top hat") it is not possible to obtain an analogous analytical result. As stressed by Yano et al. (1996), the factor of 2 obtained in the sharp k-space is correct only if the spatial correlation of the density fluctuations is neglected. In spite of the quoted problem, several authors (Efstathiou et al. 1988; Brainerd & Villumsen 1992; Lacey & Cole 1994) showed that PS analytic theory correctly agrees with N-body simulations. In particular Efstathiou et al. (1988), showed that PS theory correctly agrees with the evolution of the distribution of mass among groups and clusters of galaxies (multiplicity function). Brainerd & Villumsen (1992) studied the CDM halo mass function using a hierarchical particle mesh code. From this last work it results that PS formula fits the results of the simulation up to a mass of 10 times the characteristic 1σ fluctuation mass, M_*, being $M_* \simeq 10^{15}b^{-6/(n_l+3)}M_\odot$, where b is the bias parameter and n_l is the local slope of the power spectrum. PS theory has proven particularly useful in analyzing the number counts and redshift distributions for QSOs (Efstathiou & Rees 1988), Lyman α clouds (Bond et al. 1988) and X-ray clusters (Cavaliere & Colafrancesco 1988).

Some difficulties arise when PS theory is compared with observed distributions. To estimate the multiplicity function of real systems one needs to know the temperature-mass (T-M) relation in order to trasform the mass distribution into the temperature distribution. Theoretical uncertainty arises in this transformation because the exact relation between the mass appearing in the PS expression and the temperature of the intra-cluster gas is unknown. Under the standard assumption of the Intra-Cluster (IC) gas in hydrostatic equilibrium with the potential well of a spherically simmetric, virialized cluster, the IC gas temperature-mass relation is easily obtained by applying the virial theorem and for a flat matter-dominated Universe we have that (Evrard 1990, Evrard et al. 1996, Evrard 1997, Bartlett 1997):

$$T = (6.4h^{2/3}keV)\left(\frac{M}{10^{15}M_\odot}\right)^{2/3}(1+z) \quad (3)$$

The assumptions of perfect hydrostatic equilibrium and virialization are in reality not completely satisfied in the case of clusters. Clusters profile may depart from isothermality, with sligth temperature gradients throughout the cluster. The X-ray weighted temperature can be slightly different from the mean mass weighted virial temperature. In any case the scatter in the T-M relation given by Eq. (3) is of the order of $\simeq 10\%$ (Evrard 1991). The mass variance present in Eq. (1) can be obtained once a spectrum, $P(k)$, is fixed:

$$\sigma^2(M) = \frac{1}{2\pi^2}\int_0^\infty dk k^2 P(k) W^2(kR) \quad (4)$$

where $W(kR)$ is a top-hat smoothing function:

$$W(kR)=\frac{3}{(kR)^3}\left(\sin kR-kR\cos kR\right) \tag{5}$$

and the power spectrum $P(k)=Ak^nT^2(k)$ is fixed giving the transfer function $T(k)$:

$$T(k) = [\ln(1+4.164k)]^2\cdot(192.9+1340k+ + 1.599\cdot10^5k^2+1.78\cdot10^5k^3+3.995\cdot10^6k^4)^{-1/2} \tag{6}$$

(Ryden & Gunn 1987; BBKS) and A is the normalizing constant. The normalization of the spectrum may be obtained in several ways. One possibility is to normalize it to COBE scales using the cosmic microwave anisotropy quadrupole $Q_{rms-PS}=17\,\mu K$. This corresponds to $\sigma_8=0.95\pm0.2$ (Smoot et al. 1992; Liddle & Lyth 1993). More recent determinations give $\sigma_8=1.33$ if we use the BBKS spectrum, while $\sigma_8=1.22$ if we use the spectrum by Bond & Efstathiou (1984) (Klypin et al. 1997). Another way of fixing the normalization is via the abundance of clusters giving $\sigma_8=0.5\div0.6$ (Oukbir et al. 1996; Pen 1997; Bartlett 1997). Normalisation on scales from 10 to $50Mpc$ obtained from QDOT (Kaiser et al. 1991) and POTENT (Dekel et al. 1992) requires that σ_8 is in the range $0.7\div1.1$, which is compatible with COBE normalisation while the observations of the pairwise velocity dispersion of galaxies on scales $r\leq3Mpc$ seem to require $\sigma_8<0.5$. Our normalization, $\sigma_8=1$, is intermediate between that suggested by clusters abundance and that of COBE.

As shown by Bartlett & Silk (1993) the X-ray distribution function obtained using a standard CDM spectrum over-produces the clusters abundances data obtained by Henry & Arnaud (1991) and Edge et al. (1990). The discrepancy can be reduced taking into account the non-radial motions that originate when a cluster reaches the non-linear regime. In fact, the PS temperature distribution requires the specification of δ_c and the temperature-mass relation T-M. The presence of non-radial motions changes both δ_c and the T-M relation. Barrow & Silk (1981), Szalay & Silk (1983) and Peebles (1990) assumed that non-radial motions would be expected (within a developing protocluster) due to the tidal interaction of the irregular mass distribution around the protocluster with the neighbouring protoclusters. The kinetic energy of these non-radial motions inhibits the collapse of the protocluster enabling it to reach statistical equilibrium before the final collapse (Davis & Peebles 1977; Peebles 1990). The role of non-radial motions has been also pointed out by Antonuccio & Colafrancesco (1995). After deriving the conditional probability distribution $f_{pk}(\mathbf{v}|\nu)$ of the peculiar velocity around a peak of a Gaussian density field they showed that the regions of the protoclusters at radii $r>R_f$ contain predominantly non-radial motions. In these regions the fate of the infalling material could be influenced by the amount of tangential velocity relative to the radial one.

This can be shown writing the equation of motion of a spherically symmetric mass distribution with density $n(r)$:

$$\frac{\partial}{\partial t}n\langle v_r\rangle + \frac{\partial}{\partial r}n\langle v_r^2\rangle+\left(2\langle v_r^2\rangle-\langle v_\vartheta^2\rangle\right)\frac{n}{r} + n(r)\frac{\partial}{\partial t}\langle v_r\rangle=0 \tag{7}$$

where $\langle v_r\rangle$ and $\langle v_\vartheta\rangle$ are, respectively, the mean radial and tangential streaming velocity. Eq. (7) shows that high tangential velocity dispersion ($\langle v_\vartheta^2\rangle\geq2\langle v_r^2\rangle$) may alter the infall pattern. The expected delay in the collapse of a perturbation may be calculated using a model due to Peebles (Peebles 1993).

Let's consider an ensemble of gravitationally growing mass concentrations and suppose that the material in each system collects within the same potential well with inward pointing acceleration given by $g(r)$ (see Del Popolo & Gambera 1997, 1998a). We indicate with $dP=f(L,rv_r,t)dLdv_rdr$ the probability that a particle can be found in the proper radius range r, $r+dr$, in the radial velocity range $v_r=\dot{r}$, v_r+dv_r and with angular momentum $L=rv_\theta$ in the range dL. The radial acceleration of the particle is:

$$\frac{dv_r}{dt}=\frac{L^2(r,\nu)}{M^2r^3}-g(r) \tag{8}$$

Eq. (8) can be derived from a potential and then from Liouville's theorem it follows that the distribution function, f, satisfies the collisionless Boltzmann equation:

$$\frac{\partial f}{\partial t}+v_r\frac{\partial f}{\partial r}+\frac{\partial f}{\partial v_r}\cdot\left[\frac{L_2}{r^3}-g(r)\right]=0 \tag{9}$$

Using Gunn & Gott's (1972) notation we write the proper radius of a shell in terms of the expansion parameter, $a(r_i,t)$, where r_i is the initial radius:

$$r(r_i,t)=r_ia(r_i,t) \tag{10}$$

Remembering that $M=\frac{4\pi}{3}\rho(r_i,t)a^3(r_i,t)r_i^3$, that $\rho_{ci}=\frac{3H_i^2}{8\pi G}$, with ρ_{ci} and H_i respectively the critical mass density and the Hubble constant at the time t_i, and assuming that no shell crossing occurs so that the total mass inside each shell remains constant, $[\rho(r_i,t)=\frac{\rho_i(r_i,t_i)}{a^3(r_i,t)}]$ Eq. (8) may be written as:

$$\frac{d^2a}{dt^2}=-\frac{H_i^2(1+\overline{\delta})}{2a^2}+\frac{4G^2L^2}{H_i^4(1+\overline{\delta})^2r_i^{10}a^3} \tag{11}$$

where $\overline{\delta}=\frac{\rho_i-\rho_{ci}}{\rho_{ci}}$.

Integrating Eq. (11) we have:

$$\left(\frac{da}{dt}\right)^2 = H_i^2\left[\frac{1+\overline{\delta}}{a}\right] + \int\frac{8G^2L^2}{H_i^4r_i^{10}\left(1+\overline{\delta}\right)^2}\frac{1}{a^3}da-2C \tag{12}$$

where C is the binding energy of the shell. Integrating once more we have:

$$t_{ta}=\int_0^{a_{\max}}\frac{da}{\sqrt{H_i^2\left[\frac{1+\overline{\delta}}{a}-\frac{1+\overline{\delta}}{a_{\max}}\right]+\int_{a_{\max}}^{a}\frac{8G^2L^2}{H_i^4r_i^{10}(1+\overline{\delta})^2}a^3}} \tag{13}$$

Using Eqs. (12) and (13) it is possible to find the linear overdensity at the turn-around epoch, t_{ta}. In fact solving Eq. (13), for some epoch of interest, we may obtain the expansion parameter of the turn-around epoch. This is related to the binding energy of the shell containing mass M by Eq. (12) with $\frac{da}{dt}=0$. In turn the binding energy of a growing mode solution is uniquely given by the linear overdensity, δ_i, at time t_i. From this overdensity, using the linear theory, we may obtain that of the turn-around epoch and then that of the collapse which is given by:

$$\delta_c(\nu)=\delta_{co}\left[1+\frac{8G^2}{\Omega_o^3 H_0^6 r_i^{10}\overline{\delta}(1+\overline{\delta})^2}\int_0^{a_{\max}}\frac{L^2\cdot da}{a^3}\right] \quad (14)$$

where $\delta_{co}=1.68$ is the critical threshold for a spherical model, while H_0 and Ω_0 are respectively the Hubble constant and the density parameter at the current epoch t_0. Filtering the spectrum on clusters scales, $R_f=3h^{-1}Mpc$, we obtained the total specific angular momentum, $h(r,\nu)=L(r,\nu)/M_{sh}$, acquired during expansion, integrating the torque over time (Ryden 1988 – Eq. 36):

$$h(r,\nu) = \frac{\tau_o t_0 \overline{\delta}_o^{-5/2}}{\sqrt[3]{48}M_{sh}}\int_0^{\pi}\frac{(1-\cos\theta)^3}{(\vartheta-\sin\vartheta)^{4/3}} \times\frac{f_2(\vartheta)\cdot d\vartheta}{f_1(\vartheta)-f_2(\vartheta)\frac{\delta_o}{\overline{\delta}_o}} \quad (15)$$

where τ_o δ_o and $\overline{\delta}_o$ are respectively the torque, the mean overdensity and the mean overdensity within a sphere of radius r at the current epoch t_0. The functions $f_1(\vartheta)$, $f_2(\vartheta)$ are given by Ryden (1988 – Eq. 31):

$$f_1(\theta)=16-16\cos\theta+\sin^2\theta-9\theta\sin\theta \quad (16)$$

$$f_2(\theta)=12-12\cos\theta+3\sin^2\theta-9\theta\sin\theta \quad (17)$$

where θ is a a parameter connected to the time, t, through the following equation:

$$t=\frac{3}{4}t_0\overline{\delta}_o^{-3/2}(\theta-\sin\theta) \quad (18)$$

The mean overdensity within a sphere of radius r, $\overline{\delta}(r)$, is given by:

$$\overline{\delta}(r,\nu)=\frac{3}{r^3}\int_0^r x^2\delta(x)dx \quad (19)$$

The mass dependence of the threshold parameter, $\delta_c(\nu)$, can be found as follows: we calculate the binding radius, r_b, of the shell using Hoffmann & Shaham's criterion (1985):

$$T_c(r,\nu)\le t_0 \quad (20)$$

where $T_c(r,\nu)$ is the calculated time of collapse of a shell and t_o is the Hubble time. We find a relation between ν and M through the equation $M=4\pi\rho_b r_b^3/3$. We so obtain $\delta_c(\nu(M))$. In Fig. 1 we show the variation of the threshold parameter, $\delta_c(M)$, with the mass M. Non-radial motions influence the value of δ_c which increases for peaks of low mass and remaines unchanged for high mass peaks. As a consequence, the structure formation by low mass peaks is inhibited. In other words, in agreement with the cooperative galaxy formation theory (Bower et al. 1993), structures form more easily in over-populated regions. As we previously told, the cooperative galaxy formation is able to reconcile the CDM model with the APM correlations by assuming the threshold for galaxy formation to be modulated by large-scale density fluctuations rather than to be spatially invariant. But there exists some difficulty in finding a physical mechanism able to produce the modulation. In our model this mechanism is linked to non-radial motions.

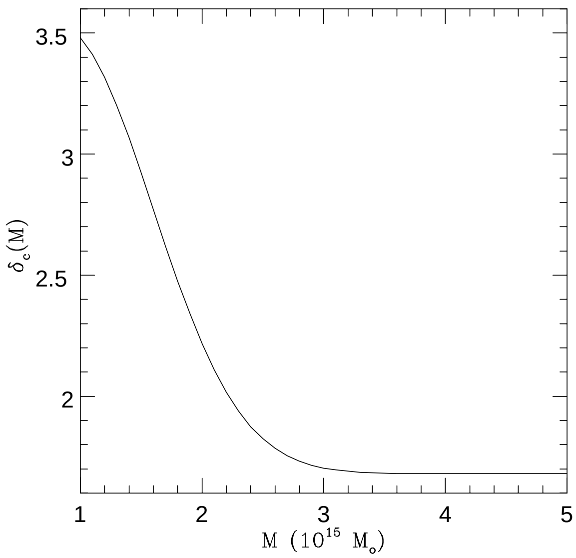

Fig. 1. The threshold δ_c as a function of the mass M, for a CDM spectrum ($\Omega_0=1$, $h=1/2$) with $R_f=3h^{-1}Mpc$, taking account of non-radial motions.

To get the temperature distribution it is necessary to know the temperature-mass relation. This can be obtained using the virial theorem, energy conservation and using Eq. (12). From the virial theorem we may write:

$$\langle K\rangle=\frac{GM}{2r_{eff}}+\int_0^{r_{eff}}\frac{L^2}{2M^2r^3}dr \quad (21)$$

while from the energy conservation:

$$-\langle K\rangle+\frac{GM}{r_{eff}}+\int_0^{r_{eff}}\frac{L^2}{M^2r^3}dr=\frac{GM}{r_{ta}}+\int_0^{r_{ta}}\frac{L^2}{M^2r^3}dr \quad (22)$$

Eq. (21) and Eq. (22) can be solved for r_{eff} and $\langle K\rangle$. We finally have that:

$$T=(6.4\,keV)\left(\frac{M\cdot h}{10^{15}M_\odot}\right)^{2/3}\left[1+\frac{\eta\psi\int_0^r\frac{L^2dr}{M^2r^3}}{(G^2\frac{H_0^2\Omega_0}{2}M^2)^{1/3}}\right] \quad (23)$$

where η is a parameter given by $\eta=r_{ta}/x_1$, being r_{ta} the radius of the turn-around epoch, while x_1 is defined by the relation $M=4\pi\rho_b x_1^3/3$ and $\psi=r_{eff}/r_{ta}$ where r_{eff} is the time-averaged radius of a mass shell. Eq. (23) was normalised to agree with Evrard's (1990) simulations for $L=0$.

3. Non-radial motions and the X-ray temperature function

The new T-M relation is Eq. (23) which differs from Eq. (3) for the presence of the term:

$$\frac{\eta\psi\int_0^r \frac{L^2 dr}{M^2 r^3}}{(G^2 \frac{H_0^2\Omega_0}{2} M^2)^{1/3}} \tag{24}$$

This last term changes the dependence of the temperature on the mass, M, in the T-M relation. Moreover the new T-M relation depends on the angular momentum, L, originating from the gravitational interaction of the quadrupole moment of the protocluster with the tidal field of the matter of the neighboring protostructures. In Fig. 2 the X-ray temperature distribution, derived using a CDM model with $\Omega_0 = 1$, $h = 1/2$ and taking into account non-radial motions, is compared with Henry & Arnaud (1991) and Edge et al. (1990) data and with a pure CDM model with $\Omega_0 = 1$, $h = 1/2$. As shown the CDM model that does not take account of the non-radial motions over-produces the clusters abundance. The introduction of non-radial motions gives a more careful description of the experimental data. As we have seen the X-ray temperature distribution function obtained taking account of non-radial motions is different from that of a pure CDM model for two reasons:

1. the variation of the threshold, δ_c, with mass, M. This is due to the change in the energetics of the collapse model produced by the introduction of another potential energy term ($\frac{L(r,\nu)^2}{M^2 r^3}$) in the equation decsribing the collapse [see Eq. (8)];
2. the modification of the T-M relation produced by the alteration of the partition of energy in virial equilibrium.

For values of mass $M = 0.5 \cdot 10^{15} M_\odot h^{-1}$ the difference between the two theoretical lines in Fig. 2 is due to the first factor for $\simeq 59\%$ and this value increases with increasing mass. The uncertainty in our model fundamentally comes from the uncertainty of the T-M relation whose value has been previously quoted.

Somebody may object that the effect here described has not been seen in some hydrodynamic simulation (Evrard & Crone 1992). The answer to this objection can be given remembering a similar problem of the previrialization conjecture (Davis & Peebles 1977; Peebles 1990), (supposing that initial asphericities and tidal interactions between neighboring density fluctuations induce significant non-radial motions, which oppose the collapse) on which our model is fundamentally based (Del Popolo & Gambera 1998a). It is known that while some N-body simulations (Villumsen & Davis 1986; Peebles 1990) appear to reproduce this effect, other simulations (for example Evrard & Crone 1992) do not. An answer to this controversy was given by Lokas et al. (1996). The problem is connected to the spectral index n used in the simulations. The "previrialization" is seen only for $n > -1$. While Peebles (1990) used simulations with $n = 0$, Evrard & Crone (1992) assumed $n = -1$. Excluding this particular case, generally the whole properties of clusters, such as their optical and X-ray luminosity functions, or their velocity and temperature distribution functions, are difficult to address directly in numerical simulations because the size of the box must be very large in order to contain a sufficient number of clusters. Then the analytical approach remains an effective alternative.

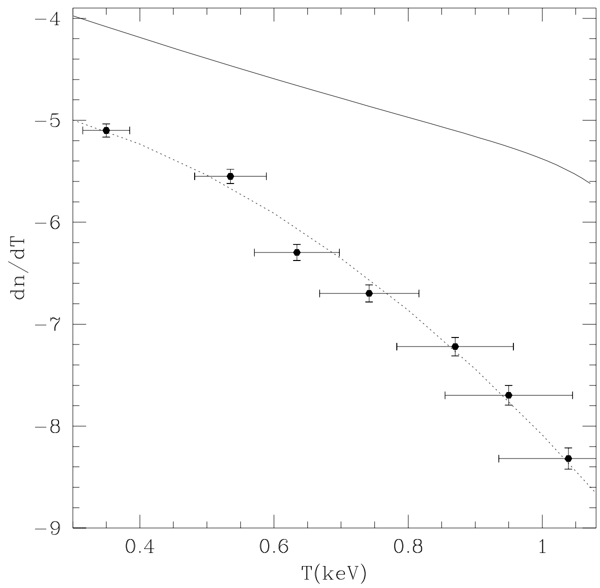

Fig. 2. X-ray temperature distribution function. The dotted line gives the temperature function for a pure CDM model ($\Omega_0 = 1$, $h = 1/2$), with $R_f = 3h^{-1}Mpc$. The solid line is the same distribution but now taking account of non-radial motions. The data are obtained by Edge et al. (1990) (dots), and Henry & Arnaud (1991) (*filled hexagons*).

4. Non-radial motions and the clusters correlation function

As previously seen in Sect. 2, in the PS theory the comoving number density of non-linear objects of mass M to $M + dM$ is simply obtained by differentiating with respect to mass the integral from δ_c to infinity of the probability distribution for fluctuations given by:

$$p[\delta(M)] = \frac{1}{\sqrt{2\pi}\sigma(M)} \exp[-\delta(M)^2/2\sigma(M)^2] \tag{25}$$

In an exactly analogous way, the probability $p(M_1, M_2, r)$ per unit volume per unit masses of finding two collapsed objects of mass M_1 and M_2 separated by a distance r is obtained by integrating both variables of the bivariant Gaussian distribution in $\delta(M_1)$ and $\delta(M_2)$, with correlation $\xi_\rho(r)$ from δ_c to infinity and then taking the partial derivatives with respect to both masses. The correlation function for collapsed objects is simply obtained from:

$$\xi_{MM} = p(M_1, M_2, r)/p(M_1)p(M_2) - 1 \tag{26}$$

which for equal masses and on scales bigger than that of the turn-around ($\xi_\rho \ll 1$) is (Kashlinsky 1987):

$$\xi_{MM} = [\delta_c^2/\sigma(M)^2]\xi_\rho(r) \tag{27}$$

where ξ_ρ is the correlation function of the matter density distribution when the density fluctuations had small amplitude.

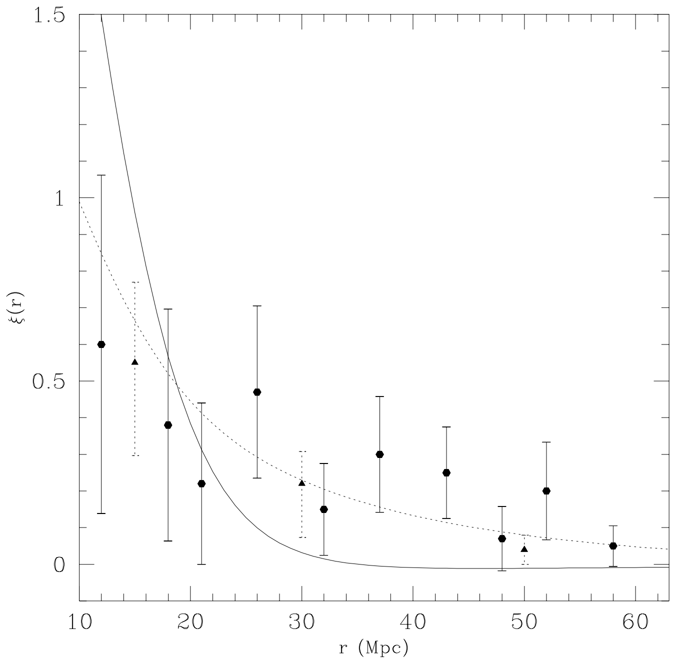

Fig. 3. Clusters of galaxies correlation function. The solid line gives the correlation function for a pure CDM model, with $R_f = 3h^{-1}Mpc$. The dotted line is the same distribution but now takes account of non-radial motions. The observational data refer to the two-point correlation function obtained by Sutherland & Efstathiou (1991) (*filled hexagons - solid errorbars*) from the analysis by Huchra et al. (1990) deep redshift survey and with the data points for the APM clusters computed by Efstathiou et al. (1992a) (*filled triangles – dashed errorbars*).

Eq. (27) shows that the correlation of collapsed objects may be enhanced with respect to the underlying mass fluctuations. This condition is usually described by the bias parameter b which is sometimes defined as $[\xi_{MM}(r)/\xi_{\rho}(r)]^{1/2} = \delta_c/\sigma(M)$.

Studies of clustering on scales $\geq 10h^{-1}Mpc$ have shown that the correlation function given in Eq. (27) is different from that obtained from observations. The most compelling data are angular correlation functions for the APM survey. These decline much less rapidly on large scales than the CDM prediction (Maddox et al. 1990a). This discrepancy can be reduced taking into account the non-radial motions that originate when a cluster reaches the non-linear regime. In fact, the calculation of the correlation function requires the specification of δ_c which is modified by non-radial motions.

The result of our calculation is showed in Fig. 3.

The observational data to which we compare the calculated correlation function are those obtained by Sutherland & Efstathiou (1991) from the analysis by Huchra et al. (1990) deep redshift survey as discussed in Geller & Huchra (1988) and the data points for the APM clusters computed by Efstathiou et al. (1992a). The deep-cluster redshift survey of Huchra et al. (1990) consists of the 145 Abell clusters with $R \geq 0$, $D \leq 6$ in the area $10^h \leq \alpha \leq 15^h$, $58^0 \leq \delta \leq 78^0$. The APM sample consists of 240 clusters with APM richness $R \geq 20$ and photometrically derived redshifts $z_\chi \leq 0.1$. The survey covers a solid angle $\omega = 1.3\,sr$ in the region of sky $21^h \leq \alpha \leq 5^h$, /, $-72.5^0 \leq \delta \leq -17.5^0$ of the APM galaxy survey. Both in Sutherland & Efstathiou (1991) and Efstathiou et al. (1992a) the correlation function was calculated from the samples using the estimator:

$$\xi(r) = F\frac{N_{cc}}{N_{cr}} - 1 \tag{28}$$

where N_{cc} and N_{cr} are the numbers of cluster pairs and cluster-random pairs having redshift-space separations in the range $(r - dr/2, r + dr/2)$. The random points are generated within the sample volume with a mean density F times that of the clusters and with a redshift distribution derived from a smoothed distribution of the redshifts for the cluster sample. The correlation function obtained was finally corrected for line-of-sight anisotropies. This correction is necessary because the clustering of Abell clusters is highly anisotropic in redshift space, providing evidence that the Abell catalogues, which were built by scanning photographic plates by eye, are affected by incompleteness on the plane of the sky which enhances the clustering amplitude measured in three dimensions (Sutherland & Efstathiou 1991; Efstathiou et al. 1992a). In particular Efstathiou et al. (1992a) presented a clear example of incompleteness in the Abell $R \geq 0$ catalogue comparing the machine-based APM survey with the Postman et al. (1992) redshift survey of Abell clusters showing that even if the clusters in these surveys have comparable space densities the redshift space correlation function of the APM sample is isotropic on large scales while the correlation function for the Abell clusters is highly anisotropic. In any case, after this correction the correlation function of Abell clusters agrees extremely well with that of APM clusters (Efstathiou et al. 1992a). We use two samples because:

a) we wanted to compare our model for the two-point correlation function with observational data and at the same time with the result obtained by Borgani (1990) who studied the effect of particular thresholds (erfc-threshold and Gaussian-threshold) on the correlation properties of clusters of galaxies and compared his result with that by Sutherland & Efstathiou (1991). For this reason we used this last sample;
b) we used the APM galaxy survey because the uniformity of the APM magnitudes, the low obscuration in the APM survey area and the use, for its construction, of a computer cluster-finding algorithm (see Maddox et al. 1990a,b) further reduce the possibility of spurious clustering on the plane of the sky.

The comparison between the correlation function and the quoted data, displayed in Fig. 3, shows that there is an evident discrepancy between pure CDM previsions and experimental data. The CDM model seems to have trouble in re-producing the behaviour of the data. In fact, the predicted two-point cluster function is too steep and rapidly goes nearly to zero for $r \simeq 30h^{-1}Mpc$, while the data show no significant anticorrelation up to $r \simeq 60h^{-1}Mpc$ (see Borgani 1990). This is a direct effect due to the non-scale invariance of the CDM spectrum (Primack & Blumenthal 1983). The introduction of non-radial motions gives a more accurate description of the experimental data showing that physical effects cannot be ignored in the study of the formation of cosmic structures. The result obtained is in agreement with that obtained by Borgani (1990) who showed how the introduction of smooth thresholds (similar to that obtained in our model) leads to two-point correlation functions in a systematically agreement with the data. Before going on we want to remember that the threshold functions are strictly connected to the concept of bias. In fact, according to the biased theory of galaxy formation, observable objects of mass $\simeq M$ arise from fluctuations of the density field, filtered on a scale R, rising over a *global* threshold, $\delta > \delta_c = \nu_t \sigma$, where σ is the rms value of δ and ν_t is the threshold height. The number density of objects, n_{pk}, that forms from peaks of density of height ν can be written following BBKS in the form:

$$n_{pk} = \int_0^\infty t(\frac{\nu}{\nu_t}) N_{pk}(\nu) d\nu \tag{29}$$

where $t(\frac{\nu}{\nu_t})$ is the threshold function, ν_t the threshold height and $N_{pk} d\nu$ the differential number density of peaks (see BBKS – Eq. 4.3). The threshold level ν_t is defined so that the probability of a peak becoming an observable object is $1/2$ when $\nu = \nu_t$. In the sharp threshold case the selection function, is a Heaviside function $t(\frac{\nu}{\nu_t}) = \theta(\nu - \nu_t)$. As previously quoted the threshold function is connected to the bias coefficient of a class of objects by (BBKS):

$$b(R_f) = \frac{\langle \tilde{\nu} \rangle}{\sigma_o} + 1 \tag{30}$$

where $\langle \tilde{\nu} \rangle$ is:

$$\langle \tilde{\nu} \rangle = \int_0^\infty \left[\nu - \frac{\gamma \theta}{1 - \gamma^2} \right] t(\frac{\nu}{\nu_t}) N_{pk}(\nu) d\nu \tag{31}$$

while, γ and ϑ are given in BBKS (respectively Eq. 4.6a; Eq. 6.14).

While in a θ threshold scheme, fluctuations below δ_c have zero probability to develop an observable object and fluctuations above δ_c have zero probability not to develop an object, the situation is totally different when an erfc-threshold, as that introduced by Borgani (1990), is used. In this case objects can also be formed from fluctuations below δ_c and there is a non-zero probability for fluctuations above δ_c to be sterile.

According with Borgani (1990), the erfc-threshold can be related to non-sphericity effects during the gravitational growing process. In fact, while in the spherical limit it is possible to find a precise relation between the time, t elapsed from the turning around and the correspondent density contrast, δ,($t \propto \delta^{-3/2}$) (Gunn & Gott 1972), when non-sphericity is introduced it is no longer possible to univocally relate the primeval density contrast and the evolutionary stage of a fluctuation because while in the spherical model an object is characterized by $\delta > \delta_c$ the non-sphericity produces a spread around the typical value δ_c. The erfc-threshold is a way to correct the quoted $t - \delta$ relation. The erfc-threshold is also linked to non-radial motions. In fact as we previously told, the tidal interaction of the irregular mass distribution within and around the protocluster, present in hierarchical models, gives rise to non-radial motions. We also told that the erfc-threshold is related to non-sphericity effects that in turn are responsible for the origin of the quoted non-radial motions.

Borgani's (1990) model, like our, is characterized by a non-θ threshold. As we showed in a previous paper, Del Popolo & Gambera 1998a, one of the effects of non-radial motions is that the threshold function differs from a Heaviside function (sharp threshold), (see Fig. 7 by Del Popolo & Gambera 1998a). In this last paper the threshold function is defined as:

$$t(\nu) = \int_{\delta_c}^\infty p\left[\overline{\delta}, \langle \overline{\delta}(r_{Mt}, \nu) \rangle, \sigma_{\overline{\delta}}(r_{Mt}, \nu)\right] d\delta \tag{32}$$

where the function

$$p\left[\overline{\delta}, \langle \overline{\delta}(r) \rangle\right] = \frac{1}{\sqrt{2\pi}\sigma_{\overline{\delta}}} \exp\left(-\frac{|\overline{\delta} - \langle \overline{\delta}(r) \rangle|^2}{2\sigma^2_{\overline{\delta}}} \right) \tag{33}$$

gives the probability that the peak overdensity is different from the average, in a Gaussian density field. As displayed, the integrand is evaluated at a radius r_{Mt} which is the typical radius of the object that we are selecting. Moreover, the threshold function $t(\nu)$ depends on the critical overdensity threshold for the collapse, δ_c, which is not constant as in a spherical model [due to the presence, in our analysis, of non-radial motions that delay the collapse of the proto-cluster - see Eq. (14)].

The fundamental difference between our and Borgani's approach is that our threshold function is physically motivated: it is simply obtained from the assumptions of a Gaussian density field and taking account of non-radial motions. Borgani's threshold functions (erfc and Gaussian threshold) are ad-hoc introduced in order to reduce the discrepancy between the observed and the CDM predicted two-point correlation functions of clusters of galaxies. The connection with the quoted non-sphericity effects, even if logical and in agreement with our results, is only a posteriori tentative to justify the choice made.

5. Conclusions

In these last years many authors (Bahcall & Soneira 1983; White et al. 1987a,b; Maddox et al. 1990a; Saunders et al. 1991; Peacock 1991; Bartlett & Silk 1993) have shown the existence of a strong discrepancy between the observed X-ray temperature distribution function of clusters and that predicted by a CDM model and the observed two-point correlation function of clusters and that predicted by the CDM model. To reduce these discrepancies several alternative models have been introduced but no model has considered the role of the non-radial motions. Here we have shown how non-radial motions may reduce both the two quoted discrepancies. To this aim we calculated the variation in the treshold parameter, δ_c, as a function of the mass M, and that of the temperature-mass relation, produced by the presence of non-radial motions in the outskirts of clusters of galaxies. We compared Edge et al. (1990) and Henry & Arnaud (1991) data with the distribution function of X-ray temperature, calculated using PS theory and Evrard's (1990) prescriptions for the mass-temperature relation. We found that the model produces a reasonable clusters temperature distribution (see Fig. 2). We also used $\delta_c(M)$ to calculate the two-point correlation of clusters of galaxies and we compared it both with that obtained by Sutherland & Efstathiou (1991) from the analysis by Huchra et al. (1990) deep redshift survey as discussed in Geller & Huchra (1988) and with the data points for the APM clusters computed by Efstathiou et al. (1992a). Our results (see Fig. 3) show how non-radial motions change the correlation length of the correlation function making it less steep than that obtained from a pure CDM model where the non-radial motions are not considered.

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