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Star formation and evolution in accretion disks around massive black holes

Star formation and evolution in accretion disks

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Abstract. We develop an exploratory model for the outer, gravitationally unstable regions of accretion disks around massive black holes. We consider black holes of mass 10^6 to 10^{10} $M_\odot$, and primeval or solar abundances. In a first step we study star formation and evolution in a purely gaseous marginally unstable disk, and we show that unstable fragments should collapse rapidly and give rise to compact objects (planets or protostars), which then accrete at a high rate and in less than 10^6 years acquire a mass of a few tens of $M_\odot$, according to a mechanism first proposed by Artymowicz et al. (1993). When these stars explode as supernovae, the supernova shells break out of the disk, producing strong outflows. We show that the gaseous disk is able to support a large number of massive stars and supernovae while staying relatively homogeneous. An interesting aspect is that the residual neutron stars can undergo other accretion phases, leading to other (presumably powerful) supernova explosions. In a second step we assume that the regions at the periphery of the disk provide a quasi stationary mass inflow during the lifetime of quasars or of their progenitors, i.e. $\sim 10^8$ yrs, and that the whole mass transport is ensured by the supernovae, which induce a transfer of angular momentum towards the exterior, as shown by the numerical simulations of Rozyczka et al. (1995). Assuming that the star formation rate is proportional to the growth rate of the gravitational instability, we solve the disk structure and determine the gas and the stellar densities, the heating being provided mainly by the stars themselves. We find self-consistent solutions in which the gas is maintained in a state very close to gravitational instability, in a ring located between 0.1 and 10 pc for a black hole mass of $10^6 M_\odot$, and between 1 and 100 pc for a black hole mass of $10^8 M_\odot$ or larger, whatever the abundances, and for relatively low accretion rates ($\leq 10\%$ of the critical accretion rate). For larger accretion rates the number of stars becomes so large that they inhibit any further star formation, and/or the rate of supernovae is so high that they distroy the homogeneity and the marginal stability of the disk. We postpone the study of this case.

Several consequences of this model can be envisioned, besides the fact that it proposes a solution to the problem of the mass transport in the intermediate region of the disk where global instabilities do not work. As a first consequence, it could explain the high velocity metal enriched outflows implied by the presence of the broad absorption lines in quasars. As a second consequence it could account for a pregalactic enrichment of the intergalactic medium, if black holes formed early in the Universe. Finally it could provide a triggering mechanism for starbursts in the central regions of galaxies. A check of the model would be to detect a supernova exploding within a few parsecs from the center of an AGN, an observation which can be performed in the near future.

Key words: accretion, accretion disks – black hole physics – stars: supernovae: general – galaxies: active – galaxies: nuclei – galaxies: quasars: general

1. Introduction

It is now a paradigm to state that quasars and Active Galactic Nuclei (AGN) harbour massive black holes in their centers (Rees 1984). Massive black holes exist also at the center of quiescent galaxies like ours. In quasars and AGN the accretion rate amounts to a fraction of the critical one. This rate of accretion of this black hole must be sustained during at least 10^8 yrs to account for the total mass of the black holes locked in quasars and for the fraction of AGN among all galaxies (cf. for instance Cavaliere & Padovani 1988 and 1989). Since the gas fueling the black hole at such a high rate cannot be produced inside the central parsec (except if there is a preexisting dense star cluster, through star collisions, tidal disruptions, or ablation) a process for supplying gas from larger distances is necessary, whatever it is. This will be a prerequisite of our study.

There is a large concensus that AGN are fueled via accretion disks. Moreover the observation of the "UV bump" (Shields 1978, Malkan & Sargent 1982, and many subsequent papers) argues in favor of geometrically thin and optically thick disks, possibly embedded in a hot X-ray emitting corona. Generally these disks are studied using the α prescription for viscosity introduced by Shakura and Sunayev (1973). It is well known

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that these "α-disks" have two serious problems at large radii: they are not able to transport rapidly enough the gas from regions located at say one parsec, and they are gravitationally unstable beyond about 0.1 parsec. They should therefore give rise to star formation, and as a consequence evolve rapidly towards stellar systems whose properties are quite different from gaseous accretion disks.

The suggestion of star formation in the self-gravitating region of such an accretion disk has been first made by Kolykhalov & Sunayev (1980). Begelman et al. (1989) and Shlosman & Begelman (1989) discussed in more details the conditions for star formation at about 1–10 pc from the black hole, and its consequences on the disk. They concluded that unless the disk can be maintained in a hot or highly turbulent state, it should transform rapidly into a flat stellar system which will be unable to build a new gaseous accretion disk and to fuel a quasar (note that they apparently did not take into account the influence of self-gravity on the scale height, which would even strengthen this conclusion). It is why they finally adopt the picture of a disk made of marginally unstable randomly moving clouds, where the "viscosity" of the disk is provided by cloud collisions. Here we adopt the opposite view that if a marginally unstable fragments begin to collapse owing to a local increase of density for instance, the collapse will continue until a protostar is formed, unless the collapse time is larger than the characteristic time for mass transport in the disk.

Farther from the center the supply of gas can be achieved by gravitational torques or by global non axisymmetric gravitational instabilities. However this does not solve the problem of the mass transport in the intermediate region where the disk is locally but not globally self-gravitating, and one cannot avoid appealing for a mechanism to transport angular momentum. This can be achieved by magnetic torques if large scale magnetic fields are anchored in the disk. We shall assume here that neither turbulent nor large scale magnetic fields are important in accretion disks, and examine whether it is possible to find a solution to the problem of the existence of a quasi continuous flow with a high accretion rate in the simple framework of star formation and its feedback to the disk.

As stated above, the outer regions of α-disks are strongly gravitationally unstable (Huré 1998). This is why in Collin & Huré (1998, hereafter referred as Paper 1) another prescription was adopted, where the self-gravitating disk is maintained in a state of marginal instability, as initially proposed by Paczyński (1978). In such a disk it is assumed that the transfer of angular momentum is not due to turbulent viscosity, but for instance to collisions between clumps and/or to gravitational dissipation, which prevent their collapse. The properties of this disk were determined (its scale height H, its midplane density and temperature, ρ and T, as functions of the radius R) in the framework of a vertically averaged stationary model. We want here to add a few comments.

First it is not clear whether a marginally unstable disk can stay homogeneous, if the marginal state is maintained by a self-regulating mechanism requiring energy liberated by the interaction of turbulent clumps or by their gravitational collapse. In this case the disk would probably be made of discrete interacting clouds. In order that the disk stays homogeneous, the mechanism providing the heating and the angular momentum transport should not perturb too much the gaseous component. An important part of this paper is devoted to verify that this condition is fulfilled in our model.

Second we stress that the assumption of stationarity is not constraining, as the properties of marginally unstable disks depend very little on the accretion rate (in particular the density profile depends only on the central mass), and moreover the model is a local one (i.e. the different rings are independent). Variations of the accretion rate with radius, due for instance to infall of gas, should simply induce small variations of the surface density and of the midplane temperature. The only assumption linked to stationarity in Paper 1 was to take the luminosity of the central source as given by the accretion rate in the gravitationally unstable region. But it was shown that this external radiation has not a strong influence on the radial structure.

It is also worth noting that in Paper 1 the disk was actually not supposed to be exactly marginally unstable, since all results were parametrized with ζ, the ratio of the disk self-gravity to the vertical component of the central attraction. The results are valid provided that this quantity is not much larger than unity (for marginal stability, $\zeta = 4.83$, cf. later). Again we stress that it is a local quantity, which can vary from place to place in the disk.

In Paper 1 the heating of the disk was assumed due to dissipation of the gravitational energy of the accreting flow (for which we use the word "viscous heating", although it might not be due to viscosity) and to the external radiative heating. For a disk made of stars and gas, other sources of heating can play a role, and one should take them into account. Note finally that the mechanism for momentum and mass transport was not specified in Paper 1.

Here our approach will be as follows. First we assume the existence of a marginally unstable gaseous disk, and discuss the formation of stars, their evolution, and the feedback of the stars. We show that the influence of the stars stays local, so they do not destroy the disk. The advantages of studying a pure gaseous disk is that the discussion can be carried out analytically, and gives a framework for studying the more complex case of a disk made of stars and gas. Second we build a self-consistent model of a disk made of stars and gas, where we assume that stars themselves provide the transport of angular momentum sufficient to maintain the accretion rate. The gas density in this model is constrained by the fact that stars can form only if the disk is not depleted in gas, i.e. if the gas is at least marginally unstable, and if the gas density is not too high, leading to a rapid destruction of the gas by star formation. It implies that the gaseous fraction of the disk is marginally unstable. On the other hand the number of stars is also regulated by the fact that too many stars would inhibit star formation by tidal effect, and would destroy the disk through supernova explosions.

In Sect. 2 we study the birth of stars inside the accretion disk, assuming that either it consists of primeval gas, or it has a solar abundance. We discuss the evolution and the ultimate fate of these stars in Sect. 3. Sect. 4 is devoted to a discussion of the feedback of the stars, and Sect. 5 to the overall behaviour of the "star-gas" disk. In Sect. 6 we envision different consequences of the model.

2. Star formation in a marginally unstable disk

One finding of Paper 1 was that a marginally unstable disk is molecular for $R \geq 0.1$ pc even if it is illuminated by a central UV-X source of radiation. Another conclusion was that the structure of disks consisting of primeval gas is different from that of disks having solar abundances. Primeval disks are optically thin (in the sense of Rosseland or Planck mean), and they are flaring. As a result they are irradiated by the UV-X photons produced by the inner regions of the disk, and the external radiative flux exceeds the gravitational flux beyond a critical radius of the order of $10^4 R_{\rm S}$ (Eq. 42 of Paper 1), where $R_{\rm S}$ is the Schwarschild radius $2GM/c^2$. In the following we shall therefore use in the zero metallicity case the self-consistent solutions found for the flaring irradiated disk (Eqs. 5, 43, 44 and 45 of Paper 1). In the solar metallicity case, the disk is optically thick and is not flaring; therefore there will be no external irradiation unless the disk is warped or a fraction of the central radiation is backscattered. Since the efficiency of backscattering and the variation of the warping angle with the radius would have to be chosen arbitrary, we prefer to use the solutions for a purely viscously heated disk (Eqs. 5, 6, 7, and 8 of Paper 1). This is justified by the fact that generally the external irradiation flux exceeds the gravitational flux only at large radii, of the order of a few tenths of a parsec, owing to the large optical thickness of the disk. We will distinguish these two cases at each step of the discussion, and to simplify, we call them the "primeval" and the "solar" case.

In Paper 1 it was also shown that simple analytical expressions can be used to model the vertically averaged structure. We shall refer to these expressions. The radius was expressed in $10^4 R_{\rm S}$, as it is a natural physical parameter for black holes, and corresponds roughly to the onset of gravitational instability. In the present paper, we prefer to express it in parsec, $R_{\rm pc}$. The black hole mass (called M_6) will be expressed in $10^6 {\rm M}_\odot$, which we take as a typical mass for small or growing black holes, and we shall often refer to the two cases $M_6 = 1$ and M_6=100 (a typical quasar or AGN black hole mass).

We recall that the expressions of Paper 1 are not valid beyond a few tenths of a parsec in the solar case for M_6 = 1, and beyond a few parsecs for M_6=100, as the temperature falls to very low values. On the contrary the expressions for the primeval disk are always valid up to a few parsecs. As we shall see in Sect. 5, these limitations do not hold for the gas-star disk, owing to the important heating induced by the stars.

Since the disk is marginally unstable, gravitationally bound fragments can form with a mass $M_{\rm frag}$ of the order of $4\rho H^3$ (Goldreich & Linden-Bell 1965), where ρ is the midplane density and H the height of the disk (equal to half its thickness). The conditions necessary for the collapse of a fragment are that the time scale for star formation, $t_{\rm form}$, and the cooling time, $t_{\rm cool}$, be smaller than the characteristic mass transport time in the disk, $t_{\rm trans}$ (in an α-disk it is the viscous time). These times are approximated in replacing derivatives by ratios of finite quantities.

We recall first that we are dealing with a non magnetized or a weakly magnetized disk, and in particular we assume that the magnetic pressure is smaller than the thermal pressure. Thus even if the time for ambipolar diffusion is large because the disk is not neutral but weakly ionized, and the magnetic field is therefore tightly bound to the gas, it is not able to support the fragment against collapse, contrary to what is generally assumed in molecular clouds. So $t_{\rm form}$ corresponds to the maximum growth rate of the gravitation instability, which is a function of the Toomre parameter Q (Toomre 1964, Goldreich & Linden-Bell 1965) defined as:

$$Q = \frac{\Omega c_{\rm s}}{\pi G \Sigma}, \tag{1}$$

where Ω is the keplerian angular velocity, Σ is the surface density, and $c_{\rm s}$ is the sound speed. Adopting the same formulation as in Paper 1:

$$\zeta = \frac{4\pi G\rho}{\Omega^2}, \tag{2}$$

Q can be written:

$$Q = \frac{2\sqrt{\zeta+1}}{\zeta}. \tag{3}$$

According to Wang & Silk (1994) $t_{\rm form}$ is:

$$t_{\rm form} = \Omega^{-1}\frac{Q}{\sqrt{1-Q^2}}. \tag{4}$$

$Q = 1$ corresponds to marginal stability. Provided Q is not too close to unity, $t_{\rm form}$ is not much larger than the freefall time $t_{\rm ff} \sim 1/\Omega$:

$$t_{\rm ff} \sim 4.5\times 10^{11} M_6^{-1/2} R_{\rm pc}^{3/2}\ {\rm s}. \tag{5}$$

For $t_{\rm cool}$, one has to distinguish between the optically thick solar case and the optically thin irradiation dominated primeval case.

In the primeval case it is simply:

$$t_{\rm cool} \sim \frac{kT}{\Lambda(T)}, \tag{6}$$

where $\Lambda(T)$ is the molecular hydrogen cooling function (Eq. 13 of Paper 1). It writes:

$$t_{\rm cool} = 1.8\times 10^7 \zeta^{0.7} (f_{-1} f_{\rm E})^{-0.7}\ {\rm s}, \tag{7}$$

where f_{-1} is a correction factor of the order of unity (see Paper 1), accounting for the flaring ($\sim 30\%$), for the proportion of the bolometric luminosity used to heat the disk (another factor $\sim 20\%$), and for a possible limb darkening effect of the UV-X source. $f_{\rm E}$ is the Eddington ratio (the bolometric to Eddington luminosity ratio: like in Paper 1 we have deliberately made the assumption that the Eddington ratio is equal to the ratio of the accretion rate to the critical accretion rate, for a mass energy conversion efficiency of 0.1; it means that we do not consider the possibility of a very low efficiency like in advection dominated disks, as these models do not seem to apply to high accretion rates).

For the solar case (i.e. gravitationally heated disks), $t_{\rm cool}$ is equal to:

$$t_{\rm cool} \sim \frac{8\pi R^3\ H}{3GM\dot{M}}\ \frac{\rho k T}{\mu m_{\rm H}} \tag{8}$$

which writes:

$$t_{\rm cool} = 1.8\times10^7 \frac{\zeta^{10/7}}{(1+\zeta)^{5/7}} f_{\rm E}^{-4/7} M_6^{-3/7} R_{\rm pc}^{-3/7} \kappa_{\rm R}^{3/7}\ {\rm s} \tag{9}$$

where $\kappa_{\rm R}$ is the mean Rosseland opacity.

The mass transport time is:

$$t_{\rm trans} \sim \frac{2\pi R^2 \rho H}{\dot{M}}. \tag{10}$$

In the primeval case it becomes:

$$t_{\rm trans} \sim 3\times10^{13} \frac{\zeta^{0.85}}{(1+\zeta)^{1/2}} f_{-1}^{0.15} f_{\rm E}^{-0.85} M_6^{-1/2} R_{\rm pc}^{1/2}\ {\rm s}, \tag{11}$$

and in the solar case:

$$t_{\rm trans} \sim 1.7\times10^{12} \frac{\zeta^{8/7}}{(1+\zeta)^{4/7}} f_{\rm E}^{-6/7} M_6^{-1/7} R_{\rm pc}^{-1/7} \kappa_{\rm R}^{1/7}\ {\rm s}. \tag{12}$$

To make the discussion easier, these times are displayed on Figs. 1, for $M = 10^6 {\rm M}_\odot$ and $M = 10^8 {\rm M}_\odot$, and for a zero and a solar metallicity. The factors f_{-1}, $\kappa_{\rm R}$ and $f_{\rm E}$, have been set equal to unity, and ζ to 4.83, the value corresponding to the marginal instability $Q = 1$.

Figs. 1 show that $t_{\rm ff}$ and $t_{\rm cool}$ are smaller than $t_{\rm trans}$, so they satisfy the requirement for star formation, up to a radius of the order of one parsec, where $t_{\rm ff}$ reaches $t_{\rm trans}$, and therefore the fragments are drawn towards the black hole more rapidly than they collapse. Two results are worth to be noted. First $t_{\rm cool}$ is much smaller than $t_{\rm ff}$ at the beginning of the collapse (it corresponds to the fact that the "equivalent" α is larger than unity). Thus a strongly gravitationally unstable disk would immediately fragment into small clumps. Second, $t_{\rm trans}$ is almost constant with radius, in contrast with α-disks, where it increases with radius and becomes larger than the lifetime of the objects at a few tenths of a parsec. It means that the mechanism for transporting angular momentum (which is not identified for the moment) must be very efficient at large radii.

One should thus verify that the radial drift velocity $V_{\rm rad}$ stays smaller than the Keplerian velocity $V_{\rm Kep}$, or the disk would not be geometrically thin. The ratio $V_{\rm rad}/V_{\rm Kep}$ is equal to:

$$\frac{V_{\rm rad}}{V_{\rm Kep}} \sim (1+\zeta)^{1/2} \frac{\dot{M}}{4\pi R^2 c_{\rm s} \rho}, \tag{13}$$

which writes, in the primeval case:

$$\frac{V_{\rm rad}}{V_{\rm Kep}} \sim 7\times10^{-3} \frac{(1+\zeta)^{1/2}}{\zeta^{0.85}} f_{-1}^{0.15} f_{\rm E}^{0.85} R_{\rm pc}, \tag{14}$$

and in the solar case:

$$\frac{V_{\rm rad}}{V_{\rm Kep}} \sim 0.13 \frac{(1+\zeta)^{9/14}}{\zeta^{8/7}} f_{\rm E}^{6/7} \kappa_{\rm R}^{-1/7} M_6^{-5/14} R_{\rm pc}^{23/14}. \tag{15}$$

The ratio $V_{\rm rad}/V_{\rm Kep}$ is therefore small up to large radii.

For an isothermal and spherical collapse the initial fragment would give rise to a dense core of mass $m_{\rm frag}$ in a time $t_{\rm collapse}$ corresponding to the growing rate (Chandrasekhar 1939):

$$\dot{M}_{\rm collapse} \sim \frac{c_{\rm s}^3}{G}. \tag{16}$$

This rate writes, in the primeval case:

$$\dot{M}_{\rm collapse} = 9\times10^{-3} \zeta^{-0.45} (f_{-1} f_{\rm E})^{0.45}\ {\rm M}_\odot {\rm yr}^{-1}. \tag{17}$$

and in the solar case:

$$\dot{M}_{\rm collapse} = 1.5\times10^{-6} \frac{\zeta^{3/7}}{(1+\zeta)^{3/14}} f_{\rm E}^{3/7} \times M_6^{15/14} R_{\rm pc}^{-27/14} \kappa_{\rm R}^{3/7}\ {\rm M}_\odot {\rm yr}^{-1} \tag{18}$$

For the initial mass of the fragment, it corresponds to a time for the collapse of the core:

$$t_{\rm collapse} \sim \frac{\zeta}{(1+\zeta)^{3/2}} \frac{1}{4\pi\Omega} \tag{19}$$

which is smaller than $1/\Omega$ for a marginally unstable gas.

Actually the collapse is not spherical, owing to the difference of disk velocity between the side facing the center and the opposite side, $\Delta V \sim H\Omega$. In a marginally unstable disk, ΔV is of the order of the sound velocity, so the collapse begins quasi spherically (this can be understood easily as the Bondi radius for spherical accretion by an object of mass m_*, $R_{\rm B} = G m_*/c_{\rm s}^2$, and the Roche radius, $R_{\rm R} = R(m_*/M)^{1/3}$, are both equal to H for the mass of the initial fragment).

In the equatorial plane the centrifugal force balances the gravitational attraction. Therefore, after having begun its collapse quasi spherically, the fragment will rapidly lead to a condensed object (let us call it a protostar, although it does not have necessarily the mass of a star, as we shall see below) and to a rotating protostellar disk.

Since the small disk is detached from the rest of the disk, it cannot give its angular momentum to the external regions like in an "infinite" accretion disk. So it must get rid of a large fraction of its angular momentum, unless the final collapsed body would rotate with an unacceptably large velocity. Indeed if the angular momentum is conserved the angular velocity of the protostar, $\Omega_{\rm final}$, is, according to the conservation of the angular momentum:

$$\Omega_{\rm final} \sim \Omega_{\rm initial} \left(\frac{\rho_{\rm final}}{\rho_{\rm initial}}\right)^{2/3} \sim 0.7 M_6^{-1/6} R_{\rm pc}^{1/2}\ {\rm s}^{-1} \tag{20}$$

where $\Omega_{\rm initial}$ is the average initial angular velocity of a fragment of radius H, assumed equal to $\Omega(R)$, $\rho_{\rm initial}$ and $\rho_{\rm final}$

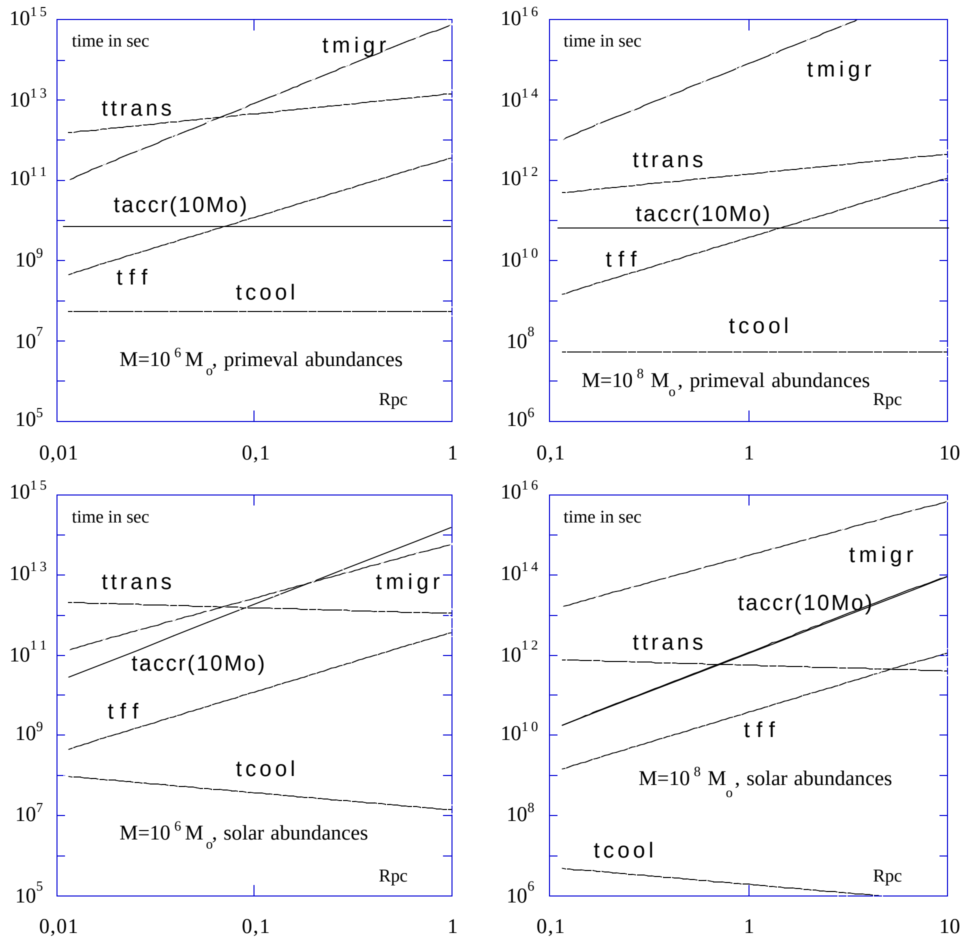

Fig. 1. different time scales in second, as functions of the radius in parsec, for $M = 10^6 \mathrm{M}_\odot$ and $M = 10^8 \mathrm{M}_\odot$, and for a zero and a solar metallicity; f_{-1}, $\kappa_{\rm R}$ and $f_{\rm E}$, have been set equal to unity, and ζ to 4.83. We recall that $t_{\rm acc}$ is actually overestimated.

are the average initial and final densities, the latter being taken equal to 1 g cm^{-3}.

The maximum angular velocity of a star, $\Omega_{\rm max}$, is of the order of 5 10^{-4} s^{-1}, for a density equal to 1 g cm^{-3}. Actually the density is higher for small mass stars, so $\Omega_{\rm max}$ is underestimated, while $\Omega_{\rm final}$ is overestimated as $\Omega_{\rm initial}$ is actually smaller than Ω. Nevertheless there are about 3 orders of magnitudes between $\Omega_{\rm final}$ and $\Omega_{\rm max}$.

A large proportion of the angular momentum is given to the protostellar disk, and in fine also to a planetary system which is likely to form around the protostar. Since $t_{\rm cool}$ is smaller than $1/\Omega$ at the beginning of the collapse, it is indeed probable that the initial cloud begins to fragment into smaller entities. Fragmentation stops when the cooling time (which increases during the collapse as the cloud becomes more optically thick), becomes of the same order as the collapse time. A fraction of the small fragments will form an ensemble of planetoids orbiting around the collapsed body, which can easily retain 99$\%$ of the angular momentum.

Another way to evacuate the angular momentum of the small disk is to transform it into orbital motion in binaries or multiple systems. One can also show that under the tidal action of the central mass the small disk is synchronized in a time of the order of the time scale for dissipative friction, i.e. in about a dynamical time. This mechanism can lead to the suppression of another large fraction of the angular momentum.

The actual final angular velocity, $\Omega_{\rm final\ real}$ is therefore at most of the order of $10^{-3}\Omega_{\rm final}$ given by Eq. 21. As the protostellar disk gets rid of a large fraction of its angular momentum, the radius down which the collapse is quasi spherical, $r_{\rm sph}$, is:

$$r_{\rm sph} \sim \left(\frac{G m_{\rm frag}}{\Omega^2_{\rm final\ real}}\right)^{1/3} \sim H \frac{M}{m_{\rm frag}} \left(\frac{H}{R}\right)^3 \sim 10^{-6} \frac{H}{\zeta} \qquad (21)$$

where we have assumed that the final angular momentum is of the order of 10^{-3} times the initial angular momentum. This equation shows that $r_{\rm sph}$ is of the order of the size of a star, so the collapse proceeds as in the spherical case with a characteristic time $t_{\rm collapse}$.

3. The accretion phase of the stars

3.1. Growth and evolution of the stars

Once the protostar and the protostellar disk are formed, they undergo a mechanism of accretion and growing proposed by Artymowicz et al. (1993). These authors consider stars orbiting around the central black hole, passing through the accretion disk, and finally being trapped in the disk. They showed that these stars would accrete matter from the disk at a high rate and become rapidly massive stars which explode as supernovae before being "swallowed" by the black hole. They invoke this process to explain the metal rich outflows observed in quasars (Broad Absorption Lines).

There are several differences between Artymowicz et al. accretion process and ours. They too adopt the marginal instability prescription, but they do not solve the radial profile of the disk, taking the scale height as a given parameter. They do not consider the effect of replenishment of the gap or of the cavity opened by the stars in the disk. Therefore the stars accrete only the gas which is initially inside the Bondi radius, and in a lesser extent inside the Roche radius.

Note first that the same remarks as made previously for the collapse phase concerning the angular momentum hold for the accretion phase, since the specific angular momentum of the matter located entering a volume $\sim H^3$ is of the same order as that of the initial fragment.

Due to the marginal instability prescription, the Roche radius $R_{\rm R}$ and the Bondi radius $R_{\rm B}$ are equal to H for the initial mass of the star. After, $R_{\rm R}$ grows as $m_*^{1/3}$. So an underestimation of the accretion rate can be obtained, assuming that it is limited to a region of radius H, and is very roughly given by the shear velocity of this region:

$$\dot{M}_{\rm accr} \sim \Delta V \rho H^2 \sim \Omega m_{\rm frag}. \tag{22}$$

The corresponding time for a star to reach $10{\rm M}_\odot$, $t_{\rm accr}(10{\rm M}_\odot)$, is equal equal to $10{\rm M}_\odot/(\Omega m_{\rm frag})$. One finds in the primordial case:

$$t_{\rm accr} \sim 10^{11}\frac{(1+\zeta)^{3/2}}{\zeta^{0.55}}(f_{-1}f_{\rm E})^{0.45}R_{\rm pc}^{-3}\ {\rm s}, \tag{23}$$

and in the solar case:

$$t_{\rm accr} \sim 6{\times}10^{14}\frac{(1+\zeta)^{2/7}}{\zeta^{10/7}}f_{\rm E}^{-3/7}M_6^{-15/14}R_{\rm pc}^{-15/14}\kappa_{\rm R}^{-3/7}\ {\rm s}. \tag{24}$$

We recall that these times are **overestimations** of the real ones since we have neglected the mass inflow towards the star coming from beyond H and swept up by the Roche lobe, which unfortunately we cannot estimate simply. Since the Roche radius is commonly equal to 10 or 100 H at the end of the accretion phase, we see that the accretion time can be overestimated by a very large factor.

Fig. 1 displays $t_{\rm accr}(10{\rm M}_\odot)$ as a function of the radius. It is always small (10^2 to 10^3 yrs) in the primordial case (owing to the large value of the scale height). In the solar case it can reach a few 10^6 yrs, i.e. be comparable to the evolution time of the star on the main sequence.

Palla & Stahler (1993) have shown that stars with masses greater than 8 $M_\odot$ have almost no pre-main sequence phase, as they are already burning hydrogen in the protostellar phase. These stars reach the main sequence in typically 10^4 yrs if they are accreting at a rate of 10^{-5} $M_\odot$ yrs^{-1}. If accretion proceeds at a rate of 10^{-4} $M_\odot$ yr^{-1}, this time is increased as stars are not thermally relaxed when the accretion is stopped, because they have larger radii.

The study of Palla & Stahler is performed for solar abundances. One can estimate very roughly the effect of a lower metal abundance. The effect of a decrease of the abundances is similar to a decrease of the accretion rate. For a metallicity 10^{-2} solar, one expects therefore that massive stars will not undergo a Kelvin-Helmoltz phase even if the accretion rate is as large as 10^{-3} $M_\odot$ yr^{-1}, and that they will reach the main sequence in less than 10^4 yrs. For very high masses the accretion rate is Eddington limited, and is at most of the order of $10^{-3}\times r_*/r_\odot$ $M_\odot$ yr^{-1} (it is smaller if electron scattering does not dominate on line absorption). Since this rate is larger than $\dot{M}_{\rm accr}$ this is actually not a limitation.

The growth of the stars is stopped when they begin to evolve on the main sequence and to drive strong radiatively accelerated winds. These winds slow down and stop the accretion, as they transport a larger outflowing momentum flux than the inflowing one. Outflowing winds are observed from Wolf-Rayet stars, up to 10^{-4} $M_\odot$ yrs^{-1}, with velocities larger than 1000 km s^{-1}, but the efficiency of radiative acceleration is probably smaller if the gas does not contain heavy elements (Meynet et al., 1994). Actually a rate of 10^{-6} $M_\odot$ yrs^{-1} with such a velocity is amply sufficient to stop the accretion. Moreover we will show in the next section that it is not the wind, but the ionizing radiation produced by the star, which stops the accretion, because it creates a cavity filled with hot and dilute gas preventing further accretion.

3.2. Gap opening and migration of the stars

To obtain supernova explosions (which are required by our model, see Sect. 5), two conditions should be fulfilled: 1. the stars must grow up to at least 10 $M_\odot$, which implies that they must not open a gap in the disk, and 2. the stars must not be swallowed by the black hole before they reach the stage of supernova. These conditions are both linked to the mechanism of tidal induced density waves discussed by Goldreich & Tremaine (1980), Ward (1986, 1988), and Lin and Papaloizou (1986a and 1986b).

Lin and Papaloizou showed that in an α-disk a gap is opened by the star if its mass exceeds $m_{*\rm gap}$, where:

$$m_{*\rm gap} \sim M\frac{40\nu}{R^2\Omega}\left(\frac{H}{R}\right)^{3/2} \sim M\frac{(40\alpha)^{1/2}}{1+\zeta}\left(\frac{c_{\rm s}}{\Omega R}\right)^{5/2}. \tag{25}$$

where ν is the kinematic viscosity. This expression is valid also in our case since the turbulent viscosity is still present and erases the gap. In the primeval case, it leads to:

$$m_{*\rm gap} \sim 4\times10^3\alpha^{1/2}\frac{\zeta^{-0.38}}{(1+\zeta)} \times (f_{-1}f_{\rm E})^{0.38}M_6^{-1/4}R_{\rm pc}^{5/4}\ {\rm M}_\odot, \tag{26}$$

and in the solar case to:

$$m_{*\rm gap} \sim 2.6\ \alpha^{1/2}\frac{\zeta^{5/14}}{(1+\zeta)^{33/28}} \tag{27}$$

$$\times\ f_{\rm E}^{5/14}\kappa_{\rm R}^{5/14}M_6^{9/14}R_{\rm pc}^{-5/14}\ {\rm M}_\odot.$$

We see that in the primeval case no gap is opened for $M_6 = 1$ and $R \geq 0.01$ pc, and for $M_6 = 100$ and $R \geq 0.03$ pc. In the solar case a gap is always opened for $M_6 = 1$, while for $M_6 = 100$ no gap is opened at any radius. In other terms, star formation is not possible for small black hole masses and solar abundances in a marginally unstable purely gaseous disk.

To know whether stars can reach the state of supernovae, one should compare the evolution time scale to the migration time towards the black hole due to the same mechanism of induced density waves. The migration proceeds towards the interior if the midplane temperature decreases with increasing radius, which is our case. The characteristic time for migration is:

$$t_{\rm migr} \sim C\ \frac{M^2}{m_*\Omega\pi R^2\Sigma}\ \left(\frac{c_{\rm s}}{\Omega R}\right)^2 \sim C\ \frac{M}{m_*}\frac{1}{\Omega}\frac{H}{R}, \tag{28}$$

where C is a factor of the order of unity. Since $t_{\rm migr}$ is inversely proportional to m_*, it means that the stars will begin to migrate only when they will have acquired large masses. For m_* = 10 ${\rm M}_\odot$, it gives, in the primeval case:

$$t_{\rm migr}(10{\rm M}_\odot) \sim 2.4\times10^{15}\frac{\zeta^{-0.15}}{(1+\zeta)^{1/2}}(f_{\rm E}f_{-1})^{0.15}R_{\rm pc}^2\ {\rm s}, \tag{29}$$

and in the solar case:

$$t_{\rm migr}(10{\rm M}_\odot)\ \sim\ 1.2\times10^{14}\frac{\zeta^{1/7}}{(1+\zeta)^{4/7}} \times\ f_{\rm E}^{1/7}M_6^{5/14}R_{\rm pc}^{19/14}\kappa_{\rm R}^{1/7}\ {\rm s}. \tag{30}$$

Fig. 1 displays $t_{\rm migr}(10{\rm M}_\odot)$ as a function of the radius. It is of the order of the evolution time of a 10 ${\rm M}_\odot$ star ($\sim$ a few 10^{14} s, see Meynet et al. 1994) only for $R \geq 1$ pc. This is a little too restrictive since star formation does not occur for $R \geq 1$ pc for a $10^6{\rm M}_\odot$ black hole. However $t_{\rm migr}$ is larger when a cavity is opened around the star, as it is the case here for massive stars, whose winds and HII regions excavate the disk during the major fraction of their life (see the next section). One can therefore consider that the migration time is not a constraint.

4. Feedback of stars and of supernovae on the disk

To be complete the above picture of star formation and accretion should take into account the effect of massive stars and of supernovae explosions on the disk. It is in particular necessary to know the distance over which the influence of a massive star or a supernova can be felt, and how long it lasts.

Many studies deal with star formation and evolution in molecular clouds. These problems should however be rediscussed in the framework of accretion disks, since the conditions are not similar to molecular clouds (the density and the temperature of the disks are higher, and they have strong Keplerian shear). Moreover Keplerian disks are not isotropic: their scale height is much smaller than their radius, so the disturbances can easily reach the surface in the "vertical" direction. In the plane of the disk the Keplerian rotation interfers with an outflow when its velocity equals the shear velocity, and it stretches the flow in the azimuthal direction.

Massive stars produce both ionizing photons which create HII regions, and winds, which create pressure driven bubbles. As a result a cavity containing a dilute gas expands around the stars. Shull (1980a) has shown that the bubble induced by the wind should dominate over the compact HII regions during the first 10^4 years of the life of the O stars. After this time, the expansion of the bubble is slowed down, and it is overcome by the HII region. Thus the supernova explodes in a low density cavity produced by the wind and the HII region. These processes have generally been considered separetely, owing to the incertainties on the involved phenomena (when in the O star life starts the wind and which is mass loss rate? Does it last until the explosion of the supernova?...). Another problem lies in the fact that when the radius of the bubble, the HII region, or the supernova remnant, is of the order of the scale height of the disk, their evolution is strongly affected by the escape of material in the "vertical" direction. We shall study these effects using very crude approximations.

4.1. HII regions

When an O star begins to shine on the main sequence, the initial radius of the HII region, R_0, is given by the equality between the flux of ionizing photons and the rate of recombinations:

$$R_0 = \left(\frac{3}{16\pi}\frac{F_{\rm ion}}{\alpha_{\rm rec}n_0^2}\right)^{1/3} \tag{31}$$

where $\alpha_{\rm rec}$ is the recombination coefficient, n_0 is the number density in the neutral medium (here molecular). Expliciting the variation of the density with the radius in the disk, it writes:

$$R_0 = 9.3\times10^{15}\ \zeta^{-2/3}F_{49}^{1/3}T_4^{1/6}M_6^{-2/3}R_{\rm pc}^2\ {\rm cm}, \tag{32}$$

where F_{49} is the flux number of ionizing photons expressed in 10^{49} cm^{-2} s^{-1}, and T_4 is the temperature of the HII region in 10^4 K. It is easy to see that this radius is always smaller than the scale height of the disk in the region of interest for us.

The pressure in the HII region being larger than the pressure in the ambient medium, after a dynamical time the Stromgren sphere begins to expand at a rate (Spitzer 1978):

$$R_{\rm i} = R_0\ \left(1+\frac{7}{4}\frac{c_{\rm s,HII}\ t}{R_0}\right)^{4/7}. \tag{33}$$

Here $c_{\rm s,HII}$ is the sound speed in the HII region. During this expansion the density decreases as $R_{\rm i}^{-3/2}$, assuming that the ionizing photon flux stays constant. Since the temperature of the HII region is constant, its pressure decreases, and when it reaches the pressure of the ambient medium, for a radius $R_{\rm i,max}$, the expansion of the HII region is stopped.

The ratio of the maximum radius $R_{\rm i,max}$ of the HII region to the scale height is thus:

$$\frac{R_{\rm i,max}}{H}\ =\ 0.46\ F_{49}^{1/3}T_4^{5/6}\frac{(1+\zeta)^{1/2}}{\zeta^{0.364}} \times\ (f_{-1}f_{\rm E})^{-0.505}M_6^{-1/6}R_{\rm pc}^{1/2} \tag{34}$$

in the primeval case, and:

$$\frac{R_{\rm i,max}}{H} = 200\, F_{49}^{1/3} T_4^{5/6} \frac{(1+\zeta)^{2/3}}{\zeta^{13/21}} \times f_{\rm E}^{-1/4} \kappa_{\rm R}^{7/21} M_6^{-1/21} R_{\rm pc}^2 \quad (35)$$

in the solar case. It is of the order of unity in the primeval case, and can be much larger in the solar case for $R \sim 0.1$ pc.

When the HII region reaches the surface of the disk (i.e. when $R_{\rm i} \sim H$), the ionized material is allowed to expand above the disk in a low density medium. This is similar to the "champagne" or "blister" model of HII regions (see Tenorio-Tagle 1979 and subsequent papers), except that in the champagne model the HII region expands in the intercloud medium whose pressure is assumed equal to that of the molecular cloud, while here it expands in a medium of very low pressure compared to the disk pressure. After a time $H/c_{\rm s}({\rm HII})$ much shorter than the life time $t_{\rm lt}$ of the star, the HII region is depressurized and maintained at the pressure of the ambient medium. **The ionization front stalls therefore at a radius** H. When it reaches the surface of the disk, matter is ejected from the disk with a velocity $c_{\rm s}({\rm HII})$. This matter is rapidly deccelerated by the gravity and returns to the disk (but not necessarily at the same place, since the ejection is not purely vertical) after about one rotation, and after having travelled on about $c_{\rm s}({\rm HII})/c_{\rm s} \times H \sim 10H$. The result is therefore very different from the champagne model, where the excavation process goes on after the HII region has opened a hole in the disk, and leads ultimately to the destruction of the whole molecular cloud.

In conclusion, the size of the HII region is equal to min$(H, R_{\rm i,max})$ and is thus of the order of H.

4.2. Winds

O stars have strong winds with terminal velocities of the order of a few 10^3 km s^{-1}, through which they can eject up to 80% of their mass, if they reach the Wolf-Rayet stage. After a rapid phase of free expansion, the evolution is dominated by the "snowplow" phase in which the ambiant gas is swept up, collapses into a thin shell and cools, and finally the flow settles into a steady state when the velocity of the shell equals the sound velocity of the ambient medium.

The evolution of a wind driven "bubble" has been studied by Castor et al. (1975) and subsequently by Shull (1980a). The major fraction of the life of the stars is spent in the snowplow phase, where a thin dense shell of radius $R_{\rm s}$ surrounds an inner region of stellar wind and of hot shocked wind. Shull has shown that after a time $t_{\rm cr} \sim 10^4 L_{36}^{-1/8} n_5^{-9/8}$yr the pressure driving the shell falls owing to cooling, and the radius of the shell increases as:

$$R_{\rm s} = R_{\rm cr} \times \left(1 + \frac{12}{5}(\frac{t}{t_{\rm cr}} - 1) \, + \, C_1(\frac{t}{t_{\rm cr}} - 1)^2\right)^{1/4} \quad (36)$$

where $R_{\rm cr}$ is the radius for $t = t_{\rm cr}$, equal to $0.165 L_{36}^{1/8} n_5^{-7/8}$ pc, and C_1 is a nondimensional factor $\sim 2.7 \times L_{36} n_5^{1/4}$. Here n_5 is the number density expressed in 10^5 cm^{-3} and L_{36} is the kinetic luminosity of the wind in 10^{36} erg s^{-1} (it corresponds to an outflow rate of 1.3×10^{-6} M$_\odot$ yr^{-1} at a velocity of 2000 km s^{-1}).

The expansion of the shell in the radial direction is stopped when its velocity is equal to the sound velocity or to the shear velocity, $\Omega \times R_{\rm s}$. The radius corresponding to the equality of the shear and of the expansion velocity, $R_{\rm s,max1}$, is given by the solution of the equation:

$$\frac{dR}{dt}(R = R_{\rm s,max1}) = \Omega \times R_{\rm s,max1}. \quad (37)$$

Using the expression for the density in the disk as a function of radius, one can easily show that only the linear term in Eq. 36 is important, and the solution can be written:

$$R_{\rm s,max1} \sim 2 \times 10^{-2} \zeta^{-19/32} L_{36}^{5/32} M_6^{-23/32} R_{\rm pc}^{69/32} \ {\rm pc} \quad (38)$$

The radius of the shell corresponding to an expansion velocity equal to the sound velocity, $R_{\rm s,max2}$, is related to $R_{\rm s,max1}$ by:

$$R_{\rm s,max2} = R_{\rm s,max1} \left(\frac{R_{\rm s,max1}}{H}\right)^{1/3} (1+\zeta)^{-1/6}. \quad (39)$$

It shows that when $R_{\rm s,max1}$ is smaller than H, $R_{\rm s,max2}$ is smaller than $R_{\rm s,max1}$, so the shell is stopped at the sound velocity and not at the shear velocity, and vice versa.

In the primeval case, one has:

$$\frac{R_{\rm s,max1}}{H} = 0.545 \frac{(1+\zeta)^{1/2}}{\zeta^{0.29}} (f_{-1} f_{\rm E})^{-0.303} \times L_{36}^{5/32} M_6^{-7/32} R_{\rm pc}^{21/32}, \quad (40)$$

and in the solar case:

$$\frac{R_{\rm s,max1}}{H} = 7.3 \frac{(1+\zeta)^{4/7}}{\zeta^{0.737}} f_{\rm E}^{-1/7} \kappa_{\rm R}^{-1/7} \times L_{36}^{5/32} M_6^{-0.576} R_{\rm pc}^{1.30}. \quad (41)$$

We see that $R_{\rm s,max1}$ is larger than H only in the solar case and for small black hole masses, for which we have already found that star formation is not possible. In the other cases the expansion in the radial direction is stopped at the sound velocity. Since it is the same in the azimuthal direction, the shell is not streched and its expansion stays quasi isotropic and confined inside the disk.

In conclusion the influence of an O star does not extend much beyond a distance H, either through its HII region or through its wind "bubble".

4.3. Supernovae

The effect of a supernova is also to excavate the disk and to create an expanding dense shell around the excavation. Several authors have studied the expansion of the shell inside a dense molecular cloud (Wheeler et al. 1980, Shull 1980b, Terlevich et al. 1992). Here is added the complication of the orbital motion and of the small scale height of the disk.

As for the winds, the first expansion phase of the supernova is followed by a pressure modified snowplow phase which dominates the evolution. The expansion of the shell proceeds then as (Shull 1980b):

$$R_{\rm s} \sim 10^{-1} E_{51}^{1/4} n_5^{-1/2} \left(\frac{t}{t_{\rm sg}}\right)^{2/7} \quad {\rm pc} \tag{42}$$

where $t_{\rm sg}$ is equal to $20 E_{51}^{1/8} n_5^{-3/4}$, and E_{51} is the kinetic energy liberated by the explosion expressed in 10^{51} ergs.

Again the expansion in the radial direction is stopped when the expansion velocity is equal either to the sound velocity or to the shear velocity at its edge. For the shear velocity, it gives:

$$R_{\rm s,max1} \sim 0.17 \zeta^{-2/7} E_{51}^{3/14} M_6^{-3/7} R_{\rm pc}^{19/14} \ {\rm pc}. \tag{43}$$

One checks that $R_{\rm s,max1}$ is larger than the scale height of the disk, both for the solar and for the primeval cases. As a consequence the radius corresponding to a velocity of the shell equal to the sound velocity is larger than $R_{\rm s,max1}$ so the sound velocity does not play any role in stopping the shell in the radial direction. The shell can still expand in the azimuthal direction and will therefore be streched and sheared by the differential rotation. It will disappear after about one rotation time.

However since the shell expands always beyond a radius H (Eq. 43), it breaks rapidly out of the disk, the interior is depressurized and its internal energy becomes negligible, so Eq. 43 does not apply. Then $R_{\rm s,max1}$ must be estimated assuming that the expansion is driven only by momentum conservation. A large fraction of that momentum escapes from the disk, and the momentum supplied to the disk by the supernova is finally:

$$P_{\rm disk} \sim P_{\rm tot} \frac{H}{R_{\rm s,max1}} \tag{44}$$

where $P_{\rm tot}$ is the total momentum carried by the supernova explosion. Since the terminal velocity is of the order of $3P_{\rm disk}/(4\pi\rho H R_{\rm s,max1}^2)$, one gets:

$$R_{\rm s,max1} \sim \frac{3P_{\rm tot}}{4\pi\rho\Omega}. \tag{45}$$

It gives:

$$R_{\rm s,max1} = 0.22 \zeta^{-1/4} P_{43}^{1/4} M_6^{-3/8} R_{\rm pc}^{9/8} \ {\rm pc} \tag{46}$$

where P_{43} is expressed in 10^{43} g cm s^{-1}. This is actually not very different from Eq. 43.

A numerical 2-D simulation of the explosion of a supernova in a thin keplerian disk has been performed by Rozyczka et al. (1995). It shows how the shell is deformed and split into two elongated edges, one corresponding to the leading hemisphere, the other corresponding to the trailing hemisphere, the radial separation between the two edges being of the order of $t\,\Delta V(R_{\rm s,max1})$. There is however an important difference with us. Rozyczka et al. computation does not depend on the scale height, but only on the surface density which is given a priori. So they assume that the disk receives the momentum corresponding to the velocity vectors inclined by less than $\pi/4$ to the mid-plane, which means $P_{\rm disk} = P_{\rm tot} \times \pi/8$. This is generally larger than our value for $P_{\rm disk}$ (as $H/R_{\rm s,max1}$ is smaller than unity in our marginally unstable model). Consequently the angular momentum supplied by one supernova to the disk is smaller in our case.

In conclusion of this section we can estimate the maximum number of stars that the disk can support at the same time without being too much perturbed: it is given by the maximum number of HII regions and wind bubbles which can fit into the disk. Let us define the number of stars per decade of radius as N_*. Since the radius of the HII regions and of the wind bubbles is of the order of or smaller than H, an **underestimation** of the maximum number of stars is then $N_{*\rm max}$.

$$N_{*,\rm max} \sim 2\pi (R/H)^2 \tag{47}$$

In the primeval case, it gives:

$$N_{*,\rm max} \sim 2 \times 10^3 (f_1 f_{\rm E})^{-0.3} (1+\zeta) \zeta^{0.3} M_6 R_{\rm pc}^{-1}, \tag{48}$$

and in the solar case:

$$N_{*,\rm max} \sim 2 \times 4 \times 10^5 f_{\rm E}^{-2/7} \kappa_{\rm R}^{-2/7} \frac{(1+\zeta)^{8/7}}{\zeta^{2/7}} M_6^{2/7} R_{\rm pc}^{2/7}. \tag{49}$$

We can also determine the maximum rate of supernovae per decade of radius supported by the disk. As the cavities created by the blast waves are replenished roughly at the shear velocity, their lifetime is $\sim 1/\Omega$, and this rate is of the order of $\Omega(R^2/R_{\rm s,max1}^2)$, or:

$$\mathcal{N}_{\rm SN,max} \sim 1.4 \times 10^{-3} P_{43}^{-1/2} \zeta^{1/2} M_6^{5/5} R_{\rm pc}^{-2} \ {\rm yr}^{-1}. \tag{50}$$

One could think that this number is overestimated because it does not take into account the stretching of the cavity in the azimuthal direction. One the contrary this estimation seems rather conservative, as the numerical simulations of Rozyczka et al. (1995) show that when the cavity and the shock wave reach their maximum radial extension after a fraction of an orbital time and become strongly elongated, they are also strongly squashed, so the surface of the perturbation seems to be rapidly decreasing.

The corresponding maximum number of stars, assuming they evolve in 3×10^6 yrs, is generally smaller than that given by Eqs. 48 and 49. So finally in most cases the supernovae, and not the HII regions and the wind bubbles, will limit the number of stars that the disk is able to sustain.

5. A model for a self-regulated disk made of stars and gas

We would like now to find whether it is possible for the stars and the gas to coexist in the disk in a quasi stationary state. To perform this study we make several assumptions. The first one, which has been discussed in the previous sections, is that when a fragment begins to collapse, it leads necessarily to the formation of a massive star. A second basic assumption is that the gaseous fraction of the disk stays quasi homogeneous. This is of course a very rough approximation considering the presence of HII cavities (although we will check that they occupy a very small volume with respect to the whole disk in the models). But the most important problem in this respect is to know what kind of perturbation supernovae induce in the disk in the long term, after their blast wave phase which is short owing to rapid erosion by the shear. It is clear that only numerical 3-D simulations would allow to answer this question.

Another assumption is that the regions at the periphery of the disk provide a quasi stationary mass inflow during the life time of quasars or of their progenitors (for instance via global gravitational instabilities induced by merging), equal to the accretion rate on the black hole. In other words we assume that there is neither infall on the inner regions of the disk, nor a strong outflowing mass rate from the disk (except that due to the supernovae, which is small with respect to the accretion rate in the following models). It would be actually possible to relax these assumptions, but we postpone such a study.

5.1. Angular momentum and mass transport

Another difference with the galactic disk is that we have to find an efficient mass transport mechanism, because the disk is an accretion one. Supernovae produce a net transfer of angular momentum towards the exterior, as discussed in the previous section and shown by the numerical simulations of Rozycska et al. (1995). This is because the leading hemisphere of the supernova shell has an excess of angular momentum compared to the disk, while the trailing hemisphere has a deficit of angular momentum. Consequently the leading hemisphere is driven towards the center and the trailing one towards the exterior, as discussed already in the previous section. The net angular momentum supplied by one supernova is equal to:

$$\Delta J \sim \frac{3}{2\pi} P_{\rm tot} R \frac{H}{R_{\rm s,max1}}. \qquad (51)$$

Rozyczka et al. (1995) estimate the angular momentum redistributed in the midplane of the disk by one supernova to be of the order of $(1/8)P_{\rm tot}R$ to the disk. We have seen that this estimation must be corrected in our case by a factor $\sim 4H/R_{\rm s,max1}$. Numerically Eq. 51 gives:

$$\Delta J \sim 2\times 10^{43} P_{43}^{3/4} \zeta_{\rm g}^{1/4} M_6^{3/8} R_{\rm pc}^{-1/8} H_{\rm cm}\ {\rm g\ cm^2\ s^{-1}}. \qquad (52)$$

(we distinguish from here on the contribution of the gas, $\zeta_{\rm g}$, and of the stars, ζ_*, to $\zeta_{\rm tot} = \zeta_{\rm g} + \zeta_*$).

In the absence of another known mechanism it is tempting to identify the mechanism of momentum transport with that induced by supernovae explosions. Let us assume therefore that **all** the angular momentum required to sustain the accretion rate is carried by these explosions. The rate of supernovae per decade of radius is then given by the relation:

$$\mathcal{N}_{\rm SN} \sim \frac{2\dot{M}R^2\Omega}{\Delta J log_{10} e} \sim \frac{2.3\times 10^{14}}{H_{\rm cm}} \zeta_{\rm g}^{-1/4} P_{43}^{-3/4} f_{\rm E} M_6^{9/8} R_{\rm pc}^{5/8}\ {\rm yr}^{-1}. \qquad (53)$$

5.2. Regulation of star formation

Now we have to look for a mechanism regulating the star formation. Such mechanisms have been studied by several authors in the context of the Galaxy (McCray & Kafatos 1987, MCKee 1989, Palous et al. 1994 among others). The idea which prevails generally is that the stars themselves provide a physical mechanism for induced star formation and regulation, via the creation of dense supernovae shells and of HII regions inhibiting the formation of new stars and even destroying the clouds. In our case supernovae may indeed induce star formation in the disk, as they lead to overdensities which should trigger gravitational instabilities. Concerning the inhibition of star formation, we will see that it is not due to the presence of HII regions which stay limited in size, but to the mass of the other stars.

A self-regulation mechanism for the gas density has been proposed for the Galactic disk by Wang & Silk (1994), through the growth time of gravitational instabilities. We adopt their view that the rate of gas transformed into stars is:

$$\frac{d\Sigma_{\rm g}}{dt} = \Sigma_{\rm g} \frac{\epsilon}{\Omega} \frac{\sqrt{1-Q^2}}{Q}, \qquad (54)$$

ϵ being an "efficiency" of star formation which "parametrizes our ignorance". In the Galaxy it is of the order of 0.1% for massive stars.

Wang and Silk have included the stellar contribution in the Toomre parameter Q of Eq. 54, as given by a two fluid approximation. Here it should not be taken into account, since the mean distance between stars is larger than the instability length H. Indeed we recall that, owing to the HII and the wind bubble cavity, a new star can form only at a distance greater than H from an existing star.

An immediate consequence of Eq. 54 is that the gas density should be close to the value corresponding to marginal instability, unless all the gas would be rapidly transformed into stars. Another consequence is that the surface density of the stars should not be too large, unless star formation would be inhibited by the tidal effect of nearby stars. This condition writes:

$$\frac{\Sigma_{\rm g}}{\Sigma_*} \geq \frac{H}{d}, \qquad (55)$$

where $\Sigma_{\rm g}$ and Σ_* are the surface densities respectively of gas and of stars and d is the mean distance between stars. A way of expressing this condition is to say that $(\Sigma_{\rm g}^2/\Sigma_*^2)$ should be larger than the ratio of the number of stars, N_*, to the maximum allowed number $N_{*,\rm max}$ given by Eq. 47.

5.3. The equations

Since the formation time and the lifetime of the stars are both small compared to the growth time of the black hole or to the active phase of a quasar, a stationary state of the disk could be established if there is a steady mass inflow from outward. We can determine then the star formation rate from the number of supernovae able to sustain the required accretion rate, and we can solve the radial disk structure in a self-consistent way.

The (vertically averaged) equations for the disk structure are the the hydrostatic equilibrium equation:

$$c_{\rm s} = \Omega H (1 + \zeta_* + \zeta_{\rm g})^{1/2}, \qquad (56)$$

where we have included the stellar contribution, and the energy equation which we shall now establish.

There are 3 heating mechanisms for the disk:

- Heating by the fraction of the stellar luminosity not absorbed in the HII regions, i.e. the near UV and visible spectrum (actually a major fraction of the bolometric luminosity of the stars). We shall make the rough approximation of an homogeneous heating, while it is clearly not the case, since the mean distance between the stars is larger than the scale height of the disk. This approximation overestimates the proportion of the stellar flux absorbed in the disk, as it does not take into account geometrical factors.
- Heating by the central source, when the disk flares. We shall see that it is now always the case, even for solar abundances, contrary to the purely gaseous disk.
- Heating due to the dissipation of kinetic energy through shock waves induced by supernova shells (it corresponds to the viscous heating in a standard disk).

The problem is to determine the proportion of photons absorbed in the disk. One of the difficulties comes from the fact that only average – grey – opacities are generally considered in this kind of problem, in particular in Paper 1. Actually it would be necessary to know the relation between the opacity in the far infrared, which gives the flux radiated by the disk, to the opacity in the optical-UV range, which gives the fraction of the radiative flux absorbed by the disk. It is out of the scope of this paper to perform detailed frequency dependent opacity computations, but clearly the model would require such a study to be fully validated. Here we shall make different assumptions in the solar and in the primeval case.

The energy equation is:

$$\sigma T^4 = \sigma T_{\rm irr}^4 + \sigma T_{\rm grav}^4. \tag{57}$$

Here $\sigma T_{\rm irr}^4 = F_{\rm ext} + F_*$, where $F_{\rm ext}$ is the external flux ($F_{\rm inc}$ in Paper 1, given by Eq. 32), and F_* the flux due to the stars. $T_{\rm grav}$ is the usual contribution of the gravitational release.

For the primeval case – we recall that the disk is then optically thin – we adopt the grey prescription as in Paper 1. The energy equation writes thus:

$$\sigma T^4 = F_{\rm ext} + \frac{4 N_* L_*}{\pi R^2} + \frac{3}{2\pi}\frac{\Omega^2 \dot{M}}{\tau}, \tag{58}$$

where τ is the optical thickness corresponding to H, N_* is the number of stars per decade of radius and L_* is their average optical-UV luminosity.

For the solar case, we use a "modified" grey approximation. In the first part of the present paper where we have assumed a purely gaseous disk, we had only to deal with the optically thick region of the disk. This is no more the case, because we would like to extend the model at larger radii, since stars maintain a temperature higher than in the pure gaseous case. The disk becomes then optically thin in the sense of the Planck mean, but the major part of the visible and UV luminosity of the stars is however absorbed by dust since the surface density of the disk is always larger than 10^{21} cm^{-2}. So finally the energy equation is, in the optically thick case:

$$\sigma T^4 = F_{\rm ext} + \frac{4 N_* L_*}{\pi R^2}\tau + \frac{9}{64\pi}\Omega^2 \dot{M}\tau, \tag{59}$$

and in the optically thin case:

$$\sigma T^4 = F_{\rm ext} + \frac{4 N_* L_*}{\pi R^2 \tau} + \frac{2}{\pi\tau}\Omega^2 \dot{M}. \tag{60}$$

Note that N_* can be expressed in terms of the supernova rate, as $N_* \sim t_{\rm lifetime}\mathcal{N}_{\rm SN}$, where $t_{\rm lifetime}$ is the mean lifetime of the stars.

We further specify the rate of star formation per decade of radius (which is equal to the supernova rate in a stationary disk):

$$\mathcal{N}_{\rm SN} = \frac{1}{m_{\rm frag}}\pi^{1/2}\rho_{\rm g} H R^2 \epsilon\, \frac{(1-Q^2)^{1/2}}{Q}\Omega\, \zeta_{\rm g}^{1/2} \tag{61}$$

where $m_{\rm frag}$ is the mass of the most unstable fragments, $\sim \rho H^3$. ϵ must take into account the gas returning to the disk through winds and escaping from the disk through supernovae, as well as the mass of neutron stars which migrate into the black hole (or are eventually ejected from the disk if the explosion is not symmetrical). We recall that Q **takes into account only the gas density**, contrary to the galactic case. Note also that the actual parameter of the problem is not Q, but $\epsilon\sqrt{1-Q^2}/Q$.

Finally the star mass density is given by:

$$\rho_* \sim \frac{\Sigma_*}{2H} \sim \frac{m_* t_{\rm lifetime} log_{10} e\ N_{\rm SN}}{4\pi H R^2} \tag{62}$$

where we assume that the stars and the gas have the same scale height.

The complete system consists of Eqs. 54, 56, and 57 (in the form of Eqs. 58, 59, or 60), 61 and 62, plus the equations defining $\zeta_{\rm g}$ and ζ_* (Eq. 2).

5.4. The results

We recall first that, due to our rough approximation of the opacity coefficient, our solutions are valid only for a molecular disk, where the cooling rate (in the primeval case) or the opacity coefficient κ (in the solar case) are close respectively to $\Lambda(T)$ and to $1\,{\rm g}^{-1}\,{\rm cm}^2$. It corresponds to the range of temperature $T \leq 1500$ K. We have also seen that our model is limited by several constraining hypothesis, such as the constancy in space and in time of the accretion rate. We will also limit the solutions to a marginally unstable gas. This will impose a condition on the efficiency factor ϵ. It is however possible that other solutions, implying for instance a larger accretion rate, exist for an unstable gas, but we do not consider this case here. Our aim is actually only to show that even in this restricted framework, one can find solutions fulfilling the conditions discussed in the previous sections.

Figs. 2 to 6 display the results of a few computations. The luminosity of the stars is set equal to 10^{38} ergs s^{-1}, their mass is 30 M$_\odot$, and we take 3 10^6 yrs for $t_{\rm lifetime}$. The total momentum given by one supernova is generally assumed equal to

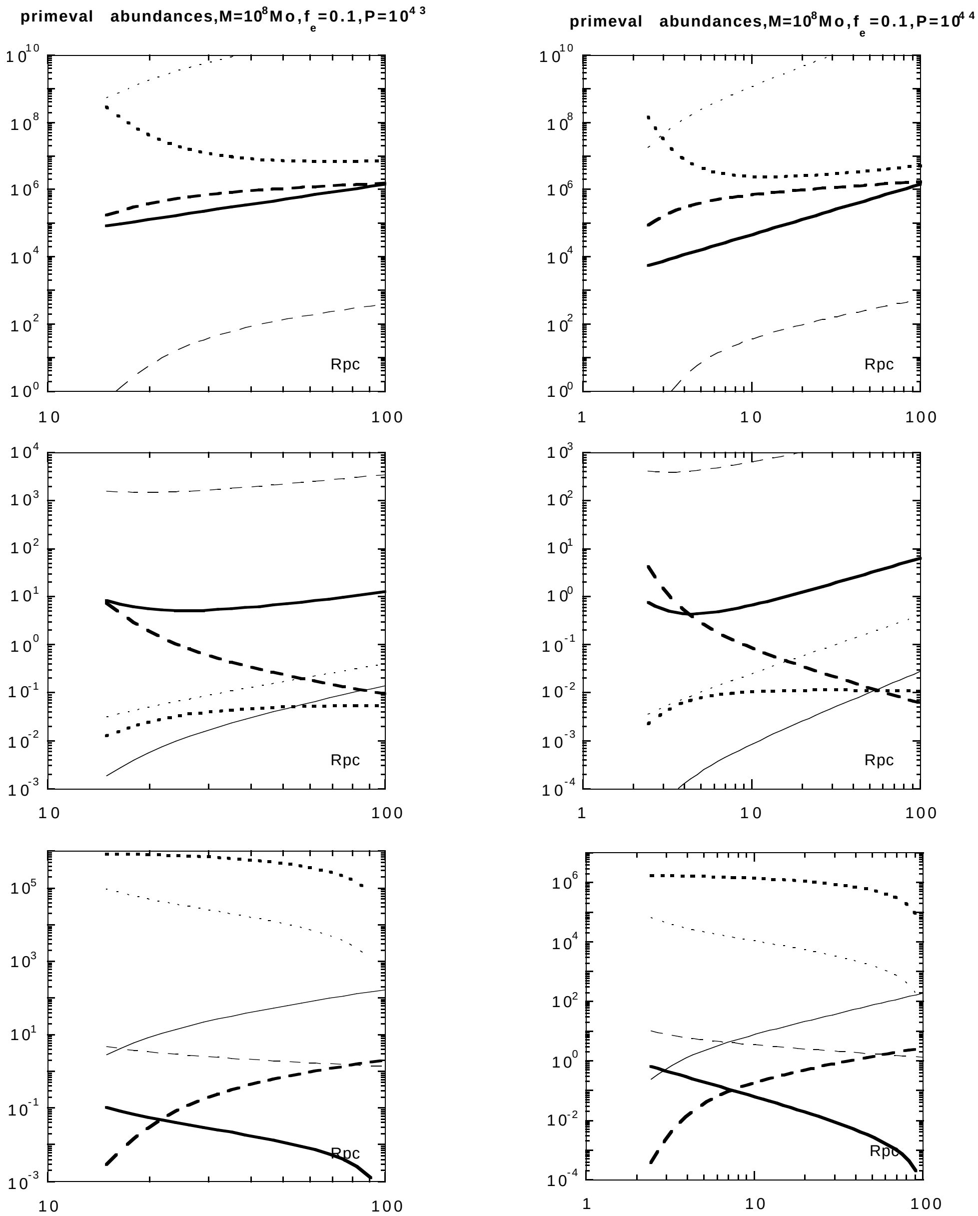

Fig. 2. Solutions for the disk made of stars and gas. In the top panels are given the characteristic times in year: bold solid lines: orbital time; bold dashed lines: $t_{\rm trans}$; bold dot lines: $t_{\rm accr}$(10M$_\odot$); thin dashed lines: $t_{\rm cool}$; thin dot lines: $t_{\rm migr}$(10M$_\odot$). In the middle panels, are given several parameters allowing to check that the conditions required by the model are fulfilled: bold solid lines: ratio of the rate of supernovae to the maximum allowed rate; bold dashed lines: ratio of the number of stars to the maximum number allowed by the tidal effect; bold dot lines: ratio of the number of stars to the maximum number allowed by the HII regions; thin solid lines: $\epsilon\sqrt{1-Q_{\rm g}}$; thin dashed line: $m_{\rm gap}$ in M$_\odot$; thin dot lines: $V_{\rm rad}/V_{\rm Kep}$. In the bottom panels, several physical parameters of the models: bold solid lines: $\mathcal{N}_{\rm SN}$ per year, integrated from the outer edge; bold dashed lines: $m_{\rm frag}$ in M$_\odot$; bold dot lines: $M_{\rm gas}$ integrated from the outer edge; thin solid lines: $H/10^{16}$ cm; thin dashed lines: $T/100\,K$; thin dot lines: N_* integrated from the outer edge.

10^{43} g cm s^{-1}, but in a few cases we use a value 10 times larger to show the influence of this fundamental parameter. For each couple black hole mass – Eddington factor, we give two figures in order to check that the required conditions are fulfilled, and one figure displaying some interesting parameters of the model.

First are displayed the characteristic times, the free fall time $1/\Omega$, $t_{\rm trans}$, $t_{\rm accr}$(10M$_\odot$) (we recall that it is an overestimation), $t_{\rm cool}$ and $t_{\rm migr}$(10M$_\odot$). The figures show that, except for small black hole masses and large accretion rates, the migration time is much larger than the typical stellar evolution time, so migration does not prevent supernovae explosions. Except for large black hole masses and accretion rates, the free fall time is smaller by about 2 orders of magnitude than the mass transport time, which means that $\sqrt{1-Q_{\rm g}^2}$ could be as small as 10^{-2} (cf. Eq. 4). The assumption that the gas is marginally unstable is therefore self-consistent. Finally the accretion time is at most of the order of a few 10^6 yrs, and we have seen that it is probably strongly overestimated.

A consequence of the large migration time is that the neutron stars left after the supernova explosions do not migrate towards the black hole. If they are not ejected from the disk (which should be the case if the explosions are asymmetrical or if they occur in binary stars), they will be able to reaccrete, and this phenomenon should lead to exotic massive stars, and probably to supernovae explosions more energetic than in normal galactic conditions.

We give then several parameters: $\epsilon\sqrt{1-Q_{\rm g}}$, $m_{\rm gap}$, $V_{\rm rad}/V_{\rm Kep}$, the ratio of the rate of supernovae to the maximum allowed rate, the ratio of the number of stars to the maximum number allowed by the tidal effect, and the ratio of the number of stars to the maximum number allowed by the HII regions. These parameters allow to check that the conditions required by the

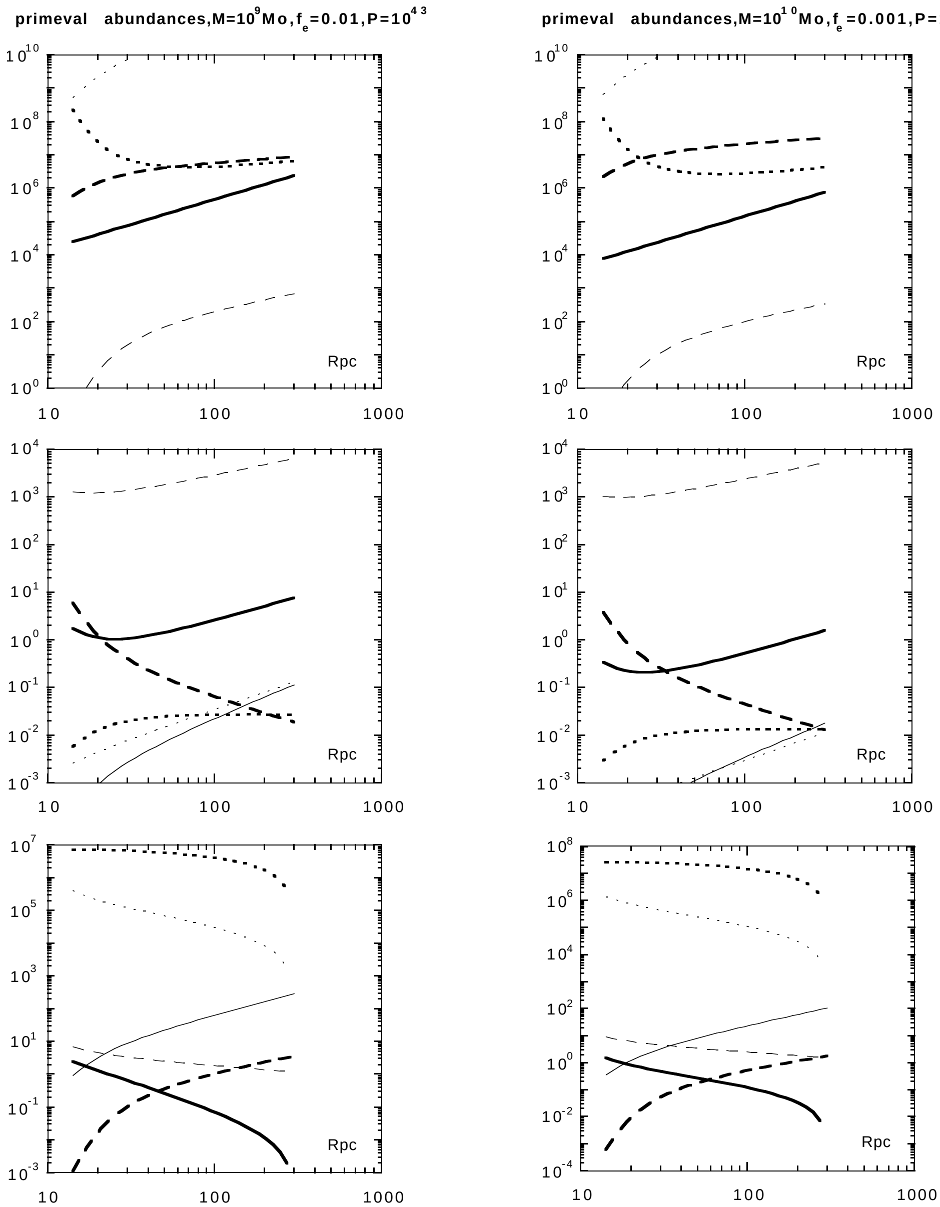

Fig. 3. Same as Fig. 2, for other cases

model are fulfilled. First we see that the minimum mass for opening a gap by induced density waves, $m_{\rm gap}$, is always very large, so no gap is opened even by large mass stars. The ratio of the radial to the Keplerian velocity is always small. One can also check from the product $\epsilon\sqrt{1-Q_{\rm g}}$ that a value of ϵ of the order of 10^{-3} is compatible with the condition $\sqrt{1-Q_{\rm g}^2} \geq 10^{-2}$. The number of stars is always smaller than the number allowed by the HII regions and by the tidal effect (except sometimes very close to the inner radius of the domain). The most constraining condition comes from the maximum allowed number of supernovae. In several cases the number of supernovae is close to this maximum number, and we show also some cases it exceeds this number in the whole domain: $M = 10^8$ M$_\odot$ with $f_{\rm E} = 0.1$ for primeval and solar abundances, $M = 10^9$ M$_\odot$ with $f_{\rm E} = 0.01$ for primeval abundances. To summarize, the cases where the number of supernovae is confortably smaller than the maximum number are only those with a small black hole mass. However we have seen that the surface covered by supernovae is probably overestimated by Eq. 52, so we are confident that the cases where the rate of supernovae is slightly larger than the maximum allowed number are also viable.

Finally the figures show some interesting physical parameters: the scale height and the midplane temperature of the disk, the mass of the initial fragments $m_{\rm frag}$, the gaseous mass of the disk, the total number of stars, and the total supernova rate. The last three quantities are integrated from the external edge.

First one can easily check that the disk is geometrically thin, with an aspect ratio H/R of the order of 10^{-3} to 10^{-2}, and is always flaring (it is why we take into account external illumination in the heating, although it is not important). The midplane temperature is of the order of a few hundred degrees, justifying our approximations for the opacity and the cooling rate. The initial mass of the fragments increases strongly with the radius. We

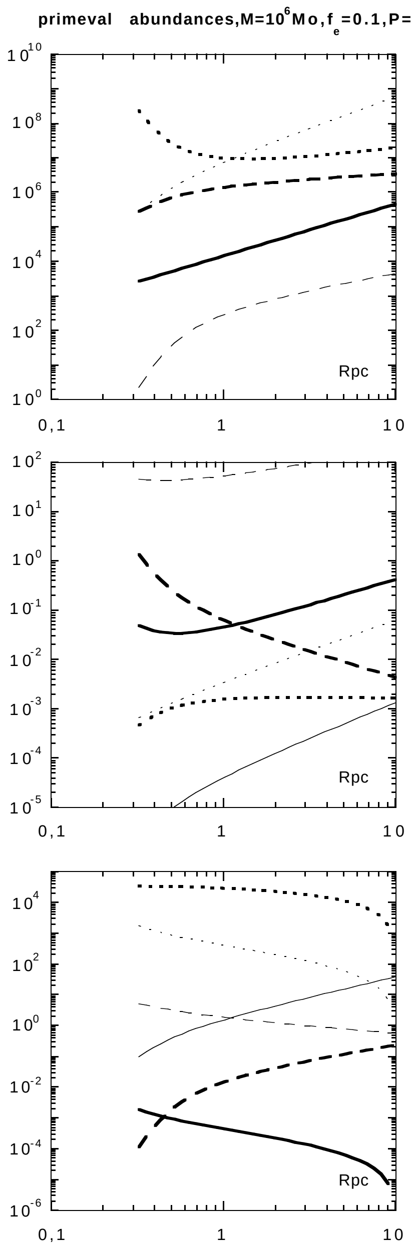

Fig. 4. Same as Fig. 2, for other cases

recall that since it depends on H^3 and due to our approximations on the opacity, it is the least correctly determined quantity, but this is of no consequence on the models. An interesting point to note is that for high black hole masses, the initial mass of the fragments reaches a few solar masses at the periphery of the domain. The total mass of the disk (actually of the domain where the model is valid) is much smaller than the black hole mass. The number of stars is small, corresponding to an amount of mass locked in stars of the order of that of the gas. Finally the total supernova rate, which is entirely dominated by the inner regions, varies from 10^{-3} to 1 $M_\odot$ per year. We shall come back to this point in the next section.

We have not mentioned the effects of the stellar winds. Using Eq. 36 one finds that they are less constraining than the supernovae in the inner regions of the disk, and generally more constraining in the outer regions. We have already mentioned that in the case of primeval abundances, stellar winds are certainly much less powerful than in the case of normal abundances. More generally the energy and momentum of the winds are certainly the least known parameters in this problem, and we prefer not to rely too much on it.

A first result is that the regions of interest are now located about 10 times further from the black hole than for the pure gaseous disk, owing to the increased heating. A second result is that the model works better when the accretion rate is smaller, and actually we have not found solutions for high mass black holes and $f_{\rm E}$ equal to unity, the reason being that the number of supernovae is then too large. The model works also better if the momentum provided by the supernovae is increased, since the number of stars required to transport the mass is smaller. Finally one must note that the model works as well with primeval or solar abundances.

6. Some implications of the model

Although the model is reminiscent of the"starburst AGN" model (cf. Terlevich & Melnick 1985 and subsequent papers), it is quite different, as the presence of the central black hole is a fundamental ingredient. One should not forget also that though the mass accretion rate is of the order of the outflowing mass due to supernovae, the luminous enrergy released in supernovae is much smaller than that of the central regions of the disk. So the only observational manifestation of the supernovae explosions would be expected in nearby low luminosity AGN, which should display an increase of flux in the optical range every 10^3 years. Since the number of known local AGN is now of the order of a few thousands, one could expect indeed to see a supernova exploding within a few parsecs from the center of an AGN, and this would be the best check of this model.

This model has several consequences. Beside the fact that it solves the problem of the mass transport in the intermediate region of the disk, it could give an explanation for the high velocity metal enriched outflows implied by the presence of the Broad Absorption Lines (BALs) in quasars. This problem is discussed in details in Collin (1998), so we recall here only a few points.

There are strong observational evidences for metallicities larger than solar (or at least solar) in the central few parsecs of quasars up to $z \geq 4$ (see the recent systematic study of Hamann 1997). This enriched material is flowing out of the central regions with a high velocity, of the order of c/30, as observed in BAL QSOs, which constitute about 10% of the total number of radio quiet QSOs. The phenomenon is generally interpreted as an outflow existing in all quasars, but limited to an opening angle $\sim 4\pi/10$. It is quite difficult to estimate the rate of outflowing mass, it is between one hundredth of the accretion rate and the accretion rate.

Comparing the observed mass rate with the rate of supernovae, each releasing about 10 $M_\odot$ of metals out of the nucleus, one sees that the mass outflow rate due to the supernovae ac-

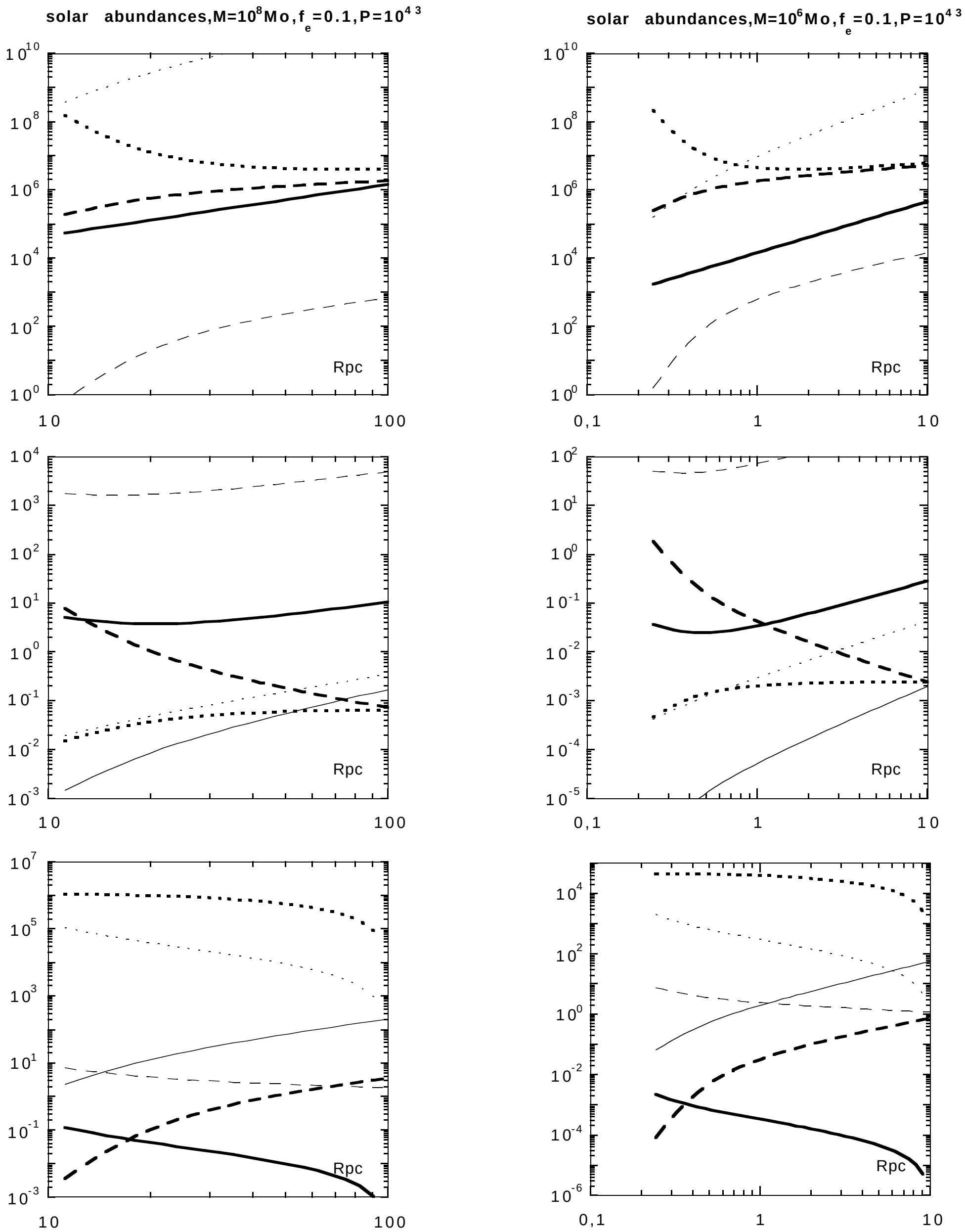

Fig. 5. Same as Fig. 2, for other cases

counts easily for the observations. Our computations show indeed that the rate of supernovae is equal to a few 10^{-2} yr^{-1} for a quasar black hole (see Fig. 6), i.e. an outflow of metals close to the accretion rate. The observed velocities are also easily accounted for by the expanding shells, and finally the location of the phenomenon is in agreement with observations. Relatively larger enrichment in some elements like N and Fe are observed, and could be explained by the oddness of the stars formed in a particular environment. It would actually be interesting to study the evolution of stars formed in such conditions to determine their final state before the explosion. Finally the fact that the opening angle of the BAL region is equal to a small fraction of 4π is easily explained, the ejection taking place mainly in a cone in the direction of the disk axis. Note that in this model the metallicity of the gas fuelling the black hole can be very small, while the observed outflow is always enriched.

A second outcome of our mechanism is to account for a pregalactic enrichment, if massive black holes are created early in the process of galaxy formation, and if galaxy formation takes place through an hierarchical scenario (Silk & Rees, 1998). Once the supernova shells are ejected from the accretion disk, their fate is determined by the mass of the host protogalaxy. Massive galaxies will retain their gas, and the supernova shells will compress the interstellar medium, trigger star formation like in the interstellar medium, and induce a starburst. This is an "inside-outside" scenario opposite to the starburst scenario of Hamann & Ferland (1993), where the central massive black hole grows by accreting already enriched gas during and after the starburst, and is therefore an "outside-inside" one. A fraction

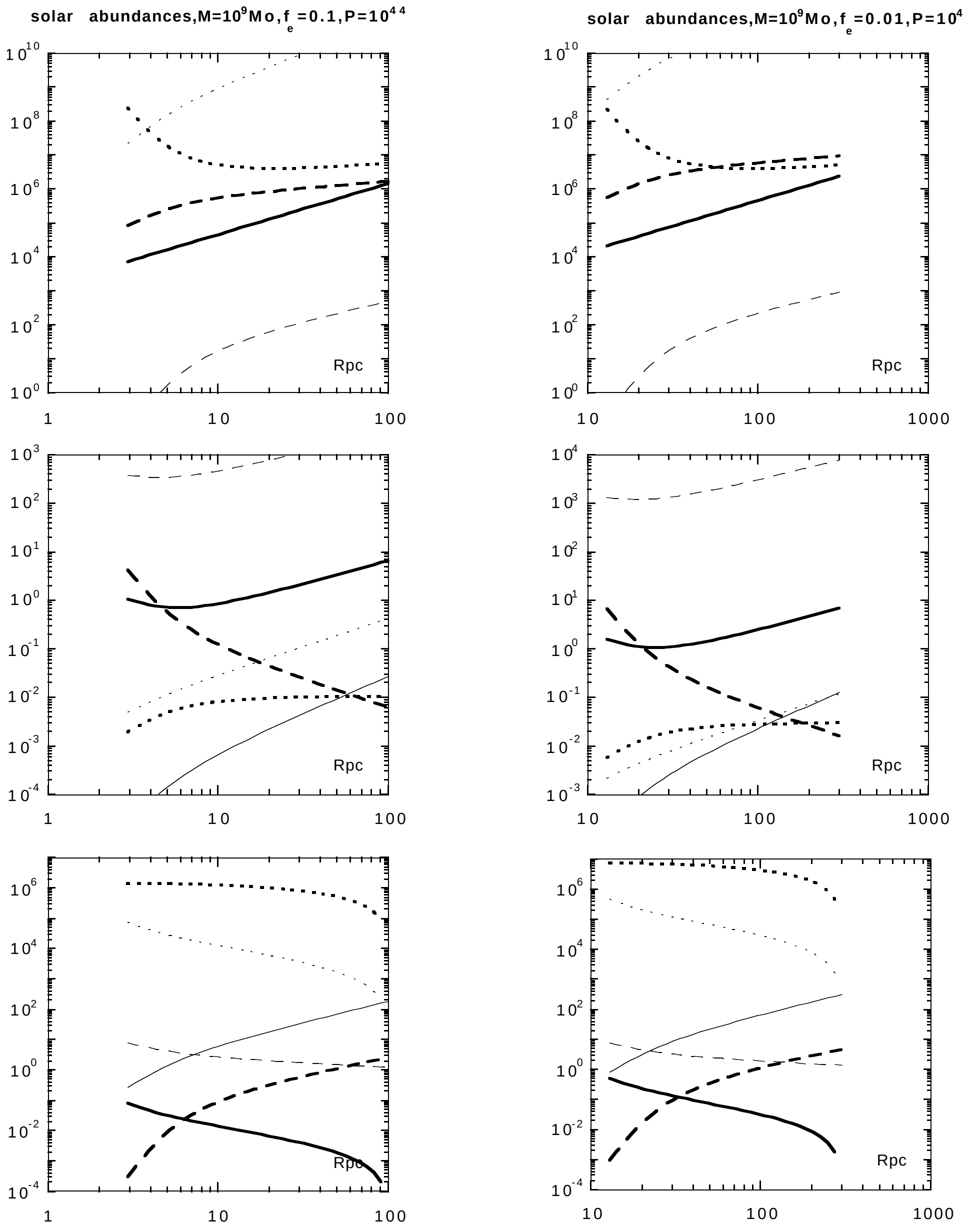

Fig. 6. Same as Fig. 2, for other cases

of the enriched gas should escape from the galaxy, not only due to the nuclear supernovae explosions, but also to the induced starburst.

Small galaxies will not retain their gas, which will escape with the enriched gas produced by the supernovae. It will pollute the intergalactic medium (IGM). In particular, if the formation of the black holes precedes the formation of galaxies, it will lead to a pregalactic enrichment of the IGM. If the universe at high redshift is dominated by a homogeneous population of compact and spheroidal galaxies (Steidel et al. 1996) which are the progenitors of massive galaxies, the enrichment of IGM induced by the black holes would be quite homogeneous and the appeal to a population III stars would not be necessary.

According to our computations the mass of metals ejected by the disk is of the order of the mass of the black hole itself. We can therefore estimate the **minimum** enrichment of IGM due to black holes, simply taking the integrated comoving mass density of **observed** quasars, which corresponds to about 10^{-6} of the closure density (Soltan, 1982, and further studies). If these black holes have a typical mass of 10^8 $M_\odot$, the present mechanism will provide about 10^{-6} of the closure density in metals, i.e. after mixing with the IGM, an average metallicity of a few 10^{-3} $\Omega(\mathrm{IGM})_{0.02} Z_\odot$, close to the metallicity observed in the Lα forest which constitutes the main fraction of the IGM.

Finally, our Galaxy is presently not active and the black hole in the center has a small accretion rate ($< 10^{-4}$ $M_\odot$ yr^{-1}), so it has most probably accreted a large fraction of its mass (2 10^6 $M_\odot$) during an early period. The previous estimation leads to an ejection of a few $\sim 10^4$ $M_\odot$ of metals. After mixing with a hydrogen halo of 10^{11} $M_\odot$, it gives a metallicity of a few 10^{-5} solar, close to that observed in the oldest halo stars.

7. Final remarks

In this paper we have explored a new model of accretion disk, made of stars and gas, with a permanent formation and death of stars, and we have shown that stationary solutions exist for primeval or solar abundances between 0.1 and 10 pc for a black hole mass of $10^6 M_\odot$, and between 1 and 100 pc for a black hole mass of $10^8 M_\odot$ or more.

However this model works only for a relatively low Eddington factors, ≤ 0.1. This raises a severe problem, as some quasars can be two orders of magnitudes more luminous than those considered here or the black hole mass would have to be very large to account for the luminosity ($\geq 10^{10}$ $M_\odot$), which seems unlikely. Moreover during the growing phase of quasars, the average accretion rate should be also at least of the order of $\dot{M}_{\rm crit}$/10 ($\dot{M}_{\rm crit}$ being the critical rate), or the growth of the black hole would take a time larger than allowed by the existence of high redshift quasars, since the e-folding time for the growing of a black hole is 4 $10^7/f_{\rm E}$ yrs (independently of the mass). In luminous quasars, the number of supernovae required to transport the angular momentum is too large, and the disk should become entirely paved with cavities surrounded by shocks. Is the disk then destroyed? Probably not, as the potential of the black hole should still maintain a fraction of the gas close to the equatorial plane. It is however possible that the thin disk is replaced by a thick disk (a "torus"). In any case the transport of mass and angular momentum by the supernovae should be very efficient, but the disk cannot be treated like in this paper as a homogeneous gas close to marginal unstability. There would be some highly compressed regions where star formation should take place (but not necessarily be followed by subsequent accretion on the protostars, so high mass stars might not dominate the mass function as in our model). Clearly this case would deserve a serious study, and numerical simulations would probably be the best way to tackle the problem.

Actually the most realistic scenario is that the mass inflow from the periphery of the disk is variable with time, as it would be the case if it is achieved by large molecular complexes comparable to those present near the Galactic center. If the mass inflow is variable with time, there will be "low states" and "high states" where the disk is alternatively "quiescent", and "active", i.e. highly perturbed by an intense supernova activity, and star formation will occur in successive "waves" propagating from the outer to the inner regions. An inflow of matter will induce an increased gas density in an outer ring. Before the stars form, accrete and evolve to supernovae, the transfer of mass will not take place, and there will be an accumulation of gas in the ring. After a few 10^6 years, supernovae will induce mass transport towards an inner ring, while the outer ring will be cleared out of its gas until a new mass inflow. Note that during the "active" phases the mass inflow at the periphery could be super Eddington. In this case our description should be modified to take into account the non-stationarity of the process, an one would expect that the corresponding averaged momentum transport (i.e. the accretion rate) could be much larger than in the stationary case, as it is not limited by a maximum allowed rate of supernovae.

Finally we should mention that an important improvement of the model would be to use better opacities, in particular to take into account the frequency dependence of the opacity. In this context it would also be interesting to study intermediate cases between a solar and a zero metallicity.

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References

Artymowicz P., Lin D.N., Wampler E.J., 1993, ApJ 409, 592
Begelman, M.C., Frank J., Shlosman I., 1989, In: Meyer F., et al. (eds.) Theory of Accretion disks. Kluwer Academic Publishers
Castor J., McCray R., Weaver R., 1975, ApJ 200, L110
Cavaliere, A., Padovani P., 1988, ApJ 333, L33
Cavaliere, A., Padovani P., 1989, ApJ 340, L5
Chandrasekhar S., 1939, An introduction to stellar structure
Collin S., Huré J.M., 1998, A&A (Paper 1), in press
Collin S., 1998, to appear in: Regev O. (ed.) The proceedings of the Conference: From atomic nuclei to galaxies, held in Haifa, Israel, Physics Reports,
Goldreich P., Lynden-Bell D., 1965, MNRAS 130, 97
Goldreich P., Tremaine S., 1980, ApJ 241, 425
Hamann F., Ferland G., 1992, ApJ 391, 53
Hamann F., 1997, ApJS 109, 279
Huré J.M., 1998, A&A, in press
Kolykhalov P.I., Sunyaev R.A., 1980, Soviet Astron. Lett. 6, 357
Lin D.N.C., Papaloizou J.C.B., 1980a, ApJ 307, 395
Lin D.N.C., Papaloizou J.C.B., 1980b, ApJ 309, 846
McCray R., Kafatos M., 1987, ApJ 317, 190
McKee C.F., 1989, ApJ 345, 782
Malkan M.A., Sargent W.L.W., 1982, ApJ 254, 22
Meynet G., Meader A., Schaller G., Schaerer D., Charbonnel C., 1994, A&AS 103, 97–105
Palla F., Stahler S.W., 1993, ApJ 418, 414
Palous J., Tenorio-Tagle G., Franco J., 1994, MNRAS 270, 75
Paczynski B., 1978, Acta Astron. 28, 91
Rees M.J., 1984, ARA&A 22, 471
Rozyczka, M., Bodenheimer P., Lin D.N.C., 1995, MNRAS 276, 597
Shakura N.I., Sunyaev R.A., 1973, A&A 24, 337
Shields, G.A., 1978, Nat 272, 423
Shields G.A., 1996, ApJ 461, L9
Shlosman I., Begelman M.C., 1989, ApJ 341, 685
Shlosman I., Frank J., Begelman M.C., 1989, Nat 338, 45
Shull J.M., 1980a, ApJ 238, 860
Shull J.M., 1980b, ApJ 237, 769
Silk J., Rees M.J., 1998, A&A 331, L1
Soltan, A., 1982, MNRAS 200, 115
Spitzer L. Jr., 1978, Physical processes in the interstellar medium. Wiley, New York
Steidel C., Giavalisco M., Pettini M., Dickinson M., Adelberger K.L., 1996, ApJ 462, L17
Tenorio-Tagle G., 1979, A&A 71, 59
Terlevich R., Melnick J., 1985, MNRAS 213, 841
Terlevich R., Tenorio-Tagle G., Franco J., Melnick J., 1992, MNRAS 255, 713
Toomre A., 1964, ApJ 139, 1217
Wang B., Silk J., 1994, ApJ 427, 759
Ward W., 1986, Icarus 67, 164
Ward W., 1988, Icarus 73, 330
Wheeler C., Mazurek T.J., Sivaramakrishnan A., 1980, ApJ 237, 781