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On features of Faraday rotation of the decametric radio emission in the Jovian magnetosphere

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Abstract. Jovian decametric emission exhibits a high degree of linear polarization. During its propagation the radiation crosses the Jovian magnetosphere and the terrestrial ionosphere which involves Faraday rotation of the polarization ellipse. We develop the basic equation necessary to estimate the amount of Faraday rotation of the polarization ellipse of the Jovian decametric emission in the different plasmas crossed by the emission, assuming that the different emission frequencies f are emitted from different regions. This assumption modifies the equation adding a new term C/f, the coefficient C which depends on the Jovian magnetospheric plasma inhomogeneity across the emission ray paths. This term being taken into consideration gives the possibility to investigate the Jovian magnetospheric plasma inhomogeneity across the emission ray paths due to the Faraday rotation measurements. Using spectropolarimeter observations performed at the Nançay Observatory (France) we derive the wave ellipse orientation allowing to get the total amount of Faraday rotation Ω between the source and the observer. We find that the amount of rotation could as well be described by two approximate formulae: the quadratic (Ω is a function of f^{-2} and f^{-4}) and the parabolic (Ω is a function of f^{-1} and f^{-2}). This ambiguous result is due to the limited number of the experimental data and the short frequency band of the observations. We calculate the rotation measure along the propagation path from the source at Jupiter to the observer and the value of the position angle of the polarization ellipse at the emission point and estimate the characteristic latitude scale of the Io torus inhomogeneity. We find that within the same data set the use of different approximate formulae leads to different estimations of the rotation measure and position angle. Besides, we show that an incorrect choice of the frame interval of the polarization ellipse angle can lead to an essential error on the estimation of this angle.

Key words: polarization – planets and satellites: individual: Jupiter – radio continuum: solar system

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1. Introduction

Jupiter's decametric (DAM) radiation is the most powerful of all radio emissions from the solar system radio planets. DAM emissions exhibit high intensity and rapid variations, both in time and frequency, which are characteristics of a nonthermal mechanism. There is a general opinion that this emission is produced as extraordinary waves at frequencies close to the local electron gyrofrequency. Contrary to other planetary radio emissions, the DAM radiation is found to be strongly elliptically polarized over a frequency band of 15 MHz (Lecacheux et al. 1992; Dulk et al. 1994). From Jupiter to the Earth, the DAM emission is travelling through three magnetized media: the magnetosphere of Jupiter, the interplanetary plasma, and the Earth's ionosphere. The elliptically polarized emission propagating through the magnetized plasma separates into two coherent components corresponding to the normal modes (extraordinary and ordinary) when a geometric-optic approximation is valid. During this propagation, in particular through the Earth's ionosphere, the phase velocity and the ray path are different for the two modes resulting in a rotation of the ellipse axis, the so-called Faraday rotation. Using linearly polarized antennae, the recorded Jovian dynamic spectra are modulated by fringes due to this rotation. Since the discovery of this phenomenon several studies have been performed to analyze the Faraday rotation since it allows, on the one hand, an estimate of the terrestrial electron content along the ray path, and on the other hand, the determination of the orientation of the ellipse axis of the emitted wave at the source region.

First studies mainly investigated the amount of Faraday rotation which is considered as proportional to the inverse emission frequency squared (linear approximation). Comparison of simultaneously measured ionosonde data with the amount of rotation deduced from dynamic spectra yielded that at least 90% of the rotation is due to the terrestrial ionosphere (Warwick & Dulk, 1964; Straka et al. 1965; Rihiimaa 1967; Parker et al. 1969). Phillips et al. (1989) analyzed six Io-B storms and found that (20–30)% of the observed Faraday rotation occurred in the Jovian magnetosphere. Dulk et al. (1992) examined decametric radio emission storm that was a mixture emission of RH and LH elliptical polarized S-bursts. They found that the orientation of

RH and LH polarized ellipses was not the same and that the difference in the orientation changed with frequency. Dulk et al. (1992) think that the features are due to the Faraday rotation in the Io plasma torus.

Lecacheux (1976) proved that the linear approximation is not a precise estimation of the real amount of Faraday rotation. Taking into account a second order term which is added to the linear approximation (quadratic approximation), Boudjada & Lecacheux (1991) showed that the previous studies under(over)estimated the Faraday rotation in the terrestrial(Jovian) ionosphere(magnetosphere). Ladreiter et al. (1995) showed that 30% of Faraday rotation in the Jovian magnetosphere found by Phillips et al. (1989) contains an error of (12–20)% due to the use of the linear approximation.

Further analysis was devoted to the estimation of the orientation of the major axis of the ellipse polarization at Jupiter. Bennett et al. (1965) and Parker et al. (1969) provided the first results of a quasi-parallel and quasi-perpendicular orientation of the major axis with regard to the local magnetic field line connected to the source region, respectively. Lecacheux (1976) interpreted the previous contradictory results as an error introduced by the use of the linear approximation. Later on, Boudjada & Lecacheux (1991) and Ladreiter et al. (1995) used the quadratic approximation and found that the orientation of the major axis is essentially along the local magnetic field in the source regions. The result of Ladreiter et al. (1995) provided an interesting convergence showing that the orientation of the major axis derived by Parker et al. (1969) is subject to a systematic error of about -30° because of the use of the linear approximation; when corrected the orientation is similar to those derived by using the quadratic approximations. The agreement between the different investigations can be regarded as a confirmation of the quasi-parallel orientation of the major axis of the polarization ellipse to the local magnetic field line associated to the source regions. However, this result is in total contradiction with the prediction of the emission mechanism where the radio emission is assumed to be generated in the X-mode where the polarization is perpendicular to the local magnetic field at the source.

In our paper we investigate and discuss features of the Faraday rotation in the Jovian magnetosphere and, in particular, in the Io torus, which could explain the disagreement between observations and the involved generation mechanism. We refuse the assumption that all frequencies are emitted from the same place as it is usually made in studies on the Faraday rotation, and assume that both the decametric radio emission source and the emission ray path heights change with frequency. We also show the possibility to use ground-based measurements of the Faraday rotation for the investigation of a latitudinal inhomogeneity of the Io plasma torus. In the second section we propose models to estimate the Faraday rotation of the major axis of the polarization ellipse from the source region until its ground-based reception. We derive the formulae for the variation of polarization ellipse orientation in the region of the Jovian magnetosphere. Using one event recorded by the Nançay radio telescope (France) we investigate, in the third section, different approximate formulae, derived by us in the previous section as well as by other authors, with the aim to recognize more precisely the description of the Faraday rotation effect in the Jovian decametric radio emission. In the last section we discuss our main findings.

2. DAM Faraday rotation from the source to the observer

Between the source and the observer, the Jovian decametric emission is mainly affected by Faraday rotation in the Jovian magnetosphere and in the terrestrial ionosphere. The approximation of quasi-longitudinal (QL) propagation occurs along all the emission ray path except a small domain close to the source region. In the interplanetary medium there is almost a complete absence of rotation due to very low plasma density and magnetic field. In the following we consider the Faraday rotation (FR) in different regions of the Jovian magnetosphere and Earth's ionosphere. Fig. 1 shows a global view of the different regions discussed below.

2.1. Faraday rotation in the Jovian magnetosphere

To investigate the propagation of DAM emissions in the Jovian magnetosphere we divide the magnetosphere into three parts: source region (S), interaction region of magnetosphere (IRM), and Io plasma torus (IPT). These regions are described hereafter with more details on modes which are generated and propagating through various regions, involving changes in the form and/or orientation of the polarization ellipse. The first and second regions are derived from a model by Shaposhnikov et al. (1997) called Paper I in the following. This model has been proposed to explain recent observations of ellipticity of the Jovian decametric emission by Lecacheux et al. (1991), Dulk et al. (1992), Dulk et al. (1994). In this model the polarized observations are interpreted as a signature of magnetospheric plasma distribution in a region of the Jovian magnetosphere close to the planet (more details in Paper I).

2.1.1. Source regions

One mode, extraordinary by definition, is generated in the sources of decametric radio emission at angles close to $\pi/2$ with regard to the direction of magnetic field lines, and the condition of quasi-transverse (QT) propagation is fulfilled in and close to the source region. In this case only the elliptical polarization of the Jovian decametric emission can be observed (Paper I). The emission is created at frequencies close to the local electron gyrofrequency $f_{\rm Be}$ (see, e.g., Shaposhnikov & Zaitsev 1996). Therefore, the sources of different emission frequencies f are located at heights which correspond to different gyrofrequency levels, $f_{\rm Be} \simeq f$. This causes a change with frequency of both the source and the emission ray path heights. As long as the geometrical optic approximation is valid, the polarization of the emission changes due to the change of the magnetic field intensity and the angle between the magnetic field and direction of wave propagation.

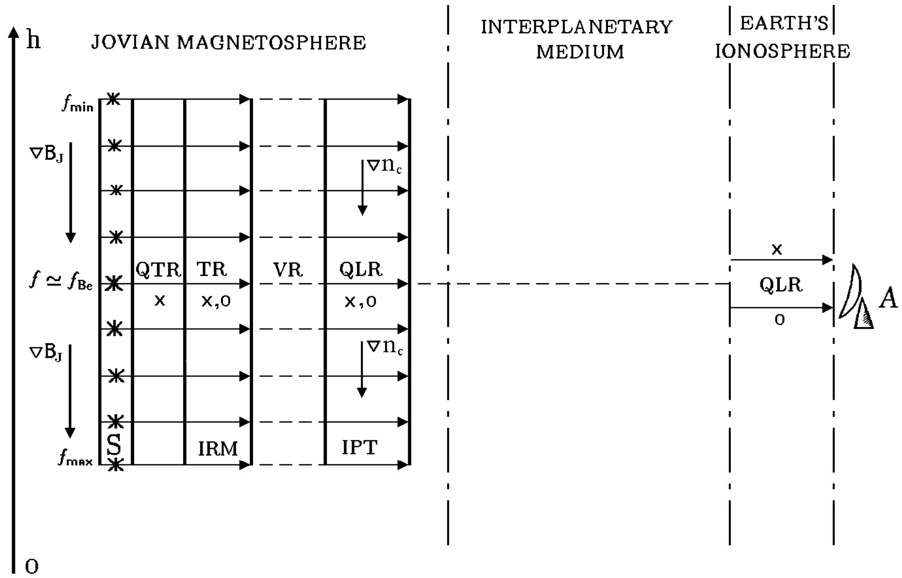

Fig. 1. A schematic model of propagation of decametric radio emission from sources at Jupiter to a ground-based observer. The "x" sign denotes the ray path of the extraordinary wave, the "o" sign corresponds to the ordinary wave, QTR and QLR are the regions of quasi-transverse and quasi-longitudinal propagation, respectively, TR is the transitional region, VR is the region of vacuum propagation, IRM and IPT are abbreviations for the interaction region of the magnetosphere and the Io plasma torus, accordingly, B_J is the planetary magnetic field, n_c is the magnetospheric plasma density, h being a height.

2.1.2. Interaction region of magnetosphere (IRM)

Results of the polarization observation of the emission allow to suppose that the geometric optic approximation is no longer valid where the polarization of the extraordinary mode is elliptical. That is in the transitional region (TR) to be a region at the emission ray between the region of QT propagation and the region where the condition of QL propagation is valid. For a given decametric radio emission storm occupying some region in frequency-time space the transitional regions are spread in a certain domain of the Jovian magnetosphere (interaction regions of the magnetosphere, IRM) due to the frequency dependence of source location and planetary rotation. In IRM, the second electromagnetic mode, the ordinary mode, is created and the linear mode coupling occurs between these two modes (Paper I). During the linear mode coupling a rotation of the resulting polarization ellipse is produced like the rotation taking place due to the Faraday effect in the geometrical optic approximation. Investigation of the rotation in IRM requires a rigid solution of the transfer equation of the polarized emission which can be found in Paper I. Following the paper we estimate the rotation which is found to be between approximately 20 and 40 degrees for different "sources" to be distinguished in the CML–Io phase diagram.

2.1.3. Io plasma torus (IPT) region

Beyond IRM the polarization remains constant along the ray path due to low level of the magnetospheric plasma density. Because of an enhanced density in the Io plasma torus the geometric optic approximation again starts here and a noticeable rotation of the polarization ellipse is expected. For example, at the frequency 18MHz Dulk et al. (1992) estimate the contribution of the Io plasma torus to the Faraday rotation from about -2π to 2π radians depending on CML.

In the geometric optic approximation the radiation propagates as two independent coherent components corresponding to the extraordinary (X-) and ordinary (O-) modes with refractive indices as given

$$n_{\mathrm{x,o}}^2 = 1 - \frac{2X(1-X)}{2(1-X) - Y^2\sin^2\theta \mp D} \tag{1}$$
$$D = \sqrt{Y^4\sin^4\theta + 4Y^2(1-X)^2\cos^2\theta}$$

where, the subscription "x" and the "minus" sign correspond to the X-mode, the subscription "o" and the "plus" sign correspond to the O-mode. The refractive indices depend on coordinates via parameters $X = f_{\mathrm{Pe}}^2/f^2$, $Y = f_{\mathrm{Be}}^2/f^2$, and θ where f_{Pe} is the electron plasma frequency, and θ is the angle between the magnetic field lines and the propagation direction. The phase difference between the ordinary and extraordinary components as they are propagating along corresponding ray paths L_{x} and L_{o} is

$$\Delta\Psi = \frac{2\pi f}{c}\left\{\int_{L_{\mathrm{o}}} n_{\mathrm{o}}\mathrm{d}l - \int_{L_{\mathrm{x}}} n_{\mathrm{x}}\mathrm{d}l\right\}. \tag{2}$$

In a cold plasma when the emission frequency is much greater than both the electron gyrofrequency and plasma frequency, $X \ll 1$ and $Y \ll 1$, Eq. (1) can be expanded in a series. One limits the series by the first order for X and Y and neglects the difference between ordinary and extraordinary ray paths, in the QL approximation we have for the rotation of the resultant ellipse (Zheleznyakov 1996)

$$\Omega(f) \simeq \frac{\Delta\Psi}{2} \simeq \frac{\pi f}{c}\int_L (n_{\mathrm{o}} - n_{\mathrm{x}})\mathrm{d}l \simeq$$
$$\frac{K}{f^2}\int_L n_{\mathrm{c}}(l)B(l)\cos\theta(l)\mathrm{d}l = \frac{K\cdot RM}{f^2} \tag{3}$$

where $\Omega(f)$ is the rotation of the emission polarization ellipse in a distance L relatively to its original position (in radians), f is measured in Hz, l in cm is a coordinate along the ray path. Further, $n_{\mathrm{c}}(l)$ in cm^{-3} and $B(l)$ in G are, respectively, the plasma

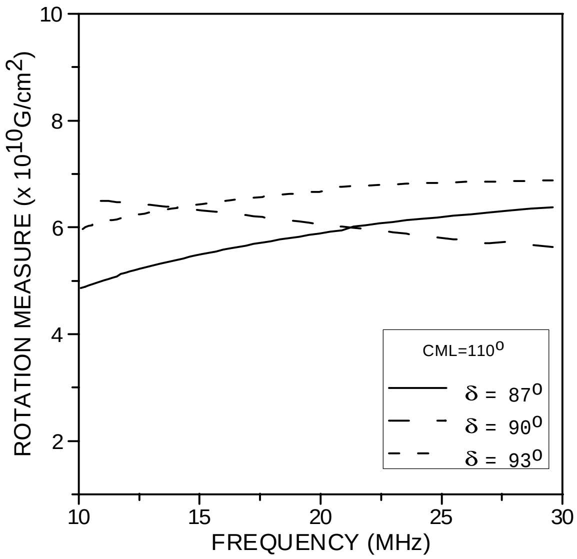

Fig. 2. Rotation measure in the Io plasma torus as a function of emission frequency for different values of inclination (δ) of the Jupiter spin axis to the Earth-Jupiter line assuming CML = $110°$.

density and magnetic field along the ray path, $K \simeq 2.4 \cdot 10^4$, and

$$RM = \int_L n_\mathrm{c}(l) B(l) \cos\theta(l) \mathrm{d}l \tag{4}$$

is the "rotation measure" which does not depend on emission frequency.

In the case of the Io plasma torus a location of the ray path L in Eqs. (3) and (4) is different for the emission at different frequencies due to the dependence of height of decametric sources on frequency. The shift of ray path location with the emission frequency variation results in a frequency dependence of quantities N_e, B, and θ under the integral sign in *RM* due to a latitude inhomogeneity of the plasma torus, so that

$$\Omega^\mathrm{I}(f) \simeq \frac{K \cdot RM^\mathrm{I}(f)}{f^2}, \tag{5}$$

where

$$RM^\mathrm{I}(f) = \int_L n_\mathrm{c}(l,f) B(l,f) \cos\theta(l,f) \mathrm{d}l. \tag{6}$$

In Eq. (5) and the following expressions the "I" superscript refers to contributions from the Io plasma torus. In order to illustrate the existence of the frequency dependence of RM due to the torus latitude inhomogeneity we have calculated RM for sources placed along the magnetic flux tube close to the limb. For simplicity we use $N_\mathrm{e}(l,f)$ as given in the torus model by Divine & Garret (1983). Fig. 2 shows a *RM* pattern in the Io plasma torus plotted as function of frequency for different values of inclination (δ) of the Jupiter spin axis to the Earth-Jupiter line for CML = $110°$. The variation of angle δ simulates a change of the characteristic scale of plasma torus inhomogeneity in a region penetrated by the ray paths for a given emission storm.

Since the characteristic scale in the Io torus is higher than the height interval occupied by the source (this interval is defined by the maximum f_max and minimum f_min frequencies observed at a fixed time), it is possible to expand the rotation measure in a series

$$\Omega^\mathrm{I}(f) = \frac{K}{f^2}\left\{ RM(f_0) + \left(\frac{\partial RM}{\partial f}\right)_{f=f_0} \cdot (f_0 - f) + \frac{1}{2}\left(\frac{\partial^2 RM}{\partial f^2}\right)_{f=f_0} \cdot (f_0-f)^2 + \cdots \right\}, \tag{7}$$

where f_0 is a point within the frequency interval $\Delta f = f_\mathrm{max} - f_\mathrm{min}$. One limits the series Eq. (7) by the first three terms, thus we have for the amount of FR in the Jovian magnetosphere

$$\Omega^\mathrm{I}(f) \simeq C_0^\mathrm{I} + \frac{C_1^\mathrm{I}}{f} + \frac{C_2^\mathrm{I}}{f^2}, \tag{8}$$

with coefficients C_j^I

$$\begin{aligned} C_0^\mathrm{I} &= K \cdot \left(\frac{\partial^2 RM^\mathrm{I}}{2\partial f^2}\right)_{f=f_0} \\ C_1^\mathrm{I} &= -K \cdot \left(\frac{\partial RM^\mathrm{I}}{\partial f} + \frac{\partial^2 RM^\mathrm{I}}{\partial f^2} f\right)_{f=f_0} \\ C_2^\mathrm{I} &= K \cdot \left(RM^\mathrm{I} + \frac{\partial RM^\mathrm{I}}{\partial f} f + \frac{\partial^2 RM^\mathrm{I}}{2\partial f^2} f^2\right)_{f=f_0} \end{aligned} \tag{9}$$

We limit ourselves to the first three terms of the series to show a possibility that a constant rotation exists in the Io torus. To estimate an error in the approximation of the amount of *RM* by the first three terms of series Eq. (7) we consider a part of the Io plasma torus, where ray paths from the sources pass through the torus. The IPT is representing an inhomogeneous plasma slab immersed in the homogeneous magnetic field B_I which is parallel to surfaces of the plasma (Fig. 1). We neglect also a curvature of the ray path and assume that they cross the slab under a constant angle to the plasma surfaces. In this case

$$RM^\mathrm{I}(f) \simeq B^\mathrm{I} \cos\theta^\mathrm{I} N_\mathrm{c}^\mathrm{I}(f), \tag{10}$$

where $N_\mathrm{c}^\mathrm{I}(f) = \int_L n_\mathrm{c}(l,f)\mathrm{d}l$ is the torus column plasma density along the emission ray path. The error in a representation of the amount of FR by Eq. (8) can be estimated as follows

$$\left|\frac{\delta\Omega}{\Omega}\right| \sim \left|\frac{1}{6}\frac{1}{N_\mathrm{c}^\mathrm{I}(f)}\frac{\partial^3 N_\mathrm{c}^\mathrm{I}(f)}{\partial f^3}\left(\frac{\Delta f}{f}\right)^3\right|_{f=f_0}. \tag{11}$$

If we approximate the plasma distribution along field lines (along an axis h in Fig. 1) by a Gaussian function with characteristic scale height H being of the order equal to the Jovian radius R_J, $N_\mathrm{c}^\mathrm{I} = N_0 \exp(-h/R_\mathrm{J})$ and take into account that $f \simeq f_\mathrm{Be} \sim (R_\mathrm{J}/h)^3$, then for an emission storm with $f_0 = f_\mathrm{max} \simeq$ 30MHz and $\Delta f \simeq$ 15 MHz we have $|\delta\Omega/\Omega| \sim 1\%$. Instead of using Eq. (7) the use of Eq. (8) introduces an error of only 1%.

2.2. *Faraday rotation in the Earth's ionosphere*

The main rotation of polarized ellipse of the Jovian decametric emission occurs in the Earth's ionosphere. Different authors give the estimation as large as about (70–90)% from the total amount of the Faraday rotation.

In the Earth's ionosphere we can neglect the frequency dependence of the emission ray paths. This is due to the size of range occupied by the ray paths of emission observed at fixed time which is much smaller than the characteristic scale of ionosphere inhomogeneity. However, the ray path splitting (separation of ordinary and extraordinary mode ray paths) can be taken into account. As shown by previous studies (Ross 1965, Lecacheux 1976), the Faraday rotation in the terrestrial ionosphere can be described by the following expression:

$$\Omega^{\mathrm{E}}(f) = \frac{C_2^{\mathrm{E}}}{f^2} + \frac{C_4^{\mathrm{E}}}{f^4}, \tag{12}$$

where

$$C_2^{\mathrm{E}} = K \cdot RM^{\mathrm{E}}, \tag{13}$$

C_4^{E} is a function of both the plasma density and angle between the ray path and magnetic field lines, the "E" superscript refers to the contribution of the Earth's ionosphere. This formula has been obtained by developing Eq. (1) to the second order for X and Y and introducing the ray splitting.

3. Analysis of Jovian decametric storm with Faraday effect

3.1. The event of Sept. 8th, 1987

In order to investigate the Faraday effect we use an event recorded on Sept. 8th, 1987, at the Nançay observatory (France). This storm represents Jovian decametric emission with Faraday fringes which had been observed between 0130 UT and 0230 UT when the ionospheric conditions have been quiet, with an absence of interferences above 15 MHz. The emission storm occupied a frequency band between 19 and 36 MHz where the signal from Jupiter is found to be much stronger than the sky background. The dynamic spectrum was recorded by the Nançay spectropolarimeter providing the four Stokes parameters of polarization (Boudjada, 1991). For this event, the receiver frequency band (from 10 to 40 MHz) was scanned each 1.25 second which is the necessary time to record 125 channels separated by 250 kHz.

In order to determine the amount of Faraday rotation we follow the same method as reported by Boudjada & Lecacheux (1991). We individually analyze ten spectra cuts, corresponding to a total of 436 data points, selected at successive times between 0210 UT and 0255 UT. For each cut we find a set of points (sample) $\{f_{\mathrm{m}}, \chi_{\mathrm{m}}\}_{m=1\ldots N_{\mathrm{k}}}$ which is associated with a set of fringes on the dynamic spectrum where k is the number of the sample and χ_m is the wave ellipse orientation relative to the antenna axis. This parameter is derived from the two Stokes parameters, Q and U, of the wave as $\chi_m = 0.5 \arctan(U_m/Q_m)$. The amount of Faraday rotation (Ω) in function of the frequency can be written as:

$$\Omega(f_m) = \chi_m + l_m \pi, \tag{14}$$

where l_m is a fringe number with the starting point $l_1 = 0$ associated, by definition, to the first fringe observed on the dynamic spectrum at high frequency.

3.2. Data fitting by different approximate formulas

From the dynamic spectrum it is possible to derive the Faraday rotation variations as function of the emitted frequency. To establish a numerical relation between the amount of Faraday rotation of the Jovian decametric radio emission and the observed frequency, we consider for regression analyis the expressions

$$\Omega^1 = C_0^1 + \frac{C_2^1}{f^2}, \tag{15}$$

$$\Omega^2 = C_0^2 + \frac{C_2^2}{f^2} + \frac{C_4^2}{f^4}, \tag{16}$$

which are known as the linear and the quadratic approximations allowing to estimate the amount of Faraday rotation, respectively. The parabolic formula developed in the previous section (see Eq. (8))

$$\Omega^3 = C_0^3 + \frac{C_1^3}{f} + \frac{C_2^3}{f^2}, \tag{17}$$

and the expression Ω^4, which is a composition of the quadratic approximate formula Eq. (16) and the parabolic formula Eq. (17) is given by

$$\Omega^4 = C_0^4 + \frac{C_1^4}{f} + \frac{C_2^4}{f^2} + \frac{C_4^4}{f^4}. \tag{18}$$

For each coefficient C_j^i we associate two indices i and j which indicate the type of approximation and the frequency power, respectively. Assuming Eq. (16) as the approximate formula we neglect either the Faraday rotation in the Io plasma torus or the latitude plasma inhomogeneity of the torus and take into account the ray path splitting in the Earth's ionosphere. Eq. (17), on the contrary, neglects the ray path splitting effect in the Earth's ionosphere but takes into consideration FR in the Io plasma torus. In Eqs. (15) and (16) coefficients $C_0^{1,2}$ are a sum of two terms, a position angle PA_{s}, which defines the position of the polarization ellipse in the emission source relative to the antenna axis and Ω_{IRM}, the constant rotations in IRM, ($C_0^{1,2} = PA_{\mathrm{s}} + \Omega_{\mathrm{IRM}}$) while in Eqs. (17) and (18) the coefficients $C_0^{3,4}$ are a sum of three terms, the two previous and C_0^{I}, the constant rotation in the Io torus ($C_0^{3,4} = PA_{\mathrm{s}} + \Omega_{\mathrm{IRM}} + C_0^{\mathrm{I}}$). The rotation in IRM does not depend on frequency because the observed ellipticity of polarization ellipse of this emission does not depend on frequency (Paper I). In Eq. (17) the coefficient C_2^3 is a sum of two terms, a constant proportional to the rotation measure in the Earth's ionosphere C_2^{E} and a constant C_2^{I} which depends on both the plasma density in the Io torus integrated along the emission ray path and the latitude plasma inhomogeneity of this torus (see Eq. (9)). The important point here is the existence of the new term in the approximate formulae Eqs. (17) and (18), $C_1^{3,4}/f$, where the coefficient $C_1^{3,4}$ depends only on the parameters of the Io plasma torus.

In Fig. 3 we display an example of scatter diagram for one sample of FR data with their regression curves. The coefficients C_j^i for these curves are computed by the data fitting using the least square method. All the analyzed curves fit, at first glance, quite good the observational points within the full frequency

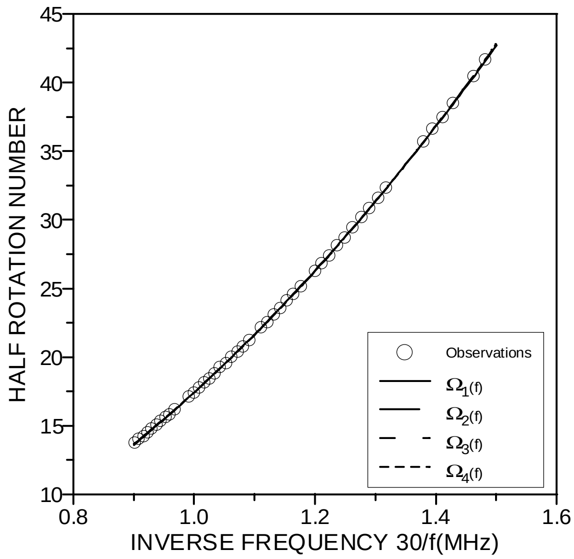

Fig. 3. Fit of different types of FR models for spectral cut corresponding to CML = 117°. All fitted curves are completely superposing.

band occupied by the storm. The coefficient of determination (r-squared) is very close to unit for each curves. However, comparison of the sum of squares residuals of all sample points shows that the new terms C_4^2/f^4 and C_1^3/f added in Eqs. (16) and (17), respectively, significantly decrease the sum of residuals with a level of significance better than 1% (see in this connection Kennedy & Neville 1986). Note here that the sums of residuals after the fit of curves Ω^2 and Ω^3 with the experimental data are found to be the same. A more precisely defined regression curve, Eq. (18), does not appear as a better fitting curve. The sum of residuals cannot be assumed to be lower than the sum of residuals for Eqs. (16) and (17) with the level of significance more than 20%. Therefore, formula (18) is not taken into consideration in the next subsection.

In Fig. 4 we plot the residuals after each fit. It is found that the fit of curve Ω^1, Eq. (15), clearly appears to be depending on the emission frequency as previously reported by Boudjada & Lecacheux (1991). The coefficient of determination is about 0.8 for the curve displayed as solid line in Fig. 4a. The residuals after the fit of curves Ω^2, Eq. (16), and Ω^3, Eq. (17), appear to be randomly distributed. The coefficients determining the fitting curves, displayed as solid lines in Figs. 4b through 4d, are close to zero. Thus, the limited set of samples data does not allow us to make a choice between those equations due to the statistical uncertainties.

3.3. Result of the approximate formulae fitting

Table 1 shows the average value coefficients of the regression curves, C_1^i, C_2^i, and C_4^i, with the corresponding standard deviation $s^i_{\langle C_j^i \rangle}$ of the average coefficient values. Here and below the angle brackets $\langle\rangle$ mean a value derived from the averaging over ten samples. The standard deviation $s^i_{\langle C_j^i \rangle}$ is calculated due to the standard least-square theory as it is given in Kennedy & Neville (1986). All values in Table 1 are given in CGS units. The average values of the coefficients C_1^i, C_2^i, and C_4^i do not depend on the choice of the starting point of the fringe number and the angle frame interval, contrary to the average value of the coefficient C_0^i. The last is due to a periodic character of the coefficient C_0^i. We return to a discussion of the problem in the next section. The coefficient C_0^i is associated with the position angle of the polarization ellipse relative to the antenna axis. At the time of observations, the Jovian spin axis was tilted approximately 22° clockwise from the antenna axis ($PA_\mathrm{s} \simeq -22°$) as reported by Lecacheux et al. (1991). Taking into consideration the central meridian longitude $\mathrm{CML} \simeq 120°$, Io-phase $\gamma_\mathrm{Io} \simeq 90°$, and the O4 model of the planetary magnetic field we estimate the angle between the Jovian rotational axis and the Io magnetic field line at 30MHz gyrofrequency level as equal to $PA_\mathrm{B} \simeq +44°$. For the selected Io-B storm the average degree of the linear polarization is $\langle r_\mathrm{l} \rangle \simeq 0.76$ that results in $\Omega_\mathrm{IRM} \simeq 25°$. Taking these values we estimate the polarization ellipse position $\langle \chi_\mathrm{s}^i \rangle$ (= $\langle C_0^i \rangle - PA_\mathrm{B} - PA_\mathrm{s} - \Omega_\mathrm{IRM}$) relative to the magnetic field lines in the emission source as it is given in Table 1. The table shows also the average values of the corresponding rotation measure $\langle RM^\mathrm{E} + RM^\mathrm{I} \rangle$ to be derived from the approximate formulae Eqs. (15) through (17). Using the value $\langle C_1^3 \rangle$, we deduce the Io torus plasma gradient across the emission ray path which is about $\mathrm{d}N_\mathrm{c}^\mathrm{I}/\mathrm{d}h \simeq 2\cdot 10^3 \mathrm{cm}^{-3}$. Using the plasma model by Divine & Garret (1983) we estimate a characteristic plasma scale as $L_\mathrm{N} \sim R_\mathrm{J}$.

4. Discussion and conclusion

Analyzing the propagation of elliptically polarized decametric radio emission in the Jovian magnetosphere we show that FR of the polarization ellipse has its own specific feature. This is due to the fact that the emission source height is frequency dependent resulting in a frequency dependence of the rotation measure in the Io torus. The frequency dependence of rotation measure can be taken into consideration by adding the term C/f in the linear approximate formula. Here the coefficient C characterizes the latitudinal torus plasma inhomogeneity. The frequency dependence of the rotation measure could modify the constant term in the approximate formulae by adding a constant which is proportional to the second derivation of the torus plasma density across the emission ray paths. Since the height interval occupied by the source of observed decametric storm is usually smaller than the characteristic scale in the Io plasma torus, this constant can safely be neglected.

Winglee (1986) has studied FR of decametric radio emission in the Io plasma torus and has also found a constant rotation in the Io plasma torus. This treatment of FR differs from ours in that: (a) the plasma of the Io torus is assumed to have both cold and energetic components and (b) the frequency dependence of the emission source position is neglected. The constant rotation founded by Winglee (1986) exists specifically for the case defined by the inequality

$$\frac{n_\mathrm{h}^2}{n_\mathrm{c}^2}\frac{f^2}{f_\mathrm{Be}^2}\beta_\mathrm{T}^4 \ll 1 \tag{19}$$

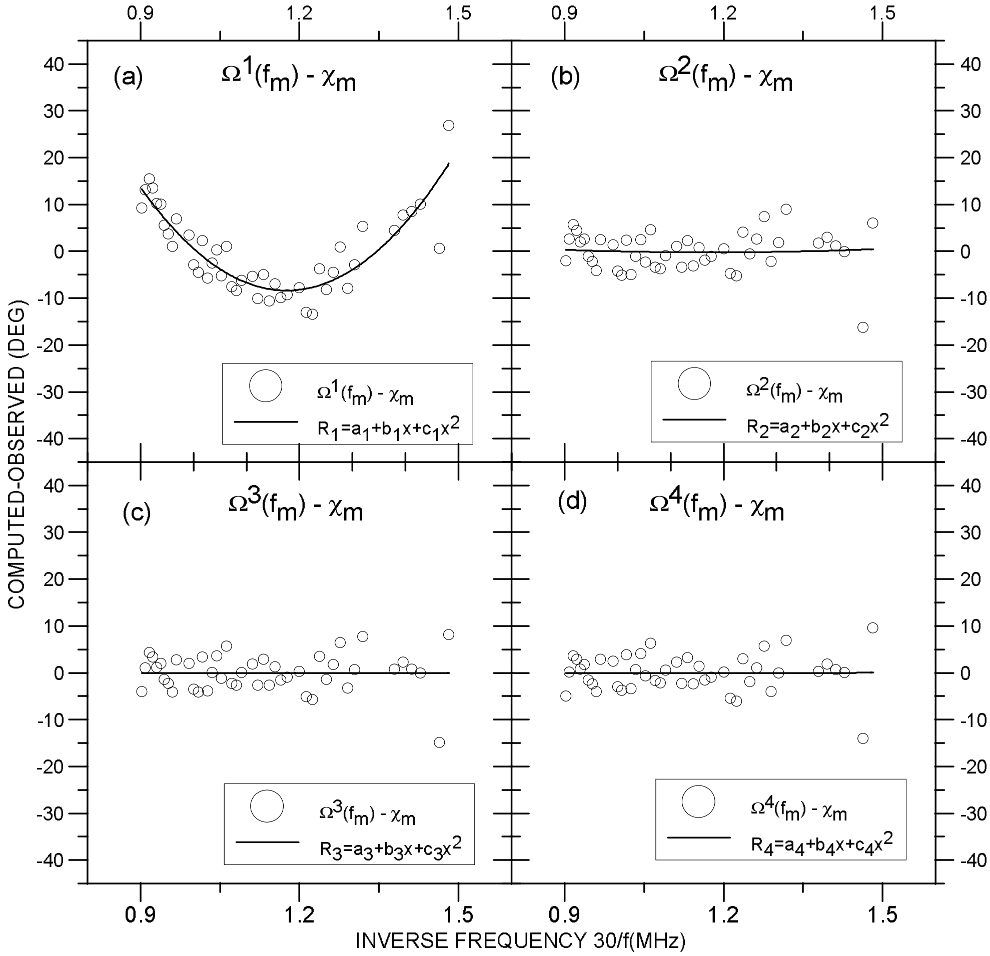

Fig. 4a–d. Residuals after the fit of different types of FR models for the spectral cut. R_i is the fitting curve and $x = 1/f$.

Table 1. The average coefficients of the sample regression curves $\langle C_j^i \rangle$, the position angle $\langle \chi_{\rm s} \rangle$, rotation measure $\langle RM^{\rm E} + RM^{\rm I} \rangle$, and their standard deviations

	$\langle C_1^i \rangle$ $\times 10^{-8}$	$s_{\langle C_1^i \rangle}$	$\langle C_2^i \rangle$ $\times 10^{-16}$	$s_{\langle C_2^i \rangle}$	$\langle C_4^i \rangle$ $\times 10^{-29}$	$s_{\langle C_4^i \rangle}$	$\langle \chi_{\rm s} \rangle$	$s_{\langle \chi_{\rm s} \rangle}$	$\langle RM^{\rm E} + RM^{\rm I} \rangle$ $\times 10^{-12}$	$s_{\langle RM \rangle}$
Ω^1			5.54	0.003			11°	2°	2.2	0.01
Ω^2			5.28	0.01	1.1	0.39	−66°	4°	2.4	0.02
Ω^3	-4.1	0.15	6.09	0.02			59°	16°	2.0	0.04

where $n_{\rm c}$ and $n_{\rm h}$ are the density of cold and energetic plasma components, respectively, $v_{\rm T} = \beta_{\rm T} c$ is the thermal velocity of energetic plasma. For this inequality (19) the left part is equal to the ratio of the constant rotation to the full amount of FR in the Io plasma torus. For the decametric radio emission the inequality (19) does not hold for a quite dense ($n_{\rm h} \sim n_{\rm c}$) and hot ($\gtrsim$ 1keV) energetic component. The existence of such a hot and dense plasma component has not yet been confirmed by observations. Moreover, if the inequality (19) is not fulfilled the presence of energetic plasma component modifies the frequency dependence of FR, and the constant rotation term does not appear in the amount of FR. Therefore, the constant rotation due to the presence of an energetic plasma component in the Io plasma torus may safely be ignored.

The statistical methods of treatment of the experimental data allow us to determine (see Table 1) the required parameters with a good accuracy in every discussed model (analytical fitting formula). However, in the different model the value of the same parameters is quite different. Therefore, it is very important to recognize the model which gives the best description of the Faraday rotation in the Jovian decametric emission. We have shown that the quadratic formula Eq. (16) and the parabolic formula Eq. (17) appear to be a better fitting with experimental data than the linear formula Eq. (15). However, these methods do not allowed us (due to both the limited number of the experimental points in the sample, about 45 points per sample, and the short frequency band of the observations, from 19MHz to 36MHz) to recognize the best fit formula among these two expressions

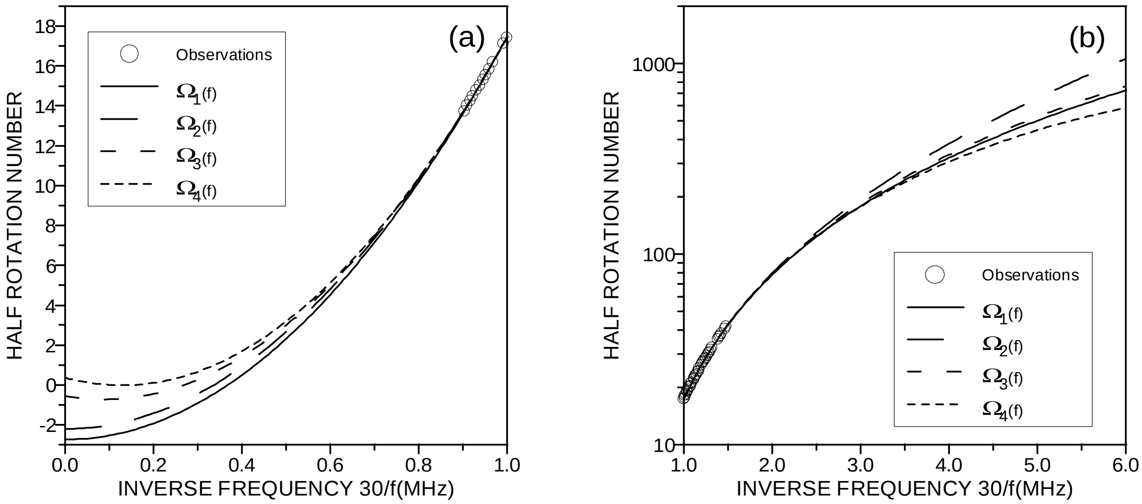

Fig. 5a and b. Extrapolation of the different fitting curves out of the measured interval

as well as to find a correct estimation of the coefficients in the "quadratic+parabolic" approximate formula Eq. (18). The latter has too many adjusting parameters for a given limited experimental data set. From Fig. 5 it is seen that the best way to make a certain choice between the last three approximate formulae based on the ground-based observations is an extension of observation frequency band. However, the natural high frequency cut-off of the Jovian decametric emission of about 39.5MHz does not allow to significantly extend the observation frequency band towards the high frequencies. From Fig. 5b we conclude that the extension of the frequency band towards the low frequencies (around 10MHz) could help us to solve the problem using on the ground-based observations. This extension requires certain improvements of measurement accuracy. The frequency separation between measured points has to be as small as about 30kHz. Solving the problem is easier when using observations outside the terrestrial ionosphere. In this case, only two approximate formulae, the linear and the parabolic, have to be tested for application. The Wind satellite, e.g., has an antennae and receiver system enabling the measurements of the full set of the polarization parameters. The large frequency band is divided into 256 channels (20 kHz bandwith, spaced every 50 kHz) in the range from 1.075MHz to 13.825MHz (Bougeret et al. 1995; Kaiser & Garcia 1997). The frequency distance between neighboring Faraday fringes is expected to be as large as about 0.5MHz at a frequency of ~ 10MHz, e.g.

We find that the standard deviations of average values of all searched parameters calculated by the approximate formula Eq. (16) are essentially smaller than the ones calculated due to the approximate formula Eq. (17). This holds in spite of the same sum of squared residuals after the fit curves Ω^2 and Ω^3 with the observational data. The differences between the standard deviations result from essentially different effective intervals of variable change (the fitting interval) on which the fit is performed. The point is, that the standard deviation is a function of inverse length of the fitting interval (Kennedy & Neville 1986). The expressions $\Omega^2 = C_0^2 + C_2^2/f^2 + C_4^2/f^4$ and $\Omega^3 = C_0^3 + C_1^3/f + C_2^3/f^2$ can be written in the same form of a second degree polynomial $P_2(x) = C_0 + C_1 x + C_2 x^2$ due to changes $(1/f)^2 = x$ and $1/f = x$, respectively. However, the intervals of the fitting equaled $\delta x = 1/f_{\min}^2 - 1/f_{\max}^2$ in the first case is larger than the interval of the fitting equaled $\delta x = 1/f_{\min} - 1/f_{\max}$ in the second case. Here $f_{\min}$ and $f_{\max}$ is a minimum and maximum frequency of the set of the observation data. In contrary, the sum of squared residuals is a function of number of the fitted points within the fitting interval rather than the length of the interval. Therefore, the sum of squared residuals is an independent criterion (not depending on a variable replacement).

Table 2. The averaged value of the coefficient C_0^i and the variance of the coefficient around its averaged value for different angle frame intervals

	$\chi_0 = 0$		$\chi_0 = -\pi/2$	
Ω^i	$\langle C_0^i \rangle$	$v_{C_0}^i$	$\langle C_0^i \rangle$	$v_{C_0}^i$
Ω^1	0.32π	0.08π	0.32π	0.08π
Ω^2	0.70π	0.30π	-0.10π	0.16π
Ω^3	0.59π	0.22π	-0.11π	0.40π

Regarding the estimation of the constant term C_0^i in the approximate formulae one has to distinguish two points. The first one is due to the uncertainty $\pm l\pi$ (l=0,1,2,3,...) when the amount of the Faraday rotation is measured at a frequency. To avoid this uncertainty the starting point has to be associated with the same Faraday fringes in different samples. The best way is the association of the frame fringe with the fringe at infinite frequency ($f = \infty$). The second point concerns a possibility of error determination of the average value of the coefficients C_0^i. The point is that the position of the polarization ellipse can be determined with equal probability by an angle χ as well as by the angle $\pi - \chi$ within the same fringe. Restriction of the variations of the angle by an interval $[\chi_0, \chi_0 + \pi]$ allows to avoid this uncertainty. However, as we demonstrate in Table 2, a shift of the frame interval (i.e. a change of the angle χ_0) could lead to a change of average values of the coefficient $\langle C_0^i \rangle$. Table 2 displays the values of the coefficient C_0^i averaged over ten samples and a dispersion $v^i_{\mathrm{C}_0}$ of the coefficients C_0^i around $\langle C_0^i \rangle$ for two from all possible frame intervals. The dispersion $v^i_{\mathrm{C}_0}$ is found as

$$v^i_{\mathrm{C}_0} = \sqrt{\frac{\sum_{l=1}^{n} (C_0^i(l) - \langle C_0^i \rangle)^2}{n-1}}. \quad (20)$$

From the table we can see that the shift of the frame interval leads to a change of both the average value of the coefficients C_0^i and the dispersion of the coefficients around their average value. This is a consequence of the periodic character of the coefficient C_0^i. We find that the error determination probability of the coefficient C_0^i as the average value over several samples is a minimum if the dispersion $v^i_{C_0}$ is the smallest. We find also that the first frame interval $0 \le C_0^i \le \pi$ is more adapted for the parabolic approximation and the second one $|C_0^i| \le \pi/2$ is more accommodated for the quadratic approximation.

From our treatment of the experimental data we conclude that the different approximate formulae give different estimations of both the rotation measure en route from the emission source till the ground-based observer and the value of the polarization angle at the emission point. Therefore, it is important to recognize the best approximate formulae. This is especially important so much the more because the parabolic approximate formula gives a possibility to monitor, additionally, the latitudinal inhomogeneity of the Io plasma torus. Using the linear approximate formula Dulk et al. (1992) have shown that the decametric storms which contain a mixture of RH and LH polarized emission being taken into consideration allow the estimate of the rotation measure in the Io plasma torus from ground-based observations. Note here, that taking a mix of polarized bursts gives us the possibility to test the validity of the parabolic formula without the expansion of the observed frequency interval. Concerning the orientation of the polarization ellipse in the source we should note that the statistical method allows us to find a function which well describes the variation of experimental data within the fitting interval. However, they do not guarantee that the found function gives a true description of the physical phenomenon outside of the interval. This case occurs when we estimate the position angle in the emission source. The constant term in the approximate formulae which is usually associated with the polarization angle is the amount of the Faraday rotation at an infinite frequency ($f \to \infty$) which is outside of the fitting interval.

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