The fundamental plane of clusters of galaxies: a quest for understanding cluster dynamics and morphology

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Abstract. We discuss implications of the fundamental plane parameters of clusters of galaxies derived from combined optical and X–ray data of a sample of 78 nearby clusters. In particular, we investigate the dependence of these parameters on the dynamical state of the cluster. We introduce a new concept of allocation of the fundamental plane of clusters derived from their intrinsic morphological properties, and put some theoretical implications of the existence of a fundamental plane into perspective.

Key words: cosmology: dark matter – galaxies: fundamental parameters – galaxies: clusters: general

1. The fundamental plane on galaxy scales

The concept of the *fundamental plane* has its origin in some monovariant relations between global observables in stellar systems. The Tully–Fisher relation for spirals and the Faber–Jackson relation for ellipticals reveal that the orbital velocity increases strongly with the luminosity. However, the residual scatter suggests the introduction of an additional observable, the effective radius, which finally leads to correlations in a three dimensional parameter space of observables, resulting in the concept of the *fundamental plane* (Dressler et al., 1987, Djorgovski & Davis 1987). Subsequently, this concept has been successfully employed to assess the physical state of an elliptical galaxy. Observationally this plane is well established in the optical energy range (e.g. Guzman et al. 1993) and the infrared energy range (e.g. Pahre et al. 1995) separately, and is defined by a multivariate correlation between characteristic parameters of elliptical galaxies. Others like spiral galaxies have been shown to populate the same plane reaching down to the dwarfs by suitably adopting the parameter space (e.g. Jablonka et al., 1996).

Beyond the claim of existence of such a plane in some (observational, but not necessarily physical) parameter space, it has proved to be a powerful concept to also investigate the physical properties as well as the evolutionary state of the galaxy (e.g. Bahcall et al., 1995). Moreover, it can be employed as a

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distance indicator (e.g. Van Albada et al., 1995). Thus, the *fundamental plane* has attained the status of a physical concept, i.e. a (conjectured) relation among physical variables.

2. Reaching out to the realm of cosmology

It is a natural question to ask whether such a concept may be extended to larger spatial scales, but there are also strong implications of such an extrapolation which have to be seriously considered.

2.1. The fundamental plane on cluster scales

Although clusters of galaxies mainly consist of elliptical galaxies, this is not a straightforward argument to expect that such a plane can be found for structures on cluster scales. Schaeffer et al. (1993) have advanced that claim, however, based on a small set of 25 optically selected clusters. Recently, Adami et al. (1998) have consolidated this concept with a sample of 29 clusters within the ESO nearby Abell cluster survey. In the present note we advocate this claim. However, we would like to stress that the luminous matter in clusters of galaxies is mainly built from the X–ray emitting intergalactic gas (10% - 30%) and not from galaxies (1% - 10%); the allocation of a *fundamental plane* should therefore be decided on the basis of X–ray data rather than optical data alone. Their combination carries additional information about the coupling between galaxies and intergalactic gas, which are both supposed to be settled down in the gravitational potential built from the hitherto unspecified 'dark matter' (70% - 90%). Based on the work by Fritsch (1997) and Fritsch & Böhringer (in prep.), which supports the existence of a *fundamental plane* in the characteristic optical and X–ray properties of galaxy clusters, we are going to investigate implications of morphometric parameters that quantify cluster substructure.

There is also a dynamical reasoning for the existence of a fundamental plane which assumes the scalar virial theorem, which was derived for isolated systems, to hold. This idealization provides a relation between the relevant physical parameters, here: the gravitational mass, the extent of the structure under consideration (measured in terms of half–light radius, see below) and the stabilizing dynamical pressure due to velocity

dispersion. We shall also consider this relation below to derive values for the mass–to–light ratio. Although simple–minded, the use of this relation stems from the common implication that an ensemble of "virialized" entities should define the *fundamental plane*. Can this be so simple?

2.2. Theoretical implications

While galaxies are easily identified as individual entities, although their dark halos may not fully follow that identification, the cluster as a structural unit is more difficult to assess. People tend to consider clusters of galaxies as the largest gravitationally bound systems in the Universe, and the wordings "decoupled from the universal expansion" and "relaxed, or virialized system" are applied to suggestively establish the possibility of isolation from the large–scale structure environment. It is clear that this can only be true for an idealized cluster; real ones are neither "relaxed", nor "decoupled" from the expansion and their environment. To write down the simplest possible formula for the relation between physical variables (the scalar virial theorem), and to use this relation as a standard of reference of a "relaxed" cluster is, at least, courageous.

What is a cluster? In general we may consider any overdense patch of matter and require some stationarity condition to hold for it *on average*. This defines the term "decoupling from the expansion". The tensor virial theorem as defined by Chandrasekhar & Lee (1968) cannot be applied *per se* to a non–isolated system. Instead a generalized cosmic virial theorem should state the relation between spatially averaged quantities for some spatial domain *embedded* into the cosmological model. This, still, requires the treatment of boundary terms that arise by averaging over an overdense portion of matter at the boundaries of the averaging domain. Within the cluster all dynamical variables in general fluctuate, and in turn the strength of the fluctuations themselves has influence on the average properties. The latter is known as the "backreaction effect" (see, e.g., Buchert & Ehlers 1997 and ref. therein). The picture to be emphasized here is that a cluster which is *embedded* into the large–scale structure environment should be subjected to a stationarity condition on average taking the internal dynamics *and* cluster boundary terms into account (compare here the so–called C–correction that has been recently applied by Girardi et al. 1998).

The merging problem: Dynamically, since we face the problem of merging, we may pick that specific clump of matter belonging to a stationary system and trace this matter back into the past by conserving its mass (the Lagrangian point of view). We so avoid that there is matter in– or outflow accross the clusters' boundary. The evolution of morphological properties of this clump of matter may then be considered, mapping the dynamical state as a function of time in terms of morphological parameters. It is here, where the idea of using morphological descriptors may provide a landmark of how to diagnose the parameter space in which we want to allocate a *fundamental plane*.

The morphology–cosmology connection: The study of the evolution of cluster morphology is a lively debated subject in connection with cosmological simulations. Especially the relation to the background cosmology is the focus of interest in this field (Evrard et al. 1993, Mohr et al. 1995, Crone et al. 1996). Still, we consider it important to understand the notion of a "relaxed" cluster from first principles. If we talk about the existence of a *fundamental plane*, then we imply this only for some asymptotic dynamical state of "relaxed" clusters for which the relation implied by the *fundamental plane* is exactly satisfied. This relation must be sought theoretically. We, here, see an intimate link between the dynamics of the cluster and a conjectured attractor in the space of the characterizing averaged variables.

We suggest to quantify the deviation from these yet unknown defining properties of "relaxed" clusters by instrinsic morphological properties such as their amount of substructure. We explain this idea now for a sample of clusters based on optical and X–ray data.

3. The optically selected and X–ray based cluster data

3.1. The sample

Using the COSMOS–/APM– and the ROSAT X–ray data robust optical and X–ray parameters for a subset of 78 clusters of galaxies from surface brightness profiles were derived in (Fritsch 1997; for details concerning the data see Fritsch & Böhringer, in prep.). To avoid the influence of evolutionary effects a homogeneous sample of clusters of galaxies in a redshift range of $0.02 \leq z \leq 0.05$ was studied. A family of parameters was so derived that, at first phenomenologically, characterize the physical state of the clusters. The most important independent parameters are the luminosities and the half–light radii R_0 (optical) and R_x (X–ray).

Fitting ellipses to each of the projected distributions gives us the optical/ X–ray centers of the clusters. The calculation of the background is based on the local galaxy/ photon distribution outside the cluster. Background corrected, differential surface brightness profiles allow us to measure the radii that contain the total light of the clusters. Finally, the half light radii were calculated from integrated surface brightness profiles. They define the radii within which half the light of the cluster is emitted. The total light L_0 of the clusters emitted from the galaxies within the optical blue band is given by all the background corrected galaxy magnitudes within a circle that contains all the light. To calculate the light below the given magnitude limit we apply the Schechter luminosity function with $M_\star = -21.8$ and $\alpha = -1.25$.

The total X–ray luminosity was calculated iteratively by using the Raymond–Smith code (Raymond & Smith 1977) for a completely ionized plasma with a typical metallicity of half the solar one, and the empirical correlation between the X–ray luminosity and the temperature $kT \propto L_x^{0.354}$ given by White (1996). The latter correlation was used because ROSAT does not provide a reasonable determination of the temperature and the literature offers too few X–ray temperatures for a correlation analysis as far as our clusters are concerned. Superpositions of other sources were detected via cross correlation with clusters stored in the NED–database (NASA/IPAC Extragalactic Data Base) and cut out, subsequently.

For all the uncertainties of the derived parameters the errors due to spatial binning and the errors of the source and background count statistics were included.

The complete list of the 79 clusters is stored in Table 2.

3.2. Morphological method of allocation

That clusters of galaxies are not arbitrarily distributed in the three–dimensional parameter space $\{L_0, L_x, R_0\}$ is not surprising, but that they lie within a fairly well–defined plane is enough reason to introduce the concept of the *fundamental plane of clusters*. To find out the physical meaning of this phenomenological "fundamental plane" is another issue. As outlined above we may approach the problem from first principles. Here, we would like to sketch a procedure which already lays down a fairly unique way to establish a diagnostic criterion of allocating the *fundamental plane*. We stress, however, that in practice the sample of clusters has to be larger than the one we are going to study in order to get statistically significant results.

Already from visual inspection we appreciate that the clusters can be classified in terms of some intrinsic structural property: the clusters should pass the test whether they are useful to define the *fundamental plane* in a physical sense. We have to find a measure of the dynamical state of the clusters of galaxies. Considering the process of structure formation and the relaxation process for the galaxies and the gas, a dynamical state may be represented by some morphological measure which characterizes the amount of substructure in the clusters. We adopt the working hypothesis that "relaxed" clusters of galaxies show a small amount of substructure and "unrelaxed" ones, which are still in the process of merging, show strong substructure. Useful substructure measures have already been proposed and employed in the literature (e.g., Crone et al. 1996 and ref. therein). We, here, base our measure on radial variations of morphological parameters derived from fitting ellipses to the projected distribution of the galaxies and X–ray photons which includes the center–of–mass shift, the ellipticity and the position angle of cluster contours (Fritsch 1997). In this line, robust structure functions based on vector–valued Minkowski functionals have been proposed recently (Beisbart & Buchert 1998) and are currently tested on simulated clusters. This new method will also help to overcome possible biases in the presently used morphological method which does not distinguish substructure from "twisted" isocontours.

3.3. Laying down the fundamental plane of clusters

Taking these substructure measures we can divide the whole sample of clusters due to the amount of their substructure into two classes with the same number of members. Now we consider the polynomial P_{EP} approximating the data for L_0: $P_{EP}(R_0, L_x) = R_0^{0.84} \; L_x^{0.21}$ (Fig. 1). It results from a two–dimensional χ^2-fit according to the Levenberg–Marquart algorithm to all the cluster data in the three–dimensional parameter space, consisting of the optical luminosity L_0, the X–ray luminosity L_x and the optical half–light–radii R_0 (we call this the
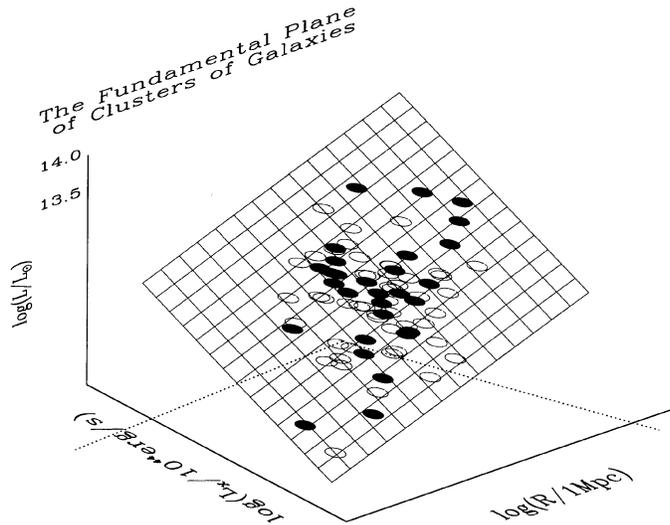

Fig. 1. The *empirical plane* for clusters of galaxies fitted to the whole sample in *face–on* projection (shown in the $\log(L_0/L_\odot) - \log(L_x/10^{44}\mathrm{erg/s}) - \log(R_0)$–space). The grid–plane results from a 2–dimensional fit to the data. The black symbols mark clusters of galaxies with less substructure in the optical and X–ray energy bands (members of the *fundamental plane*). The white symbols mark the complementary sample of clusters with more substructure.
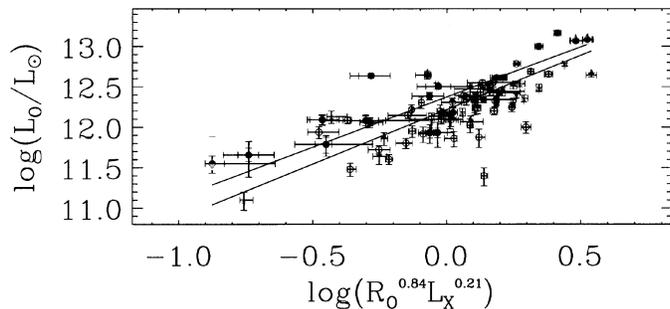

Fig. 2. The *empirical plane* ($L_0 = R_0^{0.84} \; L_x^{0.21}$) for clusters of galaxies fitted to the whole sample in *edge–on* projection (marked by the line with steeper slope). Again, the black dots mark the clusters with less substructure, whereas open circles mark clusters with more substructure in both the optical and X–ray energy bands. The line with shallower slope represents the fit to the more "relaxed" clusters defining the *fundamental plane*.

empirical plane of clusters). The orthogonal scatter of that plane is given by 24%. Applying a correlation analysis for both classes separately between the optical luminosity L_0 and the function $P_{EP}(R_0, L_x)$ we determine the probabilities for the null hypothesis that there is no correlation between $P_{EP}(R_0, L_x)$ and L_0; we obtain $\mathrm{P}(\tau_{P_{EP}; L_0}) \sim 10^{-6}$ for the clusters with less substructure, and $\mathrm{P}(\tau_{P_{EP}; L}) \sim 10^{-3}$ for the clusters with much substructure[1]. Therefore, the former class may serve as that sample which phenomenologically comprises the "relaxed" clusters (defining the *fundamental plane*) with a reduced orthogonal scatter of 21%. Additionally we find a strong correlation be-

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[1] $\tau_{P_{EP}; L_0}$ is Kendall's τ quantifying the correlation between $P_{EP}(R_0, L_x)$ and L_0 in a non–parametric way.

Table 1. Coefficients for the mass–to–light ratios for the subsamples 'EP' (*empirical plane*, all clusters), 'FP' (*fundamental plane*), 'SP' (clusters with much substructure).

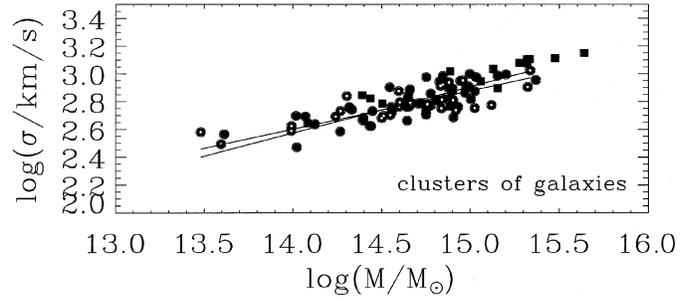| index	a	b	c
EP	448.70 ± 148.21	0.16 ± 0.17	0.09 ± 0.12
SP	613.16 ± 190.17	0.30 ± 0.14	0.08 ± 0.15
FP	318.32 ± 107.13	0.12 ± 0.18	0.10 ± 0.13

Fig. 3. The velocity dispersion versus virial mass. The velocity dispersion for the clusters is derived from the measured X–ray luminosities using the empirical relations between the X–ray luminosity and the temperature $kT \propto L_x^{0.354}$ given by White (1996). The circles denote our clusters (including a white dot, if belonging to the *fundamental plane*). The squares denote the clusters with effective radii taken from Cappi & Maurogordato (priv. comm.) and velocity dispersion taken from Struble & Rood (1991). The line with steeper slope represents the fit to all clusters corresponding to the *empirical plane*. The other line represents the fit to the more "relaxed" clusters defining the *fundamental plane*.
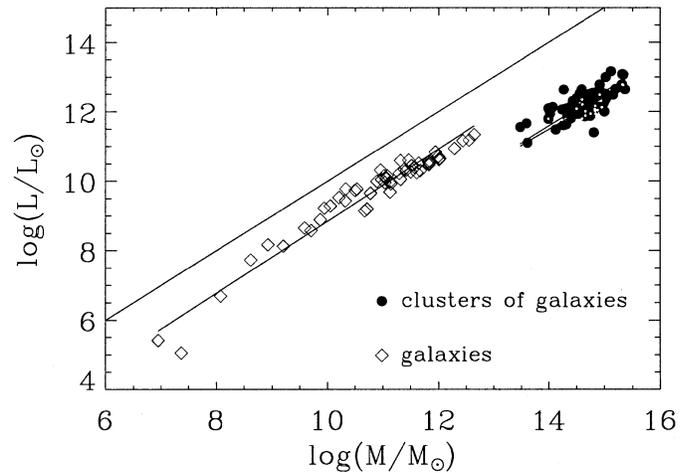
tween the distance of the clusters to the so–defined *fundamental plane* and the corresponding substructure measure (Figs. 1, 2). This correlation supports the hypothesis that the location of the *fundamental plane* is related to the feature of 'less substructure'.

Whether this morphological property uniquely relates to the virial condition, and whether the location of the *fundamental plane* is reflected correctly by the standard use of the virial relations among the parameters is not clear at all (see also Adami et al. 1998). However, in order to infer information on the mass–to–light ratio, we have to apply the virial condition in its usual, albeit naive form: to estimate the masses of the clusters of galaxies we use the empirical relations between velocity dispersion and temperature in consistency with the thermodynamical equilibrium condition ($\sigma^2 \propto T$), and between temperature and X–ray luminosity ($T \propto L_x^{0.354}$) given by White (1996). Additionally, we have to assume a density profile to find the relation between the optical half–light radius and the gravitational radius R_g. For simplicity we model the distribution of the whole matter with a King profile; note, however, that the cluster mass may be over–/underestimated by a factor of a few, if the true density profile is steeper/shallower in the core of the cluster (see: Sadat 1997). Then we start from the observed correlations between the parameters and our morphology–based allocation of the plane and transform these parameters into the physical parameter space M, σ, R_0 (Fig. 3).

3.4. Mass–to–light ratios for clusters of different morphology

Given the observed relations and the standard virial condition, $\sigma^2 = \frac{GM}{R_g}$, the *empirical plane* based on the total sample can be represented by the characteristic mass to optical light relation:

$$\frac{M}{L_0} = a \left(\frac{M}{10^{15} M_\odot} \right)^b \left(\frac{L_x}{10^{44} \text{ergs}^{-1}} \right)^c \left(\frac{M_\odot}{L_\odot} \right) \ .$$

with coefficients listed in Table 1 under 'EP'.

From Table 1 we infer that there is a clear trend of discrimination between clusters with much substructure (index 'SP') and clusters which belong to the class of more "relaxed" clusters (i.e. with less substructure in the optical and X–ray energy range) (index 'FP'). However, the respective data sets still overlap within the errors which is a consequence of the very small sample of clusters on which we base our analysis. Even if we don't admit any significant difference between the *empirical* and *fundamental* planes, there is a trend that the characteristic mass–to–light ratio for the "relaxed" clusters depends only slightly on the mass and almost does not depend on X–ray lumi-

Fig. 4. The *empirical plane* fitted to the whole sample in the $\log(L_0/L_\odot) - \log(M/M_\odot)$–projection for structures on different scales. Black circles mark clusters of galaxies and white squares mark elliptical galaxies taken from Bender et al. (1992). The upper line corresponds to objects, which would consist only of optically luminous matter.

nosity; it suggests a constant value for M/L which is a striking result of our morphology–based allocation of the *fundamental plane*. The latter would also support the assumption that the light distribution of the galaxies follows the mass distribution in "relaxed" clusters, an assumption that lies on the basis of most mass estimates (see, e.g., Girardi et al. 1998). Furthermore, in comparison with the mass–to–light ratio for elliptical galaxies (Bender et al. 1992), our data imply that the mass–to–light ratio of clusters reveals about 30 times more "non–optical luminous mass" than the elliptical galaxies which gives a hint to a certain fraction of the X–ray mass and especially to the amount of the underlying unknown dark matter (Fig. 4). It is interesting that

Table 2. List of the clusters with the luminosities L_0 (optical) and L_x (X–ray), and the optical half–light radii R_0.

cluster	L_0 $[10^{10}L_\odot]$	L_x $[10^{44}\mathrm{erg\,s^{-1}}]$	R_0 [Mpc]	cluster	L_0 $[10^{10}L_\odot]$	L_x $[10^{44}\mathrm{erg\,s^{-1}}]$	R_0 [Mpc]
a0076	143.074	1.803	0.833	a2572	105.208	2.168	1.047
a0119	451.292	11.061	1.559	a2589	89.101	3.724	0.505
a0147	157.714	1.462	1.482	a2593	176.247	3.965	1.384
a0160	220.267	0.949	1.240	a2634	226.430	2.614	1.738
a0168	210.932	2.665	1.286	a2657	116.194	5.942	0.819
a0189	99.819	1.340	2.098	a2666	200.826	0.137	1.266
a0195	134.423	0.307	0.413	a2717	1213.151	3.787	3.020
a0260	290.824	1.294	1.457	a2806	209.367	0.619	1.880
a0261	12.599	0.067	0.247	a2870	328.938	0.357	1.870
a0295	43.532	0.653	0.556	a2877	316.834	1.032	0.910
a0376	233.546	4.204	0.939	a3193	45.509	0.029	0.320
a0400	25.010	1.233	1.394	a3225	123.177	0.137	0.460
a0407	199.971	2.302	1.607	a3341	52.744	0.677	0.550
a0533	124.108	0.712	1.030	a3367	72.414	0.558	0.608
a0576	172.687	4.679	0.932	a3376	218.982	5.588	0.950
a0779	85.412	0.345	1.188	a3381	121.501	0.022	0.940
a0999	82.340	0.136	1.290	a3389	1464.532	0.815	3.270
a1100	121.335	0.688	0.940	a3390	114.425	0.309	0.610
a1139	123.110	0.706	1.138	a3395	406.866	6.350	1.050
a1142	72.391	0.720	1.170	a3554	86.541	0.326	0.358
a1177	74.972	0.513	1.645	a3558	609.462	29.090	1.440
a1185	153.499	0.938	1.193	a3560	439.546	2.673	0.640
a1213	151.170	0.925	1.088	a3577	136.677	1.230	0.940
a1228	241.229	0.266	1.165	a3706	242.537	0.693	2.242
a1314	201.711	0.990	1.063	a3716	431.446	0.492	0.550
a1367	157.068	3.069	0.705	a3736	85.228	0.327	1.110
a1644	296.538	9.702	0.720	a3744	353.983	1.265	1.360
a1656	487.000	8.000	1.409	a3747	343.648	0.285	2.900
a1736	322.886	8.790	1.150	a3816	174.029	0.882	1.380
a1983	410.034	1.237	1.713	a4038	266.279	6.633	1.070
a2052	122.214	8.781	0.600	a4049	245.423	0.215	1.750
a2063	439.425	6.862	2.711	a4059	275.124	11.593	0.960
a2107	120.115	1.755	0.379	a893004	30.240	0.159	0.590
a2147	306.000	10.023	1.448	s0141	139.191	0.219	0.981
a2148	270.398	0.779	1.504	s0316	40.202	0.086	1.020
a2151	356.908	3.300	1.198	s0585	995.286	0.628	2.890
a2152	163.925	0.488	0.833	s0639	606.948	0.530	2.400
a2162	61.319	0.091	0.530	s0892	63.376	0.273	0.910
a2197	336.479	0.463	1.093	s1065	35.631	0.082	0.170
a2199	1171.253	15.552	1.886				

our analysis suggests a tendency towards lower values for M/L in clusters as we approach the *fundamental plane*.

4. Conclusions

The data show that the nearby ($0.02 \leq z \leq 0.05$) clusters of galaxies lie preferentially within a plane in a three–dimensional parameter space built from the optical luminosity, the X–ray luminosity and the optical half–light radius. On the basis of a morphological criterion for cluster substructure and the hypothesis that "relaxed" clusters reveal less substructure, we identified a *fundamental plane* by a 2–dimensional fit to a (by a factor of 1/2) reduced sample of clusters with less substructure. The distances of the other clusters to this *fundamental plane* show strong correlations with our measure of substructure. The proposed method for allocating the *fundamental plane* should be viewed in parallel with a physical condition among spatially averaged dynamical variables, which still has to be sought. Adopting the generally held view of the standard virial relations (based on the trace of the tensor virial theorem for isolated systems), we derived the M/L values for each class of clusters implying a tendency towards lower values for more "relaxed" clusters. Our method suggests a weak dependence on mass and luminosity for clusters populating the *fundamental plane* resulting in an almost constant value for M/L of typically 300. The (orthogonal) scatter around the *empirical plane* of 24% for the sample of

all 78 clusters is reduced to a scatter around the diagnostically selected *fundamental plane* of 21% for the sample of 29 clusters with less substructure. Both are notably amplified when the L_x dependence is ignored.

For future work, this scatter can be analyzed in more detail, if one takes into account different heating mechanisms for the intergalactic gas, different profiles of the intergalactic gas and the dark matter, and different populations of galaxies within single clusters. Furthermore, the scatter around the planes may be used as an indicator of the evolution of clusters, if one applies the concept of the *fundamental plane* to samples of clusters belonging to different redshift ranges.

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