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On the mass transfer process in binary systems

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Abstract. The condition that the radius of one component of a binary star is equal to that of its Roche lobe has so far been used as a criterion for the occurrence of mass transfer. However, this criterion is only approximate. It is introduced for the simplification of the calculations from three-dimensions to one-dimension. A modified criterion is introduced, which is more strict and can also be used in one-dimensional calculations. In order to study the effect of the modified criterion on the evolution of binary systems, we calculate the evolution of a binary system consisting of a $9M_{\odot}$ star and a $6M_{\odot}$ star with mass transfer Case A and Case B. We find that the mass transfer process starts earlier and lasts longer when the modified criterion is adopted. Thus, the modified criterion for the mass transfer process has an obvious influence on the orbital period of the system, the masses and the internal structure of the component stars.

Key words: stars: binaries: close – stars: evolution

1. Introduction

The most important physical process in a binary system is the mass transfer between the two components. Many special phenomena in binary systems, such as the Algol paradox, common envelopes, accretion discs, cataclysmic variables, etc., are related to mass transfer between the components. The Roche model describes the equipotential surfaces of a binary system based on the assumptions that the masses of the two stars are concentrated at their centres and that only the effects of gravitational force and rotation are taken into account. The volume enclosed by the Roche critical equipotential surface, or briefly, the Roche lobe, is called the Roche critical volume. When a component e.g. the primary has a volume smaller than its Roche critical volume, it can be treated as an isolated body experiencing relatively small effects of the gravitational field from the secondary, and producing no mass transfer to the secondary. However, when the primary expands during the evolution to fill up its Roche critical volume, mass will flow off from the primary to the secondary through the inner Lagrangian point caused by gravitation, thus the mass transfer process starts. Therefore, the Roche critical volume is the maximum volume to which the component of a binary system can expand. Because the surface

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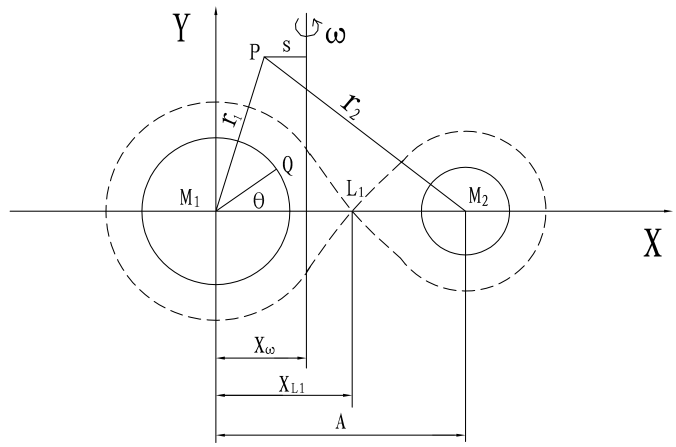

Fig. 1. Geometry of the Roche potential.

of the Roche lobe is not spherical, the calculation of the condition whether the volume of the component is equal to that of its Roche lobe, is a three-dimensional problem. However, calculations of stellar structure and evolution including binary evolution are normally one-dimensional problems. In order to simplify the calculation of the condition for the occurrence of mass transfer, the volume of the Roche lobe is then treated as an equivalent sphere, and the radius of the sphere is called the radius of the Roche lobe. This simplification has been used so far in calculations of the evolution of binary systems (Kippenhahn and Weigert 1967, Paczynski 1967a 1967b; De Loore 1980; Huang and Taam 1990; Vanbeveren 1991; De Greve 1993). The effect of the radiation pressure on the Roche lobe then on mass transfer may be important for massive binary systems (Schuerman 1972; Kondo 1974; Kondo and McClusky 1976; Vanbeveren 1976 1977 1978; Zhou and Leung 1987). However, this effect loses importance due to the large opacity during the mass transfer process (see Huang and Taam 1990). The criterion discussed above is, however, an approximate one, because the gravitational forces of the two components and the centrifugal force due to the rotation of the binary system are considered in the calculation of the radius of the Roche lobe but not in the calculation of the radius of the component star. The purpose of this paper is to introduce a new criterion for the occurrence of mass transfer process, which is stricter and can be used in one-dimensional calculations. The effect of the modified criterion on the evolution of binary systems will also be studied.

Table 1. The parameters at different evolutionary points a, b, c, d and e of the CRU1 and RAD1 sequences. The labels t, P, M_1, M_2, L, $T_{\rm eff}$, $X_1({\rm c})$, $Y_1({\rm c})$, X_1 and Y_1 denote respectively the age, orbital period, masses of the primary and of the secondary, luminosity, effective temperature, central hydrogen and helium abundance of the primary, and the hydrogen and helium abundance on the surface of the primary, respectively.

Sequence	$t(10^7{\rm yr})$	$P({\rm d})$	$M_1(M_\odot)$	$M_2(M_\odot)$	$\log(L/L_\odot)$	$\log T_{\rm eff}$	$X_1({\rm c})$	$Y_1({\rm c})$	X_1	Y_1
a										
CRU1	0.000	1.808	9.000	6.000	3.521	4.358	0.700	0.280	0.700	0.280
RAD1	0.000	1.808	9.000	6.000	3.521	4.358	0.700	0.280	0.700	0.280
b										
CRU1	2.2201	1.760	9.000	6.000	3.776	4.307	0.123	0.857	0.700	0.280
RAD1	2.2535	1.758	9.000	6.000	3.782	4.304	0.106	0.874	0.700	0.280
c										
CRU1	3.0151	8.584	2.616	12.384	2.781	3.935	0.012	0.968	0.686	0.294
RAD1	2.8805	9.002	2.564	12.436	2.792	3.930	0.014	0.966	0.679	0.301
d										
CRU1	3.0901	8.572	2.616	12.384	2.849	3.948	0.000	0.980	0.686	0.294
RAD1	2.9555	9.993	2.564	12.436	2.833	3.931	0.000	0.980	0.697	0.301
e										
CRU1	3.3435	131.235	0.931	14.069	3.376	3.763	0.000	0.975	0.121	0.859
RAD1	3.1861	101.539	1.021	13.979	3.519	3.824	0.000	0.972	0.101	0.879

Table 2. The parameters at different evolutionary points a, b, c, d and e of the CRU2 and RAD2 sequences. The labels t, P, M_1, M_2, L, $T_{\rm eff}$, $X_1({\rm c})$, $Y_1({\rm c})$, X_1 and Y_1 have the same meanings as in Table 1

Sequence	$t(10^7{\rm yr})$	$P({\rm d})$	$M_1(M_\odot)$	$M_2(M_\odot)$	$\log(L/L_\odot)$	$\log T_{\rm eff}$	$X_1({\rm c})$	$Y_1({\rm c})$	X_1	Y_1
a										
CRU2	0.000	2.835	9.000	6.000	3.521	4.358	0.700	0.280	0.700	0.280
RAD2	0.000	2.835	9.000	6.000	3.521	4.358	0.700	0.280	0.700	0.280
b										
CRU2	2.4372	2.769	9.000	6.000	3.921	4.278	0.000	0.980	0.700	0.280
RAD2	2.4378	2.765	9.000	6.000	3.923	4.274	0.000	0.980	0.700	0.280
c										
CRU2	2.4508	58.456	1.458	13.542	3.676	4.319	0.000	0.980	0.230	0.750
RAD2	2.4541	57.056	1.476	13.524	3.821	4.033	0.000	0.978	0.241	0.739

2. A modified criterion for the occurrence of mass transfer process

2.1. Roche potential

Assuming that the masses of the two stars are concentrated at their centres and taking into account gravitational forces and rotation of the system, the potential at a point P can be expressed as follows (see Fig. 1):

$$\Phi_{\rm P} = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2} - \frac{\omega^2 s^2}{2} = -\frac{GM_1}{A}\left[\frac{1}{r_1/A} + \frac{q}{r_2/A} + \frac{1}{2}(1+q)\left(\frac{s}{A}\right)^2\right] \quad (1)$$

Where r_1 and r_2 are the distances between the point P and the centres of the primary and the secondary respectively, M_1 and M_2 are the masses of the primary and the secondary, A is the orbital separation between the two components, ω the angular velocity of the system ($\omega^2 = \frac{G(M_1+M_2)}{A^3}$), $q = \frac{M_2}{M_1}$ is the mass ratio, and s is the distance of the point P from the rotation axis of the system.

2.2. Potential of the inner Lagrangian point

Using Eq. (1) the potential of the inner Lagrangian point is:

$$\Phi_{\rm L1} = -\frac{GM_1}{x_{\rm L1}} - \frac{GM_2}{(A-x_{\rm L1})} - \frac{1}{2}\omega^2(x_{\rm L1}-x_\omega)^2 = -\frac{GM_1}{A}\left[\frac{1}{x_{\rm L1}/A} + \frac{q}{1-x_{\rm L1}/A} + \frac{1}{2}(1+q)\left(\frac{x_{\rm L1}}{A} - \frac{q}{1+q}\right)^2\right] \quad (2)$$

where x_ω is the distance between the center of the primary and the mass center of the system ($x_\omega = \frac{AM_2}{M_1+M2}$), $x_{\rm L1}$ is the distance between the inner Lagrangian point and the centre of the primary. $x_{\rm L1}$ can be determined by writing that the sum of the gravitational forces and the centrifugal force at the inner Lagrangian point is zero:

$$\frac{GM_1}{x_{\rm L1}^2} - \frac{GM_2}{(A-x_{\rm L1})^2} - \omega^2(x_{\rm L1}-x_\omega) = 0 \quad (3)$$

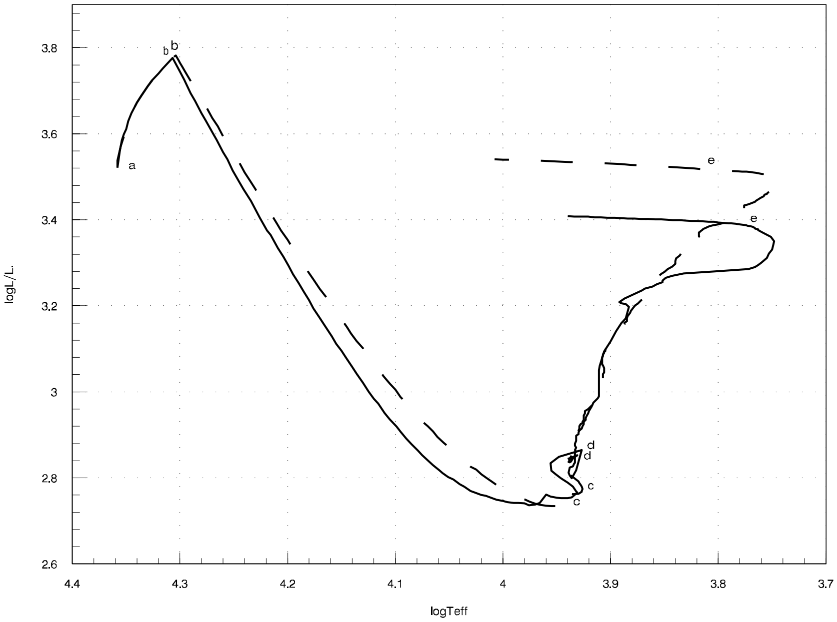

Fig. 2. Evolutionary tracks of the primary in the HR diagram for a binary system consisting of a $9M_{\odot}$ and a $6M_{\odot}$ star with Case A mass transfer. The solid and dashed curves correspond to the cases with Eq. (5) as the modified criterion for occurrence of mass transfer process and with the criterion expressed by the equilibrium of the radius of the primary to that of the Roche lobe respectively. The points a, b, c, d and e denot respectively the zero-age main sequence, the beginning of the first mass transfer phase, the end of the first mass transfer phase, the beginning of the second mass transfer phase, the end of the second mass transfer phase.

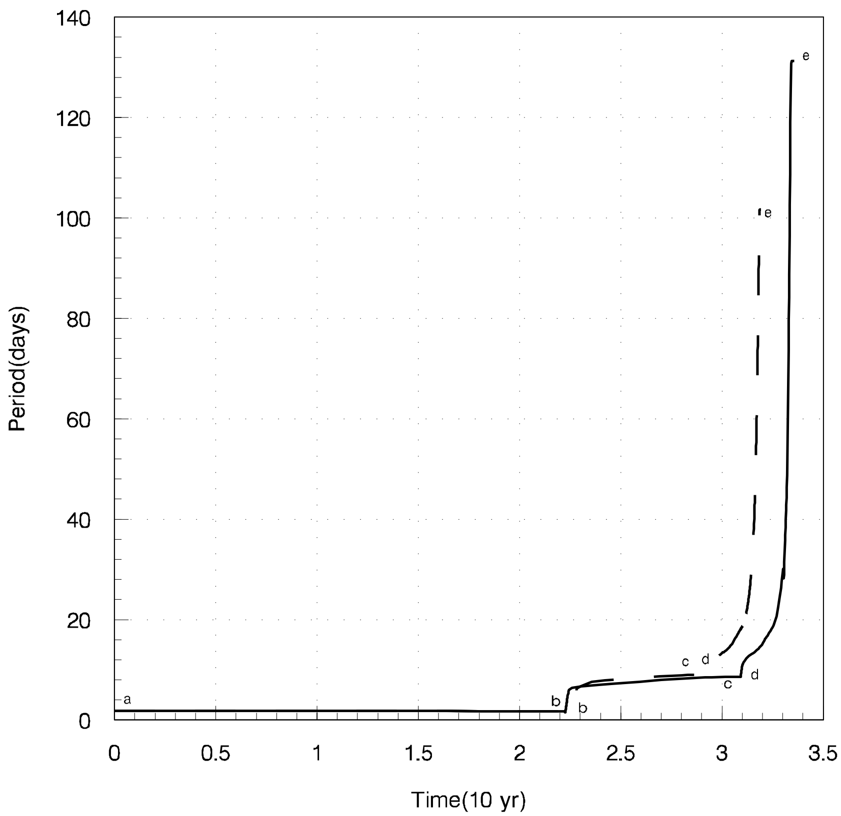

Fig. 3. The variation of the orbital period versus time for a binary system consisting of a $9M_{\odot}$ and a $6M_{\odot}$ star. The solid and dashed curves, and the points a, b, c, d and e on the curves have the same meanings as in Fig. 2

2.3. The average effective potential at the surface of the component star

Using Eq. (1) the effective potential at a point on the surface of the primary (see Fig. 1) is:

$$\Phi_{\rm Q} = -\frac{GM_1}{A}\left[\frac{1}{R_1/A} + \frac{q}{\left[1 - \frac{2R_1\cos\theta}{A} + \left(\frac{R_1}{A}\right)^2\right]^{1/2}} + \frac{1}{2}(1+q)\left(\frac{q}{1+q} - \frac{R_1}{A}\cos\theta\right)^2\right] \quad (4)$$

where R_1 is the radius of the primary. Using Eq. (4) one can calculate the effective potential at different points on the surface of the primary. Let us call $\bar{\Phi}_1$ the average effective potential of these points. It should be noticed that R_1 in Eq. (4) is not the effective radius of the primary, which considers the effects

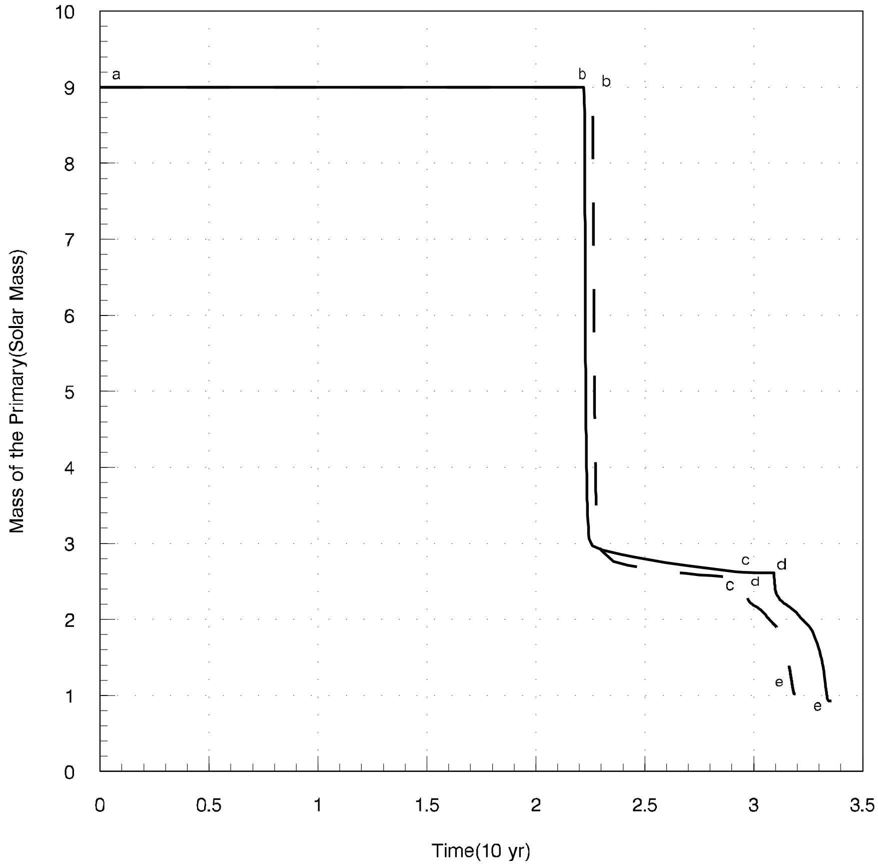

Fig. 4. The variation of mass of the primary versus time for a binary system consisting of $9M_\odot$ and a $6M_\odot$ star. The solid and dashed curves, and the points a, b, c, d and e on the curves have the same meanings as in Fig. 2

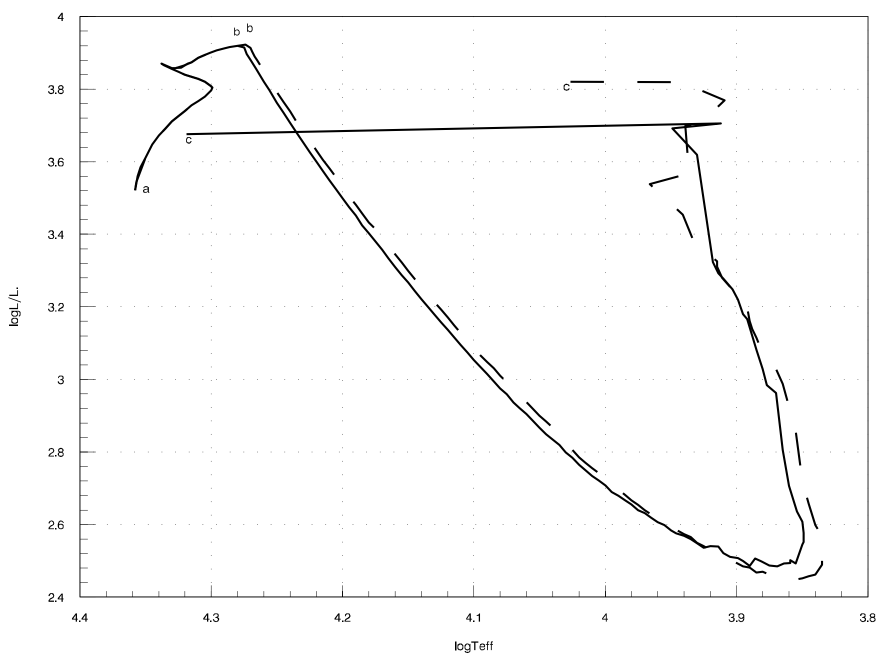

Fig. 5. The evolutionary tracks of the primary in the HR diagram for a binary system consisting of a $9M_\odot$ and a $6M_\odot$ star with Case B mass transfer. The solid and dashed curves and the points a, b and c have the same meanings as in Fig. 2

of tidal forces and rotation. If one can calculate this effective radius, $R_{1\mathrm{eff}}$, the expression of the average effective potential on the surface of the primary is simply equal to that of a single star with the radius $R_{1\mathrm{eff}}$ (i.e. $\bar{\Phi}_1 = -GM_1/R_{1\mathrm{eff}}$).

2.4. Modified criterion for the occurrence of mass transfer process

When the average effective potential on the surface of the primary is equal to the potential of the inner Lagrangian point, i.e.,

$$\bar{\Phi}_1 = \Phi_{\rm L1} \tag{5}$$

the mass will flow from the primary to the secondary. Thus, Eq. (5) can be used as the criterion for the occurrence of mass transfer. This criterion is stricter than the usual one because the gravitational forces of the two components and the centrifugal force due to the rotation of the system are considered in the calculation of both the potential of the inner Lagrangian point and the average effective potential on the surface of the primary. Furthermore, this modified criterion can be expressed in one dimension.

3. Computations and results

As an example, the evolution of a binary system consisting of a $9M_\odot$ and a $6M_\odot$ star is studied by adopting the usual and the modified criterions for the occurrence of mass transfer. The structure and evolution of the binary system is computed by a stellar structure program used for non-conservative evolution of massive binary systems (see Huang & Taam 1990) and updated to include revised opacities (cf. Rogers & Iglesias 1992; Li 1994) and energy generation rates. Owing to the fact that the rate of mass loss due to stellar wind for a $9M_\odot$ and a $6M_\odot$ star is very small, the evolution of the binary system is calculated without mass loss. The initial chemical composition of $X = 0.70$, $Z = 0.02$ is adopted for both stars. Several evolutionary sequences have been calculated for the Roche lobe overflow in Case A and Case B.

The first two sequences correspond to the evolution of Case A Roche lobe overflow with the two different criterions. The sequence denoted as CRU1 corresponds to the evolution using Eq. (5) as the modified criterion for the occurrence of mass transfer, while the sequence denoted as RAD1 assumes that the radius of the primary is equal to that of the Roche lobe as the criterion. The initial orbital separation between the components of this two sequences is taken to be $15.39R_\odot$, so that the Roche lobe overflow starts during the hydrogen-burning phase of the primary.

The parameters at different evolutionary points a, b, c, d and e of the CRU1 and RAD1 sequences are listed in Table 1. From the parameters at point b in Table 1 one finds that the mass transfer phase begins at $t = 2.2201 \times 10^7 {\rm yr}$ for the CRU1 sequence and at $2.2535 \times 10^7 {\rm yr}$ for the RAD1 sequence. Thus, the modified criterion advances the time of the onset of mass transfer by $\sim 1.5\%$. As a result, the central hydrogen content of the primary at the beginning of mass transfer phase has a value of $X_1({\rm c}) = 0.123$ for the CRU1 sequence instead of $X_1({\rm c}) = 0.106$ for the RAD1 sequence. Thus the modified criterion causes a difference of $\sim 2.5\%$ in the central H abundance. This difference in the central hydrogen contents will cause a difference in the duration of mass transfer process between the CRU1 and RAD1 sequences. The parameters at point c show that the first mass transfer phase ends at $3.0151 \times 10^7 {\rm yr}$ for the CRU1 sequence and $2.8805 \times 10^7 {\rm yr}$ for the RAD1 sequence. Thus, the first mass transfer process for the CRU1 sequence lasts $1.68 \times 10^6 {\rm yr}$ longer than that of the RAD1 sequence, because of the difference of 2.5% in the central H abundance at the onset of mass transfer. The mass of the primary at the end of the first mass transfer phase has a value of $2.62M_\odot$ for the CRU1 sequence against $2.56M_\odot$ for the RAD1 sequence. Thus, the modified criterion causes a mass difference of $\sim 2\%$. This means that the average mass transfer rate for the CRU1 sequence ($8.03 \times 10^{-7} M_\odot {\rm yr}^{-1}$) is smaller than that of the RAD1 sequence ($1.03 \times 10^{-6} M_\odot {\rm yr}^{-1}$). Furthermore, the orbital period has a value of 8.58 days for the CRU1 sequence and a value of 9.00 days for the RAD1 sequence, which corresponds to a period difference of $\sim 4.6\%$. The hydrogen and helium contents on the surface of the primary have values of $X = 0.686$ and $Y = 0.294$ for the CRU1 sequence and the values of $X = 0.679$, $Y = 0.301$ for the RAD1 sequence.

From points d to e there is a second mass transfer in both cases. The duration of the second mass transfer phase is $2.534 \times 10^6 {\rm yr}$ for the CRU1 sequence and $2.306 \times 10^6 {\rm yr}$ for the RAD1 sequence. Thus, the duration difference caused by the modified criterion increases to $\sim 9\%$ for the second mass transfer phase. From the parameters at point e in Table 1 one finds that the orbital period at the end of the second mass transfer has a value of 131.24 days for the CRU1 sequence against 101.54 days for the RAD1 sequence. The large differences in the orbital period and in the duration of the second mass transfer phase between the two evolutionary sequences show that the effect of the criterion becomes large in more advanced stages of evolution. This can be understood because the mass of the companion star increases greatly as the result of the first mass transfer phase, and thus the effect of the gravitational force from the companion on the primary becomes larger. From Table 1 and Fig. 2 we find that after the second mass transfer phase the primary will evolve to a white dwarf with a mass of $0.931M_\odot$ for the CRU1 sequence and $1.021M_\odot$ for the RAD1 sequence. Thus, the mass difference increases to $\sim 9.6\%$ in that stage of evolution. The effect may thus have some influence on the statistics for white dwarf formation through Case A mass transfer.

From Fig. 2 one sees that the evolutionary tracks of the primary for the CRU1 and RAD1 sequences are quite different since the beginning of the mass transfer phase (point b). In particular, the positions of the primary in the HR diagram for the two sequences are very different at the end of the mass transfer phase (point e). The differences in the luminosities, effective temperatures, masses and chemical abundances of the primary between the two sequences show obviously that the internal structure of the primary must be quite different.

Fig. 3 and Fig. 4 illustrate the time-dependent variation in the orbital period of the system and the mass of the primary. The solid and dashed curves in Figs. 3 and 4 correspond to the CRU1 and RAD1 sequences, respectively. The effect of the two criterions on the variations of the orbital period and mass is obvious.

The third and fourth sequences correspond to the evolution of Case B Roche lobe overflow in both cases. The sequence denoted as CRU2 corresponds to the evolution adopting Eq. (5), while the sequence denoted as RAD2 assumes that the radius of the primary is that of the Roche lobe at the onset of mass transfer. The initial orbital separation of the CRU2 and RAD2 sequences is $20.77R_\odot$.

From Fig. 5 one sees that the evolutionary tracks of the primary for the CRU2 and RAD2 sequences are quite different since the beginning of the mass transfer phase (point b). Especially,the effect on the internal structure of the primary is also very important for Case B mass transfer. The parameters at point c in Table 2 show that the period of the CRU2 sequence (58.47 days) is $\sim 2.4\%$ longer than that of the RAD2 sequence (57.06 days). The effect is less pronounced than for Case A. In conclusion, the modified criterion has an important influence on the internal structure of the primary for the evolution both of the Case A and Case B mass transfer. The effect on the orbital period and the mass of the primary is more important influence for Case A than for Case B.

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