Astron. Astrophys. 345, 256–258 (1999)

ASTRONOMY AND ASTROPHYSICS

Supernova remnants in molecular clouds: on cosmic ray electron spectra

M. Ostrowski

Obserwatorium Astronomiczne, Uniwersytet Jagielloński, ul. Orla 171, PL-30-2441 Kraków, Poland

Received 20 October 1998 / Accepted 18 December 1998

Abstract. The particle acceleration process at a shock wave, in the presence of the second-order Fermi acceleration in the turbulent medium near the shock, is discussed as an alternative explanation for the observed flat synchrotron spectra of supernova remnants (SNRs) in molecular clouds. We argue that medium Alfvén Mach number shocks considered by Chevalier (1999) for such SNRs can naturally lead to the observed spectral indices.

Key words: acceleration of particles – shock waves – ISM: cosmic rays – ISM: supernova remnants

1. Introduction

In a recent paper Chevalier (1999) discusses physical conditions in supernova remnants (SNRs) evolving inside molecular clouds. He considers a clumpy gas distribution in the clouds, where the dominant part of the cloud mass is contained in the compact dense clumps filling only 10% or less of the cloud volume and the rest of the cloud is filled with tenuous gas with a number density $N \sim 10\,\mathrm{cm}^{-3}$. Thus a SNR exploding inside a cloud evolves mostly in such low density medium. Chevalier discusses a number of consequences of such a model and compares it to observations of three SNRs: W44, IC 43 and 3C391. In his discussion the observed, very flat, cosmic ray electron distributions responsible for the radio synchrotron spectra are interpreted as shock-compressed ambient distributions radiating downstream of the radiative shock.

However, the existence of a very flat ambient electron distribution in a molecular cloud is a matter for debate. The uniform magnetic field structures observed in several clouds and efficient damping of short Alfvén waves in a partly neutral medium (Hartquist & Morfill 1984) suggests a possibility of an efficient cosmic ray exchange between clouds and a steep-spectrum galactic cosmic ray population. In the present note we point out the possibility of an acceleration of the flat spectrum electrons at a SNR shock wave, if the second-order Fermi acceleration in the vicinity of the shock is taken into account. In the next section we summarize results of an approximate analytic theory for the particle spectral indices (Ostrowski & Schlickeiser 1993, ≡ 'OS93'), with a few typing errors of the original paper corrected. Then, in section 3, we demonstrate that the physical conditions considered by Chevalier (1999) for shock waves – with the Alfvén velocity, V_A, non-negligible in comparison to the shock velocity – allow for generation of the required flat particle distributions (c.f., also, Drury 1983, Hartquist & Morfill 1983, Dröge et al. 1987; Schlickeiser & Fürst 1989). In the final section (Sect. 4) we briefly discuss an application of the present model to actual conditions in astrophysical objects.

Send offprint requests to: mio@oa.uj.edu.pl

2. Derivation of the particle spectral index

OS93 derived a simplified kinetic equation for the particle distribution formed at the parallel shock wave due to action of the first-order acceleration at the shock and the second-order acceleration in the shock turbulent vicinity. With the phase-space distribution function at the shock, $f(p) \equiv f(p, x = 0)$, an 'integral' distribution is defined as

$$F(p) = f(p) \left[\frac{\kappa_1(p)}{U_1} + \frac{\kappa_2(p)}{U_2} \right], \tag{2.1}$$

where U_i and κ_i are the plasma flow velocity in the shock frame and the spatial diffusion coefficient of cosmic ray particles, respectively ($i = 1$ upstream and 2 downstream of the shock). Analogously, the momentum diffusion coefficient is indicated as D_i ($i = 1, 2$). For the simple case of $\kappa_i = const$ ($i = 1, 2$), and $D_1 = D_2 = D$ the approximate transport equation for the function (2.1) takes the closed form:

$$- \frac{1}{p^2} \frac{\partial}{\partial p} \left\{ p^2 D(p) \frac{\partial}{\partial p} F(p) \right\} + \frac{1}{p^2} \frac{\partial}{\partial p} \left\{ p^2 \langle \frac{\Delta p}{\Delta t} \rangle F(p) \right\} + \frac{F(p)}{\tau(p)} = Q(p) \quad , \tag{2.2}$$

where we have included the source term $Q(p)$. The mean acceleration speed due to the first order process at the shock $\langle \frac{\Delta p}{\Delta t} \rangle$ and the mean escape time due to advection downstream of the shock $\tau(p)$ can be derived from the spatial diffusion equation (cf. OS93) as:

$$\left\langle \frac{\Delta p}{\Delta t} \right\rangle = \frac{R-1}{3R} \frac{U_1^2}{\kappa_1 + R\kappa_2} p \quad , \tag{2.3}$$

$$\tau(p) = \frac{R(\kappa_1 + R\kappa_2)}{U_1^2} \quad , \tag{2.4}$$

where the shock compression $R \equiv U_1/U_2$. In more general conditions with $\kappa_i = \kappa_i(p)$ the function $f(p)$ must be explicitly present in the kinetic equation:

$$\begin{aligned} &- \frac{1}{p^2}\frac{\partial}{\partial p}\left\{p^2 D_1(p)\frac{\partial}{\partial p}\left[f(p)\frac{\kappa_1(p)}{U_1}\right]\right\} \\ &- \frac{\kappa_2(p)}{U_2}\frac{1}{p^2}\frac{\partial}{\partial p}\left\{p^2 D_2(p)\frac{\partial}{\partial p}f(p)\right\} + \frac{1}{p_2}\frac{\partial}{\partial p} \\ &\left\{p^2\langle\frac{\Delta p}{\Delta t}\rangle f(p)\frac{\kappa_{ef}(p)}{U_1}\right\} + \frac{f(p)\kappa_{ef}(p)}{U_1\tau(p)} = Q(p) \; . \end{aligned} \tag{2.5}$$

Let us note in the above equation the asymmetry between the first two terms. If only the first-order acceleration takes place ($D_1 = 0 = D_2$), the known solution $f(p) \propto p^\sigma$, with $\sigma = -3R/(R-1)$, is reproduced by Eq. (2.5). Obtaining a solution for a more general situation may be a difficult task. However, if we consider momenta much above (or below) the injection momentum, and the power-law form for the diffusion coefficient $\kappa \propto p^\eta$, the solution is also a power-law. In such conditions, from Eq. (2.5) one can derive the spectral index σ. The Skilling (1975) formula is used for the momentum diffusion coefficient, relating it to the spatial diffusion, $D(p) = V_A^2 p^2/(9\kappa(p))$. For a given jump condition at the shock for the Alfvèn velocity and a given value of η, the resulting spectral index depends on only two parameters, the shock compression ratio R and the velocity ratio $V_{A,1}/U_1$.

OS93 checked the validity range of the approximate equation (2.5) with the use of numerical simulations. For $\eta \neq 0$ the equation can be used for the range of parameters preserving $V_A << U$, while, for $\kappa = const$, it provides a quite reasonable description of the particle spectrum at all $V_A < U$.

In order to find a power-law solution of Eq. (2.5) let us assume the following forms for the diffusion coefficients (i = 1, 2): $\kappa_i(p) = \kappa_{0,i}p^\eta$, $D_i(p) = D_{0,i}p^{2-\eta}$, where the constants $\kappa_{0,i}$ and $D_{0,i}$ are related according to the formula $D_{0,i} = V_{A,i}^2/(9\kappa_{0,i})$. With these formulae and the power-law form for the distribution function, $f(p) = f_0 p^\sigma$, Eq. (2.5) yields a quadratic equation for σ:

$$a\sigma^2 + b\sigma + c = 0 \quad , \tag{2.6}$$

with coefficients:

$$a = -\frac{R}{9U_1^2}(V_{A,1}^2 + RV_{A,2}^2) \quad , \tag{2.7}$$

$$b = (3+\eta)a + \frac{R-1}{3} \quad , \tag{2.8}$$

$$c = -\eta\frac{RV_{A,1}^2}{3U_1^2} + R \quad . \tag{2.9}$$

Eq. (2.6) has two solutions valid for, respectively, particle momentum much below ($\sigma > 0$) and much above ($\sigma < 0$) the injection momentum. Only the later one is of interest in the present considerations. Let us also note that the energy spectral index often appearing in the literature is $\Gamma = \sigma - 2$ and the synchrotron spectral index $\alpha = (\sigma - 3)/2 = (\Gamma - 1)/2$.

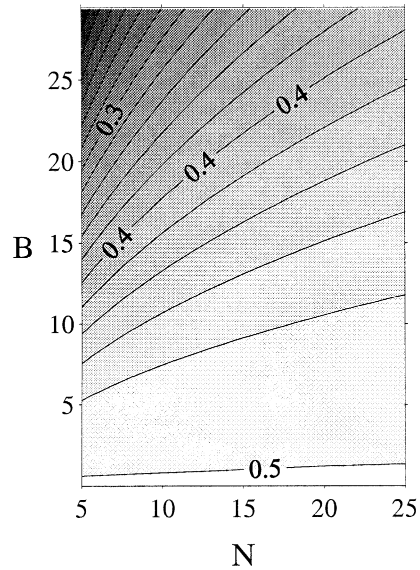

Fig. 1. A map of the synchrotron spectral index α in co-ordinates of the magnetic field B given in μG and the gas number density N in cm^{-3}, both taken upstream of the shock.

3. Spectral indices in realistic SNRs

In the SNR model discussed by Chevalier (1999) the shock wave propagates inside an inhomogeneous cloud. In the cloud, the dense clumps occupy less than 10% of the cloud volume and the shock evolution proceeds mostly in the tenuous inter-clump medium with a number density $N \sim 10\,\mathrm{cm}^{-3}$ and a magnetic field $B \approx 2 \cdot 10^{-5}$ G. Much stronger magnetic fields ($\sim 10^{-4}$ G) may occur further downstream in the radiative shock. The measured shock velocities are in the range $U_1 \approx 80 - 150$ km/s. With the notation $N_0 \equiv N/(10\mathrm{cm}^{-3})$, $B_0 \equiv B/(2 \cdot 10^{-5}\mathrm{G})$ and $U_0 \equiv U_1/(1007\,\mathrm{km/s})$ we find the ratio

$$\frac{V_{A,1}}{U_1} = 0.14 B_0 N_0^{-1/2} U_0^{-1} \quad . \tag{3.1}$$

It can be of order 0.1 for the parameters considered above. Thus, the second order acceleration process can substantially modify the energy spectrum of particles accelerated at the shock. In Fig-s 1,2 we present spectral indices derived from the formulae of the previous section for the 'canonical' Kolmogorov value $\eta = 0.67$. The shock compression ratio $R = 4.0$ for the high Mach number adiabatic shock is assumed. An efficient particle acceleration requires a high amplitude turbulence near the shock (cf. Ostrowski 1994) and oblique magnetic field configurations may occur there. Thus, the mean magnetic field downstream of the shock is $B_2 > B_1$ due to shock compression of both the uniform and the turbulent field components. Below we use an effective jump condition for the Alfvén velocity $V_{A,2} = V_{A,1}$ (in general $B_1 \leq B_2 \leq B_1 R$ and, respectively, $V_{A,1}/\sqrt{R} \leq V_{A,2} \leq V_{A,1}\sqrt{R}$). The results for $V_{A,1}/U_1 < 0.2$ are considered, where the analytic formulae provide an accurate approximation for the actual spectra.

For mean models for the SNRs discussed by Chevalier (1999) one has:

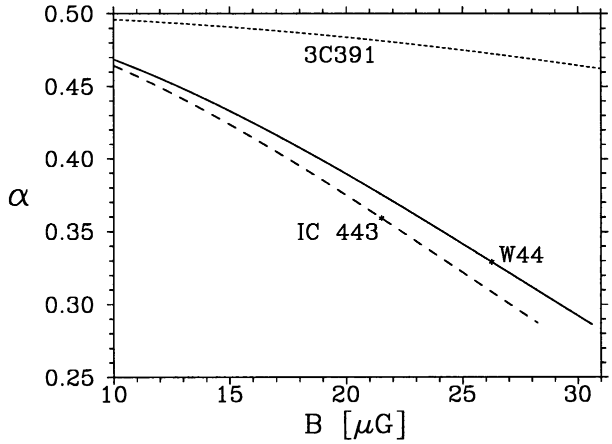

Fig. 2. The synchrotron spectral index α versus the magnetic field B for the respective values of N and U_1. The curves for different objects are indicated with the respective SNR symbol. The observed spectral indices are indicated on the curves for IC 443 and W44.

a.) for W44: $N_0 = 0.5$, $U_0 = 1.5$, $\alpha = 0.33$ ($\Gamma = 1.72$)
b.) for IC 443 (shell A): $N_0 = 1.5$, $U_0 = 0.8$, $\alpha = 0.36$ ($\Gamma = 1.66$)
c.) for 3C391: $N_0 = 1.0$, $U_0 = 3.0$, $\alpha = 0.55$ ($\Gamma = 2.1$)

With these parameters one can easily explain flat spectral indices for the cases (a) and (b) if reasonable values of the magnetic field are involved. Fig. 2 shows the spectral index α versus the magnetic field B for the above listed choices of particle density $N = N_0$ and the shock velocity $U_1 = U_0$. The measured indices are indicated on the respective curves. The steep spectrum of 3C391 can not be exactly reproduced with the use of the simple model considered in this paper, as even neglecting the second order acceleration leads, for the assumed shock compression $R = 4.0$, to a somewhat flatter spectrum with $\alpha = 0.5$. However, the observed trend in the spectral index variation is well reproduced by the model.

4. Discussion

A derivation of the particle spectral index at a shock front is presented for a situation involving the second-order acceleration process in the shock vicinity. We prove that even very weak shocks may produce very flat cosmic ray particle spectra in the presence of momentum diffusion. As a consequence, the dependence of the particle spectral index of the shock compression ratio can be weaker than that predicted for the case of pure first-order acceleration.

One should note an important feature of the acceleration process in the presence of the second-order Fermi process acting near the shock. Because the same Alfvén waves scatter particle momentum in direction and in magnitude, there exists a strict link between the first- and the second-order acceleration processes. The particle spectrum is shaped depending on $V_{A,1}/U_1$, by the compression R, the momentum dependence of κ and, possibly, by the anisotropy of Alfvén waves determining the ratio of κ to D. The last factor is not discussed here since under the present considerations we restrict ourselves to isotropic wave fields, but the role of anisotropy consists in decreasing the importance of momentum diffusion. Only through the above parameters the spectrum can be influenced by other physical characteristics of the shock and the medium in which it propagates. It is important that the Alfvén waves' amplitude, or the magnitude of the spatial diffusion coefficient related to it, do not play any substantial role in determining the spectral inclination as both, first- and second-order, processes scale with it in the same way. One could argue that an energy transfer from waves to particles can cause quick damping of the waves and will leave us with the pure first-order Fermi acceleration at the shock. From the above discussion one can infer that such an objection is not valid for the isotropic wave field. Only the presence of high amplitude one-directional wave field enables the efficient first-order shock acceleration in the absence of momentum diffusion, but a detailed calculation is not possible without a detailed model for the upstream and downstream wave fields.

In comparison to parallel shocks, the magnetic field inclined to the shock normal leads to a higher mean energy gain of particles interacting with the shock and a higher escape probability for downstream particles. Also, due to small cross-field diffusion the normal diffusive length scale near the shock decreases. As long as the shock velocity along the field is non-relativistic, the particle spectrum produced in the first-order process is not influenced by a field inclination. Because of smaller diffusive zones near the shock, the role of momentum diffusion may be of lesser importance as compared to the parallel case. This effect can be partly weakened by the presence of the magnetic field compression at the shock, which leads to higher Alfvén velocity downstream of the shock. Also the presence of high amplitude Alfvén waves (cf. Michałek & Ostrowski 1996) and an admixture of fast mode waves propagating obliquely with respect to the magnetic field (Schlickeiser & Miller 1998, Michałek et al. 1998) may lead to a more efficient acceleration than the one considered by OS93. Finaly, we would like to note a novel approach to the shock acceleration by Vainio & Schlickeiser (1999), who discuss conditions allowing for the shock generated flat spectra without action of the second order process.

Acknowledgements. I am grateful to the anonymous referee and to Horst Fichtner for valuable remarks and corrections. The present work was supported by the KBN grant PB 179/P03/96/11.

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