

## Letter to the Editor

# Neutrino emission due to proton pairing in neutron stars

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**Abstract.** We calculate the neutrino energy emission rate due to singlet-state pairing of protons in the neutron star cores taking into account the relativistic correction to the non-relativistic rate. The non-relativistic rate is numerically small, and the relativistic correction appears to be about 10–50 times larger. It plays thus the leading role, reducing great difference between the neutrino emissions due to pairing of protons and neutrons. The results are important for simulations of neutron star cooling.

**Key words:** stars: neutron – dense matter

Various mechanisms of neutrino emission in the cores of neutron stars (NSs) are important for NS cooling (e.g., Pethick 1992). In this Letter, we analyze a specific neutrino reaction associated with superfluidity of protons. The reaction consists in neutrino-pair emission  $p \rightarrow p + \nu + \bar{\nu}$  by protons whose dispersion relation contains superfluid energy gap. In theoretical studies, this process is treated as annihilation of quasiprotons  $\tilde{p}$  into neutrino pairs,  $\tilde{p} + \tilde{p} \rightarrow \nu + \bar{\nu}$ , and is referred to as *neutrino emission due to Cooper pair formation*. Similar but more powerful process takes place due to pairing of neutrons.

It is widely accepted (e.g., Takatsuka & Tamagaki 1993 and references therein), that nucleon superfluidity in the NS cores occurs mainly due to triplet-state pairing of neutrons and singlet-state pairing of protons. The neutrino emission due to singlet-state pairing of neutrons (as the simplest example) has been calculated by Flowers et al. (1976) and Voskresensky & Senatorov (1986) while the cases of triplet-state pairing of neutrons and singlet-state pairing of protons have been studied by Yakovlev et al. (1999, hereafter Paper I). The importance of neutrino emission produced by nucleon pairing has been demonstrated by simulations of NS cooling (e.g., Schaab et al. 1997, Page 1998, Paper I).

So far the neutrino energy emission rate (emissivity) due to pairing of nucleons has been calculated in the approximation of non-relativistic nucleons. We will show that the emission due to singlet-state pairing of protons is actually determined by the relativistic correction.

The process in question goes through neutral electroweak currents and is accompanied by emission of neutrinos of all flavors. As shown in Paper I, the non-relativistic term in the neutrino emissivity is produced by vector electroweak proton current and can be written as ( $\hbar = c = k_B = 1$ )

$$Q_0 = \frac{4G_F^2 m_p^* p_F}{15\pi^5} T^7 \mathcal{N}_\nu a_0 F(y), \quad (1)$$

where  $G_F$  is the Fermi weak interaction constant,  $T$  is temperature,  $m_p^*$  is an effective proton mass in dense matter (determined by the proton density of states near the Fermi level),  $p_F$  is the proton Fermi momentum, and  $\mathcal{N}_\nu = 3$  is the number of neutrino flavors. The function

$$F(y) = y^2 \int_0^\infty \frac{z^4 dx}{(e^z + 1)^2}, \quad (2)$$

depends on  $y = \Delta/T$ , where  $\Delta$  is the proton superfluid gap;  $x = \eta/T$ ,  $z = E/T = \sqrt{x^2 + y^2}$ ,  $E$  is the quasiproton energy with respect to the Fermi level,  $\eta = v_F(p - p_F)$ ,  $v_F = p_F/m_p^*$  is the proton Fermi velocity. Finally,  $a_0 = (1 - 4 \sin^2 \Theta_W)^2 \approx 0.0064$ , and  $\Theta_W$  is the Weinberg angle ( $\sin^2 \Theta_W = 0.23$ ). Numerical smallness of  $a_0$  for protons comes from their quark structure. Accordingly, the neutrino emission due to proton pairing is greatly reduced as compared with the emission due to neutron pairing; for instance, one has  $a_0 = 1$ , for singlet-state pairing of neutrons.

Let us calculate the relativistic correction  $Q_1$  to  $Q_0$  determined by the axial-vector electroweak proton currents. The corresponding interaction Hamiltonian is  $H_A = -G_F c_A J^\alpha l_\alpha / (2\sqrt{2})$ , where  $J^\alpha = \bar{\psi}_p \gamma^\alpha \gamma^5 \psi_p$  is the axial-vector proton 4-current ( $\alpha = 0, \dots, 3$ ),  $l^\alpha = (\bar{\psi}'_\nu \gamma^\alpha (1 + \gamma^5) \psi_\nu)$  is the neutrino 4-current;  $\psi_p$  is the bispinor wavefunction of quasiprotons (normalized with respect to one particle in unit volume),  $\psi_\nu$  and  $\psi'_\nu$  are the standard bispinor amplitudes of emitted neutrinos; upper bar denotes a Dirac conjugate,  $\gamma^\alpha$  and  $\gamma^5$  are Dirac matrices, and  $c_A$  is the axial-vector constant (for protons,  $c_A = -g_A = -1.26$ ). The bispinor  $\psi_p$  can be presented as combination of an upper spinor  $\phi_p$  and lower spinor  $\chi_p$ . Writing the 4-vector  $J^\alpha$  as  $(J^0, \mathbf{J})$ , we have  $J^0 = -\phi_p^+ \chi_p - \chi_p^+ \phi_p$ , and  $\mathbf{J} = -\phi_p^+ \boldsymbol{\sigma} \phi_p - \chi_p^+ \boldsymbol{\sigma} \chi_p$ , where  $\boldsymbol{\sigma}$  is the Pauli matrix.

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Let  $\hat{\Psi}$  be the non-relativistic second-quantized Bogoliubov spinor wave function of quasiprotons in superfluid matter (Lifshitz & Pitaevskii 1980). For our purpose, it is sufficient to set  $\phi_p = \hat{\Psi}$ ,  $\chi_p = -i\sigma\nabla\hat{\Psi}/(2m_p)$ , where  $m_p$  is the bare proton mass. Since the lower spinor  $\chi_p$  is small as compared to  $\phi_p$  we can neglect the term  $\chi_p^\dagger\sigma\chi_p$  in  $\mathbf{J}$ . Further considerations are similar to those in Paper I, and we omit the details.

The squared matrix elements of the interaction Hamiltonian summed over quasiproton spin states contain the tensor components  $I^{\alpha\beta} = \sum_{\text{spins}} \langle B|J^\alpha|A\rangle \langle B|J^\beta|A\rangle^*$ , where  $|A\rangle$  and  $|B\rangle$  denote, respectively, initial and final states of the quasiproton system (e.g., Paper I). Direct calculation shows that the tensor  $I^{\alpha\beta}$  is diagonal, with two nontrivial matrix elements:

$$\begin{aligned} I_t &\equiv I^{00} = (u'v + v'u)^2 |\mathbf{p} - \mathbf{p}'|^2 / (2m_p^2), \\ I_s &\equiv I^{11} = I^{22} = I^{33} = 2(u'v - v'u)^2, \end{aligned} \quad (3)$$

where  $u = \sqrt{(E + \eta)/(2E)}$  and  $v = \sqrt{(E - \eta)/(2E)}$  are the coefficients of the Bogoliubov transformation for an annihilating quasiproton with momentum  $\mathbf{p}$  and energy  $E = \sqrt{\eta^2 + \Delta^2}$ , while  $u'$  and  $v'$  are the same coefficients for a second annihilating quasiproton with momentum  $\mathbf{p}'$  and energy  $E'$ . The general expression for the neutrino emissivity  $Q_1$  due to axial-vector proton interaction  $H_A$  is readily given by Eq. (13) or (16) of Paper I, where  $c_V$  has to be formally replaced by  $c_A$ . Taking into account diagonality of  $I^{\alpha\beta}$  we obtain

$$\begin{aligned} Q_1 &= \left(\frac{G_F}{2\sqrt{2}}\right)^2 \frac{2\pi}{3} \frac{N_\nu}{(2\pi)^8} c_A^2 \int d\mathbf{p} d\mathbf{p}' f(E)f(E')\omega \\ &\times [k^2 I_t + (3\omega^2 - 2k^2) I_s]. \end{aligned} \quad (4)$$

In this case,  $\omega$  and  $\mathbf{k}$  are, respectively, energy and momentum of a neutrino pair ( $\omega = E + E'$ ,  $\mathbf{k} = \mathbf{p} + \mathbf{p}'$ ),  $f(E) = (e^{E/T} + 1)^{-1}$  is the Fermi-Dirac distribution; integration over  $\mathbf{p}$  and  $\mathbf{p}'$  is done over the kinematically allowed domain  $\omega^2 > k^2$ .

Since the proton Fermi liquid in NS cores is strongly degenerate, we set  $p = p' = p_F$  in all smooth functions under the integral. The presence of energy gaps opens the reaction kinematically in a small region of momentum space where  $\mathbf{p}$  is almost antiparallel to  $\mathbf{p}'$ . This enables us to set  $\mathbf{p}' = -\mathbf{p}$  in all smooth functions in the integrand. Then Eq. (3) gives  $I_t = 2(p_F/m_p)^2 (\Delta/E)^2$  while  $I_s \rightarrow 0$  for  $p \rightarrow p'$ . Expanding  $I_s$  in the vicinity of  $p = p'$  we obtain  $I_s = v_F^2 \Delta^2 (p - p')^2 / (2E^4)$ . After some transformations described in Paper I we come to Eq. (20) of Paper I in which again  $c_V$  should be replaced by  $c_A$ . Subsequent integration over  $p$  and  $p'$  is quite analogous, and finally we come to the expression for  $Q_1$  which can be obtained from (1) by replacing  $a_0 \rightarrow a_1$ , where

$$a_1 = c_A^2 v_F^2 [(m_p^*/m_p)^2 + 11/42]. \quad (5)$$

As seen from the derivation, both non-trivial tensor components,  $I_t$  and  $I_s$ , contribute to  $Q_1$ , producing the first and second terms in square brackets in Eq. (5), respectively. The relativistic correction to the neutrino emissivity due to singlet-state pairing of neutrons has been calculated by Flowers et al. (1976). It should be the same for neutrons and protons since the squared axial-vector constant  $c_A^2 = g_A^2$  is the same. However, the expression

obtained by Flowers et al. (1976) contains only the second term in square brackets in (5). Clearly the first term was overlooked.

Introducing  $Q = Q_0 + Q_1$  and  $a = a_0 + a_1$  we obtain the total neutrino emissivity (in the standard physical units)

$$\begin{aligned} Q &= \frac{4G_F^2 m_p^* p_F}{15\pi^5 \hbar^{10} c^6} (k_B T)^7 N_\nu a F(y) = 1.170 \times 10^{21} \\ &\times \left(\frac{m_p^*}{m_p}\right)^2 \frac{v_F}{c} T_9^7 N_\nu a F(y) \frac{\text{erg}}{\text{cm}^3 \text{s}}, \quad (6) \\ a &= 0.0064 + 1.60 (v_F/c)^2 [(m_p^*/m_p)^2 + 11/42]. \quad (7) \end{aligned}$$

Let us remind a useful relationship  $v_F/c = 0.353 (m_p/m_p^*) (n_p/n_0)^{1/3}$ , where  $n_p$  is the proton number density and  $n_0 = 0.16 \text{ fm}^{-3}$  is saturation nuclear matter density.

One can see that the relativistic correction greatly exceeds the non-relativistic term under typical conditions prevailing in the NS cores. Notice also that Eqs. (6) and (7) neglect the relativistic corrections to  $Q$  due to vector proton currents.<sup>1</sup> This is justified since the main term, in our case, is numerically small and unimportant. In the cases of any neutron pairing or triplet-state proton pairing the non-relativistic term is not numerically small and all relativistic corrections can be neglected, in practice.

The fit expression of  $F(y)$  is (Paper I)

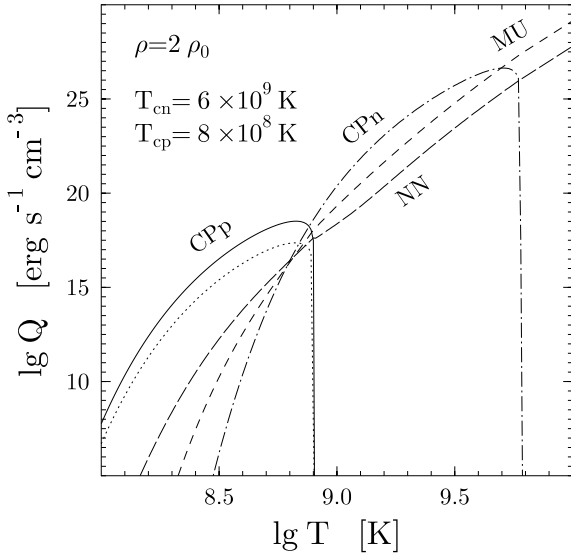
$$\begin{aligned} F(y) &= (0.602 y^2 + 0.5942 y^4 + 0.288 y^6) \\ &\times \left(0.5547 + \sqrt{(0.4453)^2 + 0.01130 y^2}\right)^{1/2} \\ &\times \exp\left(-\sqrt{4 y^2 + (2.245)^2} + 2.245\right); \quad (8) \\ y &= \sqrt{1 - \tau} \left(1.456 - 0.157 \tau^{-1/2} + 1.764 \tau^{-1}\right), \end{aligned}$$

where  $\tau = T/T_{\text{cp}} < 1$ , and  $T_{\text{cp}}$  is the superfluid critical temperature of protons. Eqs. (6) – (8) enable one to calculate the neutrino emissivity  $Q$ .

Let us discuss the importance of neutrino emission due to Cooper pairing of protons in superfluid NS cores. For illustration, we will adopt a moderately stiff equation of state proposed by Prakash et al. (1988) (the same version as used in Paper I). We assume that dense matter consists of neutrons, protons and electrons (no muons and hyperons). The equation of state allows the direct Urca process to operate at  $\rho \geq 4.64 \rho_0$ , where  $\rho_0 = 2.8 \times 10^{14} \text{ g cm}^{-3}$  is the saturation nuclear matter density. Consider two densities,  $\rho = 2 \rho_0$  and  $\rho = 5 \rho_0$ , as examples of the standard neutrino cooling (direct Urca is forbidden) and the cooling enhanced by the direct Urca process, respectively. The nucleon effective masses will be set equal to 0.7 of the masses of bare particles. Neutrons and protons are supposed to form superfluids (triplet-state neutron and singlet-state proton pairing) with critical temperatures  $T_{\text{cn}}$  and  $T_{\text{cp}}$ , respectively.

We compare the neutrino emissivity due to pairing of protons with those resulting from the main neutrino reactions in su-

<sup>1</sup> We have shown that inclusion of these corrections leads formally to Eq. (1), where  $F$  is given by Eq. (2) but with additional factor  $C$  under integral:  $C=1 + v_F^2 D/z^2$ ,  $D=-5zy^2/21 + (10z/63)(y^2 - 3x^2)/(e^z + 1) + (10/21)(z^2 + y^2) - (zm_p^*/m_p)^2$ .

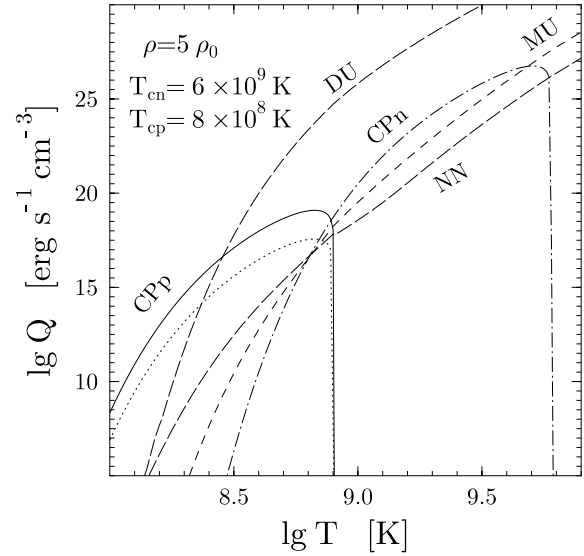


**Fig. 1.** Temperature dependence of neutrino emissivities in  $npe$  matter at  $\rho = 2\rho_0$ . The reactions are: modified Urca (MU, sum of  $n$  and  $p$  branches), nucleon-nucleon bremsstrahlung (NN, sum of  $nn$ ,  $np$  and  $pp$  branches), Cooper pairing of neutrons (CPn) and protons (CPp). The emissivity from the latter reaction is shown twice, in the non-relativistic approximation (dots), and with the relativistic correction included (solid line).

perfluid NS cores: direct and modified Urca reactions, nucleon-nucleon bremsstrahlung (as described in Yakovlev & Levenfish 1995 and Levenfish & Yakovlev 1996), and Cooper pairing of neutrons (Paper I). Our analysis shows that the emission due to pairing of protons is especially important for  $T \lesssim T_{cp} \ll T_{cn}$ . In this case, strong neutron superfluidity greatly suppresses almost all other neutrino reactions and Cooper pairing of protons becomes dominant.

Fig. 1 shows temperature dependence of the neutrino emissivities at  $\rho = 2\rho_0$ ,  $T_{cn} = 6 \times 10^9$  K and  $T_{cp} = 8 \times 10^8$  K. For  $T > T_{cn}$  matter is nonsuperfluid, and the modified Urca reaction dominates. For lower  $T$ , neutrino emission due to neutron pairing becomes most important but it dies out at  $T \lesssim 0.1 T_{cn}$ . The emission due to proton pairing dominates at  $T \lesssim T_{cp}$ . Including the relativistic correction increases its efficiency by a factor of about 10.

Fig. 2 is the same as Fig. 1 but for  $\rho = 5\rho_0$ . Now the powerful direct Urca process is allowed and dominates at  $T \gtrsim 3 \times 10^8$  K. However, in spite of its high efficiency in nonsuperfluid matter, it is strongly suppressed at lower  $T$ , so that the neutrino emission due to proton pairing becomes dominant. The relativistic correction enhances this process stronger than at  $\rho = 2\rho_0$ , by a factor  $\sim 30$ .



**Fig. 2.** Same as in Fig. 1 but for  $\rho = 5\rho_0$ . Additionally, direct Urca process (DU) is allowed.

Concluding, the relativistic correction greatly enhances the neutrino emission due to proton pairing and brings it closer to the neutrino production due to neutron pairing. This is important for NS cooling.

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