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ASTRONOMY AND ASTROPHYSICS

Relation between rotation and lightcurve of 4179 Toutatis

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Abstract. This paper presents results of modelling light variations of a freely precessing asteroid, assuming its ellipsoidal shape and a geometric light scattering law. The method is based on numerical integration of Euler equations, combined with the explicit expression of an asteroid's brightness as a function of Euler angles. Modelling is applied to simulate the lightcurve of 4179 Toutatis according to its triaxial ellipsoid shape and spin state given by Hudson & Ostro (1995). A good agreement is obtained between the frequencies of the simulated and observed lightcurves. The results explain some apparent discrepancies between the periods obtained from photometric and radar observations.

Key words: celestial mechanics, stellar dynamics – minor planets, asteroids – planets and satellites: individual: 4179 Toutatis

1. Introduction

In December 1992 the asteroid 4179 Toutatis passed within 0.0242 AU of the Earth and its photometric observations were obtained by the observers from 25 sites around the World within the international campaign. The observed lightcurve of this asteroid is very unusual and does not appear to have a single period. Lightcurve analysis carried out by Spencer et al. (1995) gave two main periods of about 7.3 and 3.1 days, however they did not explain what phenomena they are connected with. The photometric data were supplemented with radar observations of Ostro et al. (1995) performed during the 1992 approach. The periods derived from the radar data appeared to be inconsistent with the values obtained by Spencer et al. (1995). According to Hudson and Ostro (1995), the asteroid rotates in a long axis mode with periods of 5.41 days (rotation about the long axis) and 7.35 days (long axis precession about the angular momentum vector). Recently Hudson & Ostro (1998) used optical lightcurve reported by Spencer et al. (1995) and radar derived shape and spin state model to estimate Hapke parameters of this asteroid. Their synthetic lightcurves fit the optical data with rms residual of 0.12 mag.

The Fourier spectrum of the optical data of Spencer et al. (1995) has a significant noise and only the period of 7.3 days can be considerd reliable. To circumvent the insufficiency of the observational data we have developed an algorithm combining both the kinematics and the light scattering aspects of the problem. The algorithm applied to simulate the lightcurve of 4179 Toutatis has helped to resolve and explain the apparent discrepancies between the periods obtained from optical and radar data.

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2. Simulation method

The most common approximation of an asteroid's shape is a triaxial ellipsoid with the semiaxes $a \geq b \geq c$ and the principal moments of inertia $I_1 \leq I_2 \leq I_3$, respectively. Let us introduce an inertial reference frame XYZ with the origin at the ellipsoid centre. The Z axis is parallel to the angular momentum vector $\boldsymbol{M}$ of the rotating body, and the XZ plane contains a vector directed towards an observer. To simplify the situation, let us assume that the solar phase angle is equal to zero. A second reference frame $x_1x_2x_3$ has the same origin as XYZ, but the axes x_1, x_2, x_3 corresponds to the semiaxes a, b and c, respectively. The orientation of the body with respect to XYZ can be described with standard Euler angles ϕ, ψ and θ (Fig. 1).

A body is said to be in a free precession state if it rotates in the absence of external torques, so that its angular momentum vector $\boldsymbol{M}$ remains fixed in XYZ. The spin vector $\boldsymbol{\omega}$ of an ellipsoid is not parallel to $\boldsymbol{M}$ and its orientation changes in both the body-fixed $x_1x_2x_3$ and the inertial XYZ frames.

Numerical integration of Euler equations:

$$\begin{aligned} I_1 \frac{d\omega_1}{dt} + (I_3 - I_2)\,\omega_2\,\omega_3 = 0, \\ I_2 \frac{d\omega_2}{dt} + (I_1 - I_3)\,\omega_3\,\omega_1 = 0, \\ I_3 \frac{d\omega_3}{dt} + (I_2 - I_1)\,\omega_1\,\omega_2 = 0, \end{aligned} \qquad (1)$$

and

$$\begin{aligned} \frac{d\phi}{dt} &= \frac{\omega_1 \sin\psi + \omega_2 \cos\psi}{\sin\theta}, \\ \frac{d\psi}{dt} &= \omega_3 - \frac{\cos\theta\,(\omega_1 \sin\psi + \omega_2 \cos\psi)}{\sin\theta}, \\ \frac{d\theta}{dt} &= \omega_1 \cos\psi - \omega_2 \sin\psi, \end{aligned} \qquad (2)$$

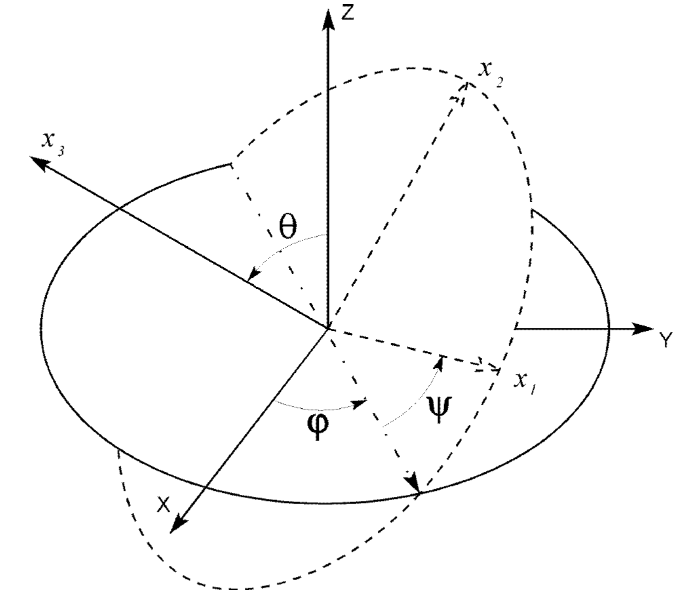

Fig. 1. Euler angles ϕ, ψ, θ describing the orientation of the body-fixed x_1, x_2, x_3 reference frame with respect to the inertial reference frame XYZ.

gives the position of an asteroid with respect to the inertial XYZ frame (Landau & Lifshitz 1976). The spin vector $\boldsymbol{\omega}$ is periodic in the body-fixed frame but not in the inertial frame, so in general the body never repeats any particular orientation.

Assuming the geometric scattering of light the brightness of an ellipsoid observed from the direction of illumination is proportional to its cross-section. It can be described by a simple equation (Connelly & Ostro 1984):

$$V = -2.5 \log \left(\pi \sqrt{\boldsymbol{e}^{\mathrm{T}}_{x_1 x_2 x_3} \, Q \, \boldsymbol{e}_{x_1 x_2 x_3} / \, |Q|} \right), \tag{3}$$

where

$$Q = \begin{pmatrix} 1/a^2 & 0 & 0 \\ 0 & 1/b^2 & 0 \\ 0 & 0 & 1/c^2 \end{pmatrix},$$

and $\boldsymbol{e}_{x_1 x_2 x_3}$ is a vector to the observer. There exists another form of Eq. (3) describing the case of non-positional geometry. It is much more complicated however, and does not introduce any changes (for phase angles smaller than $30°$) as far as basic periodicities are concerned.

Eqs. (1), (2), and (3) are sufficient to calculate the brightness V of the ellipsoid as a function of time. However, to get a better understanding of the problem, we can explicitly express V in terms of Euler angles:

$$V \;=\; -2.5 \log \left(\pi \sqrt{S(\phi,\, \psi,\, \theta\,)} \right). \tag{4}$$

Function S is a trigonometric polynomial in variables ϕ, ψ, θ, with the coefficients depending on a, b, c. Eq. (5) lists some of the terms which proved to be significant in the discussion of the Toutatis case.

$$\begin{aligned} S \;=\; & \frac{3}{8} a^2 b^2 + \frac{3}{8} a^2 c^2 + \frac{1}{4} b^2 c^2 + \\ & + \frac{1}{8} (a^2 c^2 + a^2 b^2 - 2\, b^2 c^2) \cos(2\phi) + \\ & + \frac{3}{16} a^2 (b^2 - c^2) \cos(2\phi - 2\psi) + \\ & + \frac{1}{8} a^2 (b^2 - c^2) \cos(2\psi) + \\ & + \frac{3}{16} a^2 (b^2 - c^2) \cos(2\phi + 2\psi) + \\ & + \frac{1}{16} (-a^2 b^2 - a^2 c^2 + 2\, b^2 c^2) \cos(2\phi - 2\theta) + \\ & + \frac{1}{32} a^2 (b^2 - c^2) \cos(2\phi - 2\psi - 2\theta) + ... \end{aligned} \tag{5}$$

The contribution of various terms of S to a lightcurve depends on the particular values of a, b, c. The advantage of using triaxial ellipsoid model is a simple form of Eq. (5), clearly indicating which frequencies should be present in a simulated lightcurve. On the other hand it was the only model of Toutatis available to the authors.

3. Modelled lightcurves of 4179 Toutatis

The shape of Toutatis, as revealed by radar observations, is very irregular. The triaxial ellipsoid model presented above is too simplified to describe Toutatis' lightcurve in detail. However, it is the elongation of the body that affects its brightness variation in the first place. This can be sufficiently modelled by a triaxial ellipsoid. Other effects, like nonconvex shape, scattering properties of the surface and the orientation of a body with respect to the direction to the Sun and the Earth play a secondary role at small phase angles ($< 30° - 40°$) (see e.g. review paper by Barucci & Fulchignoni 1985). Being interested only in main frequencies of the brightness variations, we assumed the ellipsoidal shape with the principal axes equal to $4.26, 2.03$ and 1.70 km (Hudson & Ostro 1995). The initial conditions for the Euler equations at the epoch 1992 Dec 11.4 were taken from Hudson & Ostro (1995). This allowed us to model the asteroid's rotation and lightcurve for a time from 20 Dec 1992 to 23 Jan 1993, covering the interval of the best photometric results available. In this period the phase angle was smaller than $40°$.

The evolution of the Euler angles is presented in Fig. 2. The precession angle ϕ circulates with an average period of 7.25 days. The rotation angle ψ also circulates with an average period of 5.4 days. These periods are in agreement with radar data. The angle θ librates around the value $130°$ with an amplitude $37°$ with a period of 7.25 days. The frequencies of rotation and precession angles are $\dot{\phi} = 0.138$ cycle/day and $\dot{\psi} = 0.185$ cycle/day.

Takingg into account all the simplifications of our model, one should not expect the amplitudes of a simulated lightcurve to be more than qualitatively correct. Indeed, the curve presented in Fig. 3 is arbitrarily scaled in brightness by factor 2. A good agreement in the epochs of the observed and modelled extrema is better visible for the vertically stretched curve. The observed brightness of Toutatis presented in Fig. 3 is reduced to zero phase angle.

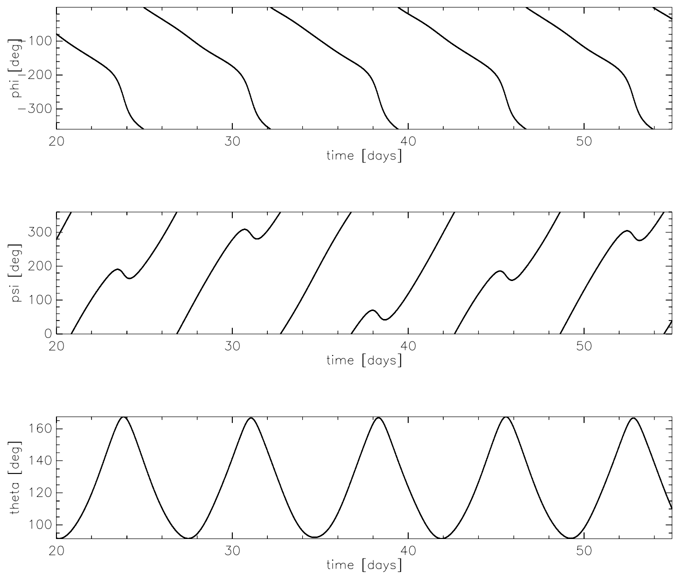

Fig. 2. Euler angles as functions of time starting from 20 Dec 95.

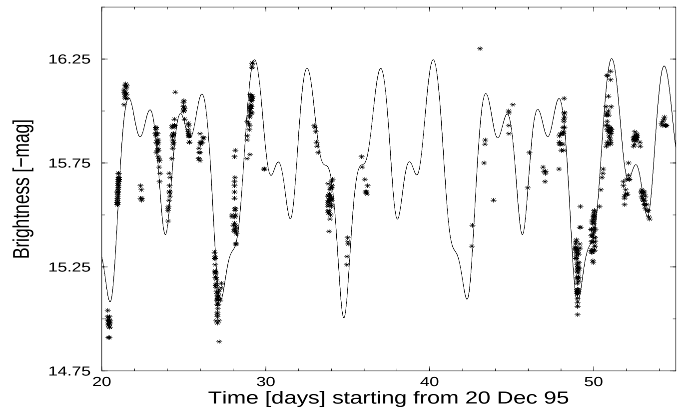

Fig. 3. Comparison of the modelled (line) and observed – stars (1995) lightcurves of Toutatis

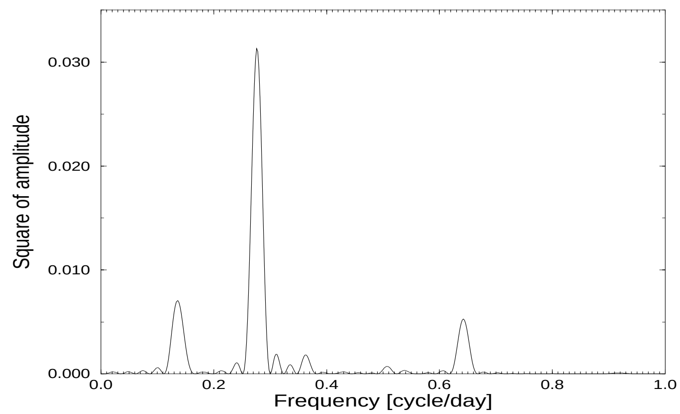

Fig. 4. Power spectrum of Toutatis simulated lightcurve.

Fourier analysis of the simulated lightcurve reveals 3 main frequencies. The power spectrum is presented in Fig. 4. We have identified the frequencies which correspond to

$$\begin{aligned} \dot{\phi} &= 0.136 \pm 0.002 \text{cycle/day}, \\ 2\dot{\phi} &= 0.276 \pm 0.002 \text{cycle/day}, \\ 2\dot{\phi} + 2\dot{\psi} &= 0.642 \pm 0.002 \text{cycle/day}. \end{aligned}$$

The first two frequencies are the precession rate and its second harmonic. Thus, the precession period is $T_1 = 7.3 \pm 0.1$ days. The last one implies a period $T_3 = 1.558 \pm 0.005$ days which is equivalent to the distance between consecutive maxima or minima. Following the tradition established in studying simpler lightcurves, Spencer et al. (1995) evaluated the periods counted between two succesive pairs of maxima or minima. Thus, to compare our results with Spencer et al. (1995) we should double T_3. The value $2T_3 \approx 3.12$ days is very close to their period of 3.1 days.

4. Conclusions

According to our results a proper physical interpretation of photometric periodicities necessarily requires the knowledge of an asteroid's rotation, at least in more complicated cases like 4179 Toutatis. We have demonstrated that the lightcurve of Toutatis is dominated by the effects of precession and the superposition of precession and rotation. The frequency of the rotation alone is not visible on the lightcurve, which makes Toutatis an interesting exception in the photometry of asteroids. Our results show that the apparent contradiction between the photometric results of Spencer et al. (1995) and the radar observations of Hudson & Ostro (1995) resulted from a misinterpretation of frequencies.

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