Thermal equilibrium of molecular clouds in cooling flow clusters

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Abstract. In many clusters of galaxies there is evidence for cooling flows in the central regions. The ultimate fate of the gas which cools is still unknown. A possibility is that a fraction of the gas forms cold molecular clouds. We investigate the minimum temperature which can be reached by clouds in cooling flows by computing the cooling function due to H_2, HD and CO molecules. As an example, we determine the minimum temperature achievable by clouds in the cooling flows of the Centaurus, Hydra and PKS 0745-191 clusters. Our results suggest that clouds can reach very low temperatures – less than $\sim 10\,\mathrm{K}$ – which would explain the non-detection of CO in these clusters.

Key words: molecular processes – galaxies: clusters: general – galaxies: cooling flows – X-rays: general

1. Introduction

In several clusters of galaxies one observes a soft X-ray excess towards the central regions. This excess is interpreted as being due to hot intracluster gas (10^7-10^8 K) with a cooling time less than the Hubble time (Fabian 1994). This gas cools and falls quasi-hydrostatically into the center of the cluster potential well (e.g. Fabian et al. 1984, 1991; Sarazin 1988).

Indeed, radial profiles of temperature and density in the intracluster gas inferred from X-ray observations show that cooler denser gas (10^6-10^7 K) is present in the central region of the cluster (Johnstone et al. 1992). Typical mass accretion rates inferred in these cooling flow clusters are ~ 100 M$_\odot$ year^{-1} and even higher in some cases (Fabian 1994). Thus cooling flows in clusters of galaxies deposit large quantities of cool gas around the central galaxy, which is still continuing to grow today. Recent extreme ultra-violet (EUV) observations revealed the presence of important amounts of a warm EUV emitting gas in the cooling flow cluster A1795 (Lieu et al. 1996a, 1996b).

The final evolution of the cool gas is not clear. It may just accumulate as cool dense clouds, indeed White et al. (1991) have observed a large amount of cold X-ray absorbing matter

distributed over the inner few 100 kpc of some clusters of galaxies and came to the conclusion that the absorbing component is likely to be either atomic or molecular (mainly hydrogen). Moreover, they suggested that the gas is very cold (< 10 K) and that it makes up a significant fraction of the total mass, which has cooled out of the cooling accretion flow during the lifetime of the cluster.

Fabian et al. (1994) have postulated that dust can form in the cold clouds embedded in cooling flows, which would explain why the clouds are almost undetectable outside the X-ray wavelength range. However, the nature of the absorbing material remains uncertain. The limits imposed by 21-cm and CO observations, suggest that it is unlikely that significant amounts of cold gas can remain undetectable. On the other hand Jaffe & Bremer (1997), using K-band spectroscopy, have found that the inner few kpc of central cluster galaxies in cooling flows have strong emission in the H_2(1-0)S(1) line. This is not seen in a comparable sample of non cooling flow galaxies. Jaffe & Bremer (1997) suggest that it is likely that a large mass of additional molecular material is in the cooling flow, producing the soft X-ray absorption. At least a fraction of the cool gas must reach the inner regions of the central cluster galaxies, where it could accumulate into molecular clouds and subsequently form stars (Cardiel et al. 1998).

The metallicity of a cluster seems to be correlated with the presence of a cooling flow (Allen & Fabian 1998). Mushotzky & Lowenstein (1997) showed that most of the enrichment of the intercluster medium occured at high redshifts. This is consistent with the semi-analytic models of galaxy formation (Kauffmann & Charlot 1998), which find indeed that the metal enrichment occurs at $z > 1$. In this context molecules such as CO can subsequently be formed in the gaseous medium. The molecular gas can cool down to very low temperatures within the intracluster environment as has been suggested by Ferland et al. (1994).

O'Dea et al. (1994) searched for molecular gas, by looking for CO emission lines in a heterogeneous sample of five radio-loud galaxies (three of which are in cooling flow clusters), using the Swedish-ESO Submillimeter Telescope in the frequency ranges of 80–116 GHz and 220–260 GHz. A positive detection of CO has been made in two of the galaxies: PKS 0634-206, a classical double radio galaxy, and possibly in NGC

4696. They come to the conclusion that in order to have escaped detection the gas has to be very cold, close to the temperature of the Cosmic Background Radiation (CBR). O'Dea et al. (1994) estimated also the expected temperature of the clouds by comparing the X-ray heating with the cooling rate for molecular gas. For the latter they adopted the cooling rate calculated by Goldsmith & Langer (1978), which includes line cooling from both molecular and atomic species likely to be abundant in molecular clouds. They thus assumed that the clouds have a metallicity similar to solar neighbourhood clouds. However, the Goldsmith & Langer cooling rates are valid in the range $\sim$ 20–60 K, below $\sim$ 20 K they are no longer reliable. In addition the possible role of HD as a cooling agent was not taken into account. Therefore, the conclusion of O'Dea et al. (1994), that the minimum temperature of the clouds is in the range 20–30 K is certainly questionable.

Recently, Bridges & Irwin (1998) have observed molecular gas in the Perseus cooling flow galaxy NGC 1275 and have mapped the central arcminute of this galaxy with the James Clerk Maxwell Telescope in $^{12}CO(2-1)$. Assuming a galactic conversion between CO intensity and the molecular H_2 gas mass they inferred a total mass $M_{H_2} \sim 1.6 \times 10^{10}$ M$_\odot$. They argue that the reason why CO has been detected in NGC 1275 is that the radio emission heats up the molecules to a detectable temperature. Indeed, NGC 1275 happens to have one of the strongest radio core of any cooling flow central galaxy. Without the heating of the radio emission the gas would stay at much lower temperatures and thus escape detection.

The aim of this paper is to investigate the minimum temperature achievable by clouds in cooling flows by improving the analysis done by O'Dea et al. (1994), in particular by considering clouds made of H_2, HD and CO molecules and by computing cooling functions which are more appropriate for temperatures below 20 K. We do not discuss the different pathways for forming H_2, HD and CO molecules. H_2 and HD can be formed at the early epoch of the recombination of the hydrogen (see Lepp & Shull 1984, Puy et al. 1993), whereas CO formation can be triggered by the radiative association of C with O after the first star formation.

The paper is organised as follows: in Sect. 2 we recall the effects of X-ray heating on the clouds using the results of Glassgold & Langer (1973) and O'Dea et al. (1994). In Sect. 3 we calculate the molecular cooling due to H_2, HD and CO, respectively, through the transition between the ground state and the first rotational level, since we are interested on what happens for very cold clouds (with temperature of order 20 K or even below). Moreover, also from the observational point of view the transition between the ground state and the first excited state gives possibly the strongest contribution at very low temperatures as suggested by the measurements by Jaffe & Bremer (1997) on the emission in the H_2(1-0) S(1) line. We then estimate in Sect. 4 the equilibrium temperatures of the clouds for the Centaurus, Hydra and PKS 0745-191 clusters, and discuss the observational consequences.

Some technical details of the calculations are given in two Appendices.

2. X-ray heating of the clouds

The clouds are embedded in the hot intracluster gas, whose emission is dominated by thermal bremsstrahlung and thus the clouds are mainly heated by X-rays. The mechanisms by which soft X-rays heat a gas cloud have been extensively studied by a number of authors: Habing & Goldsmith (1971) and Bergeron & Souffrin (1971) for H_I regions, Glassgold & Langer (1973) for molecular hydrogen clouds.

Due to the increased number of possible charged-particle reactions with molecules, the heating mechanism for molecules is different and more complicated as compared to the heating processes for atoms alone. Glassgold & Langer (1973) have given a survey of the physical processes for a pure molecular hydrogen medium. While X-ray heating of molecular hydrogen regions has been extensively studied, this has not been the case for other molecular species. Glassgold & Langer (1973) discuss in detail the energy loss processes and present a quantitative analysis which shows which part of the incident X-ray energy will go into heat. In Appendix A we recall the heating processes for H_2, HD and CO molecules which are of interest for us.

In order to evaluate the bremsstrahlung flux coming from the intracluster gas, we must define a density distribution for the electrons in the gas. A classical profile is a non-singular isothermal sphere (Jones & Forman 1984):

$$n_e(R) = \frac{n_o}{1 + \left(\frac{R}{r_c}\right)^2} \text{ cm}^{-3},$$

where $n_o \sim 10^{-3}$ cm^{-3} is the central gas number density, r_c is the core radius and R is the distance from the center of the cluster. In cooling flow clusters the central density is somewhat higher in the range $10^{-3} - 10^{-2}$ cm^{-3} (see Stewart et al. 1984).

For temperatures $T > 2$ keV the emission from the hot gas is dominated by thermal bremsstrahlung and the emissivity for an ion with charge Z and number density n_i in a gas with electron density n_e is

$$\begin{aligned}
\epsilon_\nu &= 2.4 \times 10^{-16} Z^2 n_e n_i T_{keV}^{-1/2} \exp(-E_{keV}/T_{keV}) \overline{g}_{ff} \\
&= \epsilon_\nu^o Z^2 n_e n_i \text{ (in keV cm}^{-3}\text{ s}^{-1}\text{ keV}^{-1}\text{ sr}^{-1}),
\end{aligned} \quad (1)$$

where $\overline{g}_{ff}$ is the velocity-averaged Gaunt factor, while gas temperature T and photon energy E are given in keV. The Gaunt factor for the relevant parameter range is accurately given by

$$\overline{g}_{ff} \sim 0.55 \ln(2.225) \frac{T_{keV}}{E_{keV}} . \quad (2)$$

The bremsstrahlung "flux" (i.e. $4\pi J_E$, where J_E is the mean intensity at energy E), seen by a cloud located at a radial distance R from the center of the cluster is given by O'Dea et al. (1994) in keV cm^{-2} s^{-1} keV^{-1}

$$\begin{aligned}
4\pi J_E(R) &= \frac{1.5\pi n_o^2 r_c^4}{(r_c^2 + R^2)^{3/2}} \epsilon_\nu^o \int_0^{R_b} dr \int_0^\pi d\theta \\
&\quad \times \frac{r^2 \sin\theta (r_c^2 + R^2)^{3/2}}{(r_c^2 + r^2)^2 (r^2 + R^2 - 2rR\cos\theta)} .
\end{aligned} \quad (3)$$

The factor 1.5 takes into account the presence of He and heavy elements. R_b is the outer boundary of the cluster gas distribution.

Table 1. Parameters for the X-ray heating (see O'Dea et al. 1994).

τ	γ_o	α	δ
0.01–0.5	1.2×10^{-28}	0.1	-0.65
0.5–2.5	4.6×10^{-29}	1/2	-1.1

The integral in Eq. (3) for typical values $r_c = 100\,\mathrm{kpc}$ and $R_b \sim 500\,\mathrm{kpc}$ with $0 \leq R \leq 500\,\mathrm{kpc}$ varies between 1.57 and 7. A reasonable average value for the integral is π, which for clouds located in the inner regions of the cluster is an upper bound.

The problem is much more complicated if we consider the elastic scattering of electrons from H_2 molecules. We have various dissociation and ionization channels which contribute to the heating (see Glassgold & Langer 1973). O'Dea et al. (1994) assume that the fraction of primary photoelectron's energy which goes into heating is close to half.

Moreover, O'Dea et al. (1994) consider a possible attenuating column density within a cloud, which they characterize by the parameter τ. Finally, with these different approximations they obtain for the heating rate (in $\mathrm{erg\,cm^3\,s^{-1}}$):

$$\Gamma_X(R) = \gamma_o \frac{n_{H_2} n_{-3}^2 r_{100}}{\left[1 + \left(\frac{R}{r_c}\right)^2\right]^{3/2}} T_{keV}^\alpha \tau^\delta , \qquad (4)$$

where the parameters are given in Table 1 and $n_{-3} = n_o/10^{-3}\,\mathrm{cm^{-3}}$ and $r_{100} = r_c/100\,\mathrm{kpc}$.

3. Molecular cooling

In the standard Big Bang models primordial chemistry took place around the epoch of recombination. At this stage the chemical species essentially were H, H^+, D, D^+, He and Li. Then with the adiabatic cooling of the Universe due to the expansion, different routes led to molecular formation (Lepp & Shull 1984, Black 1988, Puy et al. 1993, Galli & Palla 1998).

Heavier molecules than H_2 play an important role in the cooling of the denser clouds (Lepp & Shull 1984, Puy & Signore 1996, 1997, 1998a, 1998b) via the excitation of rotational levels. Ferland et al. (1994) have investigated the physical conditions within molecular clouds, particularly the structure of gas clouds in cooling flow and showed that molecular cooling plays an important role.

We calculate the molecular cooling in the range of temperatures ~ 3–$120\,\mathrm{K}$, where $120\,\mathrm{K}$ corresponds to the temperature of the transition between the ground state ($J = 0$) and the first rotational level ($J = 1$) for the HD molecule. In this range, molecules such as H_2, HD and CO are very efficient for cooling. HD and H_2 are the main cooling agents between 80–120 K, then HD dominates between ~ 40-80 K (H_2 cooling, in this case, turns out to be negligible). CO can be the main coolant in the range of temperature $\sim 3\,\mathrm{K}$ up to $\sim 40\,\mathrm{K}$ depending on its abundance. In order to calculate analytically the molecular cooling, we only consider the transition between the ground state and the first rotational level, although below $120\,\mathrm{K}$ it is well

possible to excite other rotational levels of CO, for instance the next higher transition ($J = 1 \rightarrow J = 2$) corresponds to a temperature of $\sim 11\,\mathrm{K}$. The reason is that we are interested on what happens with very cold clouds (with temperature of order $20\,\mathrm{K}$ or less) and by considering the higher levels the molecular cooling gets even more efficient. Thus by taking into account only the first level we will get a lower bound on cooling and, therefore, an upper bound on the equilibrium temperature.

The cooling functions for the molecules H_2, HD and CO are computed in Appendix B. We have taken into account radiative transfer effects following the treatment by Castor (1970), Goldreich & Kwan (1974) in a similar way as done in the paper of Goldsmith & Langer (1978). Thus, the rates for absorption and stimulated emission are given entirely in terms of two locally determined quantities, the source function and the escape probability β_o which depend on the molecule (see Appendix B). Finally we obtain for the cooling functions for H_2, HD and CO (in $\mathrm{erg\,cm^3\,s^{-1}}$):

$$\Lambda_{H_2} \sim 8.61 \times 10^{-24} \beta_o^{H_2} n_{H_2}^2 \sqrt{T_m} \qquad (5)$$
$$\times \frac{exp(-512/T_m)}{0.69\beta_o^{H_2} + \sqrt{T_m}n_{H_2}\left[1 + 5exp(-512/T_m)\right]} ,$$

$$\Lambda_{HD} \sim 2.66 \times 10^{-21} \beta_o^{HD} \eta_{HD} n_{H_2}^2 \sqrt{T_m} \qquad (6)$$
$$\times \frac{exp(-128.6/T_m)}{1416\beta_o^{HD} + \sqrt{T_m}n_{H_2}\left[1 + 3exp(-128.6/T_m)\right]} ,$$

$$\Lambda_{CO} \sim 1.55 \times 10^{-22} \beta_o^{CO} \eta_{CO} n_{H_2}^2 \sqrt{T_m} \qquad (7)$$
$$\times \frac{exp(-5.56/T_m)}{2659\beta_o^{CO} + \sqrt{T_m}n_{H_2}\left[1 + 3exp(-5.56/T_m)\right]} ,$$

with the ratios $\eta_{HD} = n_{HD}/n_{H_2}$ and $\eta_{CO} = n_{CO}/n_{H_2}$. The various quantities entering in the above equations are defined in Appendix B.

Combes & Wiklind (1998) have detected molecular absorption lines systems with the 15-m Swedish-ESO Submillimetre Telescope at high redshift. In particular, they have detected CO absorption lines in some galaxies or quasars. For Centaurus A they estimated the column density for CO and H_2 to be $N_{CO} = 10^{16}\,\mathrm{cm^{-2}}$ and $N_{H_2} = 2.0 \times 10^{20}\,\mathrm{cm^{-2}}$, respectively. For some clusters they could only derive an upper bound of about $6 \times 10^{15}\,\mathrm{cm^{-2}}$ for the CO column density. The values for the CO and H_2 column densities can vary substantially between different clusters.

Moreover, we do not know if the column density is due to the sum of several smaller clouds which are present along the line of sight or to a unique big cloud. It is thought that a Jean's unstable and isothermal cloud fragments into a small number of subclouds, that in turn repeat the process recursively at smaller scale as long as isothermal conditions prevail. Thus it is not unrealistic to consider very small clouds in cooling flow as a result of fragmentation (Fabian 1992). Indeed, the high pressure in a flow implies that the Jeans mass of a very cold cloud is small (Fabian 1992). We expect that the molecules are

Table 2. Escape probability β_o^{CO} for a photon emitted in a transition $J = 1 \rightarrow J = 0$ for the CO molecule, assuming a column density $N_{CO} = 10^{14}$ cm^{-2} of the cloud.

T_m	2.735	5	20	10
β_o^{CO}	0.47	0.72	0.89	0.96

mostly produced in the small clouds where the densities can be higher with respect to the surrounding regions and therefore also the cooling will be most efficient there.

These small clouds are most likely embedded in bigger less dense clouds with decreasing molecular abundance and also higher temperatures. However, the gas surrounding the small clouds will attenuate the incoming bremsstrahlung flux as taken into account by the τ parameter discussed in Sect. 2.

A scenario with successive fragmentation of the clouds in cooling flows has been proposed by Fabian & Nulsen (1994) and in another context, namely for the dark matter in the disk of a galaxy by Pfenniger & Combes (1994), whereas in the halo of our galaxy by De Paolis et al. (1995). Pfenniger & Combes (1994) have proposed that the cold H_2 clouds have a fractal structure.

We will thus in the following assume as a picture that the clouds fragment into small ones as a result of cooling processes. For definiteness we will adopt the following column densities for a typical small cloud: $N_{CO} = 10^{14}$ cm^{-2}, $N_{H_2} = 2 \times 10^{18}$ cm^{-2} with $n_{H_2} = 10^6$ cm^{-3} (density of H_2 the collisional species). We have the following ratio $\eta_{CO} \sim \frac{N_{CO}}{N_{H_2}} \sim 5 \times 10^{-5}$. O'Dea et al. 1994 instead use the value $\eta_{CO} = 10^{-4}$. For HD we take $\eta_{HD} \sim \frac{N_{HD}}{N_{H_2}} \sim 7 \times 10^{-5}$, which corresponds to the primordial ratio (Puy et al. 1993). This should possibly be an upper limit. These characteristics correspond to a size for the cloud of $L \sim 10^{-6}$ pc which is close to 30 AU as for instance proposed in the fractal model of Pfenniger & Combes (1994).

Using the above values we can then estimate the escape probability for a photon emitted in a transition in the temperature range ~ 3 K to 120 K due to the three considered molecules. The escape probabilities due to HD and H_2 are close to 1 ($\beta_o^{H_2} = \beta_o^{HD} \sim 1$) in the range where HD and H_2 are dominant ($T_m > 40$ K). In the case of CO molecule, which is supposed to be the main cooling agent at very low temperature, the probability, that the photons emitted during the transitions $J = 1 \rightarrow J = 0$ are reabsorbed, is non negligible and thus the escape probability β_o^{CO} is below 1. In Table 2 we give some values of β_o^{CO} for different temperatures. Particularly at low temperatures re-absorption has to be taken into account and leads thus to a less efficient CO cooling.

Our calculations differ from the ones obtained by O'Dea et al. (1994), since they used the cooling rates calculated by Goldsmith & Langer (1978), which include line cooling from both molecular and atomic species likely to be abundant in disk molecular clouds. Moreover, the possible role of HD as a cooling agent was not taken into account. Goldsmith & Langer (1978) considered molecular hydrogen densities in the range 10^2 to 10^5 cm^{-3}, kinetic temperatures between 10 K and 60 K

and a wide variety of molecules with fractional abundances bigger than or comparable to 10^{-9} (H_2, CO, H_2O, O_2, C_2, N_2 and hydrides). They find, like us, that below 40 K CO is the dominant coolant. Nevertheless, they approximated the cooling rate by a power law for temperatures between 10 K and 60 K. Instead, we approximate the molecular cooling by considering only the transition between the ground state and the first rotational level. We examine the equilibrium of clouds for temperatures below 10 K, a range where the power law dependence of Goldsmith & Langer (1978) is no longer valid.

4. Results and discussions

We have seen that molecular clouds in cooling flows are heated by the external X-ray flux. At equilibrium the heating and molecular cooling are equal: $\Gamma_X = \Lambda_T$, where the total cooling function is defined as $\Lambda_T = \Lambda_{H_2} + \Lambda_{HD} + \Lambda_{CO}$, see Eqs. (5), (6) and (7). The X-ray heating rate (see Eq. 4) depends on the hot gas parameters n_{-3}, r_{100} and temperature T_{keV}, for which we adopt the values (see Table 3) of three clusters considered by O'Dea et al. (1994): Hydra A, Centaurus and PKS 0745-191.

We have seen (Eq. 4) that Γ_X depends on the distance R (from the center of the cluster) at which the molecular cloud is located. Thus at the equilibrium temperature T_{eq}, where $\Gamma(R_{eq}) = \Lambda_T(T_{eq})$, we obtain

$$R_{eq} = r_c \sqrt{\left[\frac{\gamma_o n_{-3}^2 n_{H_2} r_{100} T_{keV}^\alpha \tau^\delta}{\Lambda_T}\right]^{2/3} - 1} . \qquad (8)$$

The intracluster gas is denser in the core of the cluster and, therefore, the radiative cooling time, t_{cool}, due to the emission of X-rays is shortest there. A cooling flow is formed when t_{cool} is less than the age of the system t_{sys}, roughly $t_{sys} \sim H_o^{-1}$ where H_o is the Hubble constant. We define the radius r_{cool} where $t_{cool} \sim t_{sys}$, beyond this radius we do not expect a cooling flow. Table 3 gives the cooling radius for the three clusters we consider.

In Figs. 1, 2 and 3 we plot the curves R_{eq} as a function of T_{eq} for different values of the attenuation factor τ (see Table 4) for the X-ray heating, which is due to the presence of an attenuating column density (O'Dea et al. 1994). Upper curves in the different figures correspond to low values of τ, in which case the X-ray heating is more important. Thus low equilibrium temperatures are achieved only at large distances from the cluster center. We indicate on the plots also the cooling radius r_{cool}. We notice that the curves above r_{cool} in the various figures are given just for clarity, but we do not expect them to be realistic, since the density of the clouds will decrease in the outer regions of the cluster, so that our adopted values for the density will be too high and thus no longer valid. The slope of the R_{eq} curves depends on the molecular cooling Λ_T (see Eq. 8). For all cases, H_2 cooling turns out to be negligible for equilibrium temperatures below 80 K. CO cooling is dominant in the range from $\sim$ 3 K up to 40 K, whereas HD dominates between $\sim$ 40 K and 80 K.

Fig. 1 is for the Hydra A cluster. Thermal equilibrium in the cooling flow region ($R_{eq} < r_{cool}$) is possible for different

Table 3. Parameters for the galaxy clusters we consider in the text, n_{-3} is in units of $10^{-3}\,\mathrm{cm}^{-3}$, $r_{100} = r_c/100$ where r_c is in kpc and T_{keV} in keV (see O'Dea et al. 1994). r_{cool} is the cooling radius with upper and lower limits.

Cluster	n_{-3}	r_{100}	T_{keV}	References	r_{cool}	References
Hydra A	6.5	1.45	4.5	David et al. 1990	162^{+56}_{-68}	Cardiel et al. 1998
Centaurus	9	2	2.1	Matilsky et al. 1985	87^{+7}_{-29}	Allen & Fabian 1997
PKS 0745-191	35	0.5	8.6	Arnaud et al. 1987	214^{+29}_{-25}	Cardiel et al. 1998
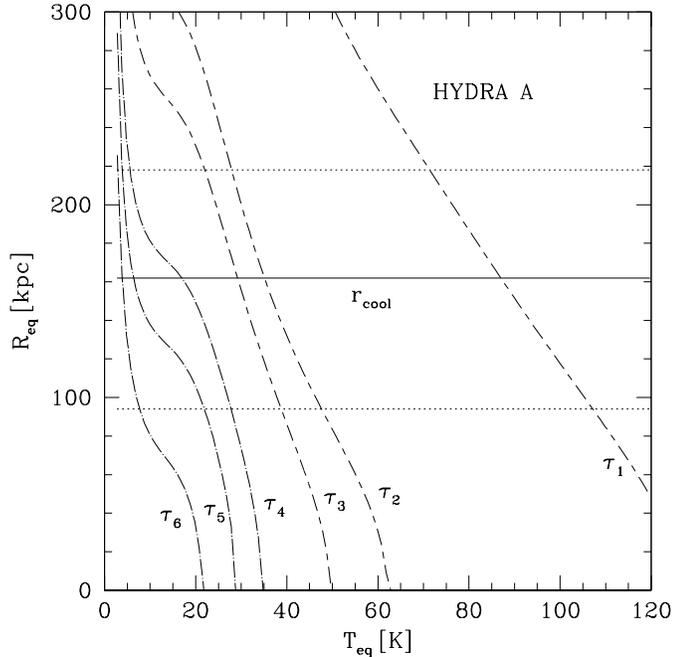
Fig. 1. Radius R_{eq} in kpc as a function of the cloud temperature T_{eq} for the Hydra cluster. Upper curves correspond to low τ values. The flat curves indicate the cooling radius (mean value with upper and lower limit), $\eta_{HD} = 7 \times 10^{-5}$, $\eta_{CO} = 5 \times 10^{-5}$ and $n_{H2} = 10^6\,\mathrm{cm}^{-3}$. For τ_1, τ_2 and τ_3 we take for γ_o, α and δ the values given in the first line of Table 1, whereas for τ_4, τ_5 and τ_6 the corresponding values are given in the second line.

Table 4. Values of the attenuation factor τ, which will be used in the text.

τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
0.01	0.25	0.5	1	1.5	2.5

values of τ. Notice, that for $\tau = \tau_5$ and $\tau = \tau_6$ the equilibrium temperature T_{eq} can be as low as $\sim 10\,\mathrm{K}$. If $R_{eq} \sim 200\,\mathrm{kpc}$ and r_{cool} close to the upper limit ($218\,\mathrm{kpc}$), it is even possible that T_{eq} is as low as $\sim 3\,\mathrm{K}$ for τ_6. O'Dea, Baum, Maloney et al. (1994) suggested, in order to explain the non-detection of CO in this cluster, that the gas is molecular and very cold with temperature close to that of the CBR. Our calculation seems to confirm this conclusion.

Fig. 2 is for the Centaurus cluster, for which the situation is quite different, since the cooling radius is smaller ($\sim 87\,\mathrm{kpc}$). For this reason the equilibrium between molecular cooling and
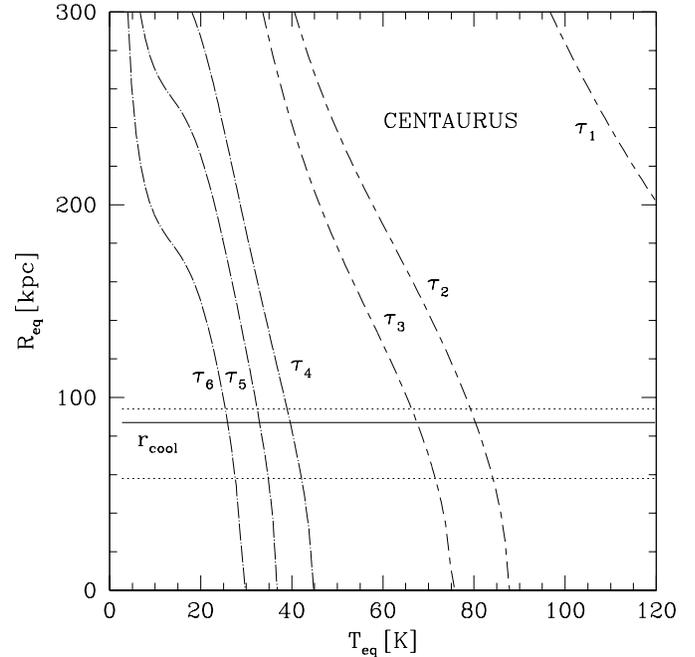

Fig. 2. Radius R_{eq} in kpc as a function of the cloud temperature T_{eq} for the Centaurus cluster. See also the caption in Fig. 1.

X-ray heating is difficult to realize at small radii. We find equilibrium solutions in the cooling flow region (not for τ_1 however) for a temperature beyond $40\,\mathrm{K}$, for which HD cooling is important. In this case the temperature of the clouds is not sufficiently low in order to explain the non-detection of CO, unless η_{CO} is higher or the total gas mass is relatively small. This latter possibility has also been mentioned by O'Dea et al. (1994) based on their observational upper limit for the $J = 0 \rightarrow J = 1$ transition in CO for NGC 4696.

In Fig. 3 we consider the PKS 0745-191 cluster. Although the cooling radius $r_{cool} \simeq 214\,\mathrm{kpc}$ is the largest one, the temperature $T_{keV} = 8.6\,\mathrm{keV}$ of the cluster is very high and thus the X-ray heating is very important. Therefore, for this reason the equilibrium between molecular cooling and X-ray heating for low temperatures would be possible only at large distances. In the region of cooling flow ($R_{eq} < r_{cool}$), we find that the equilibrium temperature is possible at very low temperature ($T_{eq} \sim 5\,\mathrm{K}$) for τ_5, τ_6 and at $T_{eq} \sim 3\,\mathrm{K}$ for τ_6 only. Thus, as for the Hydra A cluster, this could explain the non-detection of CO.

Of course it is difficult to draw firm conclusions on the existence of very cold clouds in cooling flows, since an important
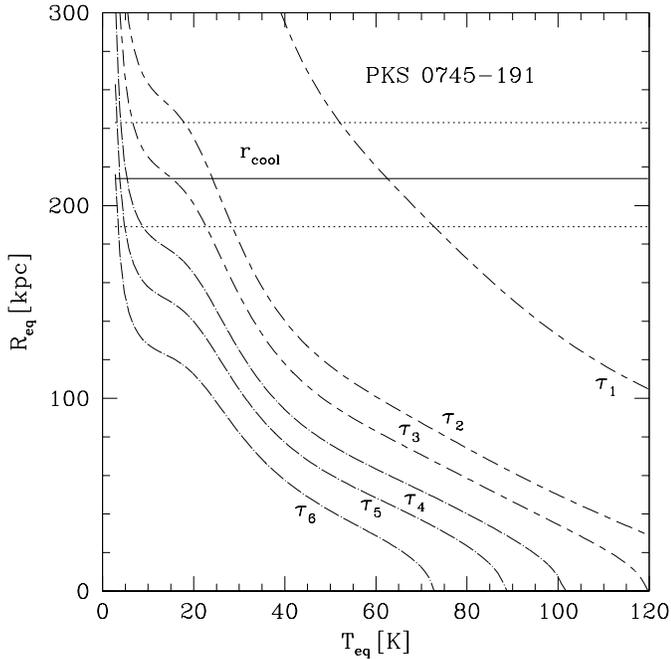
Fig. 3. Radius R_{eq} in kpc as a function of the cloud temperature T_{eq} for the PKS 0745-191 cluster. See also the caption in Fig. 1.

Fig. 4. Radius R_{eq} in kpc for the Hydra cluster for different values of η_{CO} ($\tau = \tau_6$, $N_{H_2} = 2 \times 10^{18}$ cm^{-2} and $n_{H_2} = 10^6$ cm^{-3}).

parameter like the CO abundance is quite uncertain. In Figs. 4, 5 we plotted the curves R_{eq} (for Hydra A and PKS 0745-191) as a function of T_{eq} for different values of η_{CO} (see Table 5) keeping $n_{H_2} = 10^6$ cm^{-3}, $N_{H_2} = 2 \times 10^{18}$ cm^{-2} and $\tau = \tau_6$ fixed (keeping thus the same size $L \sim 10^{-6}$ pc of the cloud and changing N_{CO}). From the figures we see that at very low temperatures R_{eq} varies by about a factor of 2 when increasing η_{CO} 10 times. If we took into account all transitions, in particular for $T_m > 10$ K, the dependence on the CO abundance would turn out to be less important. We notice that there are other values of η_{CO} for which it is possible to have an equilibrium temperature close to the CBR one. If N_{CO} is substantially smaller than 10^{14} cm^{-2} ($\eta_{CO} \ll \eta_{CO,1}$) the clouds are optically thin but cooling gets less efficient. The net effect will be an increase for the value of R_{eq}. On the other hand, if N_{CO} is bigger (i.e. $\eta_{CO} > \eta_{CO,5}$) we have only a small decrease of R_{eq} at $T_{eq} \sim 3$ K.

The hypothesis of very cold molecular gas in cooling flows seems reasonable given also the fact that with our approximations we get upper limits for the cloud temperatures. We notice, that two clusters have an important cooling flow: 400 M$_\odot$ yr^{-1} for the Hydra cluster (David et al. 1990) and 500 M$_\odot$ yr^{-1} for the PKS 0745 cluster (Arnaud et al. 1987), whereas the Centaurus cluster has a smaller cooling flow of 20 M$_\odot$ yr^{-1} (Matilsky et al. 1985). An important cooling flow could trigger thermal instabilities in the gas (David et al. 1988, White & Sarazin 1987) and could lead to the fast-growing of perturbations, which can then form objects such as molecular clouds.

Other molecules than CO could also be important, for example CN or H_2CO molecules. For CN the corresponding temperature for the transition $J = 0 \rightarrow J = 1$ is ~ 5.44 K and
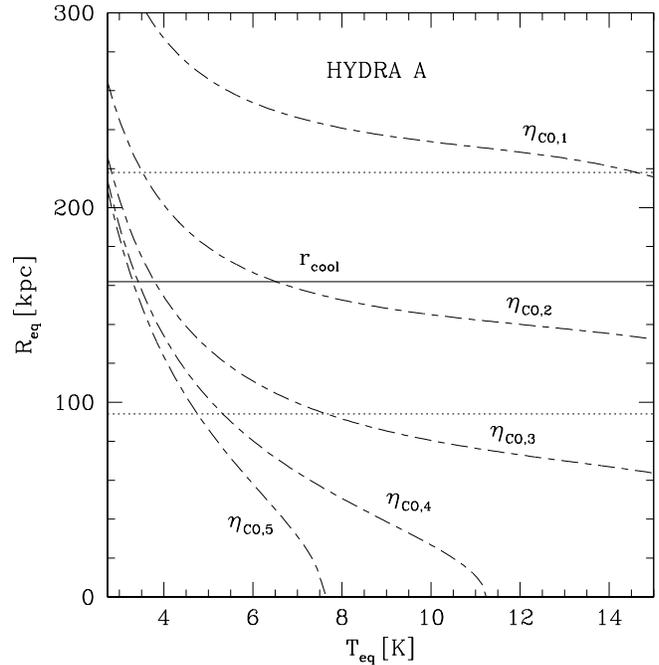
Table 5. Values of the ratio η_{CO}, used in the text.

$\eta_{CO,1}$	$\eta_{CO,2}$	$\eta_{CO,3}$	$\eta_{CO,4}$	$\eta_{CO,5}$
10^{-5}	2.5×10^{-5}	5×10^{-5}	7.5×10^{-5}	10^{-4}

for H_2CO the transition is close to 6.84 K (see Partridge 1995), so both molecules could play a role in cooling. The study of the chemistry in cooling flows could lead to important insight in the phenomenon and be of relevance in order to correctly estimate the total involved gas mass. Such a detailed study, however, is beyond the scope of the present paper.
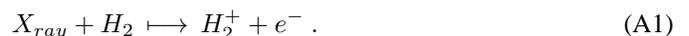
Appendix A: molecular heating

In this appendix we briefly recall the heating processes for H_2, HD and CO molecules, which occur mainly through electronic excitations.

H_2 molecules

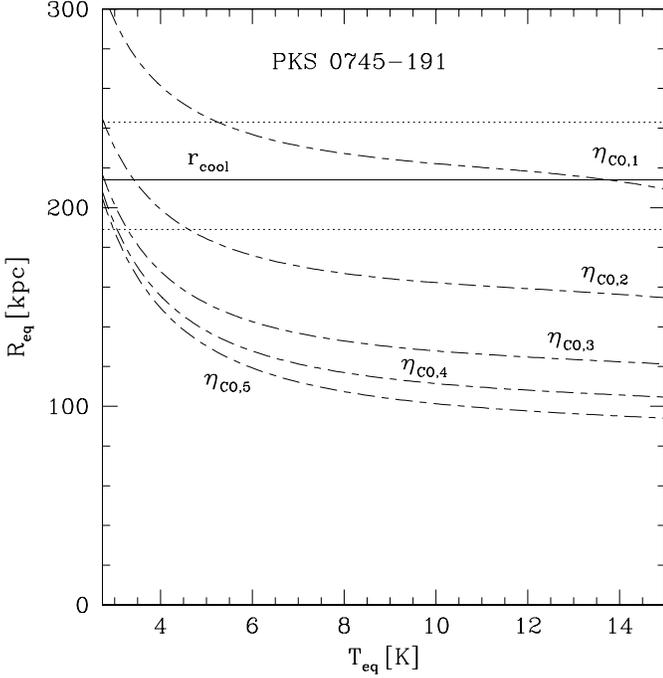The photoionization process of H_2 creates an energetic electron and an H_2^+ ion (Wishart 1979):

$$X_{ray} + H_2 \longmapsto H_2^+ + e^- . \tag{A1}$$

Fig. 5. Radius R_{eq} in kpc for the PKS 0745-191 cluster for different values of η_{CO} ($\tau = \tau_6$, $N_{H_2} = 2 \times 10^{18}$ cm^{-2} and $n_{H_2} = 10^6$ cm^{-3}).

Different routes are then possible: one of the ways is that H_2^+ recombines with a thermal electron (Nakashima et al. 1987):

$$H_2^+ + e^- \longmapsto H_2 + h\nu, \tag{A2}$$

producing a photon but no heating, since the photon can escape from the cloud. The second route is that H_2^+ collides with H_2 and gives rise to the reaction (Prasad & Huntress 1980)

$$H_2^+ + H_2 \longmapsto H_3^+ + H, \tag{A3}$$

followed by the reaction

$$H_3^+ + e^- \longmapsto H_2 + H. \tag{A4}$$

These two reactions are exothermal and lead, therefore, to a transfer of energy and a heating of 10.96 eV for every H_2 involved in the process (Glassgold & Langer 1973).

HD molecules

The fact that the HD molecule has a permanent dipole moment ($\mu_{HD} = 8.3 \times 10^{-4}$ Debye, see Abgrall et al. 1982) leads to X-ray heating processes, which are different from those for H_2. Thus, after the classical photoionization process (Von Bush & Dunn 1972):

$$X_{ray} + HD \longmapsto HD^+ + e^-, \tag{A5}$$
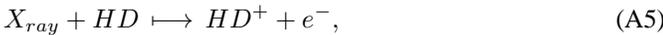
one could expect a chemistry similar to reactions (A3) and (A4) with an intermediate species H_2D^+, that takes over the role of the H_3^+ ion. The knowledge of the H_2D^+ chemistry is, however, very poor and we will not discuss it in more details. The most exothermical reaction is

$$HD^+ + e^- \longmapsto H + D, \tag{A6}$$

which gives 10.93 eV for every HD^+ (see Nakashima et al. 1987).

CO molecules

CO has a permanent dipole moment ($\mu_{CO} = 0.118$ Debye, see Lang 1974). Exothermal chemistry relies on the ion HCO^+ (Prasad & Huntress 1980, Anicich & Huntress 1986). After photoionisation which produces CO^+:

$$X_{ray} + CO \longmapsto CO^+ + e^-, \tag{A7}$$
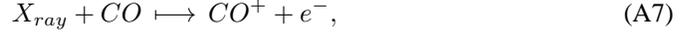
further collision with H_2 molecules can produce the intermediate species HCO^+:

$$CO^+ + H_2 \longmapsto HCO^+ + H. \tag{A8}$$
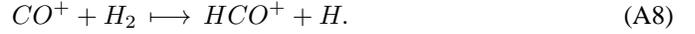
Afterwards the dissociative recombination

$$HCO^+ + e^- \longmapsto CO + H \tag{A9}$$
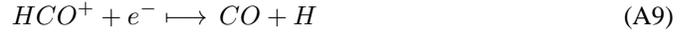
gives 7 eV for each process, which goes into heating.

Electrons

For the electrons the situation is different. The energetic primary electrons lose their energy by electronic excitation and ionisation, the latter process leading to secondary electrons:

$$e^- + H_2 \longmapsto e_1^- + e_2^- + H_2^+. \tag{A10}$$
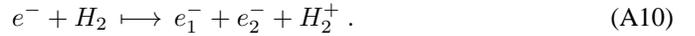
Whenever the kinetic energy of a primary or a secondary-generation electron falls below the threshold for inelastic scattering, it can lose energy only by elastic scattering on thermal electrons and thus most of his energy goes into heating.

Appendix B: molecular cooling

In this Appendix we compute the cooling functions for the various molecules.

HD molecules

The excitation of the ground state is radiative or collisional. The equations of evolution for the populations (X_o^{HD} for the ground state $J = 0$, X_1^{HD} for the first excited state $J = 1$) are given by:

$$\frac{dX_1^{HD}}{dt} = \left[C_{0,1}^{HD} + B_{0,1}^{HD}u_{0,1}^{HD}\right]X_o^{HD}$$
$$- \left[A_{1,0}^{HD} + C_{1,0}^{HD} + B_{1,0}^{HD}u_{1,0}^{HD}\right]X_1^{HD} \tag{B1}$$

$$\frac{dX_o^{HD}}{dt} = \left[A_{1,0}^{HD} + C_{1,0}^{HD} + B_{1,0}^{HD}u_{1,0}^{HD}\right]X_1^{HD}$$
$$- \left[C_{0,1}^{HD} + B_{0,1}^{HD}u_{0,1}^{HD}\right]X_o^{HD}, \tag{B2}$$

where $C_{0,1}^{HD}$ and $C_{1,0}^{HD}$ are the collisional coefficients, which characterize the collisions between HD and H_2. In the approximation of a Maxwell-Boltzmann distribution we have:

$$C_{0.1}^{HD} = 3C_{1,0}^{HD}exp\left(-\frac{T_{1,0}^{HD}}{T_m}\right)$$

with $T_{1,0}^{HD} = T_{0,1}^{HD} = 128.6$ K being the transition temperature between the states $J = 0$ and $J = 1$, T_m the matter temperature of the cloud, $B_{0,1}^{HD}$ and $B_{1,0}^{HD}$ the Einstein coefficients. The energy density of the cosmic background radiation ($T_r = 2.735$ K) at the frequency $\nu_{1,0}^{HD}$ of the transition $J = 0 \rightarrow J = 1$ is

$$u_{1,0}^{HD} = u_{0,1}^{HD} = \frac{8\pi h(\nu_{10}^{HD})^3}{c^3} \frac{1}{exp\left(\frac{T_{1,0}^{HD}}{T_r}\right) - 1} \cdot$$

Moreover,

$$A_{1,0}^{HD} = \frac{512\,\pi^4\,k_B^3}{3h^4 c^3} \frac{B_{HD}^3 \mu_{HD}^2}{3}$$

is the Einstein coefficient for the spontaneous emission (see Kutner 1984) and $B_{HD} = 64.3$ K is the rotational constant (see Herzberg 1950).

We have the following normalisation for the populations: $X_o^{HD} + X_1^{HD} = 1$. We consider a quasi-instantaneous transition, therefore, we can set

$$\frac{dX_o^{HD}}{dt} = \frac{dX_1^{HD}}{dt} = 0.$$

With these approximations we find for the populations X_1^{HD} and X_o^{HD}:

$$X_1^{HD} = \frac{B_{0,1}^{HD} u_{0,1}^{HD} + C_{0,1}^{HD}}{A_{1,0}^{HD} + u_{1,0}^{HD}(B_{0,1}^{HD} + B_{1,0}^{HD}) + C_{0,1}^{HD} + C_{1,0}^{HD}} \quad (B3)$$

and

$$X_o^{HD} = \frac{A_{1,0}^{HD} + B_{1,0}^{HD} u_{1,0}^{HD} + C_{1,0}^{HD}}{A_{1,0}^{HD} + u_{1,0}^{HD}(B_{0,1}^{HD} + B_{1,0}^{HD}) + C_{0,1}^{HD} + C_{1,0}^{HD}} \quad (B4)$$

The probability for radiative de-excitation is

$$P_r^{HD} = \frac{A_{1,0}^{HD} + B_{1,0}^{HD} u_{1,0}^{HD}}{C_{1,0}^{HD} + A_{1,0}^{HD} + B_{1,0}^{HD} u_{1,0}^{HD}} \cdot \quad (B5)$$

The molecular cooling function (collisional excitation followed by a radiative de-excitation) is given by

$$\Lambda_{HD} = n_{HD} X_o^{HD} C_{0,1}^{HD} P_r^{HD} E_{1,0}^{HD}, \quad (B6)$$

where $E_{1,0}^{HD} = k_B T_{1,0}^{HD}$ is the energy for the transition $J = 0 \rightarrow J = 1$.

With the following relation between the Einstein coefficients:

$$B_{0,1}^{HD} = 3B_{1,0}^{HD} \text{ and } B_{1,0}^{HD} = \frac{c^3}{8\pi h(\nu_{1,0}^{HD})^3} A_{1,0}^{HD},$$

we get for the HD cooling function

$$\Lambda_{HD} =$$
$$\left\{ 3C_{1,0}^{HD} exp\left(-\frac{T_{1,0}^{HD}}{T_m}\right) exp\left(\frac{T_{1,0}^{HD}}{T_r}\right) h\nu_{1,0}^{HD} n_{HD} A_{1,0}^{HD} \right\} \Bigg/$$
$$\left\{ A_{1,0}^{HD} \left[3 + exp\left(\frac{T_{1,0}^{HD}}{T_r}\right) \right] \right.$$
$$\left. + C_{1,0}^{HD} \left[1 + 3exp\left(-\frac{T_{1,0}^{HD}}{T_m}\right) \right] \left[exp\left(\frac{T_{1,0}^{HD}}{T_r}\right) - 1 \right] \right\}. \quad (B7)$$

Since $T_{10}^{HD} \gg T_r$ we can use the following approximations:

$$3 + exp\left(\frac{T_{1,0}^{HD}}{T_r}\right) \sim exp\left(\frac{T_{1,0}^{HD}}{T_r}\right)$$

and

$$exp\left(\frac{T_{1,0}^{HD}}{T_r}\right) - 1 \sim exp\left(\frac{T_{1,0}^{HD}}{T_r}\right).$$

In this way we obtain for the HD cooling function:

$$\Lambda_{HD} \sim \frac{3C_{1,0}^{HD} exp(-\frac{T_{1,0}^{HD}}{T_m}) h\nu_{1,0}^{HD} n_{HD} A_{1,0}^{HD}}{A_{1,0}^{HD} + C_{1,0}^{HD} \left[1 + 3exp(-\frac{T_{1,0}^{HD}}{T_m}) \right]} \cdot \quad (B8)$$

This result is valid for the optically thin limit. In the optically thick case one has to take into account radiative transfer effects. Goldreich & Kwan (1974) computed the average escape probability for a photon emitted in a radiative transition from the $J + 1$ to the J rotational level, denoted by β_J, using Castor's result (Castor 1970):

$$\beta_J = \left[1 - exp(-\tau_J) \right] / \tau_J \quad (B9)$$

where τ_J is the optical depth for the transition $J \rightarrow J + 1$. In our case we consider only the transition between the ground state and the first excited level, and thus we get

$$\tau_o^{HD} = \frac{8\pi^3 (\mu_o^{HD})^2}{3h\Delta v} N_{HD} X_1^{HD} \left[exp\left(\frac{T_{10}^{HD}}{T_m}\right) - 1 \right] \quad (B10)$$

and the corresponding average escape probability is

$$\beta_o^{HD} = \left[1 - exp(-\tau_o^{HD}) \right] / \tau_{o^{HD}} \cdot \quad (B11)$$

N_{HD} is the column density of HD. The linewidth Δv is assumed to be given by thermal doppler broadening

$$\Delta v = \sqrt{\frac{3kT_m}{2m_H}} \cdot \quad (B12)$$

We neglect the other possible sources of broadening such as turbulence or large-scale systematic motions.

The β_o^{HD} factor enters only in the spontaneous emission term. Therefore, the cooling function in Eq. (B8) gets modified as follows

$$\Lambda_{HD} \sim \frac{3C_{1,0}^{HD} exp(-\frac{T_{1,0}^{HD}}{T_m}) h\nu_{1,0}^{HD} n_{HD} A_{1,0}^{HD} \beta_o^{HD}}{A_{1,0}^{HD} \beta_o^{HD} + C_{1,0}^{HD} \left[1 + 3exp(-\frac{T_{1,0}^{HD}}{T_m}) \right]} \cdot \quad (B13)$$

CO molecules

The CO molecule is like the HD molecule and has a permanent dipole moment ($\mu_{CO} = 0.118$ Debye) which is much larger than the dipole moment of the HD molecule (Lang 1974). The rotational constant is low, $B_{CO} = 2.78$ K, leading to a very low temperature for the first transition $T_{1,0}^{CO} = 5.56$ K.

The expression for the cooling function of CO molecule is obtained in the same way as for HD, and we get

$$\Lambda_{CO} = \left\{ 3C_{1,0}^{CO} exp\left(-\frac{T_{1,0}^{CO}}{T_m}\right) exp\left(\frac{T_{1,0}^{CO}}{T_r}\right) h\nu_{1,0}^{CO} n_{CO} A_{1,0}^{CO} \beta_o^{CO} \right\} \bigg/$$
$$\left\{ A_{1,0}^{CO} \beta_o^{CO} \left[3 + exp\left(\frac{T_{1,0}^{CO}}{T_r}\right)\right] \right.$$
$$\left. + C_{1,0}^{CO}\left[1 + 3exp\left(-\frac{T_{1,0}^{CO}}{T_m}\right)\right]\left[exp\left(\frac{T_{1,0}^{CO}}{T_r}\right) - 1\right] \right\} , \quad (B14)$$

and β_o^{CO} si defined accordingly as in Eqs. (B9) and (B10).

H_2 molecules

The H_2 molecule differs from HD, since it has no dipole moment and the transitions are quadrupolar. We restrict our calculations to the first transition $J = 0 \rightarrow J = 2$. The other transitions can be neglected, since we consider matter temperatures which are much lower than the transition temperature of H_2 molecules (512 K for the first transition). The Einstein coefficients for the spontaneous emission and for the first transition are given by (see Kutner 1984): $A_{2,0}^{H_2} = 2.44 \times 10^{-11}$ s.

The relations between the Einstein coefficients are

$$B_{0,2}^{H_2} = 5B_{2,0}^{H_2} \text{ and } B_{2,0}^{H_2} = \frac{c^3}{8\pi(\nu_{2,0}^{H_2})^3}A_{2,0}^{H_2} .$$

The density of the cosmic background radiation at the transition is:

$$u_{2,0}^{H_2} = u_{0,2}^{H_2} = \frac{8\pi h(\nu_{2,0}^{HD})^3}{c^3} \frac{1}{exp\left(\frac{T_{0,2}^{H_2}}{T_r} - 1\right)} .$$

The collisional coefficients, in the Maxwell-Bolztmann distribution approximation, are

$$5C_{2,0}^{H_2} = C_{0,2}^{H_2} exp\left(\frac{T_{2,0}^{H_2}}{T_m}\right)$$

by analogy with the HD molecule. Because $T_{2,0}^{H_2} \gg T_r$ we can make the same approximations as for the HD molecule, and we find for the populations $X_o^{H_2}$ and $X_1^{H_2}$:

$$X_o^{H_2} \sim \frac{A_{2,0}^{H_2} + C_{2,0}^{H_2}}{A_{2,0}^{H_2} + C_{2,0}^{H_2}\left[1 + exp\left(\frac{T_{2,0}^{H_2}}{T_m}\right)\right]} ,$$

$$X_1^{H_2} \sim \frac{5C_{2,0}^{H_2} exp\left(\frac{T_{2,0}^{H_2}}{T_m}\right)}{A_{2,0}^{H_2} + C_{2,0}^{H_2}\left[1 + exp\left(\frac{T_{2,0}^{H_2}}{T_m}\right)\right]} .$$

The probability of radiative de-excitation becomes

$$P_r^{H_2} \sim \frac{A_{2,0}^{H_2}}{A_{2,0}^{H_2} + C_{2,0}^{H_2}} , \quad (B15)$$

while the molecular cooling function for the H_2 molecule is

$$\Lambda_{H_2} \sim \frac{5n_{H_2}C_{2,0}^{H_2}\beta_o^{H_2}A_{2,0}^{H_2}h\nu_{2,0}^{H_2}exp\left(-\frac{T_{2,0}^{H_2}}{T_m}\right)}{A_{2,0}^{H_2}\beta_o^{H_2} + C_{2,0}^{H_2}\left[1 + 5exp\left(-\frac{T_{2,0}^{H_2}}{T_m}\right)\right]} , \quad (B16)$$

again $\beta_o^{H_2}$ is defined accordingly as in Eqs. (B9) and (B10).

B.1. Collision rates

Since H_2 is the most abundant species, the collisional coefficients are given by

$$C_{1,0}^{HD} = C_{1,0}^{CO} = C_{2,0}^{H_2} = <\sigma.v> n_{H_2} , \quad (B17)$$

where $\sigma \sim 1 \text{Å}^2$ is the collisional cross section, n_{H_2} the number density of H_2 and $v \sim \sqrt{\frac{3kT_m}{2m_H}}$, with m_H being the hydrogen mass.

References

Abgrall H., Roueff E., Viala Y., 1982, A&AS 50, 505
Allen S.W., Fabian A.C., 1997, MNRAS 286, 583
Allen S.W., Fabian A.C., 1998, MNRAS 297, L67
Anicich A., Huntress W., 1986, ApJS 62, 553
Arnaud K., Johnstone R., Fabian A., et al., 1987, MNRAS 227, 241
Bergeron J., Souffrin S., 1971, A&A 11, 40
Black J., 1988, Molecular Astrophysics. Hartsqvist, 473
Bridges T., Irwin J., 1998, MNRAS 300, 967
Cardiel N., Gorgas J., Aragón-Salamenca A., 1998, MNRAS 298, 977
Castor J., 1970, MNRAS 149, 111
Combes F., Wiklind T., 1998, The ESO Messenger 91, 29
David L., Arnaud K., Forman W., Jones C., 1990, ApJ 356, 32
David L., Bregman J., Seab G., 1988, ApJ 329, 66
De Paolis F., Ingrosso G., Jetzer P., Roncadelli M., 1995, A&A 295, 567
Fabian A.C., Nulsen P., Canizares C., 1984, Nat 310, 733
Fabian A.C., Nulsen P., Canizares C., 1991, A&AR 2, 191
Fabian A.C., 1992, Clusters and Superclusters of Galaxies. Kluwer, 151
Fabian A.C., Johnstone R., Daines, 1994, MNRAS 271, 737
Fabian A.C., 1994, ARA&A 32, 277
Fabian A.C., Nulsen P., 1994, Cosmol. aspects of X-ray clust. of gal., Kluwer, 163
Ferland G., Fabian A.C., Johnstone R., 1994, MNRAS 266, 399
Galli D., Palla F., 1998, preprint, astro-ph/9803315
Glassgold A., Langer W., 1973, ApJ 186, 859
Goldreich P., Kwan J., 1974, ApJ 189, 441
Goldsmith P., Langer W., 1978, ApJ 222, 881
Habing H., Goldsmith D., 1971, ApJ 166, 525
Herzberg G., 1950, Spectra of diatomic molecules. Van Nostrand
Jaffe W., Bremer M., 1997, MNRAS 284, L1
Johnstone R., Fabian A., Edge A., Thomas P., 1992, MNRAS 255, 431
Jones C., Forman W., 1984, ApJ 276, 38
Kauffmann G., Charlot S., 1998, MNRAS 294, 705
Kutner M., 1984, Fund. Cosm. Phys 9, 233
Lang K., 1974, Astrophysical data. Springer Verlag
Lepp S., Shull M., 1984, ApJ 280, 465
Lieu R., Mittaz J., Bowyer S., et al., 1996a, ApJ 458, L5
Lieu R., Mittaz J., Bowyer S., et al., 1996b, Sci 274, 1335

Matilsky T., Jones C., Forman W., 1985, ApJ 291, 621

Mushotsky R., Lowenstein M., 1997, ApJ 481, L63

Nakashima K., Takanagi H., Nakamura H., 1987, J. Chem. Phys. 86, 726

O'Dea C., Baum S., Maloney P., Tacconi L., Sparks W., 1994, ApJ 422, 467

Partridge R., 1995, 3K: The cosmic microwave background radiation. Cambridge University Press

Pfenniger D., Combes F., 1994, A&A 285, 94

Prasad S., Huntress W., 1980, ApJS 43, 1

Puy D., Alecian G., Lebourlot J., Leorat J., Pineau des Forets G., 1993, A&A 267, 337

Puy D., Signore M., 1996, A&A 305, 371

Puy D., Signore M., 1997, New Astronomy 2, 299

Puy D., Signore M., 1998a, New Astronomy 3, 37

Puy D., Signore M., 1998b, New Astronomy 3, 247

Sarazin C., 1988, X-ray Emission from clusters of galaxies. Cambridge University Press

Stewart G., Fabian A., Jones C., Forman W., 1984, ApJ 285, 1

Von Bush F., Dunn G.H., 1972, Phys. Rev. A 5, 1726 252, 72

White D., Fabian A., Johnstone R., Mushotzky R., Arnaud K., 1991, MNRAS 252, 72

White R., Sarazin C., 1987, ApJ 318, 612

Wishart A.W., 1979, MNRAS 189, 59