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Pulsational mode of gravitational collapse and its impact on the star formation

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Abstract. The gravitational condensation of a collapsing dust mass is always inhibited by the acoustic viscosity, restricting thereby the unstable scale size. In this paper we show that, if some of the dust particles (in the collapsing mass) are ionised due to plasma environment, a new mode of Jeans condensation namely the Pulsational Mode of the Gravitational Collapse (PMGC) seems to be a likely phenomenon around a fluctuation scale size of the order of the critical Jeans length. This novel mode of the gravitational collapse is a consequence of the finite linear gravito-electrostatic coupling due to partial ionisation of the collapsing mass. It may be interpreted as a time synchronised resonant linear superposition of a purely hybrid oscillator-like mode (gravito-electrostatic oscillator) and a purely growing (Jeans-like) mode over a levitational kind of equilibrium around the critical Jeans length.

Key words: plasmas – stars: formation – ISM: dust, extinction

1. Introduction

The dark interstellar clouds, are believed to be the ideal sites for star formation. These clouds are in general sub-divided into clumps of size $\sim 10\,pc$ and mass 10^3 to $10^4 M_O$, which undergo gravitational collapse towards the formation of low mass stars. A recent work by Nakano et al. (1995) discusses in detail the mass of star formed in cloud core. Whether a cloud will successfully collapse into a protostar requires the condition that the gravitational energy should exceed the sum of thermal, rotational and magnetic energies.

Sometimes near the dark interstellar clouds, we have new born stars, associated emission nebula or other ionising sources. As a result some parts of the gas in these clouds are always ionised and dusts present there pick up charges from the plasma environment. Therefore, we have the dust grains which are charged and the electrostatic forces between these charged dust grains also come into existence. In this paper we make an attempt to find whether, the presence of such forces plays any role in the energy balance mechanism and make calculations to understand its role in the star formation process. In the beginning we note that we use the words 'dust', 'dust grains' and 'grains' to mean one and the same.

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2. Charged dust in molecular clouds

In the above interstellar clouds a typical value of the degree of ionisation has been estimated to be $\sim 10^{-7}$ (Shu et al. 1987). The dusts present in these clouds are actually embedded in very weekly ionized plasma. These dust particles pick up charges from the plasma environment. The equilibrium dust charge can be estimated by the capacitor charging model, first addressed by Goertz & Ip (1948). Besides there are numerous other works in literature (Hazelton & Yadlowsky 1994; Cui & Goree, 1994; Walch et al. 1995) which describe the charging of dust grains in plasma. The amount of charge collected by a spherical dust of radius a, will be

$$q_d = 4\pi\epsilon_0 a\phi_f \tag{1}$$

where ϕ_f will be the surface potential of dust grain. Accordingly we get about 700 electrons per volt for a grain of 1 micron radius, which is assumed to be a typical size for the grains in interstellar medium. However the amount of charge decreases with increase in hydrogen density in the cloud and also depends upon various other factors including sources of ionising radiations, properties of dust grains present in the cloud etc. (Nakano 1998). The value of ϕ_f is obtained by solving the equation for floating condition

$$\frac{dq_d}{dt} = -a^2(J_e + J_i - J_{sec})/\epsilon_0 \tag{2}$$

where J_e, J_i the electron and ion current densities to the surface of the grain are functions of ϕ_f. J_{sec} is the density of electron current that is emitted out of the grain, by photoemission and/or other secondary emission processes. By neglecting the J_{sec} one can obtain for hydrogen plasma (where $T_i = T_e$) (Spitzer 1978):

$$\phi_f \sim -2.5 * T_e/e$$

where T_e and T_i are the electron and ion temperatures in ev.

The electron temperature in a typical emission nebula can be 7000 K (Allen 1983), which can be definitely an upper limit for the corresponding value of the dark cloud. Thus taking T_e for dark cloud to be, $\sim$ 0.5 ev, we get

$$\phi_f \sim -1.25V$$

and finally $q_d \sim$ 900 electrons.

Now in order that the electrostatic force plays any significant role, it should be of the order of gravitational force, which requires $Gm_d^2/q_d^2 \sim O(1)$, where m_d and q_d are the mass and charge of the dust grains.

Thus a 1 micron size dust grain of density 1 gm/cc demands q_d to be $\sim 10^{-6}$ electrons. This falls much below the value of 900 electrons as calculated above. However, for a grain of 1 cm diameter we can show that these two forces become comparable. As we know the grain size always increases towards the central condensation and sizes of this order can be found only in the inner part of proto-stellar disk. Further we have neglected the secondary emission current (like photo-emission etc.) from the grain surface, which otherwise can significantly lower the value of ϕ_f and q_d.

It has been discussed by Umebayashi and Nakano (1980) that grains with electric charge are scarce in dense molecular clouds shielded from uv radiations. As a result Nishi et al. (1991) have considered grains with charges only $\pm$ 2e, $\pm$ 1e and 0 in their works on star forming clouds. As we can see for such a value of charge, the size of the grain has to be few tens of microns, in order that the gravitational and electrostatic forces are of the same order. However, Nishi et al. (1991) have considered much smaller sizes of the grains with four models, having power law distributions and sizes ranging between 0.0001 micron to $\sim$ 1 micron.

As it is difficult to ascertain the exact size of the grains in a cloud, we keep an open mind towards grain size distribution and feel that it may be worth exploring the role of neutral and charged dust grains in the process of star formation.

Recently Pandey et al. (1994) have carried out linear & nonlinear theoretical analysis of the gravitating dust mass with high ($\sim$ 1) degree of ionisation. However in reality the degree of ionisation in these clouds can be as low as 10^{-7}. Further the dust and plasma in these clouds may not be homogeneously mixed. Also due to the closer proximity to some ionising source some parts of the clouds may be more ionised, producing relatively a higher number of charged dust grains there. Also there can be local (small scale) instabilities and fluctuations of the plasma. The net result can be only a part of the dust population is getting charged in the cloud.

As mentioned above Nishi et al. (1991) have considered a composite of four different grain models with various grain sizes and compositions and they have worked out the relative abundances of different charged grains. With hydrogen density n_H varying between 10^3–10^{13} per cc, they showed that the number density of neutral grains can vary between 10^{-6}–10^{-10} times of n_H; whereas the corresponding value for singly charged grains can vary between 10^{-8}–10^{-14} times of n_H. The abundances of grains with higher charges are still lower. Therefore the ratio of the abundances of charged to neutral grains can vary between 10^{-2}–10^{-4} or lower.

However, in our calculations we consider the grains which are bigger in sizes than those considered by Nishi et al. (1991).

We further don't try to derive any value for this ratio (say η) of number densities of charged to neutral dust particles. Below we try to model such a cloud, to find the role of charged dust grains in the star formation processes.

3. Cloud with charged dusts

For the above clouds, we shall consider the frictional coupling between the neutral and charged components of the dust grains. However, for simplicity and insight into the subject, the analysis has been carried out by ignoring the frictional force term in the neutral dust dynamics. This can be justified for the Jeans mode frequency $\omega \gg \nu_{nc}$ with $\frac{\nu_{cn}}{\nu_{nc}} = \frac{n_{dno}}{n_{dco}} \gg 1$. Here ν_{cn} represents the binary collisional rate of momentum transfer from charged grains to neutral grains, whereas ν_{nc} represents the opposite case. These rates can be estimated as

$$\nu_{cn} \sim \pi a^2 n_{dno} v_{td} \tag{3a}$$

$$\nu_{nc} \sim \pi a^2 n_{dco} v_{td} \tag{3b}$$

where πa^2 corresponds to the geometrical cross section and v_{td} corresponds to the thermal speed of the dust grains. n_{dno} and n_{dco} respectively denote the density populations of the neutral dust grains and charged dust grains. Further simplification has been introduced by ignoring the Coulomb scattering of the plasma charge particles by the charged dust grains. The effective Coulomb cross section could be estimated as (Sodha et al. 1971):

$$\sigma_{eff} = \pi a^2 \left(1 - 2\frac{Z_j Z_d e^2}{m_j v_{tj}^2 a}\right) \tag{4}$$

where v_{tj} represents the thermal velocity of the plasma charge species. $Z_j e$ and $Z_d e$ are the plasma particle charge and the dust grain charge. Under the approximation $2Z_d e^2/(aT_j) \ll 1$ (Vladimirov 1994), the frictional coupling between the plasma particles and the dust grains could be significantly weak over the dust-dust binary interactions and hence can be ignored. The binary collisions between the plasma species and the dust grains are related as

$$\nu_{ed} \sim \pi a^2 v_{te} n_{dno} \tag{5a}$$

$$\nu_{id} \sim \nu_{cd} v_{ti}/v_{te} \tag{5b}$$

It may be argued that high rate of collisional momentum transfer from plasma species to the dust grains may thermalise the dusty plasma with neutral dust grains. For the time being, we have neglected the drag effects and other force field effects. Though these approximations may not be always realistic, but within some limit of the possible degrees of ionisation of the dust clouds (Draine & Saltpeter 1979), our proposed theoretical model reveals a physically rich idea, about the Jeans collapse of a self gravitating dust mass with part of the dust population being charged.

The charged dust grains in general exhibit the dynamical behaviour of the dust charge either due to the plasma turbulence (Jana et al. 1993) or due to the statistical fluctuations (Cui & Goree 1994). The dust charging mechanism requires a certain time scale for the build up of the maximum equilibrium charge in a given plasma environment. The rough estimation of dust charging time scale τ_c results in $\tau_c \sim \omega_{pl}^{-1} \frac{\lambda_D}{a}$, λ_D being the plasma Debye length (Tsytovich & Havnes 1993).

The present treatment of the gravitational collapse problem excludes the dynamical effect of the dust charge. Its justification for $Gm_d^2/q_d^2 \sim 1$ has been described by Dwivedi et al. (1996), and hence the restriction on the dust plasma parameters is imposed for the constant dust charge model.

We now develop the mathematical formulation of the proposed theoretical problem. The equilibrium is defined as the static distribution of the multifluid consisting of electrons, ions, neutral gas, and the neutral dust grains with partial ionisation. Their thermal distributions are supposed to be Maxwellian. For simplicity it is assumed that, the neutral gas particles form the background which is weakly coupled with the collapsing dust plasma mass. Due to higher inertial mass of the dust grains the gravitational decoupling of the background neutral particles can be justified. However, the frictional coupling of the neutral gas particles with the electron-ion plasma component may modify the Boltzmannian distributions. Nevertheless, to begin with it may be useful to retain the same, which off course seems to be an ideal approximations, for discussing the physics of the Jeans collapse. Assuming all the dust grains of the same size, all the charged dust grains will have the same amount of charge in the given plasma environment. For any physical phenomenon on dust inertial time scale, the electrons and ions could be assumed to follow the Boltzmann distributions:

$$n_e \approx n_{eo} exp\left(e\phi/T_e\right) \quad (6)$$

$$n_i \approx n_{io} exp\left(-\frac{T_e}{T_i} e\phi/T_e\right) \quad (7)$$

The dynamical behaviour of the cold neutral dust grains could be well described by the full inertial response without frictional coupling with any other fluids as discussed earlier.

$$\frac{\partial \boldsymbol{v}_{dn}}{\partial t} + \left(\boldsymbol{v}_{dn}.\boldsymbol{\nabla}\right)\boldsymbol{v}_{dn} = -\boldsymbol{\nabla}\psi \quad (8)$$

$$\frac{\partial n_{dn}}{\partial t} + \boldsymbol{\nabla}.\left(n_{dn}\boldsymbol{v}_{dn}\right) = 0 \quad (9)$$

Now the inertial response of the charged dust fluid with frictional force term can be described as:

$$\frac{\partial \boldsymbol{v}_{dc}}{\partial t} + \left(v_{dc}.\boldsymbol{\nabla}\right)\boldsymbol{v}_{dc} = -\frac{q_d}{m_d}\nabla\phi - \nabla\psi - \nu_{cn}\left(\boldsymbol{v}_{dc} - \boldsymbol{v}_{dn}\right) \quad (10)$$

$$\frac{\partial n_{dc}}{\partial t} + \boldsymbol{\nabla}.\left(n_{dc}\boldsymbol{v}_{dc}\right) = 0 \quad (11)$$

This is to note that the frequency range for which the gravitational collapse of the proposed compositional model holds good is characterised by $\nu_{cn} \sim \omega \gg \nu_{nc}$ and other characteristic frequencies. The scale size of the fluctuations of interest λ is assumed to be shorter than all the characteristic mean free paths but longer than the dust size and the inter grain separation. These scalings have been derived under the conditions that the normalised fluctuation variables are of the same order within an order of magnitude. Finally, the Poisson equations for the electrostatic field potential (ϕ) and the gravitational field potential (ψ) close the set of required dynamical equations to describe the gravitational collapse dynamics of a neutral dust mass which is embedded in a plasma background. This leads to the charging of a part of the dust population. Now the Poisson's equations read as;

$$\nabla^2\phi = 4\pi e\left(n_e - n_i - q_d * n_{dc}/e\right) \quad (12)$$

$$\nabla^2\psi = 4\pi G m_d\left(n_{dc} + n_{dn} - n_{do}\right) \quad (13)$$

where $n_{do} = n_{dco} + n_{dno}$ models the Jeans swindle of the equilibrium gravitational force field. This is in fact a kind of local approximation for equilibrium mass distribution as in the case of quasi-neutral equilibrium charge distribution. However, the quasi-neutral approximation is a local approximation which is characterised by the characteristic screening length ie plasma Debye length. Whereas, no such screening length exists in the case of gravitational mass distribution due to unipolar nature of the gravitational force field. Nevertheless, within the domain of uniform mass distribution, the equilibrium force of gravity could be unimportant to affect the equilibrium dynamics. Now if this symmetry of uniformity is broken either due to some mass fluctuations (as in the case of a purely neutral dust mass) or even due to the charge fluctuations (as in the case of the charged mass), the gravitational collapse will set in and evolve in a different manners than those as discussed in the literature (Kolb & Turner 1990; Pandey et al. 1994 etc.).

Now assuming the spherical symmetry of the gravito-electrostatic fluid distribution and considering the 1-D fluctuation response in radial direction as a mathematical analogue of X-direction in plane geometry approximation ($f \sim exp(-i\omega t + ikx)$), the Fourier analysis of the linear basic governing equations results in:

$$\tilde{n}_e \sim n_{eo}\frac{e\tilde{\phi}}{T_e}\ ; \quad (14)$$

$$\tilde{n}_i \sim -n_{io}\frac{e\tilde{\phi}}{T_i}\ ; \quad (15)$$

$$\tilde{n}_{dn} \sim n_{dno}\frac{\tilde{\psi}}{(\omega^2/k^2)} \quad (16)$$

$$\tilde{n}_{dc} \sim n_{dco}\left(\tilde{\psi} + \frac{q_d}{m_d}\left(1 + i\frac{\nu_{cn}}{\omega}\right)^{-1}\tilde{\phi}\right)/\left(\omega^2/k^2\right) \quad (17)$$

The use of quasi-neutrality approximations for the electrostatic potential field fluctuations and full Poisson equation for the gravitational potential field yields the following dispersion relation:

$$1 + \frac{\omega_J^2}{\omega^2} - \frac{k^2 C_{scam}^2}{\omega^2}\left(1 + i\frac{\nu_{cn}}{\omega}\right)^{-1} - \frac{k^2 C_{scam}^2}{\omega^2}\left(1 + i\frac{\nu_{cn}}{\omega}\right)^{-1}\frac{\left(\omega_J^2 - \omega_{Jd}^2\right)}{\omega^2} = 0 \quad (18)$$

where $\omega_J^2 = 4\pi G m_d(n_{dno} + n_{dco})$; $\omega_{Jd}^2 = 4\pi G m_d n_{dco}$, $C_{scam}^2 = (\frac{q_d}{e})^2 \frac{n_{dco}T_i}{n_{io}m_d}$ for $\frac{\lambda_{De}^2}{\lambda_{Di}^2} \gg 1$ corresponds to the speed of low frequency characteristic acoustic mode of a dusty-like multispecies plasma namely the So-Called Acoustic Mode (SCAM) (Dwivedi et al. 1989; Dwivedi 1997). Equation (18) can be also written in the form:

$$\left(\frac{\omega}{\omega_J}\right)^4 + i\frac{\nu_{cn}}{\omega_J}\left(\frac{\omega}{\omega_J}\right)^3 + \left(1 - \frac{k^2 C_{scam}^2}{\omega_J^2}\right)\left(\frac{\omega}{\omega_J}\right)^2 + i\frac{\nu_{cn}}{\omega_J}\left(\frac{\omega}{\omega_J}\right) - \frac{k^2 C_{scam}^2}{\omega_J^2}\left(\frac{1}{1+\eta}\right) = 0 \quad (19)$$

This is a fourth degree polynomial which is to be solved for ω. We have used here $\eta = \frac{n_{dco}}{n_{dno}}$, as discussed earlier. It is obvious to see that in the limiting case of the ideal charge model of the charged dust grains ie for $\omega_J = \omega_{Jd}$ or $n_{dno} \to 0$ (without any frictional force ie $\nu_{cn} \to 0$) the above dispersion relation (18) reduces to the case of the usual Jeans mode as discussed by Pandey et al. (1994). Similarly in the other limiting case of purely neutral mass ie $n_{dco} \to 0$ one recovers the usual Jeans mode (Kolb & Turner 1990). This is interesting to note that the coupling of the gravito-electrostatic forces produces a gravitational screening-like scale length characterised by the critical Jeans length (as it arises in the case of neutral dust mass collapse due to thermal coupling with the gravity), which may be treated as an equivalent to usual plasma Debye length.

The stability behaviour of the above dust mass distribution can now be discussed in two extreme limits. In the case of frictionless dust mass distribution ($\nu_{cn} \to 0$), if we look at roots' characteristics of Eq. (18) near the levitational equilibrium point where the critical Jeans length $\lambda_{Jcr} \sim \frac{2\pi C_{scam}}{\omega_J}$, two distinct roots are found to exist:

$$\omega_{1r} \sim \omega_J (1+\eta)^{-1/4} \quad \text{with} \quad \omega_{1i} \sim 0\,. \quad (20a)$$

and

$$\omega_{2r} \sim 0 \quad \text{with} \quad \omega_{2i} \sim \omega_J (1+\eta)^{-1/4} \quad (20b)$$

where ω_{1r}, ω_{2r} are the real and ω_{1i}, ω_{2i} are the imaginary roots. These derivations are valid under the approximations $k^2 C_{scam}^2 \sim w_J^2$. These two roots correspond respectively to the hybrid type oscillatory mode (gravito-electrostatic mode) and hybrid type Jeans-like growing mode. The characteristic time scales of the oscillatory mode and that of the purely growing modes are almost equal and exist on the equilibrium levitational scale length of the total dust mass. If we utilise the linear superposition principle, it can be argued that a new mode of gravitational collapse namely the Pulsational Mode of Gravitational Collapse (PMGC) could exist in a self gravitating model dust cloud which has its size of the order of the critical Jeans length. This leads to conclude that a possibility of pulsational mode of star birth in some star forming dust clouds with partial charging could not be ruled out. Physically speaking, the pulsational mode of the gravitational collapse arises due to partial charging of the dust grains' population, which leads to the survival of finite linear gravito-electrostatic coupling. The oscillation sets in due to electrostatic slippage of the charged dust fluid over the neutral dust mass around the critical Jeans length of the total dust mass distribution. The gravitational force acts as a restoring force for the oscillation to occur. This is basically an effort of the collapsing gravito-electrostatic fluid to maintain a quasi-hydrostatic equilibrium on Jeans critical length corresponding to total dust mass and thus the charged mass oscillates around the collapsing hydrostatic equilibrium.

Let us now analyse Eq. (18) in the extreme limit of frictional coupling ie $\frac{\nu_{cn}}{\omega} \to \infty$. In this case, the dispersion relation (18) is reduced to

$$1 + \frac{\omega_J^2}{\omega^2} - i\frac{k^2 C_{scam}^2}{\omega \nu_{cn}}\left(1 + \frac{\omega_J^2 - \omega_{Jd}^2}{\omega^2}\right) = 0 \quad (21)$$

It can be shown that only one unstable root exist near $\omega^2 \sim -\omega_J^2$ with growth rate

$$r_J \approx \omega_J \left(1 + \frac{k^2 C_{scam}^2}{2\omega_J^2}\frac{\omega_{Jd}^2}{\nu_{cn}\omega_J}\right) \quad (22)$$

It seems that the frictional damping of the acoustic mode which itself acts as a dissipative agent to produce acoustic resistance to be termed as acoustic viscosity for the Jeans collapse, further enhances the gravitational collapse. The other roots are found to be purely damped. Thus in the high collisional limit, the pulsational mode seems to be suppressed and eventually disappears and only modified hybrid type Jeans-like purely growing mode survives with enhanced growth rate.

4. Conclusions

The proposed theoretical model here, for the gravitational collapse of the neutral dust mass (with part of the dust population being charged) leads to the possibility of a qualitatively new mode of Jeans collapse namely the PMGC mode. It is predicted that a collapsing dust cloud with its size of the order of the critical Jeans length should exhibit a pulsational mode of star formation. Moreover, as discussed in recent papers by Nakano and his collaborators, the actual compositions of the star forming clouds require more realistic calculations to improve the proposed theoretical model for meaningful discussions about the star forming mechanism. Thus, the consideration of all possible drag effects of the background neutral gas and also the magnetic field effect are required to improve our model calculations in the context of more realistic situations so that the validity of calculations can be verified for observational data.

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