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On the determination of carbon isotopic ratios in cool carbon stars

II. Accuracy of the iso-intensity method for crowded spectra

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Abstract. de Laverny & Gustafsson (1998, Paper I) questioned the *iso-intensity* method used by Ohnaka & Tsuji (1996) to derive carbon isotopic ratios in cool carbon stars. As a reply, Ohnaka & Tsuji (1998) pointed out that the method criticized in Paper I differs from the original one and conclude that our results are unreliable. No comparison of the two methods was, however, made. The *iso-intensity* method used in Paper I and OT96 do slighty differ in their definition. In investigating the consequences of this difference we have found that they are rather insignificant. We therefore confirm that the *iso-intensity* method is not reliable when applied to crowded spectra of cool stars. That reinforces the conclusions presented in Paper I.

Key words: stars: abundances – stars: late-type – stars: AGB and post-AGB – stars: carbon – molecular processes

1. Introduction

The determination of carbon isotopic ratios (^{12}C/^{13}C) in cool carbon stars is of considerable interest for C-stars interior studies and Galactic evolution (see for references de Laverny & Gustafsson, 1998, Paper I).

This ratio, as well as several other elemental abundances, has been determined by Lambert et al. (1986) in 30 Galactic N-stars. They derived rather high carbon isotopic ratios ($30 < {}^{12}\mathrm{C}/{}^{13}\mathrm{C} < 100$). More recently, Ohnaka & Tsuji (1996, OT96 hereafter) found much lower ^{12}C/^{13}C ratios in 62 similar stars. 20 stars were independtly studied by the two groups and for all of them the carbon isotopic ratios derived by OT96 are the smallest (up to 3 times smaller for several stars).

We showed in Paper I that the *iso-intensity* method used by OT96 to derive the ^{12}C/^{13}C ratios is risky when applied to crowded spectra. Much of the uncertainties could indeed be explained by blend effects and errors due to model atmosphere, carbon enrichment and effective temperature. However, Ohnaka & Tsuji (1998, OT98) questioned the discussion of Paper I about the *iso-intensity* method. They pointed out that our simulation of this method differs from the one they actually used. They concluded that our study is unreliable but did not study the magnitude of the effects of this discrepancy.

We prefer to concentrate this paper to a discussion about the inaccuracy of the *iso-intensity* method when applied to complex spectra of cool stars and do not want to reply to the whole discussion of OT98 about blending, spectral synthesis applied to cool carbon stars, etc. We indeed already discussed these points in Paper I and reached similar conclusions. We present a comparison of the different *iso-intensity* methods in the next section. We show that they lead to similar results and therefore confirm the conclusions of Paper I which are summarized in Sect. 3.

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2. Comparison of the *iso-intensity* methods of Paper I and OT96.

The *iso-intensity* method was developed by Cayrel & Jugaku (1963) to derive chemical abundances in main-sequence stars. It consists in plotting calculated strengths of the lines versus their observed central depths, and therefore can be called *curves of growth of central depth intensity* since line intensities are measured instead of equivalent widths as in more classical studies. OT96 favoured this *iso-intensity* method since the spectra of cool carbon stars are crowded by millions of molecular lines and blends are very important. OT96 indeed cannot find enough isolated lines to accurately measure their equivalent widths. Abundance determinations by spectral synthesis are also difficult for such complex spectra (see Paper I).

The abscissa of these curves of growth for central depth intensities ($\log gf\Gamma$ with g and f being the statistical weight of the lower level and the oscillator strength of the transition, respectively, and Γ being a function of the model atmosphere) is clearly defined by Cayrel & Jugaku (1963) and applied by OT96 in the weak line approximation. In Paper I, we used a different abscissa for simplicity. We actually checked in the preparation of this paper that the abscissa of OT96 could be reasonably well approximated by $\log(gf\lambda) - \theta\chi - \log(\kappa_\nu)$, where λ is the wavelength of the transition, χ the excitation energy of its lower level and θ and κ_ν the reciprocal temperature and continuous absorption coefficient determined at $\tau_{\mathrm{line\ center}} = 0.1$,

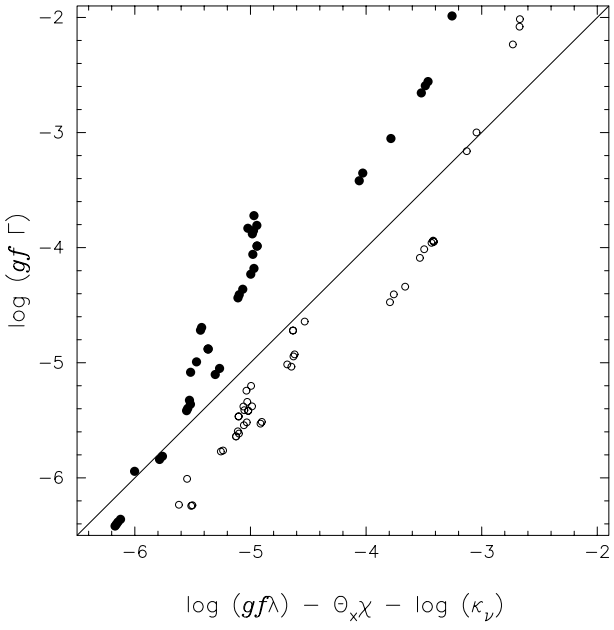

Fig. 1. Comparison of the abscissa of the curve of growth for central depth intensities in the *iso-intensity* method considered in Paper I ($\log(gf\lambda) - \theta\chi - \log(\kappa_\nu)$) and OT96 ($\log gf\Gamma$). ^{12}CN and ^{13}CN lines are represented by filled and unfilled circles, respectively. Γ depends on the ^{12}C/^{13}C ratio, which explains the shift of the ^{12}CN relative to the ^{13}CN lines. Model atmosphere parameters are $T_{\rm eff}$ = 3 000 K, log (g) = 0, [Fe/H] = 0, C/O = 1.1, $\xi_{\rm micro}$ = 2 km s^{-1}and ^{12}C/^{13}C= 10. To faciliate the comparison between the two abscissae, we also plot the line corresponding to $\log gf\Gamma = \log(gf\lambda) - \theta\chi - \log(\kappa_\nu)$ (see text).

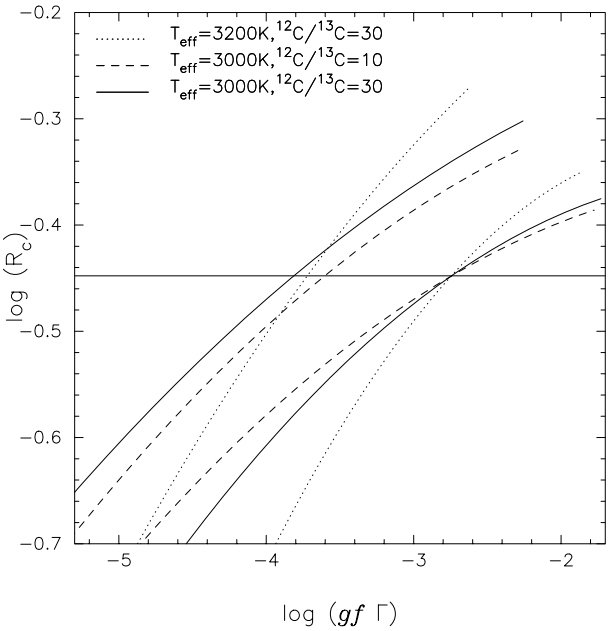

Fig. 2. Curves of growth of central depths intensities mimicking the procedure of OT96. Model parameters are those of Fig. 1 except for the effective temperature ($T_{\rm eff}$ = 3 000 K and 3 200 K) and the carbon isotopic ratio (^{12}C/^{13}C = 10 and 30). The depth of the CN lines analyzed by OT96 has been fixed to a given value (as if they have been derived from an observed spectrum) when calculating the curves of growth. It can be seen that an uncertainty of +200 K in $T_{\rm eff}$ leads to a smaller horizontal distance between the ^{12}CN (right) and ^{13}CN (left) curves at the typical depth of the ^{13}CN lines (horizontal line in the figure). Therefore, a smaller ^{12}C/^{13}C ratio can be derived if $T_{\rm eff}$ is kept constant at 3 000 K as already demonstrated in Fig. 3 of Paper I.

respectively. The comparison of the two abscissa formulations is discussed below.

We have calculated the abscissa of the curves of growth for the central depth intensities, as defined by OT98, ($\log gf\Gamma$) for all the ^{12}CN and ^{13}CN lines considered in Paper I. In Fig. 1, we compare them with calculations made with the definition of Paper I. First, one can note that there is a rather moderate dispersion of the ^{12}CN and ^{13}CN points since they form two well separated curves. In this figure, every line of a given molecular band of ^{12}CN or ^{13}CN has the same behaviour and form a compact group. For example, all ^{12}CN lines with $\log(gf\lambda) - \theta\chi - \log(\kappa_\nu) \simeq -5$ are transitions of the (5-2) band. The (rather small) scatter of the points around the two curves can be partially explained by numerical effects due to the stratification of the atmosphere. Uncertainties in the molecular constants used when computing the data for a given band (see Paper I) could also lead to some scatter from band to band.

Furthermore, it can be seen in this figure that there is almost a linear relation between the abscissa we considered in Paper I and that of OT96, for the ^{12}CN lines as well as for the ^{13}CN ones. Moreover, the relations for the ^{12}CN and ^{13}CN transitions are almost parallel and have a slope close to unity. Therefore, curves of growth for central depth intensities with the abscissa of Paper I are only slightly horizontally shifted with respect to curves of growth computed as OT96. Since this affects both the ^{12}CN and ^{13}CN curves of growth, the discussion of Paper I on the horizontal distance between these curves (directly proportional to the carbon isotopic ratio ^{12}C/^{13}C) is valid. We can check this by comparing how great the variation is of the abscissae computed with the two methods for the five lines used by OT96 to derive their ^{12}C/^{13}C ratios. We recall that the error analysis in Paper I (and mimicking the procedure of OT96) was also made from the estimate of the effects of model-parameter uncertainties on the behaviour of these lines. Indeed, Table 1 clearly shows that every line is shifted in the same direction and with almost the same magnitude. From this, we find that the *maximum effect* on ^{12}C/^{13}C values derived using the different abscissae can not be larger than about 0.1 dex.

This is confirmed in Fig. 2 where some curves of growth of central depth intensities for different effective temperatures and carbon isotopic ratios are plotted. First of all, we can see that the horizontal distance between the ^{12}CN and ^{13}CN curves of growth increases with ^{12}C/^{13}C as already shown in Paper I where a different abscissa was considered. Furthermore, this figure is actually similar to Fig. 3 of Paper I although the abscissae differ. Indeed, although the shape of the curves has slightly changed, the horizontal distance between them (indicator of the ^{12}C/^{13}C ratio) is proportionally as large as in Fig. 3 of Paper I (see Table 2 for a quantitative comparison). From Table 2 we

Table 1. Shift of the abscissa of the curve of growth for central depth intensities for the five lines used by OT96 when they are calculated as in Paper I (log $(gf\lambda) - \theta\chi - \log(\kappa_\nu)$) and as in OT96 (log $gf\Gamma$), respectively. In both cases corrections for the ^{12}C/^{13}C ratio have been made.

^{12}CN (7,4) lines			^{13}CN (2,0) lines	
$Q_1(34)$	$Q_1(35)$	$P_1(29)$	$P_1(16)$	$P_2(19)$
+0.70	+0.69	+0.57	+0.59	+0.65

Table 2. Comparison of the horizontal distance (at the depth of the ^{13}CN lines) between the ^{12}CN and ^{13}CN curves of growth for central depth intensities shown in Fig. 3 of Paper I (computed with log $(gf\lambda) - \theta\chi - \log(\kappa_\nu)$) and Fig. 2 (this work, computed with log $gf\Gamma$).

	Fig. 3 (Paper I)	Fig. 2
$T_{\rm eff}$ = 3 000 K $\rightarrow$ 3 200 K	-0.10 dex	-0.09 dex
^{12}C/^{13}C= 30 $\rightarrow$ 10	-0.25 dex	-0.22 dex

also confirm that an uncertainty on the effective temperature leads to a large error on the carbon isotopic ratios derived with the *iso-intensity* method. Finally, we find that the change of the estimated ^{12}C/^{13}C ratio when applying our version of the *iso-intensity* method as compared with that of OT96 will be a shift of typically 0.01–0.07 dex. E.g., for the $T_{\rm eff}$ /log g/[Fe/H]/^{12}C/^{13}C = 3 000./0.0/0.0/30. case, the ^{12}C/^{13}C ratio estimated by our method will be $\sim 8\%$ larger.

Therefore, we conclude that the simulation of the *iso-intensity* method of Paper I is valid. This comes from the fact that the abscissa as defined in Paper I is actually derived from the model atmosphere, contrary to the claim of OT98. We never considered a unique excitation temperature for every line to compute the abscissae, although we admit that the atmospheric structure was considered in a less sophisticated way than in OT96. An equivalent "excitation temperature" (θ) is indeed derived for each individual line at $\tau_{\rm line\,center} = 0.1$. And, since different lines are formed more or less deep in the atmosphere, θ varies from line to line and is close to the "reciprocal excitation temperature" $\langle\theta_{\rm ex}\rangle$ defined by OT96 as log Γ_λ = log $\Gamma_\lambda(0) - \chi\langle\theta_{\rm ex}\rangle$.

3. Conclusion

We have compared the abscissa of the *iso-intensity* method used in Paper I with the original one considered by OT96. We have shown that choosing one formulation rather than the other for the computation of these abscissae of the curves of growth for the central intensities slightly shifts the location of the ^{12}CN and ^{13}CN curves. However, the horizontal distance between these curves of growth, and thus the derived carbon isotopic ratios, is not affected by these different formulations for the abscissa. Moreover, this distance is still rather sensitive to model parameters. We have therefore clearly shown that the result of the two methods do not strongly differ and actually lead to very similar results. The conclusion of Paper I is verified: the *iso-intensity* method should be avoided in the analysis of complex spectra of cool stars since it is rather sensitive to model parameters and blends. The rather low carbon isotopic ratios derived by OT96 should therefore still be questioned.

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