Letter to the Editor

On the ultimate fate of AM Her stars

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Abstract. We suggest, that the magnetic field of the white dwarf in AM Her systems loses coupling to the secondary star when the latter becomes non-magnetic at the transition from a late main sequence star to a cool degenerate brown dwarf. This leads to spin-up of the primary white dwarf. After synchronous rotation is lost the systems do not appear as AM Her stars anymore. We discuss the further evolution of such systems.

1. Introduction

AM Her stars (polars) are magnetic cataclysmic variables where the white dwarf primary and the Roche lobe filling low-mass secondary star rotate synchronously with the orbit. During secular evolution the secondary star loses mass to the primary. The matter flow is channeled by the magnetic field and accretes via accretion columns near magnetic poles (King 1995). The magnetic coupling of white dwarf and secondary star causes the synchronous rotation.

Secular evolution is the same for magnetic and non-magnetic systems. It is commonly accepted that below the period gap gravitational radiation causes the loss of angular momentum from the binary system and the decrease of the orbital period. For stars of mass below about $0.2 M_\odot$ the effective temperatur drops, for around $0.06 M_\odot$ a transition to a cooling brown dwarf occurs. The mean density reaches a maximum and decreases again when the star becomes degenerate. Correspondingly Roche lobe and orbital period increase again (Paczyński & Sienkiewicz 1981, Rappaport et al. 1982, Ritter 1986). For recent stellar modelling see Baraffe et al. (1998), Allard et al. (1996) and Allard et al. (1997).

But stellar structure calculations consistently yielded a minimum period of 70 minutes (most recently Kolb & Baraffe 1999), significantly lower than the observed cut off in the orbital period distribution of cataclysmic variables (CVs) (Ritter & Kolb 1998), near 80 minutes. To explain the missing (non-magnetic) dwarf nova systems with periods between 80 and 70 minutes

a selection effect (rare or no outbursts) has been invoked. For the (magnetic) AM Her systems the shortest period observed is 77.8 minutes (RXJ0132.7-6554, Burwitz et al. 1997). Kolb & Baraffe (1999) found that the theoretical period minimum can be raised to 80 minutes if the braking would be four times the gravitational wave value. This might point to a residual braking from the secondary star's magnetic field. We note that Patterson (1998) pointed out a discrepancy between the predictions of CV evolution and observations: far too few CVs are observed which have evolved past period minimum.

We here propose the disappearance of AM Her systems after the period turning point. We argue that the magnetic field of the white dwarf loses coupling to the secondary star when the secondary becomes non-magnetic at the transition from a late main-sequence star to a brown dwarf. This leads to spin-up of the primary white dwarf and the binary does no longer appear as an AM Her system. Such a spin-up could lead to ejection of mass and magnetic flux in a "propeller phase", a process which would finally stop further spin-up. It is an interesting question how the stars will appear after this metamorphosis.

In Sect. 2 we discuss the mechanisms of magnetic coupling and the loss of coupling with the transition of the secondary to the brown dwarf state. In Sect. 3 and 4 we consider the spin-up phase and a possible propeller phase. In Sect. 5 the further evolution is discussed.

2. Magnetic coupling

Various mechanisms have been suggested for the magnetic coupling. Generation of a magnetic torque by a small degree of asynchronism, dipole-dipole interaction, and conductive connection of field lines between the two stars were considered. Even if no secondary star magnetic field would exist convective mixing-in of primary fields has been appealed to. (Campbell 1985,1986, 1989, Lamb 1985, Lamb & Melia 1988). We discuss in the following why these conditions for coupling disappear in the late state of secular binary evolution.

2.1. Disappearance of the secondary's magnetic field

In the transition from a low-mass main-sequence star to a brown dwarf the overadiabatic structure and convection disappear. No

convection-based dynamo exists and no containment of captured flux in a convective zone (Meyer 1994) is possible anymore. In both cases the magnetic field of the secondary disappears.

A further hint that the field vanishes when the secondary becomes a brown dwarf comes from the outburst behaviour of the dwarf novae in late secular evolution. The modelling of the extremely bright outbursts and the recurrence time of decades of WZ Sge stars requires an extremely low viscosity in the quiescent accretion disk (Meyer-Hofmeister et al. 1998). If the accretion disk viscosity in quiescence is caused by the secondary's magnetic field (Meyer & Meyer-Hofmeister 1999,) this low viscosity value can be interpreted as due to the disappearance of the secondary star's magnetic field.

2.2. Loss of coupling

If the magnetic field of the secondary disappears no dipole-dipole interaction and no conductive tying of primary field to secondary field are possible. The primary field cannot couple to a non-magnetic secondary because buoyancy tends to expell magnetic fields from the star and there is no convective mixing to counteract this effect. Ohmic diffusion on the hotter irradiated side does not suffice for efficient penetration. On the cool surface of the backside conductivity is too low for magnetic coupling (Meyer & Meyer-Hofmeister 1999).

3. The spin-up phase

When magnetic coupling is lost the white dwarf starts to spin up. No angular momentum is returned to the secondary and thereby to the orbit.

We investigate how the mass transfer develops. For the structure of the secondary we take a polytrope with pressure p and density ρ

$$p = K\rho^\gamma \; ; \;\; \gamma = 5/3 . \tag{1}$$

K depends on the secondary's mass M_2 and radius R_2 (Emden 1907). We assume that the star fills its Roche lobe $R_{R,2}$. We use Paczyński's (1971) approximation for small mass ratios $q = M_2/M_1$ (M_1 mass of the primary)

$$\frac{R_{R,2}}{a} = 0.462 \left(\frac{q}{1+q}\right)^{1/3} , \tag{2}$$

where a is the separation of the binary stars. The separation is related to the orbital period $P = 2\pi(a^3/GM)^{1/2}$, $M = M_1 + M_2$, G gravitational constant. The mean density $\bar{\rho}_2$ is then related to the orbital period P, $\bar{\rho}_2 = 110/(P/\mathrm{h})^2$ (Frank et al. 1985). For a Roche lobe filling secondary the constant K can be expressed

$$K = 10^{13.08} \left(P_{80} \frac{M_2}{0.07 M_\odot}\right)^{2/3} , \tag{3}$$

where P_{80} is the orbital period in units of 80 minutes (a completely degenerate star of cosmic abundance would have K=$10^{12.85}$).

The rate of mass transfer through the Lagrangian point L_1 is given by the product of density, sound speed and effective cross section (see Kolb & Ritter 1990). For a polytrope with $\gamma = 5/3$ we obtain

$$\dot{M} = \frac{2\pi a^3 K^{3/2}}{k(q)GM} 8.545\, \bar{\rho}_2^2 \left[\frac{R_2 - R_{R,2}}{R_2}\right]^3 \tag{4}$$

with $k(q)$ from the Roche geometry (compare Meyer & Meyer-Hofmeister 1983), $k(q)$=5.97 for q=0.1.

For $q = 0.1$ and $M_2 = 0.07 M_\odot$ follows:

$$\dot{M} = 10^{29.93} \frac{M_2/(0.07 M_\odot)}{P_{80}} \left[\frac{R_2 - R_{R,2}}{R_2}\right]^3 . \tag{5}$$

We determine the change of $\dot{M}$. We call β the fraction of the transferred matter accreted on the primary, $\dot{M}_1 = \beta\dot{M}$, in the spin-up phase $\beta = 1$.

The change of the Roche radius $R_{R,2}$ depends on the change of the quantities a and q.

$$\frac{1}{R_{R,2}} \frac{dR_{R,2}}{dt} = \frac{1}{a}\frac{da}{dt} + \frac{1}{q(1+q)}\frac{dq}{dt} . \tag{6}$$

The time derivative of a is related to the derivative of the angular momentum J

$$J = \sqrt{GMa}\frac{M_1 M_2}{M} . \tag{7}$$

We consider orbital angular momentum loss due to gravitational radiation, spin-up of the white dwarf and, in the propeller phase, expulsion of matter

$$\dot{J} = \dot{J}_{\mathrm{GW}} + \dot{J}_{\mathrm{spin-up}} \;\; \mathrm{spin-up\; phase}, \tag{8}$$

$$\dot{J} = \dot{J}_{\mathrm{GW}} + \dot{J}_{\mathrm{propeller}} \;\; \mathrm{propeller\; phase}.$$

We take $\dot{J}_{\mathrm{GW}}$ according to Misner et al. (1973). Additional braking, as considered by Kolb & Baraffe (1999) can be taken into account by applying a corresponding factor to $\dot{J}_{\mathrm{GW}}$ and $\dot{M}_{\mathrm{GW}}$. $\dot{J}_{\mathrm{spin-up}}$ results from the accreted angular momentum used for spin-up and lost from the orbit

$$\dot{J}_{\mathrm{spin-up}} = \sqrt{GM_1 a} \left(\frac{r_{\mathrm{LS}}}{a}\right)^{1/2} \dot{M}, \tag{9}$$

where $r_{\mathrm{LS}}/a = \tilde{\omega}_{\mathrm{d}}$ was evaluated by Lubow & Shu (1975), 0.23 for q=0.1.

Eqs. (5) to (9) finally yield the equation for the change of $\dot{M}$. The quantity X collects all terms proportional to $\dot{M}$ and is decisive for the evolution.

$$\frac{d\dot{M}}{dt} = C\dot{M}^{2/3} \left(\dot{M}_{\mathrm{GW}} - X\dot{M}\right), \tag{10}$$

$$C = \frac{10^{-21.43}}{k(q)^{1/3}} \left(\frac{M_2}{(0.07 M_\odot)}\right)^{-2/3} P_{80}^{-1/3}\, \mathrm{g}^{-2/3}\mathrm{s}^{-1/3},$$

$$\dot{M}_{\mathrm{GW}} = \frac{10^{14.79}}{q^{2/3}(1-q)^{1/3}} \cdot \left(\frac{M_2}{0.07 M_\odot}\right)^{8/3} \cdot P_{80}^{-8/3} \ \mathrm{g\,s^{-1}},$$

$$X = \zeta - \frac{1+\beta q}{3(1+q)} - 2f\sqrt{(1+q)(\frac{r_{\mathrm{LS}}}{a})}$$
$$+2(1-\beta q) - (1-\beta)\frac{q}{1+q} \ .$$

The factor f is the ratio of specific angular momentum lost from the orbit by the matter flow (either accreted on the white dwarf or expelled from the system) to the specific angular momentum carried by the aarriving ccretion stream. For the spin-up phase f is equal to 1. The mass-radius exponent ζ=dlnR/dlnM changes from about 0.8 to -1/3 as the secondaries evolve from a late main sequence star to a (partially) degenerate cool brown dwarf (Ritter 1986, Kolb & Baraffe 1999).

For positive value of X the solution of Eq. (10) tends to the stable point $\dot{M} = \dot{M}_{\mathrm{GW}}/X$. Due to the change of X with the start of the spin-up the mass transfer rate $\dot{M}$ increases by more than a factor of ten. We assume that the matter falling towards the white dwarf in the rotating magnetosphere behaves diamagnetically and experiences a braking force from the relative motion between field and fluid (King 1993, Wynn & King 1995). The strongest interaction occurs at the point where the free fall path would reach closest approach and the magnetic field is largest. If the speed of the rotating magnetic field becomes faster than that of the material there a new phase involving expulsion of matter can set in.

For our standard case $M_1 = 0.7 M_\odot$, $M_2 = 0.07 M_\odot$, P_{orbital}=80 minutes, the amount of matter required to reach critical rotation of the white dwarf is ΔM=0.002$M_\odot$, accumulated in about $6\,10^6$ys.

4. The propeller phase

The falling matter enters the magnetic field of the rotating dipole. For an inclined dipole the braking and acceleration force experienced by the matter depends on the rotational phase of the dipole. Thus one part of the matter reaches the strong interaction a bit farther away from the primary and the other comes closer in. The latter would experience braking and will finally be accreted. The former can be accelerated outward and flung out of the system. A propeller effect was already discussed by Illarionov & Sunyaev (1975). The interaction of the magnetic field and the matter stream is complex. We adapt here results obtained from a model for AE Aquarii by Wynn et al. (1997).

The intermediate polar AE Aqr is observed to spin down at a rate of $\dot{P}_{\mathrm{spin}} = 5.64 \times 10^{-14}$. In the model of Wynn et al. (1997) the angular momentum given off by the white dwarf adds to the angular momentum of the stream itself to expel nearly all of the matter transferred from the secondary. The interpretation of the observed Doppler tomogram supports this expulsion of matter. Along these lines we now estimate the orbital angular momentum loss $\dot{J}_{\mathrm{propeller}}$ involved in the expulsion of matter in our case. We obtain an estimate for the fraction of matter 1-β expelled from the system by the following consideration.

The matter arriving at the distance of closest approach $r_{\min}$ needs additional angular momentum to be accelerated above escape speed. This is provided by the angular momentum of the accreted fraction β.

$$(1-\beta)\left[\sqrt{(1+b^2)}\sqrt{2GM_1 r_{\min}} - \sqrt{GM_1 r_{\mathrm{LS}}}\right] \quad (11)$$

$$= \beta\sqrt{GM_1 r_{\mathrm{LS}}}.$$

We take for $r_{\min}$ the value determined by Lubow & Shu (1975) $r_{\min} = a \cdot \tilde{\omega}_{\min}$. Assuming that the matter arrives at infinity with velocity one half of the escape speed at distance $r_{\min}$ from the white dwarf, $b = v_\infty/v_{\mathrm{escape}} = 1/2$ one obtains the estimate

$$\beta = 1 - \frac{1}{\sqrt{2 r_{\min}/r_{\mathrm{LS}}}} = 0.2. \quad (12)$$

The numerical value results for q=0.1.

The orbital angular momentum loss in the propeller phase has to be taken with respect to the binary's center of gravity. The forces that produce the roughly $90°$ swing around at the primary before ejection retard the primary's orbital motion and thereby extract orbital angular momentum. We use the computed trajectories from Lubow & Shu, interpolated for q=0.1 and write the estimate for $\dot{J}_{\mathrm{propeller}}$

$$\dot{J}_{\mathrm{propeller}} = (1-\beta)\dot{M} \cdot \quad (13)$$

$$\left[\sqrt{v_{\mathrm{escape}}^2 + v_\infty^2} \cdot r_{\min} + v_\infty \frac{q}{1+q}a\right],$$

where we assume that the expelled matter keeps its angular momentum with respect to the primary as it rapidly climbs out of the gravitational potential. The last term on the right side accounts for the distance between the primary and the binary's center of mass. This yields

$$\dot{J}_{\mathrm{propeller}} = \dot{M} f \sqrt{GM_1 r_{\mathrm{LS}}} \quad (14)$$

with

$$f = (1-\beta)\sqrt{\frac{2 r_{\min}/a}{r_{\mathrm{LS}}/a}}\sqrt{1+b^2} \quad (15)$$

$$\cdot \left(1 + \frac{q}{q+1}\frac{b}{\sqrt{1+b^2}}\frac{1}{r_{\min}/a}\right).$$

For q=0.1 and β=0.2 one obtains f=1.3. A graphical evaluation of trajectories calculated by Wynn et al. (1997) for AE Aqr taking into account the smaller mass ratio leads to f=1.5. We emphasize that these estimates are rough. With such values of f the increase of orbital angular momentum loss compared to $\dot{J}_{\mathrm{spin-up}}$ changes the sign of X and then leads to an accelerated growth of the mass transfer rate [Eq. (10)]. Whether this occurs depends on the detailed process during the swing around the white dwarf. For f=1.3 and our standard case the time to reach arbitrarily large $\dot{M}$ is only $10^{4.6}$ys. The rate cannot grow indefinitely, finally the magnetic field becomes unable to handle the ever growing mass transfer. The efficiency of acceleration diminishes and angular momentum loss from the system gets limited.

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5. Further evolution

The question arises how such systems develop. If the system stabilizes at a high transfer rate the secondary may lose all its mass in a relatively short time and a single fast rotating magnetic white dwarf remains. The observation of RE J0317-853 (Burleigh et al. 1999) with these unusual characteristics seems to support such a possibility. Another path of evolution might lead to the formation of an accretion disk, which results in return of angular momentum back to the secondary, connected initially with a strong increase of accretion onto the white dwarf, as β becomes 1, and then a rapid decrease of $\dot{M}$. But the strong magnetic pressure of the white dwarf removes such a disk before the mass accretion rate has dropped to the low stable value $\dot{M}_{GW}/X$ [Eq. (10)]. This could lead to a cyclic spin-up/spin-down on short timescales, involving phases of high $\dot{M}$ (compare the model for AE Aqr by Wynn et al. 1997) and also rapid depletion of the secondary. These phases are probably too short to be observable. If expelled matter would form a circumbinary disk, such a disk could extract significant angular momentum from the orbit and thereby even lead to a supersoft X-ray source - state (van Teeseling & King 1998).

6. Conclusions

The loss of magnetic coupling and the degeneracy of the secondary stars are the important features of the late evolution of AM Her systems. We conclude that no AM Her systems should exist beyond the period turning point (except the brown dwarf would be young, hot and still magnetic - searches for the secondary stars in AM Her systems near period mimimum would be very desirable). Instead many of the unusual systems at low orbital periods might be descendants of the AM Her systems. There is one parameter, the strength of the white dwarf magnetic field, which may decide which way the later evolution goes.

Dwarf nova systems do not partake in these evolutionary phases. When they lose magnetic braking and settle to gravitational wave braking only their mass transfer rate could well become too low for allowing any outburst (see the marginal case of WZ Sge, Meyer-Hofmeister et al. 1998) and thus fade from view.

The low viscosity needed to model the outbursts cycles of the dwarf nova systems in late secular evolution and the disappearance of the AM Her systems could thus both result from the loss of the magnetic field of the companion star, when the star evolves to a cool brown dwarf.

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