Colour transformations for isochrones in the VI-plane

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Abstract. Reliable temperature–colour transformations are a necessary ingredient for isochrones to be compared with observed colour-magnitude-diagrams of globular clusters. We show that both theoretical and empirical published transformations to $(V - I)$ to a large extend exhibit significant differences between them. Based on these comparisons we argue for particular transformations for dwarfs and giants to be preferred. We then show that our selected combination of transformations results in fits of V-$(V - I)$-CMDs of a quality which is comparable to that of our earlier V-$(B - V)$ isochrones for a wide range of cluster metallicities. The cluster parameters, such as reddening, are consistent with those derived in $(B - V)$. Therefore, at least in the case of the fit with our own isochrones – based on the particular distance scale provided by our own horizontal branch models, and on the treatment of convection by the mixing-length theory having l/H_p calibrated on our solar model – the chosen transformations appear to lead to self-consistent $(V - I)$ isochrones. Our isochrones are now well tested and self-consistent for B, V and I photometric data.

1. Introduction

The last step in comparing theoretical isochrones to globular cluster colour-magnitude diagrams (CMD) is the transformation of luminosity and effective temperature into brightness and colour. While there are a number of both theoretical and empirical sets of transformations available, this last step is, nonetheless, difficult and critical. Known shortcomings in the theoretical transformations which are obtained from theoretical non-grey model atmospheres calculations are, e.g., incomplete line lists for the opacity computation which affect the derived spectra and broad-band colours, the treatment of atmospheric convection which affects the colour determination as well (Gratton et al. 1996), and an incomplete coverage of the $\log g$–$\log T_{\mathrm{eff}}$ and composition parameter space, while empirical relations are naturally based on a

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limited number of stars. The transformations of Kurucz and coworkers (Kurucz 1992; Castelli et al. 1997) have well-known problems with the colours of red giants (Gratton et al. 1996; Salaris et al. 1997; Grundahl et al. 1998), but are nevertheless widely used. It is generally believed that the bolometric corrections, which are more important for most methods of cluster age determinations, are reliable, at least in their differential properties, and indeed results based on different transformations are in reasonable agreement (Salaris et al. 1997). As long as there are no unified stellar models available, which treat the stellar interior and the non-grey atmosphere together in one complete and realistic model (but see Bernkopf 1998 for a first unified solar model), such transformations are the only way to produce observable photometric quantities.

There is also the persistent concern about the unsolved problem of superadiabatic convection in the envelopes of cool stars, which determines surface temperature and thus colour. In our own calculations we have used the standard mixing-length theory with one constant value for the mixing-length parameter, because even with this very simplistic choice we are able to match both the solar radius and the effective temperature of metal-poor giants (Salaris & Weiss 1998). The existence of a rather constant value for this parameter is also supported by comparing stellar models with the structure of convective envelopes computed by means of 2-dimensional hydrodynamical models (Ludwig et al. 1997; Freytag 1998). However, there are other convection theories possible (Canuto & Mazzitelli 1991), which lead to different T_{eff} along an isochrone compared to the standard mixing-length approach. The discussion in this paper is therefore restricted to our own treatment of convection, and the conclusions could be different for a different choice.

The confidence in ages derived from theoretical isochrones is higher if the isochrone is able to reproduce the complete CMD from the lower main sequence (MS) to the tip of the red giant branch (RGB) to high accuracy, even if in practice only single points along the isochrone may be used for determining the cluster age (e.g. turn-off and horizontal branch). Reliable colours become crucial for methods that use colour differences, e.g. that between turn-off (TO) and RGB, for (absolute) age determinations, for determining distance and age from the MS-fitting technique, or for deriving correct integrated colours from theoretical isochrones, which constitutes an necessary step for studying old stellar populations in unresolved galaxies.

In our previous papers on this subject (Salaris et al. 1997; Salaris & Weiss 1997, "SW97"; Salaris & Weiss 1998, "SW98") we have taken great care to find transformations from the theoretical Hertzsprung-Russell-Diagram (HRD) to the CMD in $V - (B - V)$, which give a satisfying fit for *all* metallicities ranging now from $[\mathrm{Fe/H}] = -2.3$ to -0.6. We found that a combination of the transformations of Buser & Kurucz (1978) and Buser & Kurucz (1992) (with an appropriate colour shift of the latter ones to enforce continuity in (B-V) at the common temperature of $6000\,\mathrm{K}$) satisfied our needs.

Work on globular cluster (GC) dating during the last few years has received new attention. Due to improved models and methods and lately due to HIPPARCOS-based distances to clusters, much lower ages are obtained than before, such that, at this time, no serious conflict between cluster ages and the age of the universe exists (SW98; Chaboyer 1998). In order to extend our work to a larger sample of galactic halo clusters and in particular to those of the bulge (both needed for questions about the formation of the Galaxy), it is necessary to work in the $V - (V - I)$–CMD[1], because there is now an increasing number of high-quality observations of clusters in $V - (V - I)$. However, up to now, well-tested isochrones for this colour have not been available.

In this paper, we demonstrate that existing empirical and theoretical transformations provide very different $T_{\mathrm{eff}} - (V - I)$ relationships with only very few being in agreement with each other. For the choice as to which transformation we apply to our isochrones, we want to see the following requirements being fulfilled: (i) the transformation should give RGB colours consistent with the determined age; (ii) an isochrone that fits a cluster well in $(B - V)$, should do so in $(V - I)$, too, for $E(V-I)$ values compatible with the corresponding $E(B-V)$. In Sect. 2 we will present a number of recently published theoretical and empirical transformations and apply them to our theoretical isochrones. We will show that considerable differences in the resulting colours exist and that a priori there is no preferred transformation to be selected. However, at the same time the reliability of the bolometric correction in V will be reconfirmed. We will provide arguments for our selection of transformations for MS and RGB. In Sect. 3, we will then present tests verifying its usefulness for the case of our own isochrones applied to globular clusters of various metallicities. By no means this rather pragmatic approach is what one really wants for isochrone fittings, but at present it appears to be the only approach promising convincing results. Finally, a discussion and summary closes the paper (Sect. 4).

2. A comparison of different published colour-transformations

2.1. Transformation sources

For the purpose of this investigation we have used a number of recently published transformations, which we briefly describe in the following:

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[1] If not stated differently, the I-band refers to the Cousins system

– Buser & Kurucz (1978; 1992, "BK92"): This is the source for synthetic colours we have been using in our previous papers, showing that an appropriate combination (henceforth called "BK") yields satisfactory CMD-fits for all metallicities in the $V-(B-V)$ plane. The transformations are based on theoretical atmospheres and are available for all metallicities and both for dwarfs and giants. $(V - I)$ colours are available only for the cooler stars on the lower MS and on the RGB (BK92).

– ATLAS9: This is a set of spectra and colours obtained from a model atmosphere grid computed with the ATLAS9 code (Kurucz 1992), covering the range of O-K stars. The first set of ATLAS9 colours was discussed in Kurucz (1992). The convective models of this set were later revised ("K95"; see Bessell et al. 1998); the influence of the treatment of convection in the ATLAS9 model atmospheres on some colour indices has been discussed by Castelli et al. (1997). The latest grid of ATLAS9 colours, computed from "no-overshoot" models is presented in Bessell et al. (1998) for the solar metallicity. In this paper we will use this latest set of colours (hereinafter "BCP98") extended to lower metallicities by Castelli (1997, private communication).

– Lejeune et al. (1997, 1998; "LCB"): Synthetic spectra and colours based on various sets of theoretical atmospheres, among them K95 (covering almost completely the range of gravities and T_{eff} spanned by our isochrones). The theoretical spectra for the solar metallicity have been corrected to match empirical colour-T_{eff} relations; since comprehensive empirical calibration data have only been available for the full temperature sequences of solar-abundance giant and dwarf stars, the spectra (and derived colours) of lower metallicity models have been consequently corrected in such a way as to preserve the monochromatic flux ratios predicted by the models.

– Hauschildt et al. (1999; "Next-Gen"): Grid of theoretical model atmospheres, colours and bolometric corrections covering the main sequence of globular clusters, with T_{eff} ranging between 3000 and 10000 K, $\log g$ between 3.5 and 5.5 and [M/H] between -4.0 and 0.0. The atomic line list is the same as in Kurucz (1992), but the treatment of molecular, bound-free and free-free opacity sources, as well as the equation of state and the selected solar metal distribution are different. For convection, standard mixing-length theory with a constant parameter of 1.0 was used.

Note that all theoretical work up to now is for solar metal ratios only, i.e. no enhancement of the α-elements has been considered. Work along this line is in progress (Castelli 1999, in preparation).

– Alonso et al. (1996; "AAM96"): Empirical determination of the T_{eff}-colours relation for main sequence stars as a function of metallicity, using a large sample of dwarfs and subdwarfs. Since AAM96 use the Johnson–I-band, we have transformed Johnson–$(V-I)$ into Johnson–Cousin–$(V-I)$ colours according to Fernie (1983) throughout this paper.

- Montegriffo et al. (1998; "M98"): Empirical determination of bolometric correction-colour, colour-colour and $(V - K) - T_{\rm eff}$ relations obtained from photometric observations of 6500 GC giants covering metallicities from $[{\rm Fe/H}] = -2.2$ to 0.0. The results are averaged over two mean metallicity ranges below ("poor") and above ("rich") $[{\rm Fe/H}] = -1.0$.

- von Braun et al. (1998; "BCM98"): Empirical colour–$(V - K)$ relations for giants in GC of low and intermediate metallicity as well as one open cluster. These relations, together with an empirical $T_{\rm eff} - (V - K)$ relation taken from the literature, provide direct $T_{\rm eff}$-colour transformations for GC giants, taking into account their dependence on metallicity.

In the empirical work, α-element enhancement is considered only as far as the observed stars show it. There is no systematic differentiation between solar-type and α-enhanced populations.

2.2. Isochrones in the $V - (B - V)$–CMD and the bolometric correction

The theoretical isochrones used throughout this paper were computed for metallicities ranging from $[{\rm Fe/H}] = -2.3$ up to $[{\rm Fe/H}] = -0.6$, and are based on the same stellar evolutionary tracks as in our previous papers (e.g. SW98), except for the fact that meanwhile we have extended all the calculations to the tip of the RGB, i.e. to the onset of the core helium flash for all masses considered. The ZAHB models have been calculated from RGB-tip models. All isochrones[2] used here are for α-element enhanced mixtures ($\langle[\alpha/{\rm Fe}]\rangle$=0.4).

As previously mentioned, our source for $(B - V)$ colours and bolometric corrections (BC_V) is BK. In the previous papers (e.g. SW98) we have shown that by using these colours we were able to obtain a good fit to all clusters sequences covered by our isochrones, to derive GC reddenings in good agreement with the results from Zinn (1985), and to reproduce the HIPPARCOS subdwarf colours given by Gratton et al. (1997). As for the BC_V scale, the zero point was calibrated to the empirical scale by Alonso et al. (1995, 1996) for metal-poor MS stars; it reproduces the empirical BC_V-values for the whole range of metallicities and MS temperatures covered by the isochrones within ±0.02 mag. We required in particular that the solar V magnitude ($V_\odot = 4.82$) is reproduced.

With all the isochrones extended now up to the RGB-tip, we can also compare our BC_V scale for RGB stars with the empirical results by Frogel et al. (1981). In Fig. 1 (panels a-c) we show the comparison between the empirical BC_V values derived for three template clusters (M92, NGC3201, and 47 Tuc) spanning almost the whole metallicity range of galactic GC and the BC_V scale from our theoretical isochrones. After correcting for the slightly different (by 0.04 mag) zero points (i.e. to identical solar bolometric correction), we find very good agreement between the two sets of BC_V. Also the BC_V scales derived from the ATLAS9, Next-Gen (for the main sequence) and LCB transformations agree well with the BK one. In particular, once

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[2] The isochrones are available on request from the authors.
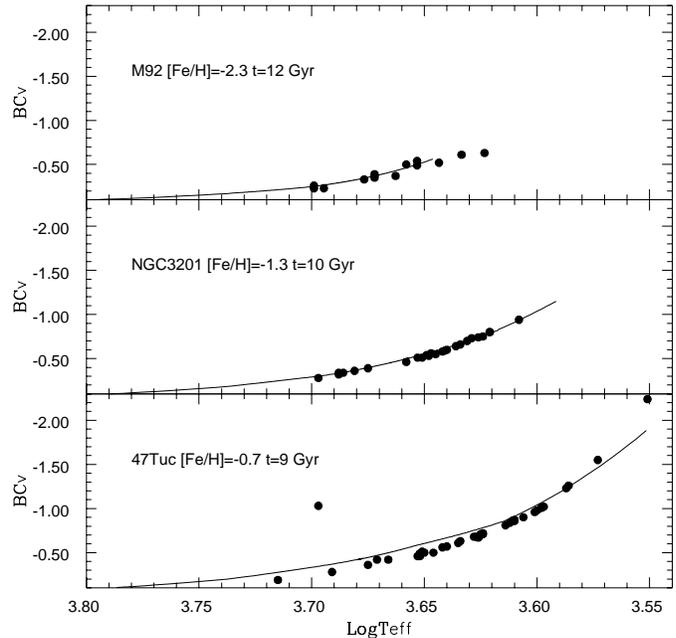
Fig. 1. Comparison of BC_V from BK with empirical data by Frogel et al. (1981) (see text). The outlier in panel c is in the original data

the BC_V zero point is calibrated as we did for BK (see above and SW98), the different scales agree within 0.05 mag all along the isochrones and on the ZAHB; moreover, the brightness differences between TO and ZAHB for fixed age and metallicity agree again within 0.05 mag. This confirms once more the consistency of the absolute ages derived in SW97 and SW98.

As a final demonstration of the usefulness of the BK-transformations applied to our isochrones, we show in Figs. 2 and 3 the fits to M68 and 47 Tuc in the V-$(B-V)$ plane with our extended isochrones (in SW97 and SW98 only fits up to the HB-brightness were displayed). The data for M68 (Walker 1994) are the same as in our previous papers, while we used the latest photometry by Kaluzny et al. (1998) for 47 Tuc. We briefly recall that the ages of the clusters are derived from the brightness difference between the ZAHB (defined as the lower envelope of the observed HB stellar distribution; see SW97 for more details) and TO; this permits to derive the cluster age independently of the knowledge of the cluster distance and reddening (due to the horizontal nature of the HB), but of course the theoretical ZAHB models provide a distance. Therefore, the distance modulus is derived by comparing the theoretical ZAHB level to the observed one, while the reddening is obtained from fitting the unevolved theoretical MS to the cluster one. For 47 Tuc we recover with the new photometric data exactly the same age, reddening and distance modulus as in SW98, where we were using the older composite photometry by Hesser et al. (1987). Note that we have used a rather high helium content of $Y = 0.273$ for 47 Tuc; this is, at least partially, based on the requirement of consistent vertical and horizontal (colour-dependent) age indicators (see SW98 for a full discussion on this matter). It is
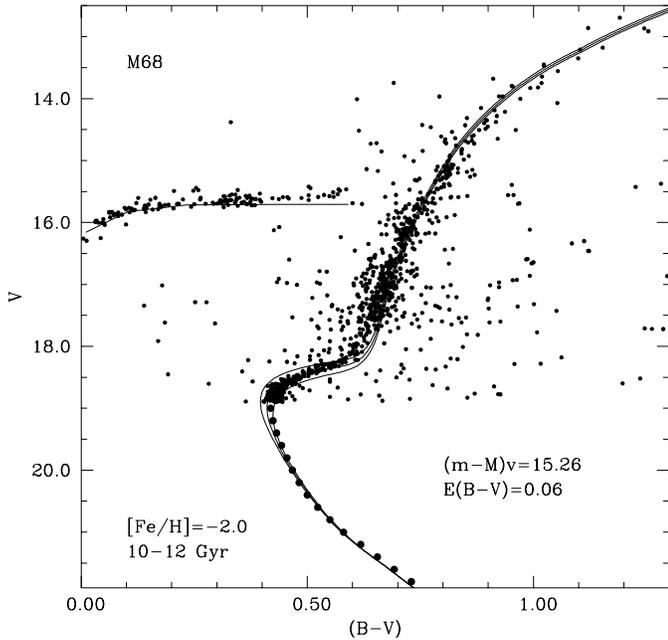
Fig. 2. The $V - (B - V)$–CMD of M68 (data from Walker 1994) and isochrones of 10–12 Gyr. For stars fainter than 19th mag only the ridge line is shown; for stars brighter than that only the data points
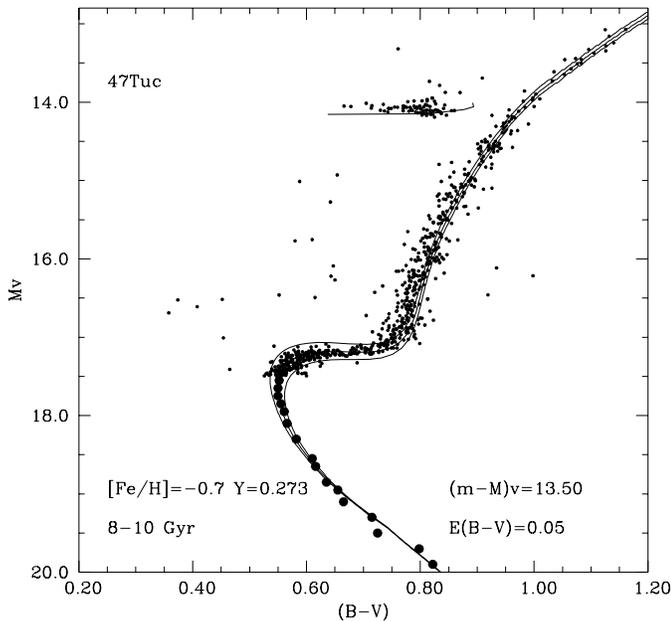

Fig. 3. The V-$(B-V)$-CMD of 47 Tuc (data from Kaluzny et al. 1998) with isochrones of 8–10 Gyr. For MS stars only the ridge line is shown.

evident from the pictures that the extended isochrones reproduce well the observed RGB of both clusters.

2.3. Isochrones in the $V - (V - I)$ CMD

For transforming our isochrones to the $V - (V - I)$ plane we have tested all transformations mentioned above. The BK transformations to $(V - I)$ colours do not extend to temperatures
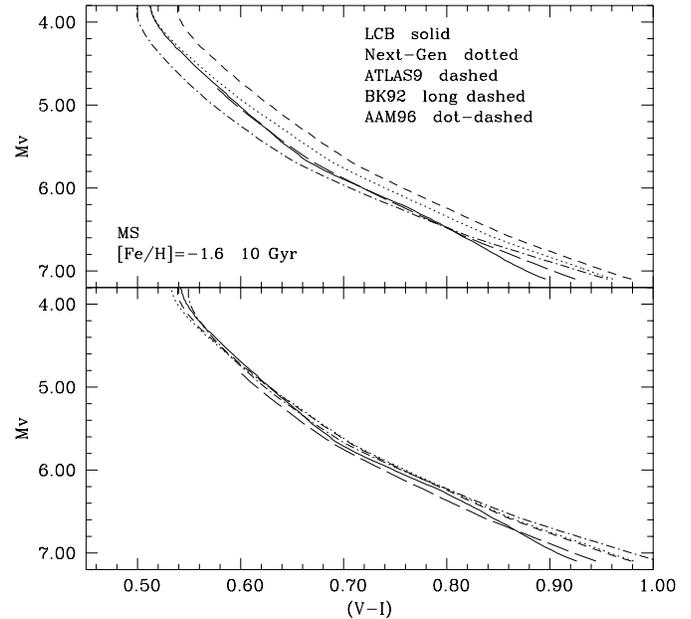

Fig. 4. *Upper panel:* Result of various colour-transformations (indicated in the figure) applied to a theoretical isochrone of $[Fe/H] = -1.6$ and 10 Gyr; shown is the main-sequence part. The various labels are explained in Sect. 2.1. *Lower panel:* The same comparison, but now after adding a constant to the various relations such that they reproduce $(V - I)_\odot = 0.73$

higher than 6000 K such that neither ZAHB nor TO models are covered for all metallicities and ages. Moreover, it is not guaranteed automatically that if this set of models reproduces well the observations in one colour band, the same is true for any other colour.

To exemplify the comparison between the different $T_{\rm eff} - (V - I)$ relations, we have used a representative isochrone ($[Fe/H] = -1.6$/10 Gyr), transformed to the M_V–$(V - I)$ plane in Fig. 4 (upper panel) for the MS and in Fig. 5 for the RGB. In the case of the BCM98 empirical transformations, we have used the empirical $T_{\rm eff} - (V - K)$ relation they give (Ridgway et al. 1980) together with their empirically determined $(V - I) - (V - K)$ relations. As they discuss in that paper, the Ridgway et al. (1980) relation is obtained for solar metallicity stars, but there are good indications that it is not affected by the metallicity. As for M98, the authors derive an empirical $T_{\rm eff} - (V - K)$ relation for metal-rich stars; following the arguments in BCM98, it appears to be safe to use the same relation for the most metal-poor stars, too. In conjunction with the empirical $(V - I) - (V - K)$ relation we obtain again a direct $T_{\rm eff} - (V - I)$ transformation which is displayed in Fig. 5.

It immediately is evident from these illustrations that colour differences up to between 0.05 and 0.10 mag exist all along the isochrone and that there are no two transformations yielding the same result for all evolutionary stages. This problem persists for other metallicities as well and is, of course, not restricted to our isochrones, but would be similar for any other. Part of the mismatch between the different transformations might be attributed to differences in the composition (of model atmo-
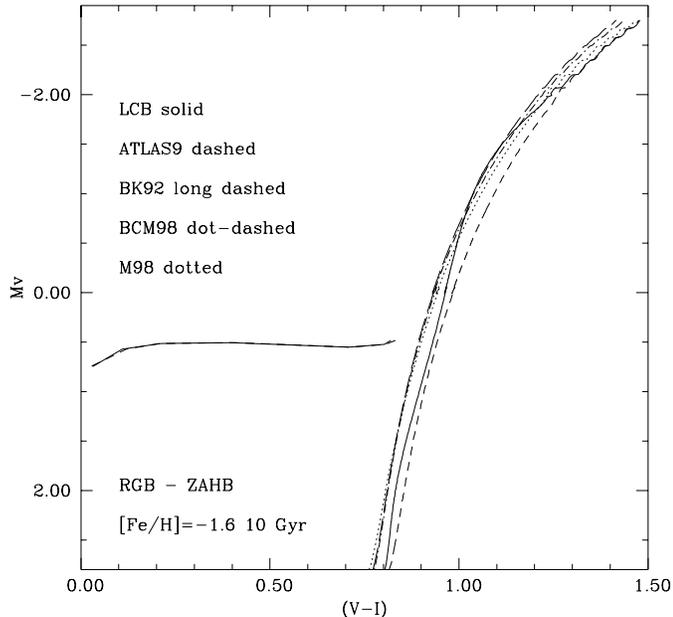
Fig. 5. As the upper panel of Fig. 4, but for the red giant branch and ZAHB
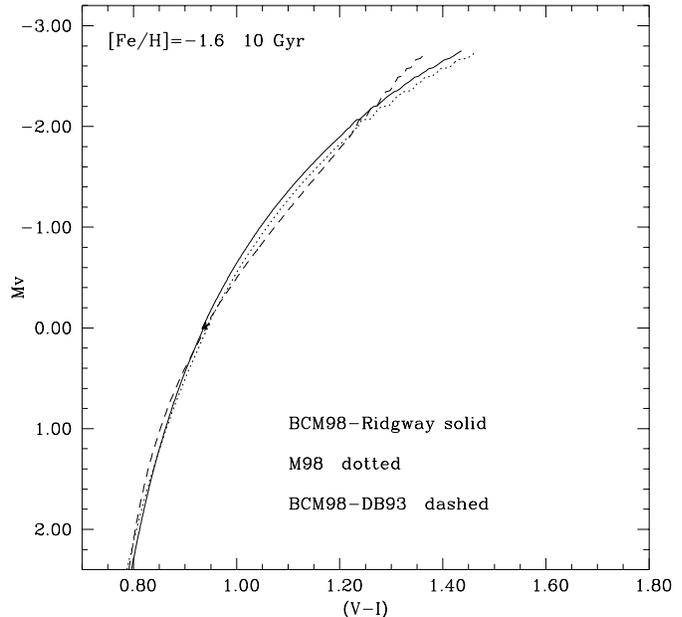
Fig. 6. Comparison of RGB empirical $T_{\mathrm{eff}} - (V - I)$ relations applied to our isochrones (see text)

spheres or observed stellar samples), to different or inappropriate assumptions in theory (e.g. convection theory, line lists, LTE-conditions) and to the internal spread in the empirical relations. Since the width of the upper MS and the RGB in high-quality photometry is at most as large as 0.05 mag, it is evident that whatever transformation is chosen as it was published, isochrones can deviate from the CMD structures by at least the width of the branches in colour. *It is therefore an illusion to assume that a perfect isochrone fitting is a strong indication for excellent theoretical stellar models or isochrones!*

Recently, Bessell et al. (1998) have independently compared a number of theoretical colour transformations with empirical relations for MS stars between spectral types O and M. Since they were interested in aspects of population synthesis, they concluded that there is in general an excellent agreement between theoretical and empirical T_{eff}–colour, bolometric correction–colour, and colour–colour relations. However, inspecting their figures (e.g. Fig. 6, where the $T_{\mathrm{eff}} - (V - I)$ relation for the lower MS is shown) one realizes that the deviations between the different sources are of same order as in our case. The same spread is also visible in the empirical $T_{\mathrm{eff}} - (V - I)$ relation (Fig. 2 as corrected in their Erratum). Only due to the enormous scale of the colour axis the agreement appears to be excellent. Globally, over all temperatures, the agreement is indeed very good, but not sufficient for such narrow structures as the RGB in galactic GC.

Along the MS, the empirical relation by Alonso et al. (1996) does not support any theoretical transformation. The Next-Gen and ATLAS9 (BCP98) colours are the most similar, the average differences being within 0.02 mag. Their $T_{\mathrm{eff}} - (B - V)$ relations, however, are almost identical in the range of T_{eff} we are dealing with, and in very good agreement with the $T_{\mathrm{eff}} - (B - V)$

relation adopted in SW97 and SW98, at least for T_{eff} larger than ≈ 5300 K.

Assuming $(V - I)_{\odot} = 0.73$ as in Bessell et al. (1998) we find that this value is reproduced within 0.01 mag by BCP98, while Next-Gen, BK92, AAM96 and LCB yield colours bluer by, respectively, ≈ 0.02, 0.02, 0.05, and 0.03 mag. Within the uncertainty of solar colours (see, e.g. the discussions in Chmielewski 1981 and Taylor 1994) all these values are probably acceptable; nevertheless, for gaining more insight about the differences among the $T_{\mathrm{eff}} - (V - I)$ relations under discussion, we normalized the different colour transformations to the same value $(V - I)_{\odot} = 0.73$, by correcting for the mentioned differences. After performing the necessary shifts one obtains the lower panel of Fig. 4 (we have also tested the case with $[\mathrm{Fe/H}] = -0.7$, with similar results). BCP98, Next-Gen (theoretical relations) and AAM96 (an empirical one) now agree rather well with each other along the whole MS, while the other colours differ from these by different amounts at different metallicities. This latter fact implies that the derivative $\delta(V - I)/\delta[Fe/H]$ at a fixed value of the MS brightness, a fundamental quantity needed for applying the MS–fitting method in the V-$(V - I)$ plane, does depend on the particular colour transformation used.

From the results of these comparisons for the MS phase (which are independent of the underlying theoretical isochrones) it results that BCP98, Next-Gen and AAM96 $T_{\mathrm{eff}} - (V - I)$ relations reproduce acceptably well the solar constraint and, moreover, show the same differential behaviour. If we base our 'objective' choice of the colour transformations on the consistency with empirical constraints and with independent theoretical determinations, we have to conclude that BCP98, Next-Gen or AAM96 fulfill these criteria for the MS.

While on the MS the isochrones transformed using different $T_{\rm eff} - (V - I)$ relations run more or less parallel to each other (except for LCB), along the RGB they differ also in slopes. However, we find that the two empirical relations tested on the RGB (M98 and BCM98) agree with each other rather well. von Braun et al. (1998) used the $T_{\rm eff}-(V - K)$ relation by Ridgway et al. (1980) and their empirical $(V - I)$–$(V - K)$ relation. These we combined to obtain the $T_{\rm eff}-(V - I)$ transformation. In Fig. 6 we show how this relation ("BCM98-Ridgway") compares to one using the more recent di Benedetto (1993; "DB93") $T_{\rm eff}-(V - K)$ relation instead and to the empirical one by Montegriffo et al. (1998). The agreement is, except for the brightest RGB part, better than 0.02 mag. The very good reciprocal consistency between these two purely empirical and independently determined relations is appealing, and on this ground we decided to try to use them for our isochrones. In particular, we selected the BCM98-Ridgway relation which contains an explicit (albeit very weak) metallicity dependence.

We finally need a transformation for our HB models. We have derived an empirical $(B - V) - (V - I)$ relation using multicolor photometries of clusters spanning the relevant range of metallicities. We used HB data for M68 (Walker 1994), M3 (Ferraro et al. 1997), M5 (Sandquist et al. 1996), and 47 Tuc (Kaluzny et al. 1998), and we applied this relation to our ZAHB models in the $(B - V)$ plane. In Fig. 7 the comparison between the ZAHB transformed by using this empirical colour-colour relation and two theoretical transformations is shown for two different metallicities. Evidently, the theoretical relations agree quite well with each other in predicting the turn-down of the ZAHB in the blue. But there are differences not visible due to the horizontal nature of the HB. The predicted colour along the HB differs in some cases by 0.05–0.10 mag, just as for the RGB. Also, the ATLAS9 transformation yields consistently redder stars at the cool edge (similar to the known effect on the RGB) with increasing metallicity. The empirical relation deviates at the blue end, possibly because of the effect that evolved stars show up in the relation. Since observed HBs usually have a large scatter at the blue, less luminous end, this part is not putting any strong constraint on the ZAHB level, such that the difference between the transformations as demonstrated in Fig. 7 is not really significant. Except for the mentioned problem of too red colours from the ATLAS9 transformation, we consider all relations as being effectively equivalent. Since we have used empirical data from those clusters we will discuss below, we will use the empirical relation throughout the remainder of this paper.

3. Globular cluster test cases

In the last section we presented a possible choice of colour transformations for all CMD-branches. This choice was made on the basis of which relations agree best with each other. The real justification of this choice, however, will be whether our transformation successfully fulfills the following requirements we are imposing: (i) for a given cluster, the fits in $(B - V)$ and $(V - I)$ should be of the same quality; (ii) the reddening
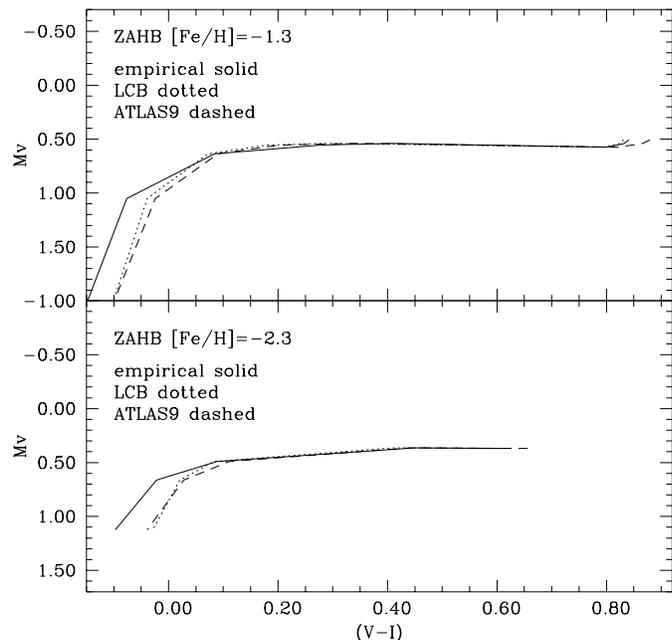

Fig. 7. The ZAHB at two metallicities in $(V - I)$: shown is our own relation ("empirical"; see text) and two other transformations. Except for the most extreme colours the agreement is good

obtained from both colours should be consistent; (iii) derived quantities as age and distance should not depend on the colour used. This implies that the final choice of transformations depends on the set of isochrones used. In the following tests we have employed the BCP98 colours for the MS; between MS and RGB we smoothly switch from the BCP98 $T_{\rm eff}-(V - I)$ relation to the BCM98-Ridgway one, which overlap over a sufficiently wide range. There is no need for any additional correction or shift. For lowest metallicities they almost agree (at the level of ≈ 0.01 mag).

To test our $T_{\rm eff}-(V - I)$-relation we use four GC of different metallicity for which we have determined the age already in our previous papers and for which BVI data exist. We transform the corresponding isochrones into $(V - I)$ by our new relation and determine the reddening $E(V - I)$. For the transformation of reddenings we take the extinction law by Cardelli et al. (1989), which gives $E(V - I) = 1.3 \, E(B - V)$.

As an example for metal-poor clusters we use M68 ([Fe/H] $= -1.99 \pm 0.10$; Carretta & Gratton 1997); the $(V - I)$ data are from Walker (1994). The $(B - V)$-CMD has been shown in Fig. 2 and the corresponding $(V - I)$-CMD is displayed in Fig. 8 (for sake of clarity the ridge-line only is displayed for the MS). The age determined in SW98 from the V brightness difference between TO and ZAHB is 11.4 ± 1.0 Gyr, in good agreement with the 11 Gyr isochrone displayed Fig. 8. The MS ridge-line was determined by us from the original data by taking the mode of the colour-distribution within brightness bins. The transformed isochrones appear to become too blue in the brightest RGB parts, while the corresponding part in $(B - V)$
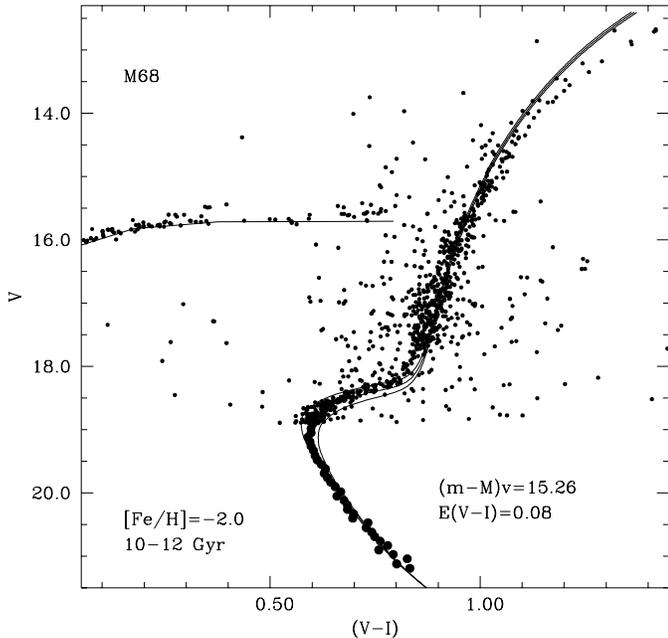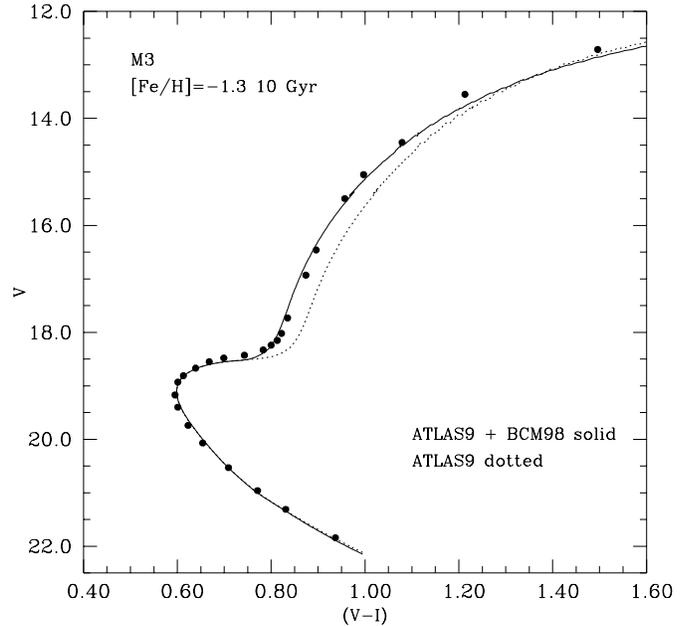
Fig. 8. The $V - (V - I)$ isochrones for M68. Data are from Walker (1994) and isochrone parameters as in Salaris & Weiss (1998). For simplicity, only data points brighter than $V = 18.5$ are shown. Large filled circles mark the ridge-line

Fig. 9. The $V–(V - I)$ 10-Gyr isochrone and the ridge line for M3. Data are from Johnson & Bolte (1998). Also displayed (dashed) is the same isochrone transformed by using the ATLAS9 colours for the RGB as well

is fitting very well[3]. The reddening (obtained, as described in Salaris & Weiss 1998) is 0.06 mag in $(B - V)$ and 0.08 mag in $(V - I)$; their ratio is as predicted by Cardelli et al. (1989).

As an example of an intermediate metal-poor cluster, Fig. 9 shows the comparison between the ridge-line from Johnson & Bolte (1998) and the 10 Gyr-isochrone for M3 ([Fe/H] $= -1.34 \pm 0.06$; Carretta & Gratton 1997), for which SW98 determined differentially the age from $(B - V)$-data (Ferraro et al. 1997) to be 10.1 ± 1.1 Gyr. The agreement is excellent except for a small deviation in the upper part of the RGB. The derived $E(V - I)$-reddening is 0.03 mag, in perfect agreement with $E(B - V) = 0.02$ (SW98; see also Buonanno et al. 1994). Using the ATLAS9 $T_{\mathrm{eff}} - (V - I)$ relation for the RGB as well results in a discrepancy of ≈ 0.05 mag for the larger part of the upper part of the CMD. This difference, if taken seriously, could be taken as evidence that the cluster should be older.

From the third of our metallicity groups (see SW97) we have selected M5; the absolute cluster age as derived from the $V - (B - V)$ diagram (Sandquist et al. 1996) is 9.9 ± 0.7 Gyr (SW98). Very recently Stetson et al. (1999) have published a new $V - (V - I)$ diagram of M5, and found some discrepancy with respect to the scale and zero points of the Sandquist et al. (1996) V and I magnitudes. However, their estimated TO-luminosity agrees with the one given by Sandquist et al. (1996) within 0.01 mag, and the same is true

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[3] Worthey (1998, private communication) points out that the Walker (1994) RGB-data are outliers in $(V - I)$ vs. $(V - K)$ when compared to the standard giant branches of Da Costa & Armandroff (1990) and the BCM98 data
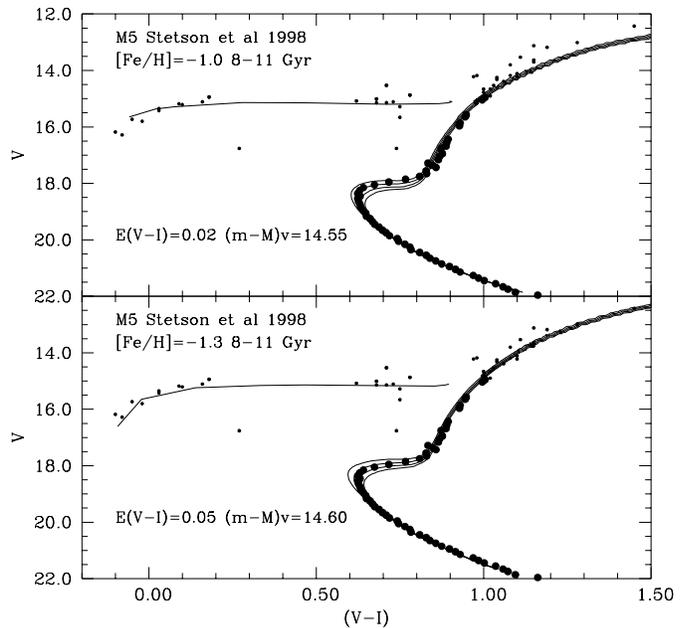

Fig. 10. The $V–(V - I)$ ridge-line for M5 (from Stetson et al. 1999) and isochrones of 8, 9 and 10 Gyr; upper panel: [Fe/H] $= -1.0$; lower: [Fe/H] $= -1.3$

for the V brightness of the HB and RGB stars in common (with V ranging between 14.5 and 16.5 mag). Therefore, the absolute age as determined from the V difference between ZAHB and TO remains basically unchanged, and the same is true for the cluster distance modulus determined from the fit of theoretical ZAHB sequences to the observed one (in the following we will
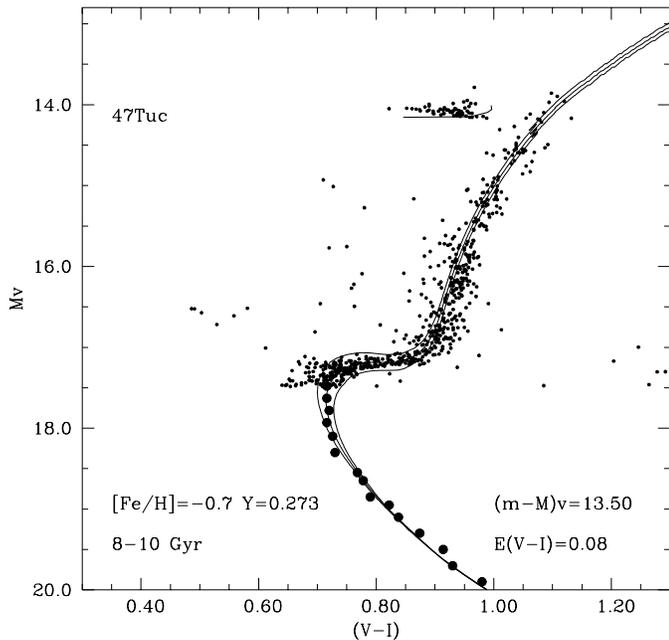
Fig. 11. The $V - (V - I)$-CMD of 47 Tuc (data from Kaluzny et al. 1998) with the same isochrones as in Fig. 3

therefore use the observed ZAHB-V brightness as determined by SW98 employing the more populated HB in the diagram by Sandquist et al. 1996). The Stetson et al. (1999) $V - (V - I)$ diagram (they provide the ridge-line) is shown in Fig. 10. The two panels contain the comparison with two of our isochrones, whose metallicities bracket the cluster metallicity [Fe/H] $= -1.11 \pm 0.11$, as determined by Carretta & Gratton (1997); the distance modulus is fixed by the observed ZAHB brightness. The quality of the fit is good in both cases; there is only a difference in the upper part of the RGB, where the more metal-rich isochrones are too red (as expected). By interpolating for the value of the cluster metallicity we get $E(V - I) = 0.03$, in good agreement with $E(B - V) = 0.02$ as determined by SW98 (due to a misprint in Table 2 of SW98, the apparent cluster distance modulus was quoted $(m - M_V) = 14.53$, while the correct value is $(m - M_V) = 14.57$).

Finally, we compare the data by Kaluzny et al. (1998) for 47 Tuc ([Fe/H] $= -0.70 \pm 0.07$; Carretta & Gratton 1997) with our isochrones. Since in SW98 we used the data by Hesser et al. (1987) for determining the age from the $(B - V)$ data, we showed the same fit for the new data in Fig. 3 (the ridge-line shown for the cluster MS is derived as previously described). The theoretical isochrones are identical to those in SW98 (including the higher helium content of $Y = 0.273$) except for the fact that they are now extended to the tip of the RGB. The newly derived age, distance and reddening are the same as in SW98. In Fig. 11 the corresponding $(V - I)$-diagram is shown. The reddening determined in this colour is 0.08, which, when transformed to $E(B - V)$ is 0.06, differing from the expected value of 0.05 by only 0.01 mag. Note that while in $(V - I)$ the isochrone appears to be a bit too blue for the lower RGB, it is too red by the same amount in $(B - V)$. The fit is of the same

quality in both colours and slightly better than that shown in SW98 for the Hesser et al. (1987) data.

Inspecting the HB of 47 Tuc in $(V - I)$, one has the impression that is inclined towards the blue. This effect is not so evident in $(B - V)$ (Fig. 3). We think it results from the fact that not all stars have both $(B - V)$ and $(V - I)$ colours and that in $(V - I)$ a number of redder HB stars are missing, thus making the inclination, which we ascribe to evolution on the HB, more apparent. In any case, the inclination could only be a brightness effect, independent of colour.

To conclude this section, we have demonstrated that the T_{eff}–$(V - I)$ transformation we have constructed on the basis of two existing transformations, if applied to our own isochrones (SW98), results in cluster-CMD fits in $(V - I)$ which are equally good as in $(B - V)$ for all metallicities and yield consistent reddenings in the two colours. Thus our requirements formulated at the end of the introduction and the beginning of this section are fulfilled.

4. Conclusions

In this paper we tried to approach the problem of fitting theoretical isochrones to globular cluster data in $(V - I)$. This is important since more and more I-band photometric data are becoming available and for some objects (e.g. bulge clusters) will be the only data of quality possible. Absolute ages of clusters determined by the turn-off brightness or the $\triangle V$-method (see SW98) depend almost exclusively on the bolometric correction BC_V. Theoretical values agree, when properly calibrated, with empirical values and also between different transformation sources.

On the other hand, transformations of effective temperatures to colours differ from source to source by a rather constant level of 0.05–0.10 mag. From the comparisons we have shown it is evident that (i) there is no straightforward way to decide which transformation is the correct or best one and (ii) that the accuracy with which an isochrone fits a colour-magnitude-diagram does not indicate the quality of the underlying stellar evolution calculations, if the mismatch is of the order of 0.05–0.1 mag. While in Sect. 2 we displayed $(V - I)$–fits, the situation is the same in $(B - V)$ and corresponding examples and conclusions can be found in the literature (e.g. Grundahl et al. 1998; D'Antona et al. 1997).

The problem of inaccurate colours is not just a cosmetic one, but can influence absolute age determinations which make use of colour differences (e.g. that between turn-off and red giant branch), distances derived by means of the main-sequence–fitting technique, or integrated colours of unresolved stellar populations. Finally, *if* a set of transformation was available which is known to reproduce colours accurately, any deviation of an isochrone from the observed CMD will indicate a real inconsistency in the isochrone. As an example we repeat that a colour mismatch of the RGB indicates (but does not prove) a higher helium content for 47 Tuc (SW98). Thus, reliable colour transformations are in principle a source for obtaining additional knowledge about cluster stars. We therefore attempted to find

a preliminary solution to this problem, such that we can continue our work about cluster dating with $(V - I)$ data, expecting improved theoretical transformations in the meantime.

We found that for MS stars the BCP98, Next-Gen and AAM96 $T_{\rm eff} - (V - I)$ relations reproduce acceptably well the solar constraint and, moreover, show the same differential behaviour with a rather constant offset of a few hundredths of a magnitude. We selected the BCP98 colours for our models.

On the red giant branch, no theoretical transformation is in agreement with empirical data over all the metallicity range spanned by our isochrones, but two recent empirical sources (M98 and BCM98) confirm each other quite well. Since BCM98 provide metallicity-dependent relations, we use this one for the RGB. Both parts (MS and RGB) can be connected smoothly along the subgiant branch.

For the ZAHB colours we have derived an empirical $(B - V) - (V - I)$ relation using multicolor photometries of clusters spanning the relevant range of metallicities, and we applied this relation to our ZAHB models in the $(B - V)$ plane.

While the arguments just given have only led the way to our rather pragmatic combination of two $T_{\rm eff}$–$(V - I)$ transformations, the justification for the combined and final relation comes from the tests we have performed using our own isochrones. For each of the four metallicity ranges defined in SW97, which reach from the most metal-poor halo to the half-solar metallicity disk clusters, we selected one cluster with BVI-photometry (three of them had already been used for absolute age determinations in SW98). We then showed successfully that (i) the quality of the isochrone fit is comparable in both colours and (ii) the reddenings determined independently from these two fits fulfill the reddening law by Cardelli et al. (1989) either exactly or within 0.01 mag.

We therefore conclude that the combination of the (theoretical) transformation of BCP98 for the main-sequence and the (empirical) one by BCM98 for giants is of sufficient accuracy as to allow us isochrone fits to globular cluster colour-magnitude-diagrams in V–$(V - I)$.

In a forthcoming paper we will determine cluster ages based on VI-photometry, both for clusters we already have investigated in our previous papers and for clusters with VI-data only. Although $(V - I)$ is less sensitive to metallicity than $(B - V)$, we found that $\triangle(V - I)$ (the colour difference between turn-off and red giant branch, which we need for relative age determinations) at fixed age is actually *more* metal-sensitive. Our approach of grouping clusters into four metallicity bins will therefore not be accurate enough. Either a finer metallicity grouping is needed (implying a higher number of clusters for absolute age determination), or a purely relative age approach is needed as in Pulone et al. (1998).

Independent of this, we are still waiting for improved theoretical colour transformations, because for more metal-rich α-enhanced clusters (e.g. bulge clusters) the BCM98 results cannot be applied straightforwardly. The study of such clusters therefore actually requires to some extent an extrapolation of our adopted colour transformations.

References

Alonso A., Arribas S., Martínez-Roger C., 1995, A&A 297, 197
Alonso A., Arribas S., Martínez-Roger C., 1996, A&AS 117, 227
Bernkopf J., 1998, A&A 332, 127
Bessell M.S., Castelli F., Plez B., 1998, A&A 333, 231, Erratum A&A 337, 321
Buonanno R., Corsi C.E., Buzzoni A., et al., 1994, A&A 290, 69
Buser R., Kurucz R.L., 1978, A&A 70, 555
Buser R., Kurucz R.L., 1992, A&A 264, 557
Canuto V., Mazzitelli I., 1991, ApJ 370, 295
Cardelli J.A., Clayton G.., Mathis J.S., 1989, ApJ 345, 245
Carretta E., Gratton R., 1997, A&AS 121, 95
Castelli F., Gratton R.G., Kurucz R.L., 1997, A&A 318, 841
Chaboyer B., 1998, preprint astro-ph/9808200
Chmielewski Y., 1981, A&A 93, 334
Da Costa G.S., Armandroff T.E., 1990, AJ 100, 162
D'Antona F., Caloi V., Mazzitelli I., 1997, ApJ 477, 519
di Benedetto G.P., 1993, A&A 270, 315
Fernie F.D., 1983, PASP 95, 782
Ferraro F.R., Carretta E., Corsi C.E., et al., 1997, A&A 320, 757
Freytag B., 1998, In: Schielicke R. (ed.), Annual meeting of the Astronomische Gesellschaft 1998, Abstract Series 14. Astronomische Gesellschaft, Hamburg, p. 113
Frogel J.., Persson S.E., Cohen J.G., 1981, ApJ 246, 842
Gratton R., Carretta E., Castelli F., 1996, A&A 314, 191
Gratton R.G., Fusi Pecci F., Carretta E., et al., 1997, ApJ 491, 749
Grundahl F., VandenBerg D.A., Andersen M.I., 1998, ApJ 500, L179
Hauschildt P.H., Allard F., Baron E., 1999, ApJ 512, 377
Hesser J.E., Harris W.E., VandenBerg D.A., et al., 1987, AJ 99, 739
Johnson J.A., Bolte M., 1998, AJ 115, 693
Kaluzny J., Wysocka A., Stanek K.Z., Krzemiński W., 1998, Acta Astron. 48, 439
Kurucz R.L., 1992, In: Barbuy B., Renzini A. (eds.) The Stellar Populations of Galaxies. IAU Symp. n. 149, Kluwer, Dordrecht, p. 225
Lejeune T., Cuisinier V., Buser R., 1997, A&AS 125, 229
Lejeune T., Cuisinier V., Buser R., 1998, A&AS 130, 65
Ludwig H.G., Freytag B., Steffen M., 1997, In: F. P. F.P., Christensen-Dalsgaard J., Rosenthal C. (eds.) SCORe'96: Solar Convection and Oscillations and their Relationship. Astrophysics and Space Science Library Vol. 225, Kluwer, Dordrecht, p. 59
Montegriffo P., Ferraro F.R., Origlia L., Fusi Pecci F., 1998, MNRAS 297, 872
Pulone L., Salaris M., Weiss A., Buonanno R., 1998, A&A 336, L77
Ridgway S.T., Joyce R.R., White N.M., Wing R.F., 1980, ApJ 235, 126
Salaris M., Degl'Innocenti S., Weiss A., 1997, ApJ 479, 665
Salaris M., Weiss A., 1997, A&A 327, 107
Salaris M., Weiss A., 1998, A&A 335, 943
Sandquist E.L., Bolte M., Stetson P.B., Hesser J.E., 1996, ApJ 470, 910
Stetson P.B., Bolte M., Harris W.E., et al., 1999, AJ 117, 247
Taylor B.J., 1994, PASP 106, 444
von Braun K., Chiboucas K., Minske J.K., Salgado J.F., Worthey G., 1998, PASP 110, 810
Walker A.R., 1994, AJ 108, 555
Zinn R., 1985, ApJ 293, 424