

*Letter to the Editor***Radio spectrum of the Crab nebula as an evidence for fast initial spin of its pulsar**A.M. Atoyan^{1,2}¹ Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, D-69117 Heidelberg, Germany² Yerevan Physics Institute, 375036 Yerevan, Armenia

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Abstract. The origin of relativistic electrons in the Crab nebula which are producing the broad-band flat radio spectrum of this prototype plerion has proved difficult to understand. Here I show that these electrons can be naturally explained as a relic population of the pulsar wind electrons that have lost most of their energy in the expanding nebula because of intensive radiative and adiabatic cooling in the past. The observed radio spectrum suggests that the initial slowing-down time of the pulsar was $\tau_{sd} \leq 30$ yr, which implies that it has been born with a spin period $P_0 \sim 3\text{--}5$ ms, several times shorter than presently believed. Consistency of these results with the current data and the historical records is discussed.

Key words: stars: pulsars: general – ISM: individual objects: Crab Nebula – ISM: supernova remnants – radio continuum: ISM

1. Introduction

Flat radio spectra, with power-law indices $0 \leq \alpha_r \leq 0.3$ (for $S_\nu \propto \nu^{-\alpha}$) represent one of the key features of the plerions (Weiler & Panagia 1978), that distinguish these filled-center supernova remnants (SNRs) from a typical shell-type SNR with the mean $\alpha_r \sim 0.5$ (e.g. Green 1991). Meanwhile, the origin of the electrons producing these unusual radio spectra is not yet understood (e.g. Green & Scheuer 1992, Woltjer et al. 1997).

Radio electrons appear particularly enigmatic for the Crab Nebula (see Kennel & Coroniti 1984 – hereafter KC84) where the synchrotron spectrum extends with $\alpha_r \simeq 0.3$ (Baars et al. 1977) from 10^7 Hz to 10^{13} Hz, and then it steepens to $\alpha_{opt} \sim 0.8$ in the IR/optical band (Marsden et al. 1984) This requires an energy distribution of the electrons $N(\gamma) \propto \gamma^{-\alpha_e}$ with $\alpha_e = 1 + 2\alpha_r \simeq 1.6$ extending over 3 decades of energy (Lorentz factor) γ from ~ 300 to 3×10^5 , which then steepens to $\alpha_e \simeq 2.6$. For the mean magnetic field $B_* \simeq 0.3$ mG and the known age t_* of the Crab Nebula, born in 1054, this steepening seems to agree with the radiative break $\Delta\alpha = 0.5$ (Kardashev 1962) both in the magnitude and position of the break. One might thus suppose that all relativistic electrons in

the nebula, including the ones with γ below the radiative break energy $\gamma_{*br} \sim 3 \times 10^5$ (at present), called “radio electrons”, represent a single population of particles currently accelerated at the pulsar wind termination shock (Rees & Gunn 1974 – RG74) that would produce an injection spectrum $Q(\gamma)$ with $\alpha_{inj} = 1.6$ at $\gamma \leq 3 \times 10^6$, and $\alpha_{inj} \simeq 2.3$ at higher energies (in order to explain the X-ray fluxes with $\alpha_x \simeq 1.1\text{--}1.2$). However, the mean electron energy in such a spectrum $\bar{\gamma} \leq 10^4$, which is much less than the energy of the wind electrons upstream of the shock, $\gamma_{*w} \sim 10^6$ (e.g. Kundt & Krotscheck 1980 – KK80, KC84, Arons 1996 – Ar96), whereas the energy and particle number flux conservation laws across the shock require that $\bar{\gamma} \sim \gamma_{*w}$. An idea of current acceleration of radio electrons in the main nebula beyond the wind termination shock appears also problematic, because radio observations do not show any significant variation of the radio spectral index, implying an absence of effective electron acceleration sites there (Bietenholz & Kronberg 1992, Bietenholz et al. 1997).

The remaining alternative is to assume that radio electrons in the Crab nebula are relics of the history of its evolution (Shklovskii 1977, KK80, KC84). The model proposed below reconsiders the evolution of the electrons injected into the expanding nebula at the pulsar wind shock.

2. Results

Previous studies of expanding plerions (Pacini & Salvati 1973 – PS73; Reynolds & Chevalier 1984 – RC84) have assumed an injection spectrum $Q(\gamma, t) \propto \gamma^{-\alpha_{inj}}$ with $\alpha_{inj} = 1 + 2\alpha_r$ starting from $\gamma \geq 1$. Thus, the origin of such a flat spectrum was not considered. In agreement with theoretical predictions (KK80, KC84, Ar96, Atoyan & Aharonian 1996), here I assume an injection spectrum with a profound deficit of electrons below some γ_c , for example in the form $Q(\gamma, t) \propto x^2(1+x)^{-2-\alpha_{inj}} \exp(-\gamma/\gamma_{max})$ where $x = \gamma/\gamma_c$. For $\alpha_{inj} \approx 2.3$, the mean energy of electrons in this spectrum $\bar{\gamma} \simeq (8\text{--}10)\gamma_c$. This defines γ_w of the pulsar driven wind of electromagnetic fields and particles which powers the plerion (RG74). The electrons are injected into the nebula with a power $L_e = \eta_e L_{sd}$ where $L_{sd} = L_0/(1+t/\tau_{sd})^k$ is the

spin-down luminosity, and $k = (n + 1)/(n - 1)$ in terms of the pulsar braking index n . A significant, if not the dominant ($\eta_e \rightarrow 1$, KC84), fraction of L_{sd} is injected in the electrons, so $\gamma_w(t) \propto L_{sd}/\dot{N}$ where $\dot{N}(t)$ is the production rate of $e^+ - e^-$ pairs by the pulsar. Very generally, \dot{N} can depend mainly on the magnetic field B_n of the neutron star and its spin frequency $\Omega(t) = \Omega_0(1 + t/\tau_{sd})^{-1/(n-1)}$. Because the basic time dependent parameter is Ω , I approximate $\gamma_w \propto \Omega^p(t)$ with a parameter p . This results in $\gamma_w \propto (1 + t/\tau_{sd})^{-s}$ with¹ a model parameter $s = p/(n - 1)$.

Because of intensive synchrotron losses in the past, electrons were brought to energies much smaller than γ_{*br} at present. For the energy losses $P_s \equiv -(\partial\gamma/\partial t)_s = b\gamma^2$ where $b \approx 0.04(B/1 \text{ G})^2 \text{ yr}^{-1}$, and magnetic field in the expanding nebula declining in time as $B = B_*(t_*/t)^m$, typically with $m \sim 1.3-2$ (RC84), the equation $\gamma/P_s = t$ gives the radiative brake energy

$$\gamma_{br}(t) = 3 \times 10^5 (B_*/0.3 \text{ mG})^{-2} (t/t_*)^{2m-1}. \quad (1)$$

All electrons injected into the nebula at times t with energies higher than $\gamma_{br}(t)$ are rapidly cooled down to this energy. Electrons of smaller energies are affected only by adiabatic energy losses which do not change a power-law shape of $N(\gamma)$ once established.

The origin of the flat distribution of radio electrons in plerions is readily understood if we neglect, for a moment, the adiabatic energy losses. Then Eq. (1) predicts that all electrons that enter the nebula between times t and $t + \Delta t$ with energies above $\gamma_{br}(t)$ will be soon found in the energy region between $\gamma \approx \gamma_{br}$ and $\gamma + \Delta\gamma$, where $\Delta\gamma \propto t^{2m-2}\Delta t$. Since injection occurs mostly at high energies, a δ -functional approximation $Q(\gamma, t) = \dot{N}\delta(\gamma - \gamma_w)$, where $\dot{N} = \dot{N}_0(1 + t/\tau_{sd})^{s-k}$, can be used to find the number of electrons $\Delta N = \dot{N}\Delta t$ accumulated in the interval $\Delta\gamma$. Eq. (1) connects the injection time of electrons with their energy after radiative cooling as $t \propto \gamma^{1/(2m-1)}$. Then the distribution $\Delta N/\Delta\gamma \rightarrow N(\gamma)$ of electrons that have been injected at times $t \ll \tau_{sd}$ acquires a power-law slope $\propto \gamma^{-\alpha_e}$ with $\alpha_e = 1 - 1/(2m - 1)$. At time $t \sim \tau_{sd}$ the injection rate $\dot{N}(t)$ starts to decline, resulting in an increase of the slope to $\alpha_e \sim 1$. This can explain the spectra of plerions like 3C 58 or G21.5-0.9 where $\alpha_r \sim 0$, and suggests that such plerions are powered by pulsars with τ_{sd} not much less than their age. However, radio spectra with $\alpha_r \simeq (0.2-0.3)$ are produced by the electrons injected at times $t \gg \tau_{sd}$, when $\dot{N}(t)$ rapidly declines, and $\alpha_e \rightarrow 1 + (k - s - 1)/(2m - 1)$. This slope extends up to the current radiative break energy γ_{*br} , if the energy of electrons in the wind presently $\gamma_{*w} > \gamma_{*br}$, as in Crab nebula, and to a smaller energy otherwise.

This qualitative analysis demonstrates the role of the radiative losses for the formation of flat spectra of radio electrons in plerions. The results do not significantly change after we take into account also the adiabatic energy losses of electrons,

¹ Note that this relation predicts $\gamma_w \simeq \text{const}$ for times $t \ll \tau_{sd}$. This seems to be correct also generally, allowing a reasonable variation (e.g. a gradual decline) of $B_n(t)$, although this would somewhat change the relation $s = p/(n - 1)$.

$P_{adb} = av\gamma/R$. In the approximation of a homogeneous spherical source of a radius R which expands with a velocity v , the parameter $a = 1$ (Kardashev 1962). For an arbitrary a , the energy distribution of electrons reads (Atoyan & Aharonian 1999):

$$N(\gamma, t) = \int_0^t \left(\frac{R_1}{R}\right)^a \frac{\Gamma_{t_1}^2}{\gamma^2} Q(\Gamma_{t_1}, t_1) dt_1, \quad (2)$$

where $R_1 \equiv R(t_1)$, and $\Gamma_{t_1} \equiv \Gamma(\gamma, t, t_1)$ is the particle energy at an instant $t_1 \leq t$:

$$\Gamma(\gamma, t, t_1) = \gamma \left(\frac{R}{R_1}\right)^a \left[1 - \gamma \int_{t_1}^t b(t_2) \left(\frac{R}{R_2}\right)^a dt_2\right]^{-1}. \quad (3)$$

Expansion of the Crab nebula is approximated as $v = v_*(t/t_*)^\delta$ with acceleration parameter $\delta \approx 0.1$, as observed (Trimble 1968). In the region $\gamma_{sd} \ll \gamma \leq \gamma_{*br}$, where $\gamma_{sd} \approx \gamma_{*br} (\tau_{sd}/t_*)^{2m-1+a+a\delta}$ is the contemporary energy of the electrons injected at time $t = \tau_{sd}$, the δ -functional approximation for $Q(\gamma)$ results in $N(\gamma, t_*) \propto \gamma^{-\alpha_e}$ with

$$\alpha_e = 1 + (k - s - 1)/(2m - 1 + a + a\delta), \quad (4)$$

which confirms the result derived qualitatively above for $a = 0$. The spectral index $\alpha_r = (\alpha_e - 1)/2$ then implies

$$s = k - 1 - 2\alpha_r(2m - 1 + a + a\delta). \quad (5)$$

The energy distribution of radio electrons in the Crab nebula should extend with the same index α_e over 3 decades of energy below γ_{*br} . This requires that $10^{-3}\gamma_{*br} \gg \gamma_{sd}$, therefore the radio spectrum of the Crab nebula would suggest $\tau_{sd} \ll 10^{-3/(2m-1+a+a\delta)}t_* \simeq 100 \text{ yr}$ (!).

Numerical calculations confirm this very unexpected and far reaching conclusion. It may seem at the first glance to Fig. 1 that all curves explain equally well the measured synchrotron fluxes from radio to optical, and even further to X-ray and γ -ray regions (where the fluxes can be sensitive to spatially inhomogeneous distribution of the nebular magnetic field). However, the radio fluxes of the Crab nebula are known, and must be explained, with high accuracy $\leq 10\%$. A closer look to the fluxes S_ν shown on the inset in Fig. 1 reveals that an explanation of the data at low frequencies is acceptable only if $\tau_{sd} \sim 30 \text{ yr}$ or smaller. This implies that the spin frequency Ω of the Crab pulsar initially was $(1 + t_*/\tau_{sd})^{1/(n-1)} \approx 10$ times higher than presently. For the spin period $P = 2\pi/\Omega$ this corresponds to the initial $P_0 \approx 3 \text{ ms}$. It could be a bit larger, $P_0 \leq 5 \text{ ms}$, for the canonical value of the pulsar braking index $n = 3$ (the radio data then require $\tau_{sd} \leq 20 \text{ yr}$). In any case, the derived initial spin period of the Crab pulsar is much shorter than $P_0 \approx 19 \text{ ms}$ predicted previously (see Manchester & Taylor 1977). Obviously, the reason for such puzzling discrepancy is to be understood prior to consider the model as realistic.

3. Discussion

This puzzle is solved if we remember that P_0 has been previously derived from the relation

$$t_* = \tau_* [1 - (\Omega_*/\Omega_0)^{n-1}], \quad (6)$$

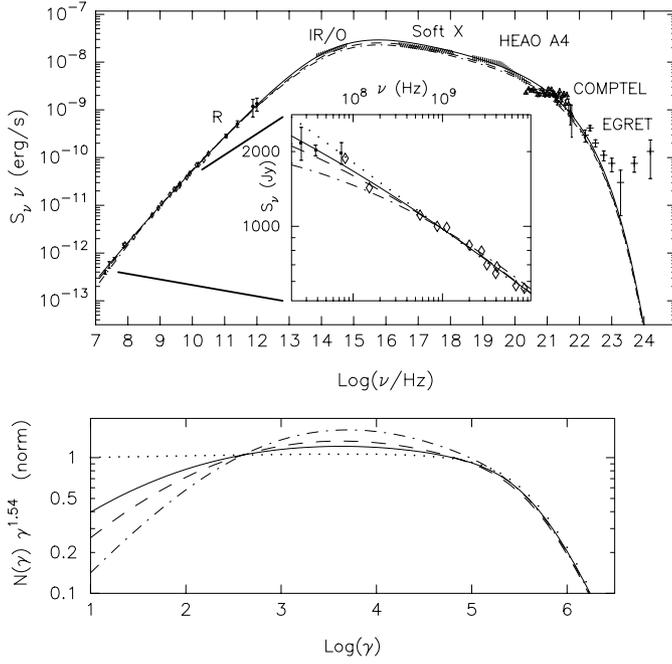


Fig. 1. The radiation fluxes (*top*) and the distribution of electrons (*bottom*) calculated for different τ_{sd} . The parameter s has been adjusted to give the best possible fit to radio data: $s = -0.7$ for $\tau_{sd} = 20$ yr (dashed); $s = -1.1$ for $\tau_{sd} = 40$ yr (solid); $s = -2$ for $\tau_{sd} = 100$ yr (dot-dashed). The dotted lines are calculated in the formal case of $\tau_{sd} = 1$ yr assuming $s = -0.3$, in agreement with Eq. (5). Other model parameters are fixed at $m = 1.4$, $a = 1$, $k = 2.3$, $\delta = 0.1$. The fit at the optical and higher frequencies is reached assuming $\alpha_{inj} = 2.3$, $\gamma_{max} = 2.2 \times 10^9$, and $\gamma_c = 1.9 \times 10^5$ (the principal parameter actually is the mean $\bar{\gamma} \approx 1.6 \times 10^6$). Compilation of data is from Atoyan & Aharonian (1996) and references therein. The size of the diamonds in the inset corresponds to a typical accuracy $\pm 6\%$ of the fluxes reported by Baars et al. (1977). Note that the negative values of s correspond to a gradual *increase* of γ_w in time. Qualitatively, this can be explained as a result of very high efficiency of the cascade (Sturrock 1971) of $e^+ - e^-$ pairs and γ -rays in the pulsar magnetosphere. In the past, when the pulsar rotated faster, this cascade was developing in a more compact ($\propto c/\Omega$) and tense magnetosphere, where electrons were effectively produced in the thus much higher magnetic fields. Fast increase of N would then result in a smaller γ_w .

with $\tau_* = \Omega_*/(n-1)\dot{\Omega}_*$, which is found after integration of the torque equation $\dot{\Omega} = -K\Omega^n$ assuming that n and K do not depend on time. In that case the braking index n can be expressed through the spin frequency Ω_* and its time derivatives at present as $n = \ddot{\Omega}_*\Omega_*/(\dot{\Omega}_*)^2$. For the Crab pulsar the measurements result in $n = 2.5$ (Groth 1975), so for $t_* \sim 900$ yr and $\tau_* = 1640$ yr Eq. (6) predicts $P_0 \approx 19$ ms, which then implies also that $\tau_{sd} \approx 700$ yr.

Reliability of these predictions, however, is questionable if we keep in mind that Eq. (6) is only an *approximate* formula for the pulsar age, and as such, it tells us that the model of a rotating magnetic dipole (Pacini 1967, Gunn & Ostriker 1969) does provide a good agreement with reality because $t_* \geq 0.6\tau_*$ for any Ω_0 . However, attempts to treat this formula as a strict equation appropriate for *derivation* of Ω_0 would

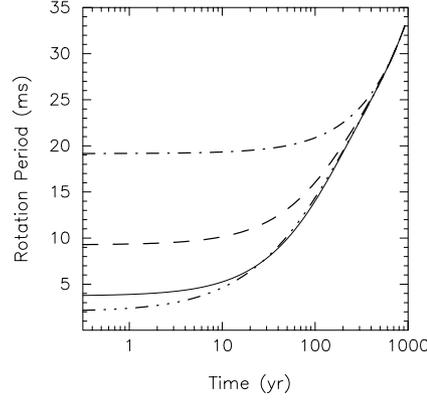


Fig. 2. The rotation period $P(t)$ of the pulsar resulting from numerical integration of the equation $dX/dt = -KX^n$ for the normalized frequency $X \equiv \Omega/\Omega_*$, assuming $K(t) = K_0[1 + A \exp(-t/t_K)]$ with different values of parameters A and t_K , and for different braking indices n : $A = 0$ and $n = 2.5$ (dot-dashed); $A = 4.1$, $t_K = 170$ yr and $n = 2.25$ (dashed); $A = 6.2$, $t_K = 155$ yr and $n = 2.25$ (solid). The 3-dot-dashed line is calculated for $A = 4.1$, $t_K = 170$ yr, but assuming a time-dependent $n(t)$ linearly decreasing from the initial $n_0 = 2.6$ to $n_* = 2.25$. In all cases the current value of the parameter $\ddot{\Omega}_*\Omega_*/(\dot{\Omega}_*)^2 = 2.5$ is provided.

overestimate its real accuracy, and can be largely misleading because it becomes practically independent of Ω_0 already at times when $\Omega_* < \Omega_0/2$. Actually, the *equation* (6) has to push Ω_0 close to the current Ω_* in order to be able to formally compensate the difference between τ_* and the known age t_* . Instead of that, a mathematically correct approach is to attribute this difference to the correction terms due in the approximate formula (6). For example, assuming that $n = \text{const}$ but $K = K(t)$, we would derive $t_* = \tau_*[1 - (\Omega_*/\Omega_0)^{n-1}] \times C_*$ where $C_* = t_*K(t_*)/\int_0^{t_*} K(t)dt$. For $K(t)$ gradually declining in time, e.g. because of decreasing magnetic field $B_n(t)$ of the neutron star ($K \propto B^2$), a correction term $C_* \geq 0.6$ would readily allow any $\Omega_0 \gg \Omega_*$. Some formal examples are shown in Fig. 2. Thus, the knowledge of Ω_* , $\dot{\Omega}_*$ and $\ddot{\Omega}_*$ at present may not be sufficient for determination of Ω_0 significantly larger than Ω_* . Information about the initial parameters of pulsars can be found in the radio spectra of plerions which they create.

The results found for the Crab nebula allow important conclusions about its history. The initial rotation energy of the neutron star, and hence also of the explosion that produced the star, reach the level $\geq 10^{51}$ erg. This confirms the earlier suggestion of Chevalier (1977) that Crab Nebula was born in a normal Type II supernova event.

Meanwhile, the total energy residing in the Crab nebula presently is only $\approx 4 \times 10^{49}$ erg (e.g. Davidson & Fesen 1985). Where is then the initial 10^{51} ergs of the pulsar? The answer becomes clear if we take into account that this energy has been injected into the nebula mostly by electrons ($\eta_e \approx 1$, KC84) with a low-energy ‘cutoff’ (!) and the mean energy $\bar{\gamma} \approx \gamma_w(t)$ that at times $t \ll t_*$ was much higher than $\gamma_{br}(t)$ down to which they have radiatively cooled on timescale $\Delta t \leq t$. Because of that, basically all of the injected energy has been radiated away,

and only a tiny fraction $\varepsilon(t) \sim \gamma_{\text{br}}/\gamma_{\text{w}} \ll 1$ of the pulsar spin-down power $L_{\text{sd}}(t)$ was depositing in the nebula. The total energy accumulated in the nebula by time t would be then $W(t) \propto t^{2m}(1 + t/\tau_{\text{sd}})^{s-k}$. Remarkably, because γ_{*w} is still larger than γ_{*br} , this energy might be *increasing* even currently if $2m - k + s > 0$. In particular, in the case of $m = m_{\text{eq}} \simeq 1.4$ predicted by the model for the magnetic field evolving close to equipartition with the electrons, $2m - k + s > 0$ if $s > -0.5$. Numerical calculations confirm this unexpected result. The increase of $W(t)$ at times $t \sim t_* \gg \tau_{\text{sd}}$ (to be compared with RC84) could explain the expansion of the nebula with a noticeable recent (Trimble 1968), or perhaps even current, acceleration.

The principal suggestion of RG74 that the magnetic field energy W_{B} in the Crab nebula is presently close to equipartition with W_{e} of the electrons not just accidentally, but because magnetic fields can effectively reenergize the electrons when W_{B} exceeds W_{e} , becomes important for explanation of the observed small decline rate of radio fluxes, $\simeq (0.16\text{--}0.19)\%$ per year (Aller & Reynolds 1985, Vinyaikin & Razin 1979). For $m = m_{\text{eq}} \simeq 1.4$, the calculated decline rate

$$-d \ln S_{\nu}/dt \approx [m(1 + \alpha_r) + 2a\alpha_r(1 + \delta)]/t_* \quad (7)$$

is in a reasonable agreement with observations only if $a \ll 1$. Adiabatic reenergization of radio electrons in the regions of magnetic field compression could significantly compensate the expansion losses, effectively reducing the adiabatic parameter from $a = 1$ relevant for a simple homogeneous model considered here, to much smaller values in the case of a spatially inhomogeneous study. Note that the regions of magnetic field compression, like those around numerous dense optical filaments in the nebula, may be distinguished by an enhanced radio intensity, e.g. as diffuse *radio* filaments (Swinbank 1980), but importantly, they will have practically the same $\alpha_r \simeq 0.27$ as the rest of the nebula (Bietenholz et al. 1997), because the adiabatic energy gain (or loss) process does not change the power-law index of the electrons.

For the Crab pulsar, a speed of rotation 10 times higher than presently implies a very high initial spin-down luminosity $L_0 \sim 10^{42}$ erg/s. The pulsar thus was injecting into the young nebula an energy about 3×10^{49} ergs per year, which was emitted mostly in the form of X-rays and γ -rays that could not be detected at that time. However, during the first years after the explosion, a significant part of this energy should be absorbed in the then opaque fast expanding shell of supernova ejecta between the nebula and an observer. This additional energy input into the thermally emitting shell can be the reason why the light curve of the supernova 1054 was declining much slower than that of a typical Type II event (Minkowski 1971).

The high initial luminosity of the pulsar could have also a strong impact on the dynamics of the ejecta outside the nebula. Interesting consequences for the possible progenitor of the Crab Nebula could be deduced from the high angular momentum of the pulsar, and hence of the progenitor star itself. Thus, the fast initial spin of the Crab pulsar as appears evident from the radio spectrum of the Crab nebula, suggests new approaches for understanding the origin and history of this prototypic plerion.

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