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Letter to the Editor

Improving the signal-to-noise ratio in solar oscillation spectra

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Abstract. We describe a data analysis technique for helioseismology that provides a reduction in the contamination of the solar oscillation spectrum from incoherent noise. We show that the technique allows: (i) a significant improvement in the signal-to-noise ratio for the modes in the oscillation power spectrum, and (ii) the solar velocity background spectrum to be observed at low frequencies using ground-based observations.

Key words: Sun: oscillations – methods: data analysis

Over the last thirty years much has been learnt about the internal structure and dynamics of the Sun through the analysis and interpretation of the solar oscillation power spectrum. This spectrum is densely populated with peaks, due to the Sun's normal modes, superimposed on a smoothly varying background whose amplitude increases with decreasing frequency. The background spectrum is always contaminated by a non-solar component due to noise from the Earth's atmosphere and/or the measurement instrument. Since the precision with which the frequency of an oscillation can be determined depends on the signal-to-background ratio of its peak in the oscillation spectrum (Libbrecht 1992), it is important to keep the amplitude of the non-solar background component to a minimum: especially as the low-frequency modes tend to have small amplitudes in comparison to the true solar background signal.

One can reduce the adverse effects of noise by making two independent measurements and using cross-spectrum techniques to extract the common signal. The cross-spectrum (S), defined as the average of the complex product of the Fourier transform of one data set, A, by the complex conjugate of the Fourier transform of the other, B, i.e.,

$$S(\nu) = \langle A(\nu) \cdot B^*(\nu) \rangle \ , \tag{1}$$

enhances any coherent signals between the two data sets. The degree of coherence between the signals is measured using the coherency function (C)

$$C(\nu) \equiv \frac{\langle A(\nu) \cdot B^*(\nu) \rangle}{\left(\langle A(\nu) \cdot A^*(\nu) \rangle \cdot \langle B(\nu) \cdot B^*(\nu) \rangle \right)^{1/2}} \ , \tag{2}$$

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which is equal to zero for completely incoherent signals and unity for completely coherent signals. The first application of cross-spectral analysis to helioseismic data was by Elsworth et al. (1995) who used data gathered simultaneously at two observing sites to measure the behaviour of the solar continuum spectrum in velocity. Since the observing sites were widely separated, cross-spectral analysis provided a reduction in both the instrumental and atmospheric noise levels at high frequencies. García et al. (1998) used the cross-spectrum technique to significantly reduce the incoherent photon noise between the two photomultiplier tubes of the GOLF instrument on board the ESA/NASA SOHO spacecraft (Domingo et al. 1995): the noise power at high frequencies in the cross spectrum was nearly an order of magnitude smaller than in the traditional power spectrum. This allowed the first detection of high-frequency interference peaks for the low-degree modes. Unfortunately, most helioseismology experiments do not use two detectors, and the overlap of simultaneous observations from different sites in a network of stations is limited. Thus, traditional cross-spectral analysis methods appear to be of limited use. Here we show that this is not necessarily the case.

Two data sets can be generated from one by splitting the original data into the odd and even indexed observations: i.e., use every other observation to make a new series. The phase offset of the even indexed data, with respect to the odd indexed data, is removed using the Fourier shift theorem (Bracewell 1987):

$$x(t-dt) = FT^{-1}\left\{ e^{-i2\pi dt\nu} FT(x(t)) \right\} \ , \tag{3}$$

where dt is the sampling interval for the observations, ν is the temporal frequency, FT and FT^{-1} are the forward and inverse Fourier transform operators, respectively. Splitting up the data in this way halves the Nyquist frequency for the observations and is therefore only practical if the resulting aliasing of the high-frequency components of the oscillation spectrum will not cause problems. We will refer to the modulus of the cross spectrum generated using the odd and even indexed data sets as the *Interleaved-Shifted-Cross-Spectrum* (hereafter ISCS).

To test the validity of the ISCS technique we have used it in the analysis of disk-integrated velocity data from the "Mark-I"

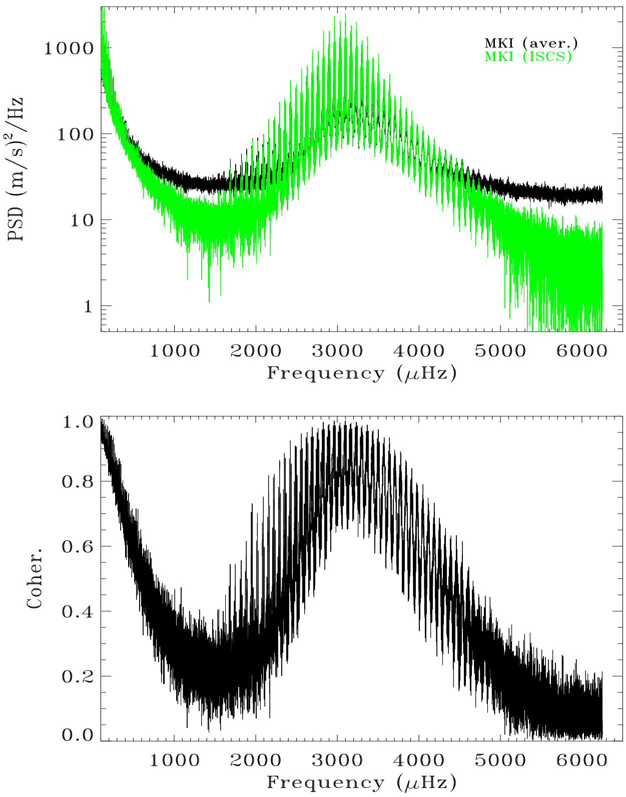

Fig. 1. *Top:* Averaged power spectrum of 136 subseries of 50 days (black line) and the resultant ISCS (gray line). The reduction of noise in the latter is a factor of $\sim$ 3 around 1.5 mHz and $\sim$ 7 at 6.2 mHz. *Bottom:* ISCS coherency function. Most of the signal at 1.5 mHz and above 5 mHz is incoherent. At the middle of the p-modes envelope there is still 20% gain in the signal to background of the ISCS signal as the coherency of the modes is ~ 1 but the inter-peak region is less than 0.85.

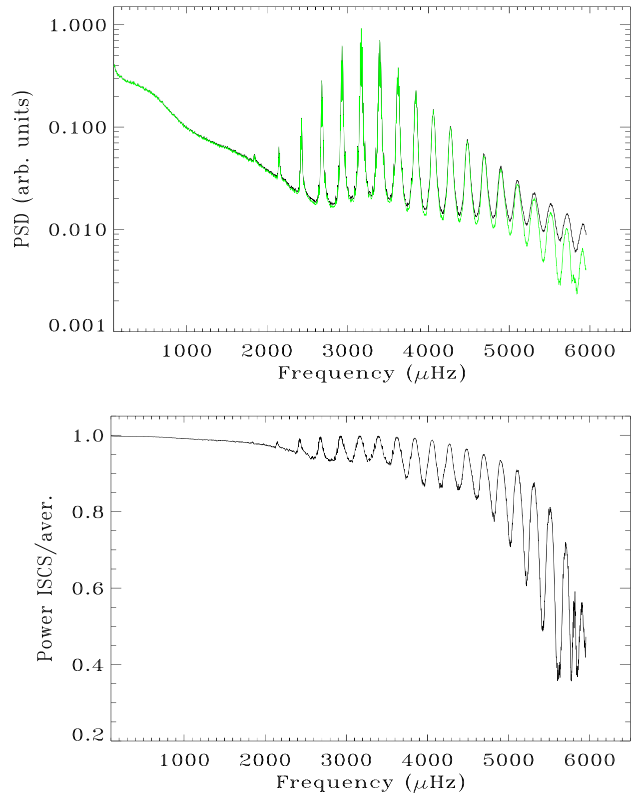

Fig. 2. *Top:* The rotation corrected spectrum for $\ell = 100$, computed from 61.8 days of full-disk Ca II K-line intensity images. Average power spectrum, AVER (black line), and the ISCS (gray line). *Bottom:* The ratio ISCS / AVER. The improvement in the peak-to-background ratio for the ISCS is readily visible above 2 mHz.

instrument of the BiSON network operated at the Observatorio del Teide, Tenerife (Chaplin et al. 1998) and full-disk Ca II K-line intensity data from the South Pole (Jefferies & Harvey 1995). These data suffer from both atmospheric and instrumental noise contamination. The Mark-I instrument, which uses the resonant scattering technique to measure the integrated radial velocity between the Sun and the instrument, is extremely stable (Pallé et al. 1999) and has operated unchanged since its deployment fifteen years ago. Our analysis uses Mark-I data for the period April 19, 1984, to October 14, 1998. The data set has a duty cycle of 25.75 %. The velocity residuals were obtained by a standard procedure (see Pallé et al. 1993) and integrated from the original 2 second samples to 40 s. From this ~ 14.5 years of data we have built 210 subseries of 50 days where adjacent series overlap by 50 %. Using only those series with a duty cycle higher than 20 % (136 out of 210), we multiplied each series by a Hanning window and then applied the ISCS method. For comparison, we have also made a “traditional” average power spectrum of the same data (see Fig. 1). The Nyquist frequency for the resulting spectra, 6.25 mHz, is sufficiently high to avoid problems from aliasing of the high-frequency signal. The Ca II K-line intensity data comprise 61.8 days of full-disk images having a duty cycle of 59% and recorded every 42 seconds. The images were decomposed into time series of spherical harmonic coefficients (Brown, 1985). The resulting time series of coefficients were then divided into 41 overlapping subseries of 3.1 days before computing the average power spectrum and the ISCS. Fig. 2 shows the results for $\ell = 100$ after correction for the effects of solar rotation.

The reduction in the background signal in the ISCS, in comparison to the level observed in the power spectrum, can be understood by considering the observed signal to comprise three components: a coherent signal, coherent noise, and incoherent noise. Since the ISCS technique reduces any incoherent noise, the ISCS shows significant differences from the power spectrum at frequencies where the incoherent noise component is a sizeable fraction of the total signal. This can be seen by comparing the photospheric velocity spectra in Fig. 1 with the chromospheric intensity spectra in Fig. 2. Because intensity observations inherently have a higher (coherent) solar background signal at low frequencies, there is not such a dramatic gain in the ISCS at these frequencies. However, the ratio of the ISCS and the power spectrum for the intensity data shows that there is still an important gain in signal-to-background afforded by the ISCS.

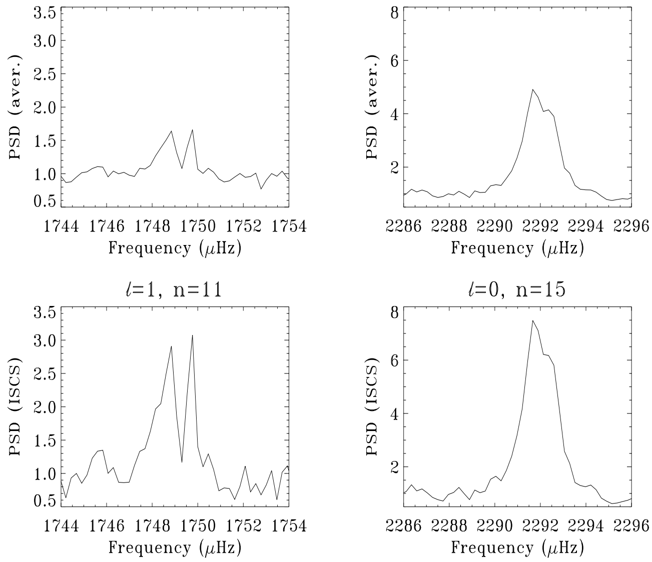

Fig. 3. A comparison between the average power spectrum and the ISCS for the modes: $\ell = 1, n = 11$ at $\nu = 1749 \pm 0.027\,\mu$Hz and $\ell = 0, n = 15$ at $\nu = 2228.895 \pm 0.021\,\mu$Hz. The improvement in the signal-to-background ratio in the ISCS is clearly visible. This allows a more accurate determination of the mode parameters. Notice that the background of the power spectra has been normalized to one.

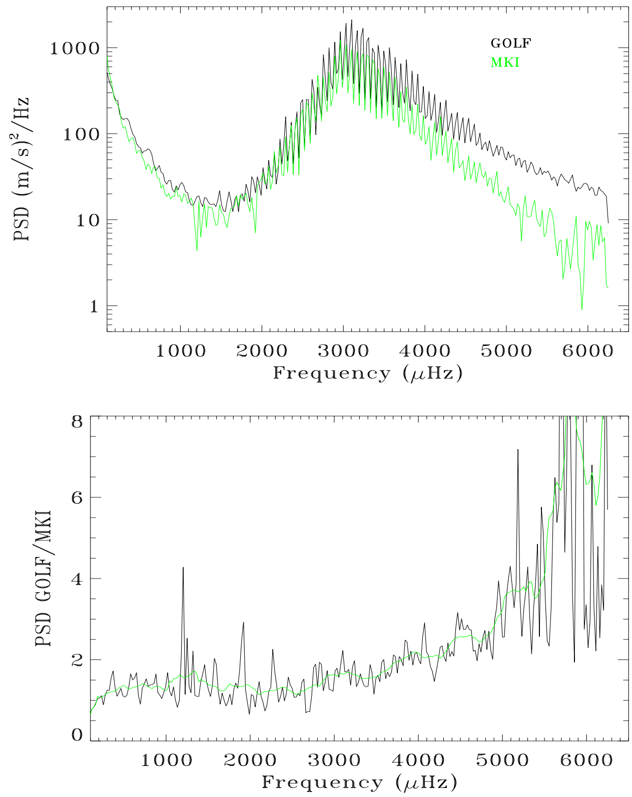

Fig. 4. *Top:* Modulus of 124 cross spectrum of GOLF data (black line), and the same number of ISCS spectra of Mark-I data, generated from subseries of 12 hours (gray line). *Bottom:* ratio between the GOLF and Mark-I spectrum. Up to 3 mHz, the ratio is roughly constant (~ 1.32), but above this frequency there is an excess of power in the GOLF instrument due to the different line used for the observations (Na for GOLF and K for Mark-I). The light curve is a smooth with a boxcar of 15 points.

To try and better compare the quality of the ISCS and the average power spectrum, we have used a maximum-likelihood algorithm (Appourchaux et al. 1998) to fit the spectral peaks for the modes ℓ=0,1 and n <15 in both spectra. Before fitting the spectral peaks we normalized both power spectra using a model of the velocity background (Harvey 1985) with two components, granulation and supergranulation; this was fit to the logarithm of the data between 0.6 and 2 mHz (see Fig. 3).

The frequency estimates based on the ISCS as well as the other fitted parameters are more precise than those based on the averaged power spectrum following the increase in the signal to background ratio (Libbrecht 1992).

To determine the quality of the noise supression achieved in the ISCS, we have compared the ISCS for the Mark-I data with the cross spectrum of GOLF data. Up to the present time, the GOLF instrument has worked in three different configurations: using both wings of the sodium profile, only the blue wing (Gabriel et al. 1997), and only the red wing. We have just considered the two-wing mode for which 73 days of data are available. In order to compare the two data sets using series that have a duty cycle greater than 95 %, we have to consider series of 12 hours. With this limitation we have only 124 series from both instruments. After forcing the window functions for the two data sets to be identical we computed the ISCS for the Mark-I data and the cross spectrum between the two photomultipliers for the GOLF data (see Fig. 4). From this figure, it can be seen that the ISCS reaches the solar background level at frequencies ~ 1.5 mHz. Above 3 mHz there is a progressive difference between the Mark-I and GOLF data due to the different lines where they are observing (Na for GOLF and K for Mark-I). This difference is in good agreement, up to 4 mHz, with previous comparisons between observations made using the sodium and potassium spectral lines (Isaak et al. 1989, Pallé et al. 1992). Above this frequency the reduction in the incoherent noise modifies the ratio and the comparison is no longer possible.

The ISCS technique provides a significant improvement in spectral quality, consequently, it will have an impact on the sampling rate used in future experiments. Simulations show that a further (small) reduction in the incoherent signal can be obtained by using every n^{th} data point to generate n time series subsets and thus $n!/2(n-2)!$ cross spectra. In principle, therefore, one could select a sampling rate high enough to not only reduce contamination of the ISCS from temporal aliasing, but also to maximize the reduction of any incoherent noise present in the measured signal. We note, however, that there will be a trade-off between the signal-to-noise per measurement and the sampling rate (i.e., as sampling rate increases, the signal-to-noise decreases). Therefore, even with the ISCS technique, there will be a limiting sampling rate beyond which no further gains will be made.

Finally, although we have used the ISCS technique with helioseismic data, in principle, it should be useful for any equally sampled time series data that is contaminated by incoherent noise.

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