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LETTER

Letter to the Editor

Close approaches of asteroid 1999 AN_{10}: resonant and non-resonant returns

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Abstract. The Earth passes very close to the orbit of the asteroid 1999 AN_{10} twice per year, but whether or not this asteroid can have a close approach depends upon the timing of its passage across the ecliptic plane. Among the possible orbits there are some with a close approach in 2027. The period of the asteroid may be perturbed in such a way that it returns to an approach to the Earth at either of the possible encounter points. We have developed a theory which successfully predicts the 25 possible such returns up to 2040. We have also identified 6 more close approaches resulting from the cascade of successive returns. Because of this extremely chaotic behaviour there is no way to predict all possible approaches for more than a few decades after any close encounter, but the orbit will remain dangerously close to the orbit of the Earth for about 600 years.

Key words: minor planets, asteroids – chaos

1. The 2027 encounter with 1999 AN_{10}

The asteroid 1999 AN_{10} was discovered by the LINEAR telescope on 13 January 1999 (MPEC 1999-B03). The discovery was somewhat unusual in that the declination was $+70°$. The nominal orbit computed by our online information service (NEODyS), that is the solution of the least squares fit to 94 observations (with one outlier removed, RMS of the residuals 0.59 arc-sec), is as follows (J2000): $a = 1.458432$ AU, $e = 0.562093$, $I = 39°.932$, $\Omega = 314°.556$, $\omega = 268°.255$, $M = 321°.958$, for epoch $JD\ 2\,451\,206.805$. The absolute magnitude is estimated at 17.9 ± 0.5; given that the albedo is unknown, this object could be between 0.5 and 2 km in diameter. The ascending node is only $0.000\,25$ AU closer to the Sun than a point where the Earth is in early August; the descending node is $0.004\,78$ AU inward from a position of the Earth in early February. This means that, whenever the asteroid and the Earth are in phase at each node, close approaches are possible. Indeed a close approach is possible in August 2027. To analyse the 2027 encounter, we need to consider not only the nominal solution, but also all the solutions compatible with the observations, that is resulting in residuals which are not much larger than the ones of the nominal orbit (Milani 1999). The region of these compatible solutions can be approximated by an ellipsoid of confidence, which, for a σ value up to 3, contains solutions with RMS of the residuals up to 0.63 arc-sec.

The nominal solution undergoes a close approach in August 2027 with a minimum distance from Earth's center of $0.025\,7$ AU. The plane normal to the geocentric velocity at closest approach is the Modified Target Plane (Milani & Valsecchi 1999). The hypothetical objects filling the confidence region evolve along a bundle of orbits; their intersections with the MTP define the confidence region of the encounter. If the Earth is not touched by this confidence region, then a collision can be ruled out. The confidence regions are very thin, the width being only $2\,600$ km, and 0.42 AU long. Thus the occurrence of a very close approach is not very likely: the true orbit could be anywhere along a very long line, including long stretches corresponding to very shallow encounters.

In conclusion, the 2027 encounter could be a shallow approach, or could be, with a low, non-negligible probability, very close. In any case it cannot result in an impact. But the case for a possible dangerous encounter is not closed after 2027; indeed, it is just opened.

2. Resonant and non-resonant returns

Resonant returns after a close approach have been discussed in different contexts, e.g., the close approaches of comet Lexell to Jupiter (Leverrier 1844) and the repeated visits to Mercury of the Mariner 10 spacecraft. B. G. Marsden recently applied this idea to the asteroid 1997 XF_{11} in the assumption that the 1990 precovery observations had not been discovered (Marsden 1999).

We can formulate the basic theory of resonant returns as follows. When an asteroid undergoes a close approach in the future, decades after the last available observation, the confidence region on the MTP is thin, with a width much less than the diameter of the Earth and very long; thus it is enough to perform the analysis on the long axis of the confidence region, which we call the Line Of Variation (LOV) (Milani 1999). The alternate solutions along this line undergo different degrees of perturbations, as a result of the close approach. The elements after the encounter describe a curve in the orbital elements space, e.g., in the (a, e) plane; the shape of such curves can be understood by using Öpik's piecewise two body approximation (Greenberg et al. 1988). These curves are almost closed, they go back to nearly the unperturbed values when the encounter is shallow, on both extremes of the LOV (Valsecchi & Manara 1997). Let $P_{\rm min}$ and $P_{\rm max}$ be the corresponding minimum and maximum orbital periods; every rational number in the interval between them corresponds to at least two resonant returns. If the period $P = h/k$ years with h, k integers, then after h years the asteroid has completed k orbits, the Earth has completed h orbits, and both return to nearly the same position. As an example, 1999 $\rm AN_{10}$ can have several different $7/4$ resonant returns in 2034, resulting in an approach potentially closer than the one in 2027, down to $0.000\,10$ AU.

However, two refinements must be taken into account. First, the amount of time by which the first encounter has been missed needs to be recovered to make the second encounter a close approach. If Δt is the amount of time by which the asteroid is early for an encounter, the condition to be satisfied for a resonant return at the minimum distance is $h + \Delta t = k\,P$, where Δt and P are in years. Thus the resonant returns are described, in the $(\Delta t, P)$ plane, by lines which are somewhat slanted with respect the $P = h/k$ lines. Fig. 1 depicts these resonant lines for the returns of 1999 $\rm AN_{10}$ after 2027 and for $h \leq 13$. Where these resonance lines intersect the LOV, one finds a resonant return leading to a close approach. In the figure the LOV has been traced by using the multiple solutions algorithm of (Milani 1999), Sec. 5. We have used $1\,001$ solutions equally spaced along the σ axis between -3 and $+3$; we have added a denser sampling of solutions along the σ axis in the region near the 2027 closest approach. The intersections with the resonant lines can be counted from the figure; the resonances not touching the LOV, e.g., the $P = 5/3$ resonance, cannot result in deep encounters.

The other refinement is to consider the Minimum Orbital Intersection Distance (MOID), the minimum distance between the two osculating ellipses representing the orbit of the Earth and of the asteroid. Even if the asteroid were exactly on time at the rendezvous with the Earth, the unperturbed close approach distance cannot be less than the MOID. For 1999 $\rm AN_{10}$, there are in fact two local MOIDs, one per node; each is $\simeq 0.74$ times the minimum of distance at the respective node. If the MOID were to remain small forever, since every real number P is approximated arbitrarily well by a rational number h/k a resonant return after h years would be always possible.

What is the evolution in time of the local MOIDs of 1999 $\rm AN_{10}$? It is not enough to compute the evolution of the MOIDs along the nominal solution, because the close approaches can change them: in particular, an encounter near the *ascending* node (in August) can reduce the distance at the descending node, and make possible a closer approach at the *descending* node (in February). We have asked G.F. Gronchi to compute the evolution of the mean orbital elements, 'averaged' in the sense of (Gronchi & Milani 1998), (Gronchi & Milani 1999), in a way accounting also for the secular effects of the close approaches.

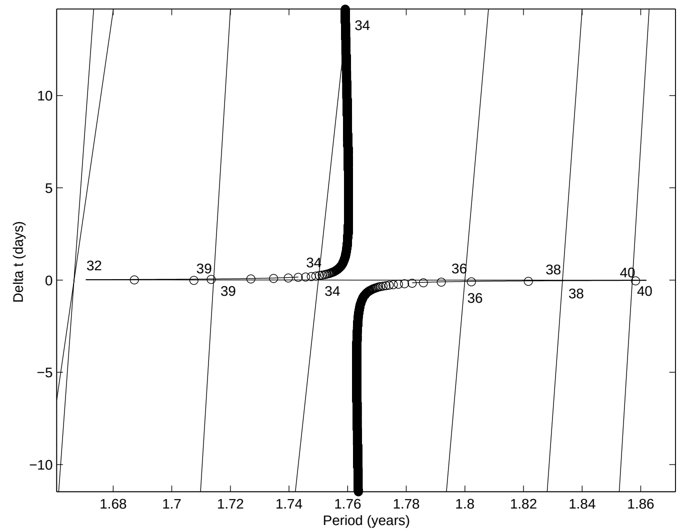

Fig. 1. Resonant returns of 1999 $\rm AN_{10}$ after the 2027 encounter, taking place until August 2040. The circles represent $1\,001$ alternate orbits along the LOV. The solid line represents other solutions in the region of highest stretching. The resonant returns are labeled with the year (after 2000) in which the return takes place.

The answer is that 1999 $\rm AN_{10}$ will continue to have a very low distance at both nodes for about 600 years. Thus it is simply not possible to perform close approach analyses in the sense of (Milani & Valsecchi 1999) for all possible resonant returns: there are hundreds of them.

Because of the low nodal distance also at the descending node, there is the possibility of a *non-resonant* return. This can occur if the Earth completes $h + 1/2$ revolutions while the asteroid completes $k + 1/2$ revolutions, so that they are both at the descending node at the same time. Taking into account the eccentricities of both orbits, the time required to go from the ascending node to the descending node is $t_{\rm E}$ for the Earth (not exactly half a year), and $t_{\rm A}$ for the asteroid (much more than half a period). Again allowing for the timing of the 2027 encounter, the condition to be satisfied for an encounter at the descending node is $h + t_{\rm E} + \Delta t = k\,P + t_{\rm A}$. If we add the condition that the distance is zero at both nodes, we have 4 conditions on the 5 variables (a, e, ω, u_1, u_2), where u_1, u_2 are the eccentric anomalies at the nodes, and we can explicitly compute $t_{\rm A}$ as a function of a. Thus the above condition defines a curve in the $(P, \Delta t)$ plane, as in the resonant case. Note that this analysis would equally apply even if the first encounter were with another planet. Fig. 2 shows all the possible non-resonant returns to the Earth after the 2027 encounter with $h \leq 12$.

3. Global return analysis

Combining the 11 resonant returns of Fig. 1 and the 14 non-resonant ones of Fig. 2, our theory predicts 25 close approach solutions; for each of these we could perform a detailed close approach analysis to determine the minimum distance possible. However, this is not necessary because the minimum distance is essentially the local MOID near the relevant node; this is

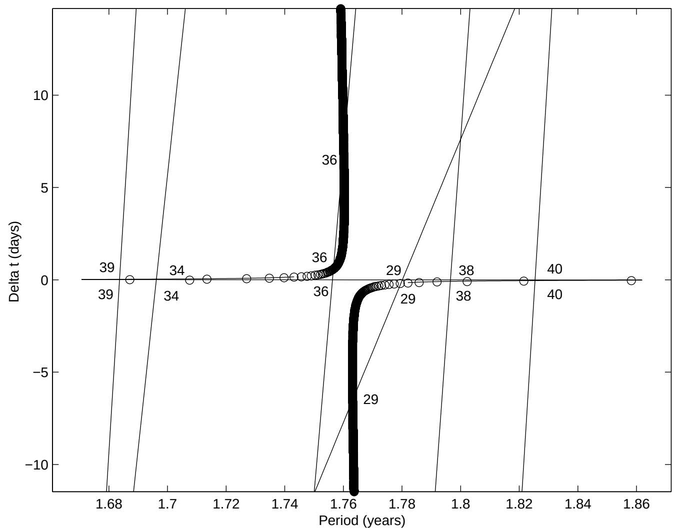

Fig. 2. Non-resonant returns of 1999 AN_{10} taking place until February 2040, in the same style as the previous figure.

also not sufficient to identify all the possible returns, because a secondary return from a previous one is possible, and so on. For this reason we have devised a global method to find returns.

We started from the same catalog of $1\,001$ alternate orbital solutions used for the figures. Each solution was propagated forward from the 2027 encounter, recording the position and the nodal distances every time the Earth is passing at the nodes. We determine if there was a crossing of the relevant node (near that time) by the changes of sign of the z coordinate in an ecliptic reference frame. We interpolate between these adjacent solutions to find the σ value corresponding to the node crossing at the time when the Earth is there. We similarly obtain the minimum distance between the orbit of the Earth and of the asteroid around the relevant node. By a continuity argument, if z changes sign between two solutions at σ_1 and σ_2, there is an intermediate value of σ for which z is zero, that is at least one solution along the LOV always exists that passes at a distance from the Earth as low as the local MOID (even slightly less, due to gravitational focusing).

This argument cannot be applied for values of $|z|$ too large, otherwise the two consecutive solutions could be out of phase by more than one period. Thus the limit of the method is the stretching Γ, which is the ratio between the distance in physical space of two orbit solutions at some time and the distance $\Delta\sigma$ of the corresponding values of σ, which parametrises the LOV. For $\Delta\sigma = 0.006$, as in our $1\,001$ solutions catalog, $\Gamma \simeq 1\,500$ AU would result in two consecutive orbits being out of phase by 1 revolution; we can reliably detect a close approach only up to $\Gamma \simeq 200$. (P. Chodas, private communication, has found another return in August 2039 which has escaped our search because it has $\Gamma \simeq 400$.) After a very close approach such values of Γ do occur, and even more after a sequence of close approaches. For this reason we have densified our sampling of the LOV in the region of high stretching around the solution with the closest approach in 2027, namely for $-0.46 \leq \sigma \leq -0.26$, by computing another $2\,001$ alternate orbits. With $\Delta\sigma = 0.000\,1$, even returns with $\Gamma > 10\,000$ AU can be detected.

Table 1. Earth close approaches possible through 2040.

Date	MOID (10^{-3} AU)	$\Gamma_{\rm MTP}$ (AU/1 σ)	% prob. w/in L.D.	σ
Aug. 2027	0.26	0.071	1.2	−0.358
Feb. 2029	3.0	0.065	–	−1.801
	6.5	4.5	–	−0.388
	42.8	240.	–	−0.362
Feb. 2034	21.4	110.	–	−0.352
	2.0	650.	0.000 08	−0.360
Aug. 2034	0.12	0.077	1.1	+2.734
	0.10	3.8	0.023	−0.299
	0.16	1100.	0.000 08	−0.361
Feb. 2036	2.8	0.074	–	+1.317
	4.5	0.91	–	−0.226
	4.5	4.8	–	+2.695[a]
	6.8	760.	–	−0.299[1]
	29.6	1400.	–	−0.361
Aug. 2036	0.15	52.	0.001 7	−0.379
	0.17	1300.	0.000 07	−0.362
	0.54	10000.	0.000 009	−0.299[b]
Feb. 2038	10.9	52.	–	−0.380
	42.9	1600.	–	−0.362
	8.1	4600.	–	−0.379[c]
	7.2	5600.	–	−0.379[3]
Aug. 2038	0.09	195.	0.000 4	−0.370
	0.16	1100.	0.000 08	−0.363
Feb. 2039	23.	220.	–	−0.355
	6.0	710.	–	−0.359
Aug. 2039	0.073	110.	0.000 8	−0.348
	0.15	1500.	0.000 06	−0.360
	0.011	15000.	0.000 006	−0.299[d]
Feb. 2040	24.1	200.	–	−0.371
	49.8	1300.	–	−0.363
Aug. 2040	0.080	220.	0.000 4	−0.366
	0.051	450.	0.000 2	−0.364

[a] Secondary non-resonant return from Aug. 2034
[b] Secondary resonant return from Aug. 2034
[c] Secondary non-resonant return from Aug. 2036
[d] Secondary resonant return from Aug. 2034

Table 1 presents all the returns up to August 2040 that we have found with this method, using both the $\Delta\sigma = 0.006$ catalog and the denser $\Delta\sigma = 0.000\,1$ catalog. The stretching $\Gamma_{\rm MTP}$ in the Table is not Γ, computed with distances in the 3-D space, but its projection upon the MTP, which is in a fixed ratio to Γ. That is, we use the product of the time difference in the node crossing and the relative encounter velocity divided by $\Delta\sigma$. $\Gamma_{\rm MTP}$ allows one to compute the size of the interval along the LOV, in σ units, where approaches within a given distance occur. Given a probability density function on the LOV, the probability of such an event can be determined. But, there is no such thing as a unique probability of an event involving an orbit obtained by a least squares fit: it depends upon assumptions on the statistical distribution of the observational errors. In the Table we have used a uniform probability density along the LOV for $\sigma \leq 3$, to estimate the probability of an encounter within the mean distance of the Moon. Note that the lower the stretching, the higher the probability of an encounter within a given distance; thus shallow encounters can be more effective in generating likely returns than the deep ones.

Each of the 25 returns predicted by our theory appear in the Table, with $\Gamma_{\rm MTP} \leq 1\,500$. 6 solutions not predicted by the figures appear; they can all be interpreted as secondary returns. Both the $5/3$ and $2/1$ returns become possible after the 2034 encounter. Among these secondary returns there is one in August 2039 for which the interpolated MOID is less than the radius of the Earth. Since the stretching is extreme, we have checked by performing close approach analysis: a collision solution does exist. But $\Gamma_{\rm MTP}$ appears as divisor in the formula for the probability, so the probability for this impact is of the order of 10^{-9}. If the probability of an impact by an undiscovered 1 km asteroid is of the order of 10^{-5} per year (Chapman & Morrison 1994), the probability of impact in 2039 is less than the probability of being hit by an unknown asteroid of this size within the next few hours. In any case the asteroid orbit will soon be refined by further observations and this possible solution may be ruled out.

The stretching coefficients used here are related to the dimensionless stretching used in the computations of the Lyapounov characteristic exponents: they differ only by a constant factor. Thus the data in the Table indicate the level of chaos of each return orbit. The cascade of successive returns could be described by a symbolic dynamics, as in other chaotic celestial mechanics problems (Zare & Chesley 1998).

4. In the long run

Asteroid 1999 AN_{10} was observed until the angular distance from the sun became $< 70°$. It will be again at $> 60°$ from the Sun after the beginning of June; by that time the uncertainty of its position on the sky will grow to 1.5 arc minutes (corresponding to $\sigma = 3$), that is any new observation will significantly contribute to an improvement of the orbit. It is very likely that the observations made in the second half of this year will constrain the orbit well enough to predict accurately the 2027 encounter. This implies that some of the returns of the Table will be discarded as incompatible with the observations; in fact, most of them if the 2027 encounter is not very deep.

The problem, however, will not go away, because all along the LOV there are possible encounters occurring later, almost every six months. We have analysed with our global method the same $1\,001$ multiple solutions over a time span of 50 years after 2027, and found 165 possible returns, out of which 117 in the moderate to low stretching region. This situation is qualitatively stable: whatever the actual orbit is, it will not be possible to predict with certainty the returns after the next close approach for a time span longer than a few decades.

Since at least one node of 1999 AN_{10} will remain close to the orbit of the Earth for centuries, this asteroid shall have to be monitored, by observations and computations, for a very long time. It is conceivable that at some time in the future a decision could be made to deflect it; but, a deflection decreasing the depth of some specific close approach could increase the impact risk at a later date. Thus before such a decision can be contemplated we need to better understand the theory of resonant and non-resonant returns, which has only been outlined in this paper.

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