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Research Note

Fitting formulae for cross sections of tidal capture binary formation

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Abstract. Tidal captures can produce objects that are observationally and dynamically important in dense stellar systems. Recent discoveries of compact young clusters in and out of the Galaxy have prompted the studies of dynamics of star clusters with a large range in stellar masses. The tidal interactions between high and low mass stars are found to be rather frequent in such clusters. In this Research Note, we present fitting formulae for the cross sections of tidal capture binary formation between two stars with a large mass ratio. We present the cases between two main-sequence stars, and between a degenerate star and a main-sequence star.

Key words: methods: numerical – celestial mechanics, stellar dynamics – stars: binaries: general – Galaxy: globular clusters: general

1. Introduction

A relatively large number of X-ray sources in globular clusters was first pointed out by Katz (1975). These objects are thought to be close binary systems. As a mechanism for the formation of these binaries, the tidal capture of a normal star by a degenerate star was suggested by Clark (1975) and Fabian et al. (1975). In addition, tidally captured binaries can play an important role in globular cluster dynamics (e.g., Ostriker 1985, Kim et al. 1998).

A precise mechanism for the dissipational tidal capture process was introduced by Fabian et al. (1975), and a detailed computation for the amount of energy deposited into oscillatory modes during a close encounter was performed by Press & Teukolsky (1977) for an $n = 3$ polytropic model. This work was extended by various authors to other polytropes (Lee & Ostriker 1986; abbreviated as LO hereafter, Ray et al. 1987), and to realistic stellar models (McMillan et al. 1987). Some further considerations to the subsequent dynamical evolution of tidal capture binaries are presented by Kochanek (1992) and Mardling (1995a, 1995b). Many numerical simulations for close stellar encounters, whose products include tidal capture binaries, have been performed (e.g. Benz & Hills 1987; Davies et al. 1991).

The interests for tidal capture process have been mainly concentrated on old stellar systems such as globular clusters and galactic nuclei whose densities are known to be very high. Therefore, the cross sections were calculated only for the cases applicable to those systems. The range of mass in these systems is considerably smaller than that in young stellar systems.

Recent advances in infrared astronomy lead to the discoveries of compact young clusters near the Galactic Center (Okuda et al. 1990; Nagata et al. 1995), which have been also observed in detail using the *HST* (Figer et al. 1999). These clusters are found to be as dense as some globular clusters. The sharp image quality of the *HST* also enabled to find star clusters, whose age can be deduced from the population synthesis technique, from galaxies at considerable distances (e.g., Östlin et al. 1998).

The evolution of young clusters near the Galactic center region becomes an important issue in understanding the evolutionary history of the bulge of our galaxy, because these clusters would have rather short evaporation times. The main reason for fast dissolution is the strong tidal field environment. The dynamical process is further influenced by close encounters between stars, including tidal captures. The young clusters found in external galaxies are also of great interest in view of the general evolution of the galaxies. Again, the dynamical evolution and evaporation process are the key in these problems. Two-body relaxation drives the dynamical evolution and close interactions between two stars modify the course of evolution significantly (see, for example, Meylan & Heggie 1997).

The effect of tidal interaction during stellar encounter can be incorporated in different ways depending on the methods of the study of dynamical evolution of stellar systems. In direct N-body calculations, one should know exactly when the tidal capture takes place. In statistical methods such as Fokker-Planck models, the capture cross section as a function of other kinematic parameters (usually velocity dispersion) is necessary.

In the present Research Note, we present convenient formulae for cross sections for tidal capture between two stars as a function of relative velocity at infinity. These formulae can be easily incorporated into statistical methods such as Fokker-

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Planck models or gaseous models for the studies of dynamics of star clusters. They will also be useful in making estimates for the interaction rates for a given cluster parameters.

2. Cross sections

Previous studies presented tidal cross sections for the limited ranges of stars. For example, the mass ratios between encountering stars considered in LO ranged only from 1 to 8 for normal-normal star pairs, and 0.5 to 1.5 for normal-degenerate pairs. Also, $R_1/M_1 = R_2/M_2$ was assumed and only encounters between the same polytropes were considered in LO, which may not be appropriate for encounters between stars with large mass ratios. While an $n = 1.5$ polytrope may well represent the structure of low-mass stars, the outer structure of intermediate to massive stars ($M \gtrsim 1\, M_\odot$) may be better represented by a $n = 3$ polytrope. Thus a consideration for the encounter between $n = 1.5$ and 3 polytropes is necessary for encounters with a large mass difference. Here we extend the work of LO to obtain the cross sections for i) encounters between stars with a very large mass ratio, ii) encounters with mass-radius relation that deviates from conventional $R \propto M$ relation (i.e., $[R_2/M_2]/[R_1/M_1]$ values other than 1), and iii) encounters between $n = 1.5$ and 3 polytropes, and present the results in the form of convenient fitting formulae for cross sections and critical periastron distances.

The amount of orbital energy deposited to the stellar envelope is a very steep function of the distance between stars. The relative orbit can be described as a function of energy and angular momentum. However, the relative orbit near the periastron passage can be approximated by a parabolic orbit which can be specified by only one parameter: periastron distance $R_{\rm min}$. There exists a critical $R_{\rm min}$ below which the tidal interaction transforms the initial unbound system into a bound one for a given set of mass and radius pair, and the relative velocity at infinity, v_∞.

Assuming a parabolic relative orbit for the encounter, Press & Teukolsky (1977) expressed the deposition of kinetic energy into stellar oscillations of a star with M_1 and R_1 due to a perturbing star with M_2 and R_2 by

$$\Delta E_1 = \left(\frac{GM_1}{R_1}\right)^2 \left(\frac{M_2}{M_1}\right)^2 \sum_{l=2}^{\infty} \left(\frac{R_1}{R_{\rm min}}\right)^{2l+2} T_l(\eta_1), \quad (1)$$

where l is the spherical harmonic index, $R_{\rm min}$ the apocenter radius, and the contribution of the summation behind $l = 3$ to ΔE is negligible. The dimensionless parameter η is defined by

$$\eta_1 \equiv \left(\frac{M_1}{M_1 + M_2}\right)^{1/2} \left(\frac{R_{\rm min}}{R_1}\right)^{1.5}. \quad (2)$$

Expression for ΔE_2 may be obtained by exchanging subscripts 1 and 2 in Eqs. (1) and (2). For $T_2(\eta)$ and $T_3(\eta)$ values, we use Portegies Zwart & Meinen's (1993) fifth order fitting polynomials to the numerical calculations by LO.

Tidal capture takes place when the deposition of kinetic energy during the encounter $\Delta E = \Delta E_1 + \Delta E_2$ is larger than the initially positive orbital energy $E_{\rm orb} = \frac{1}{2}\mu v_\infty^2$, where μ is the reduced mass of M_1 and M_2 pair. The critical $R_{\rm min}$ below which the tidal energy exceeds the orbital energy depends on v_∞ via η.

After obtaining critical $R_{\rm min}$ by requiring $\Delta E = \frac{1}{2}\mu v_\infty^2$, we can compute the critical impact parameter R_0 that leads to the critical $R_{\rm min}$ (assuming the orbit does not change in the presence of tidal interaction), as a function of $R_{\rm min}$ and v_∞:

$$R_0(v_\infty) = R_{\rm min} \left(1 + \frac{2GM_{\rm T}}{R_{\rm min} v_\infty^2}\right)^{1/2}, \quad (3)$$

where $M_T = M_1 + M_2$. The capture cross section, $\sigma(v_\infty)$, is simply the area of the target, πR_0^2. The velocity dependence of cross section is expected to be close to v_∞^{-2} since the second term in the parenthesis of Eq. (3) is much greater than 1, and $R_{\rm min}$ is only weakly dependent on v_∞.

3. Fitting formulae

The cross section depends on v_∞ in a somewhat complex way. However, for the limited range of v_∞, it can be approximated as a power law on the tidal capture cross section. Following LO, we provide σ as a function of R_1 and escape velocity at the surface of star 1, $v_{*1} \equiv (2GM_1/R_1)^{1/2}$:

$$\sigma = a \left(\frac{v_\infty}{v_{*1}}\right)^{-\beta} R_1^2. \quad (4)$$

We obtain constants a and β by fitting the power law curve to $\sigma(v_\infty)$ at $v_\infty = 10\,{\rm km\,s^{-1}}$, which is typical velocity range in the globular clusters and compact young clusters. The galactic nuclei are also thought to be dense enough for tidal interactions, but the direct collision is more probable because of high velocity dispersion (Lee & Nelson 1988).

We assumed $R_1/M_1 = R_\odot/M_\odot$ for the calculation of v_{*1}, but a and β values are nearly insensitive to the choice of R_1/M_1 value because the above power law holds for a wide range of v_∞ near $10\,{\rm km\,s^{-1}}$.

3.1. Normal-degenerate encounters

For encounters between a normal and a degenerate star, we obtain a and β for $0.01 \le M_2/M_1 \le 100$. In this subsection, subscript 1 is for the normal star and 2 for the degenerate star. While a is a steep function of M_2/M_1 (see Fig. 1), β ranges only from 2.24 ($M_2/M_1 = 0.01$) to 2.13 ($M_2/M_1 = 100$) for $n = 1.5$, and from 2.24 to 2.19 for $n = 3$. Thus $\beta = 2.2$ would be a good choice. We find that a is well fit with a sum of two power law curves:

$$a_{\rm ND} = 6.60 \left(\tfrac{M_2}{M_1}\right)^{0.242} + 5.06 \left(\tfrac{M_2}{M_1}\right)^{1.33} \quad \text{for } n = 1.5;$$
$$a_{\rm ND} = 3.66 \left(\tfrac{M_2}{M_1}\right)^{0.200} + 2.94 \left(\tfrac{M_2}{M_1}\right)^{1.32} \quad \text{for } n = 3, \quad (5)$$

where subscript ND is for normal-degenerate star encounters. Eq. (5) fits our calculated $a_{\rm ND}$ values with a relative error (|fit-data|/data) better than 4%.

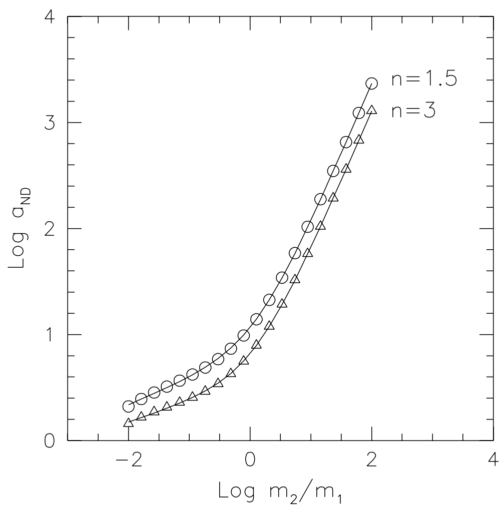

Fig. 1. Calculated $a_{\rm ND}$ for $n = 1.5$ (circles) and $n = 3$ (triangles) as a function of m_2/m_1. Lines represent the fitting formulae of Eq. 5.

3.2. Normal-normal encounters

For encounters between normal stars, we consider M_2/M_1 of 1 through 100. In this subsection, subscript 1 is for less massive star. Encounters between normal stars involve two more parameters, R_2 and M_2, but only one parameter, $\gamma \equiv \log(R_2/R_1)/\log(M_2/M_1)$, is enough in adding the second normal star. For $0.5 \le \gamma \le 1$, it is found that β ranges from 2.12 ($M_2/M_1 = 1$) to 2.24 ($M_2/M_1 = 100$) for $n = 1.5 : 1.5$ and $n = 1.5 : 3$, and from 2.18 to 2.24 for $n = 3$. Again, $\beta = 2.2$ would be a good approximation. Fig. 2 shows a for three different γ values. We find that a simple modification to the form of Eq. 5 can fit a as a function of both M_2/M_1 and γ:

$$\begin{aligned}
a_{\rm NN} = &\ 6.05 \left(\frac{M_2}{M_1}\right)^{0.835\ \ln\gamma + 0.468} \\
&+ 6.50 \left(\frac{M_2}{M_1}\right)^{0.563\ \ln\gamma + 1.75} \quad \text{for } n = 1.5 : 1.5; \\
a_{\rm NN} = &\ 3.50 \left(\frac{M_2}{M_1}\right)^{0.814\ \ln\gamma + 0.551} \\
&+ 3.53 \left(\frac{M_2}{M_1}\right)^{0.598\ \ln\gamma + 1.80} \quad \text{for } n = 3 : 3; \\
a_{\rm NN} = &\ 7.98 \left(\frac{M_2}{M_1}\right)^{-1.23\ \ln\gamma - 0.232} \\
&+ 3.57 \left(\frac{M_2}{M_1}\right)^{0.625\ \ln\gamma + 1.81} \quad \text{for } n = 1.5 : 3, \qquad (6)
\end{aligned}$$

where subscript NN is for normal-normal star encounters, and star 1 has $n = 1.5$ in case of n=1.5:3 encounters. Eq. (6) fits our calculated $a_{\rm NN}$ values with a relative error better than 10%.

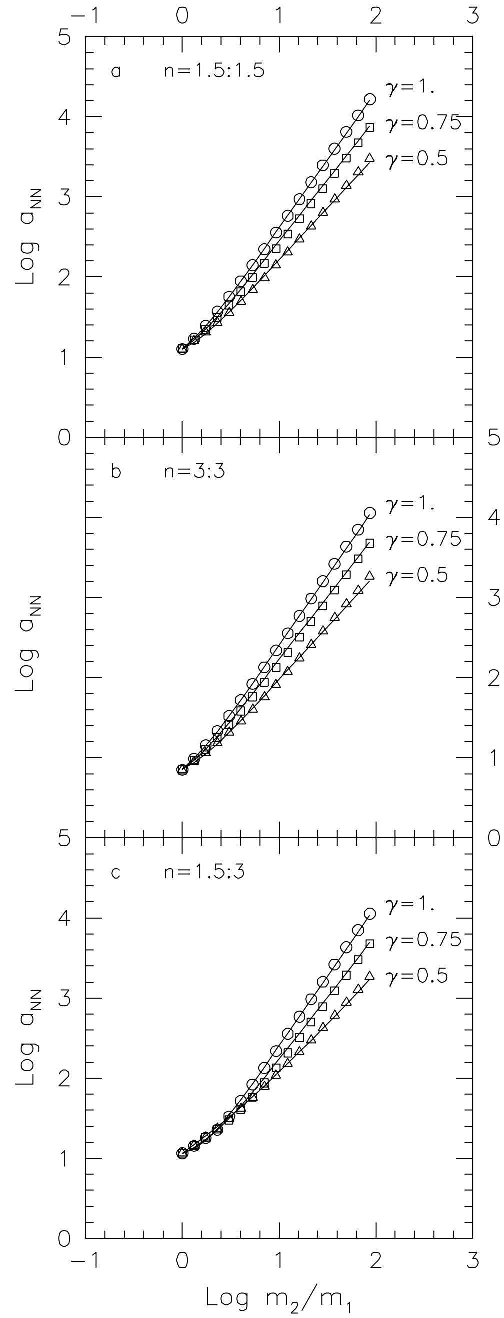

Fig. 2a–c. Calculated $a_{\rm NN}$ for $n = 1.5 : 1.5$ **a**, $n = 3 : 3$ **b**, and $n = 1.5 : 3$ **c** as a function of m_2/m_1. Circles are for $\gamma = 1$, squares for $\gamma = 0.75$, and triangles for $\gamma = 0.5$. Lines represent the fitting formulae of Eq. 6.

4. Discussion

We expressed σ in terms of v_{*1} and R_1 following LO in Sect. 3, but when $\sigma_{\rm NN}$ is expressed in terms of v_{*2} and R_2 such that

$$\sigma_{\rm NN} = a'_{\rm NN} \left(\frac{v_\infty}{v_{*2}}\right)^{-\beta} R_2^2, \qquad (7)$$

the γ dependence of $a'_{\rm NN}$ becomes smaller than that of $a_{\rm NN}$. Also note that $a'_{\rm NN}(M_2/M_1)$ for $\gamma = 1$ is nearly the same as $a_{\rm ND}(M_1/M_2)$ with only a slight difference near $M_2/M_1 \simeq 1$.

Critical $R_{\rm min}$ for tidal captures is also frequently useful for some studies such as the ones with N-body methods. One finds the approximate critical $R_{\rm min}$ as

$$R_{\rm min} \simeq \frac{a}{\pi} \left(\frac{R_1}{1 + M_2/M_1}\right) \left(\frac{v_\infty}{v_{*1}}\right)^{2-\beta} \qquad (8)$$

for the velocities that satisfy

$$\left(\frac{v_\infty}{v_{*1}}\right)^{\beta-4} \gg \frac{4a}{\pi}\left(\frac{1}{1+M_2/M_1}\right)^2, \qquad (9)$$

where the right-hand-side does not vary much from 10.

Heating by tidal capture binaries is incorporated in Fokker-Planck models by calculating $\langle\sigma v\rangle$, where brackets indicate the average over velocity. With the Maxwellian velocity distribution for stars, we obtain $\langle\sigma v\rangle \propto \langle v^{-1.2}\rangle \simeq 1.5 v_{\rm rms}^{-1.2}$ where $v_{\rm rms}$ is the root-mean-square relative velocity between two stellar mass groups. The σ presented here also includes encounters that will lead to a merge before encountering a third star. We will not, however, attempt to go over this issue because it is beyond our scope in this Research Note. We find that the maximum velocity beyond which $R_0 \leq R_1$ is significantly larger than 100 $\rm km\,s^{-1}$ for our parameter regime.

In this Research Note, we merely gave the cross sections. The subsequent evolution of tidally captured systems is a very important subject, but is a rather difficult to follow. If the energy deposited to the envelope of stars can be quickly radiated away before two stars become close, the final orbit of the binary will be a circle whose radius is twice of the initial $R_{\rm min}$ (LO). The stellar rotation induced by the tidal interactions during the circularization process can reduce the separation of circularized orbits. Therefore, the tidal products are usually stellar mergers or tight binaries.

However, there are possibilities of resonant interactions between the tides and stellar orbits. In such an environment, the tidal energy can also be transferred back to the orbital energy. The final product of such a case is a rather wide binary, although it is still “hard” binary in terms of cluster dynamics (Kochanek 1992, Mardling 1995a, 1995b). Therefore, the tidal capture could produce dynamically and observationally interesting objects in dense stellar environments. The cross sections presented here can be useful in estimating the frequencies of such interactions.

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