Astron. Astrophys. 347, 366–369 (1999)

ASTRONOMY
AND
ASTROPHYSICS

Comet Hale-Bopp as a free-rotation rigid body

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Received 28 October 1998 / Accepted 5 March 1999

Abstract. After considering the advantages of using such a simple mechanical model, the nucleus of Comet Hale-Bopp is treated as a free rotation symmetrical top. Although most authors have rejected the periodicity of more than 20 days, we considered in our analysis a possible complex rotational state with the 11.30 hour and the 20–24 day periods. From our model, a complex rotation could be possible if the large period is the spin around the axis of minimum moment of inertia (long axis) and the accepted 11.30 hour period corresponds to a free precession motion. The motion of precession should be nearly perpendicular to the direction of the long axis of the Comet, and then it should be in conflict with the observed stability of the shell structures. Because this result is independent of the axes ratio for reasonable values, we conclude that the complex rotational state is not compatible with observations.

Key words: comets: individual: C/1995 01 Hale-Bopp

1. Introduction

The significance of knowledge of the rotational state of comets is clearly shown in the excellent review by Belton (1991) (see also references therein). Briefly, the dynamical history, cometary activity, nuclear structure and energy dissipation, and the relationship between comet and asteroids, are, among others, topics that benefit from cometary rotation studies. As indicated by Schleicher et al. (1998), periods of rotation have been measured with convinciing accuracy for only about a dozen comets. Undoubtedly, the most studied rotation of a comet was that of Comet Halley (see Belton et al., 1991, and references therein). Nevertheless, two rotation periods or only one rotation period for that comet is a controversy that remains nowadays. In spite of the arguments given by Smith et al. (1987) rejecting a 7.4-day period, is highly doubtful to consider the 2.2-day period as the unique period for Comet Halley (see, for instance, Wilhem (1987)). The problem is that the determination of the rotational state is a troublesome task because the comets are hidden completely by their comae when they are near the Sun during the observations. Notwithstanding this, a combination of observational results and theoretical models provides interesting information concerning the rotation of comets as well as its relationship with the structure and dimensions of their nuclei.

Comet Hale-Bopp is the most distant comet from the Sun ever discovered visually (7.3 AU). This fact clearly indicates the high brightness and the important outgassing and dust rate production of that comet. A lot of interesting results (to be published in a special issue of Earth, Moon and Planets) were recently presented at the Tenerife " First International Conference on Comet Hale-Bopp" (February 1998), hereafter referred to as HBT-98.

The purpose of this work is to apply the classical rigid body theory to the Comet Hale-Bopp after considering some known results such as body periodicities or the estimated nuclear radius. In Sect. 2 we discuss the arguments in favor of the consideration the comet nucleus as a torque-free symmetrical top, and we have also taken into account some observational constraints such as periods, size and nuclear shape in order to obtain physical information. The conclusions inferred from the obtained results are shown in Sect. 3. The work is completed with Appendix A, where the aspects of the rigid body theory used in the study are given in detail and shown in a homogeneous terminology and notation.

2. Nucleus of comet Hale-Bopp as a torque-free symmetrical top

We consider the very simple situation of a torque-free symmetric rigid body as a model for the nucleus of the Comet Hale-Bop. Obviously, the reaction torque from a jet of ejected material affects the state of rotation of a comet. Peale and Lissauer (1989) wrote an interesting paper analyzing the effects of the reaction torque on the rotation of Comet Haley. Nevertheless, there are more uncertainties in the case of Comet Hale-Bopp; for example, the size and the shape (that also affect the rotation state) are not known as well as in the case of Comet Halley. Thus, we believe that free rotation is a good first approximation. Besides, the stability of the structures observed in the inner coma suggests free rotation (see, for instance, Kidger et al., 1998; Licandro et al., 1998).

We assume a homogeneous rigid symmetric ellipsoid (minor semi-axes $a = b$) for the nucleus of the comet. The principal moments of inertia are given by $I_x = I_y = \frac{m}{5}(a^2 + c^2)$ (henceforth I) and $I_z = \frac{2}{5}ma^2$, where m is the mass of the ellipsoid and c ($c > a = b$, prolate body) is the different semi-axis of

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the body along the z body-fixed coordinate. As the body is considered torque-free, the Euler's equations (see the rigid body chapter in any classical mechanics book, for instance, Marion (1970)), can be written

$$\begin{aligned} I\dot{\omega}_x &= (I - I_z)\omega_y\omega_z \\ I\dot{\omega}_y &= -(I - I_z)\omega_x\omega_z \\ I\dot{\omega}_z &= 0 \end{aligned} \tag{1}$$

being ω_x, ω_y, and ω_z the angular velocity components in the body system coordinates (x,y,z) and a dot above a symbol denotes the time derivative.

Using the Euler angles, (see Appendix A) we can write

$$\begin{aligned} \dot{\theta} &= 0 \\ \dot{\phi} &= \frac{L}{I} \\ \dot{\psi} &= \frac{I - I_z}{II_z} L\cos\theta \end{aligned} \tag{2}$$

where L is the angular momentum (oriented along Z coordinate of the inertial system coordinates (X,Y,Z)) and the Euler angles θ,ϕ,ψ have the usual meaning (see Appendix A). $\dot{\phi}$ is the $\overrightarrow{\omega}$ (angular velocity vector) free-precession around the Z-axis and $\dot{\psi}$ is the rotation (spin) of $\overrightarrow{\omega}$ around the z-axis.

From the last two equations, it follows

$$\dot{\psi} = \frac{I - I_z}{I_z}\dot{\phi}\cos\theta \tag{3}$$

and, in terms of semi-axes values, we obtain

$$\cos\theta = \frac{\frac{2\dot{\psi}}{\dot{\phi}}}{(\frac{c}{a})^2 - 1} \tag{4}$$

Unlike the Comet Halley, there are no estimated values for the axes of the Hale-Bopp's nucleus. We assume an axial ratio between the long and short axes c/a between 1.5 to 2.0 as reasonable values. The commonly accepted value of the rotation period of the comet is 11.3 hour. This was determined, among other by, Fernández et al. (1998) thanks to the infrared imaging of changes in the inner coma; Licandro et al. (1998b) with narrowband near infrared images; Farnham et al. (1998) using the motions of the dust arcs, at HBT-98. However, some workers obtained a periodicity greater than 20-day from some Comet Hale-Bopp features (Licandro et al., 1998a; Serra-Ricart et al., (1998) 24-day period and Manzini et al., (1998) 20–22-day period, both at HBT-98, Kidger et al., 1998), but further analysis of data made most of the authors retract the large period. Using the above mentioned observational results, the expression (4) gives no solution if we identify the 11.30 hour period as the rotation about the long axis ($\dot{\psi}$ =2π/0.47 day^{-1}) and the 24 -day period as the free-precession about the Z-axis ($\dot{\phi}$=2π /24 day^{-1}). Relation (4) only gives solutions if $\dot{\psi}$ = 2/24 day $^{-1}$ and $\dot{\phi}$ = 2/0.47 day^{-1}, and then, θ =88–89°, which is roughly 90°. Therefore, the angular momentum L direction must be nearly perpendicular to the major axis body, and this explains the elusive 24-day period.

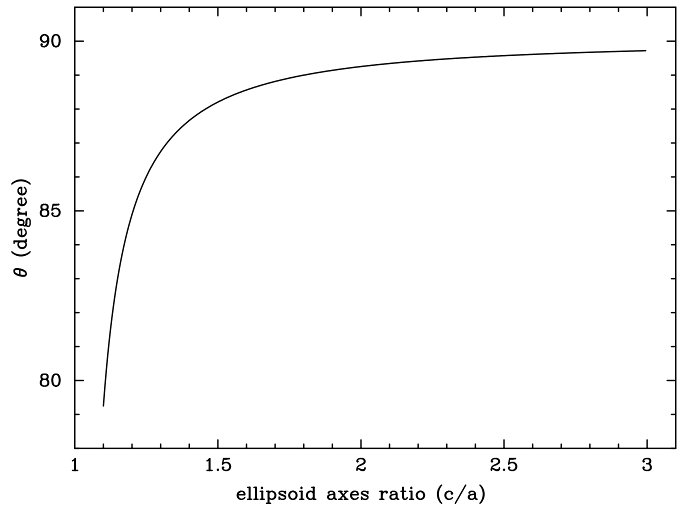

Fig. 1. Precession angle (in degrees) versus ellipsoid axes ratio (c/a) after considering the nucleus of the comet as a symmetric top.

Samarasinha et al. (1998) set limits on the spin state of comet Hale-Bopp based on coma morphology. They reported a possible complex rotational state with a large ratio between the component periods as one of the three cases compatible with near perihelion observations. From their Fig. 2, a precessional angle of 88° is obtained for a ratio (c/a)=2 if the 24 day and 11.30 hour are the considered periods. Licandro et al. (1998a) indicated that a strong precession of the spin axis would imply large variations of the position of the North Pole, and because of the stability of the structures observed they concluded that the angle between the spin axis and the angular momentum should be small. According to our model, if both periods (24 and 0.47 day) are assumed to be correct, then this would result in a large value for θ (nearly 90°). This would conflict with observations: for example the long axis should have rotated 90 degrees in less than six days and rather dramatic changes in the morphology of the shell structure should have been seen. As no such phenomenon was observed, we infer that either the ratio between the long and the short axes is close to 1 or a complex rotational state is unrealistic. If the last option is considered, the 20–24 day periodicity must be rejected as a precessional period. A different explanation could be that the assumed revolution body spins around an axis in the same direction as the angular momentum which is perpendicular to the minimum moment of inertia. Thus, $\dot{\psi}$ should be equal to zero after considering Eq. (3) and only the respective to the $\dot{\phi}$ periodicity is non-vanish. Thus, $\dot{\phi}$ is not a free-precession and it should be considered as the unique real rotation, that fits accordingly with most observational results. In Fig. 1 we show θ versus the c /a ratio. With the adopted values for both periods, it is easily noted that the result is essentially the same for any axes values of an axisymmetric ellipsoid.

3. Conclusions

We assume that the Comet Hale-Bopp is rotating as a force-free axisymmetric rigid body. Obviously this should be considered only as a reasonable first approximation. The Classical Rigid-Body Theory was used to analyze the comet rotation and two observational periods, 11.3 hours and 24 days were used as possible periods on which the analysis was based. If a complex rotational state is assumed, the Comet should rotate around the axis for which the moment of inertia is distinct from a 24 day period (the most elusive period) as well as from a precessional movement nearly perpendicular to the angular momentum vector with an 11.30 hour period (confirmed most convincingly by observation). This possibility must be rejected (and hence the 20–24 day period) because it is not compatible with the stability of the structures observed. This result does not depend on the different ratios between the long and the short axis length which were considered.

Appendix A

It is known that dynamic equations are always easier to solve when they are based in a fixed body reference system (x,y,z) but we must also determine the equations when they are based in an inertial reference system (X,Y,Z). The Rigid Body theory uses the Euler angles (θ, ϕ, ψ) for that purpose (see any Classical Mechanic book: for instance, Marion (1970)). The three rotations of the Euler angles are the three components of an angular velocity vector $\vec{\omega}$: $\omega_\phi = \dot{\phi}$; $\omega_\theta = \dot{\theta}$; $\omega_\psi = \dot{\psi}$; and $\vec{\dot{\phi}}$, called precession, has the Z direction; $\vec{\dot{\theta}}$, called nutation, has the nodes line direction; and $\vec{\dot{\psi}}$, called rotation or spin, has the z direction. Then, the three components of $\vec{\omega}$ in the fixed body system (hereafter principal axes directions) can be written in the following way:

$$\begin{aligned} \omega_x &= \dot{\phi}_x + \dot{\theta}_x + \dot{\psi}_x = \dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi \\ \omega_y &= \dot{\phi}_y + \dot{\theta}_y + \dot{\psi}_y = \dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi \\ \omega_z &= \dot{\phi}_z + \dot{\theta}_z + \dot{\psi}_z = \dot{\phi}\cos\theta + \dot{\psi} \end{aligned} \quad \text{(A1)}$$

Since the components of the angular momentum vector $\vec{L}$ are $L_i = I_i\omega_i$, and the fundamental rotation equation is $\vec{M} = \frac{d\vec{L}}{dt}$, ($\vec{M}$, representing the torque), the Euler equations can be written as follows:

$$\begin{aligned} I_x\dot{\omega}_x + (I_z - I_y)\omega_y\omega_z &= M_x \\ I_y\dot{\omega}_y + (I_x - I_z)\omega_x\omega_z &= M_y \\ I_z\dot{\omega}_z + (I_y - I_x)\omega_x\omega_y &= M_z \end{aligned} \quad \text{(A2)}$$

In free rotation, $\vec{M} = 0$, thus $\vec{L} = constant$. Choosing the Z axis in the $\vec{L}$ direction, we can write:

$$\begin{aligned} L_x &= I_x\omega_x = L\sin\theta\sin\psi\omega_x = \frac{L}{I_x}\sin\theta\sin\psi \\ L_y &= I_y\omega_y = L\sin\theta\cos\psi\omega_y = \frac{L}{I_y}\sin\theta\cos\psi \\ L_z &= I_z\omega_z = L\cos\theta\omega_z = \frac{L}{I_z}\cos\theta \end{aligned} \quad \text{(A3)}$$

Substituting (A3) in the Eqs. (A1), we obtain

$$(\dot{\phi} - \frac{L}{I_x})\sin\theta\sin\psi + \dot{\theta}\cos\psi = 0 \quad \text{(A4-1)}$$

$$(\dot{\phi} - \frac{L}{I_y})\sin\theta\cos\psi - \dot{\theta}\sin\psi = 0 \quad \text{(A4-2)}$$

$$(\dot{\phi} - \frac{L}{I_z})\cos\theta + \dot{\psi} = 0 \quad \text{(A4-3)}$$

Multiplying the Eq. (A4-1) by $\cos\psi$, and Eq. (A4-2) by $-\sin\psi$, and after adding both resulting expressions, we obtain

$$\dot{\theta} = (\frac{L}{I_x} - \frac{L}{I_y})\sin\theta\cos\psi\sin\psi \quad \text{(A5-1)}$$

Working in a similar way, multiplying the Eq. (A4-1) by $\sin\psi$, and Eq. (A4-2) by $\cos\psi$, and after adding both resulting expressions, we obtain

$$\dot{\phi} = \frac{L}{I_x}\sin^2\psi + \frac{L}{I_y}\cos^2\psi \quad \text{(A5-2)}$$

Eq. (A4-3) gives $\dot{\psi} = (\frac{L}{I_z} - \dot{\phi})\cos\theta$, and using the expression (A5-2) we obtain

$$\dot{\psi} = (\frac{L}{I_z} - \frac{L}{I_x}\sin^2\psi - \frac{L}{I_y}\cos^2\psi)\cos\theta \quad \text{(A5-3)}$$

If the solid is axisymmetric, $I_x = I_y = I$, and the Eqs. (A5) result in

$$\dot{\theta} = 0 \quad \text{(A6-1)}$$

$$\dot{\phi} = \frac{L}{I} \quad \text{(A6-2)}$$

$$\dot{\psi} = \frac{I - I_z}{II_z}L\cos\theta, \quad \text{(A6-3)}$$

which, after integrating, can be written as follows:

$$\theta = \theta_\circ \quad \text{(A7-1)}$$

$$\phi = \frac{L}{I}t + \phi_\circ \quad \text{(A7-2)}$$

$$\psi = \frac{I - I_z}{II_z}Lt\cos\theta + \psi_\circ \quad \text{(A7-3)}$$

Acknowledgements. We thank Angel Delgado (Univ. of Granada) for numerous discussions. This work was supported by the Comisión Nacional de Ciencia y Tecnología under contracts ESP97-1536, ESP97-1788, and ESP97-1773-C03-01.

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