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Letter to the Editor

Expansion of solar magnetic flux tubes large and small

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Abstract. In the solar photosphere the magnetic field of magnetic elements and sunspots is known to expand with height. In the case of sunspots this expansion is known to be very rapid, with the field forming an almost horizontal canopy. In this contribution we present new results on the superpenumbral canopy of sunspots based on fits to Stokes I and V profiles of infrared spectral lines. The new models take pressure balance across the boundary of the canopy field into account, which leads to significantly lower canopy base heights than previously determined from similar data.

Due to the lower canopy base height, the density above the canopy base is larger, so that estimates of the mass transported by the Evershed effect in the canopy need to be revised upwards: approximately 15–50% of the mass flowing through the penumbra travels beyond the sunspot boundary above the canopy base.

A comparison with small flux tubes leads to the surprising result that although the two types of features have magnetic fluxes that differ by 5–6 orders of magnitude, their relative rate of expansion with height is very similar, suggesting that at least in this respect sunspots can be described by the thin-tube approximation.

The remaining small differences between the relative expansion of the two types of flux tubes is qualitatively compatible with the presence of magnetic flux that returns into the solar interior at the spot boundary, as has been proposed by Westendorp Plaza et al. (1997).

Key words: magnetic fields – Sun: sunspots – Sun: faculae, plages – Sun: infrared

1. Introduction

The magnetic field of the sun is largely concentrated into features that resemble flux tubes (Stenflo 1994, Schüssler 1992, Solanki 1998) in the sense that they harbour most of the magnetic energy in photospheric layers. The cross-sectional area of individual flux tubes varies from an estimated 5×10^{-10} of the area of the visible solar hemisphere (elements of the intranetwork field) to 2×10^{-3} (sunspots), i.e. by over 6 orders of magnitude. These areas correspond roughly to diameters of 50 km to 70000 km. The magnetic flux carried by a flux tube also exhibits a similarly large range. However, the intrinsic field strength, B, of features with an estimated diameter $\gtrsim 100$ km changes by less than a factor of two if *averaged over the cross-section of the flux tube*, as was first pointed out by Solanki & Schmidt (1993).

In this paper we look for additional similarities in the magnetic structure of such disparate features as magnetic elements and sunspots. In particular, we want to compare how the cross-sectional area of each evolves with height.

2. Magnetic elements: the thin-tube approximation

Consider first the slender magnetic elements. For our purposes their magnetic field can be adequately described by the thin-tube approximation (Defouw 1976, Ferriz Mas & Schüssler 1989), which assumes that B is constant over the flux tube's cross-section and that it satisfies horizontal pressure balance:

$$B^2/(8\pi) = p_\mathrm{e} - p_\mathrm{i}\,, \qquad (1)$$

where p_i is the internal and p_e the external gas pressure. The environment of the flux tube is assumed to be field free.

The vertical stratification of p_i and p_e follows from hydrostatic equilibrium. Accordingly, they drop approximately exponentially with height, z (but not exactly for realistic temperature stratifications). According to Eq. (1) B then also drops roughly exponentially with height, although with a scale height double that of the pressure. Finally, magnetic flux conservation requires that the radius of the tube increases roughly exponentially with z with a scale height that is four times larger than the pressure scale height.

From an observational point of view there is a variety of evidence in favour of the thin-tube approximation as a description of magnetic elements:

1. The field strength of magnetic elements revealed by Zeeman-sensitive lines formed at different heights – such

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as Fe I 1.5648 μm, Fe I 5250.2 Å and Mg I 12.32 μm, which together span almost the whole photospheric height range – is completely consistent with the thin-tube approximation (Zayer et al. 1989, Zirin & Popp 1989, Rüedi et al. 1992).

2. The widths and shapes of the Stokes V profiles of Zeeman sensitive infrared lines require field-strength gradients that are again consistent with the thin-tube description (Zayer et al. 1989, Rüedi et al. 1992, Bruls & Solanki 1995).
3. The expansion of flux tubes predicted by the thin-tube approximation also reproduces the line shapes of the Stokes I and V profiles of the Mg Ib line at 5172.3 Å (Briand & Solanki 1995).

3. Sunspots

In sunspots the field strength drops from the centre of the spot to its edge by a factor of between 2 and 4 (e.g., Solanki & Schmidt 1993, Keppens & Martínez Pillet 1996). Additionally, there is strong evidence for small-scale magnetic structure in the penumbra (e.g., Title et al. 1993), so that considerable *local* currents are present. Hence, a theoretical description of sunspot magnetic fields requires complex models.

It is therefore not surprising that no complete description of the sunspot field is currently available. Even the most advanced models, which take body and boundary sheet currents into account (e.g., Jahn 1989) only satisfy global observational constraints and cannot reproduce observations of the fine-scale structure. Descriptions of the small-scale structure of the sunspot magnetic field usually consider it as a perturbation on the large-scale field (e.g., Martens et al. 1996).

The expansion with height of the sunspot's magnetic field provides an additional constraint on the models. Above the quiet-sun $\tau = 1$ level the expansion can be determined by following the boundary of the sunspot's magnetic field beyond its white-light boundary.

The magnetic field of a sunspot forms an almost horizontal canopy outside its white-light boundary. The height of the lower edge of this canopy as a function of distance from the sunspot provides a measure of its expansion. The canopy has been seen in magnetograms near the limb (Livingston, priv. communication, Giovanelli 1980), in infrared Stokes I and V spectra (Solanki et al. 1992, 1994, Bruls et al. 1995), and with the help of vector polarimetric observations (Lites et al. 1993, Adams et al. 1993). Here we investigate canopies using infrared data – specifically Stokes I and V profiles of Fe I 1.5648 μm (Landé factor, $g = 3$) and Fe I 1.5652 μm ($g_{\rm eff} = 1.53$) – which allow the base height, $z_{\rm c}$, of the canopy and its field strength to be determined anywhere on the solar disc.

A major shortcoming of earlier determinations of the canopy base height based on infrared spectra is that they neglected magnetic pressure. Here we take magnetic pressure into account in the force balance across the canopy base:

$$B_{\rm c}{}^2/(8\pi) + p(z_{\rm c} + \delta z) = p(z_{\rm c} - \delta z), \qquad (2)$$

where $B_{\rm c}$ is the field strength just above the canopy baseheight $z_{\rm c}$, i.e., at the boundary of the flux tube, and $\delta z \ll z_{\rm c}$. We then apply an inversion code to iteratively find the $(z_{\rm c}, B_{\rm c})$ pair providing the best fit to the observed Stokes I and V profiles. The code is an extended version of the one described by Solanki et al. (1992).

We have derived $z_{\rm c}$ and $B_{\rm c}$ of three sunspots. Two of the analysed sunspots are the same as those investigated by Solanki et al. (1992, 1994). The observations of the third sunspot were obtained on 12 Sept. 1994 with the McMath-Pierce facility, the main spectrograph and the Baboquivari detector (Livingston 1991). This sunspot was observed close to the solar limb at $\mu = \cos\theta = 0.22$. A total of 11 spectra showed signatures of the magnetic canopy in this sunspot. The other two sunspots for which we (re-)derived $z_{\rm c}$ and $B_{\rm c}$ were located at $\mu = 0.98$ and $\mu = 0.5$ respectively (Solanki et al. 1992 and 1994 respectively).

An uncertainty of around ± 50 G is found for the field strength at the canopy base. The derived $z_{\rm c}$ is accurate to within ± 10 km. The sensitivity to $z_{\rm c}$ comes from the fact that at heights $z \gtrsim 100$ km the contribution functions of the 1.56 μm lines drop very rapidly with height. Thus, a small change in $z_{\rm c}$ produces a large relative change in both Stokes I and V amplitudes.

Possible systematic errors may, however, be considerably larger than this, so that we do not consider our $z_{\rm c}$ values to be more accurate than ± 50 km and B than ± 100 G. See Solanki et al. (1992) for a discussion.

All $z_{\rm c}$ values obtained in the course of this investigation are plotted in Fig. 1 vs. r/R_0. Here r is the radial coordinate centred on the axis of the sunspot and R_0 is the radius of the white-light sunspot. The choice of r/R_0 as abscissa provides a natural relative scale on which to directly compare the canopies of sunspots or, more generally, flux tubes of different sizes. The two smooth solid lines are chosen to bracket most of the determined $z_{\rm c}(r/R_0)$ values and are described by simple analytical functions. The difference between them reflects the scatter of these values.

The $z_{\rm c}$ values plotted in Fig. 1 are 100–200 km lower than older ones obtained by, e.g., Solanki et al. (1994), who neglected magnetic pressure. The reason is that the magnetic pressure lowers the gas density above the canopy base. Hence the spectral line obtains a smaller contribution there. Only by lowering $z_{\rm c}$ does the total contribution become sufficiently large to reproduce the observed Stokes V and I profiles (including the Stokes V amplitude).

Fig. 2 shows $B_{\rm c}$ vs. r/R_0. The smooth solid lines serve the same purpose as in Fig. 1.

4. Large and small flux tubes

We are now in a position to compare the expansion of large and small flux tubes. We have calculated $z_{\rm c}(r/R_0)$ for different empirical models of slender flux-tube temperature stratification due to Zayer et al. (1990), Solanki & Brigljević (1992), Briand & Solanki (1995) and Frutiger & Solanki (1998). The two extreme $z_{\rm c}(r/R_0)$ resulting from these models are plotted in Fig. 3

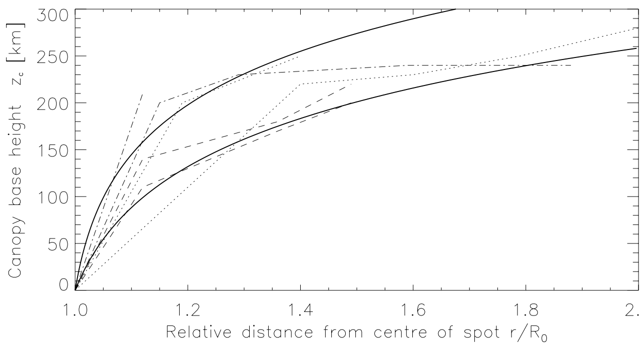

Fig. 1. Canopy base height, $z_{\rm c}$, vs. r/R_0, the distance from the centre of the sunspot, normalised to the radius of the sunspot R_0. Thin curves: $z_{\rm c}(r/R_0)$ determined from the observations along different slices. Dashed, dotted and dot-dashed curves distinguish between the three sunspots. The smooth solid curves are analytical and indicate the range of $z_{\rm c}$ values

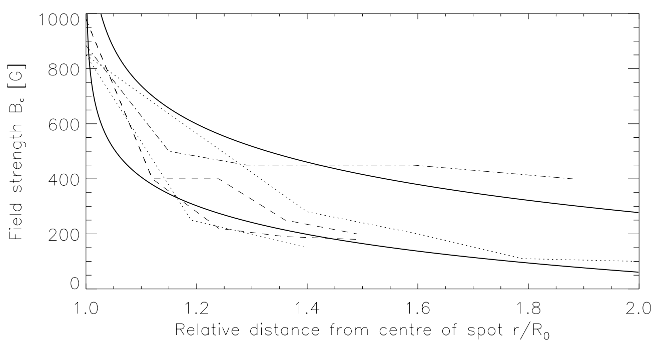

Fig. 2. Field strength at the canopy base, $B_{\rm c}$, vs. r/R_0. The line styles of the curves have the same meaning as in Fig. 1

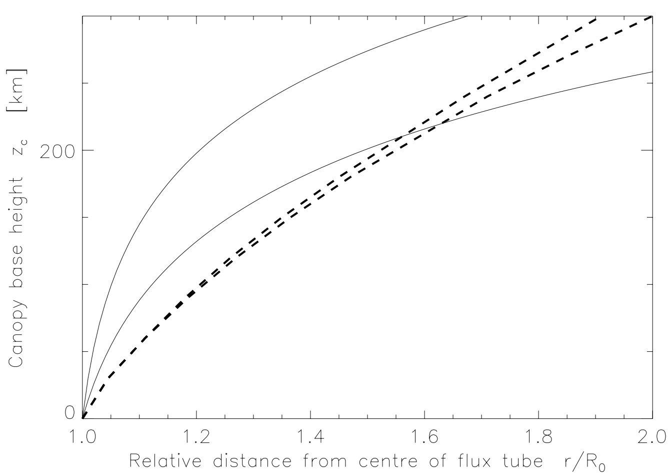

Fig. 3. $z_{\rm c}$, vs. r/R_0. Plotted are the two analytical curves from Fig. 1 (solid lines), as well as two curves describing the expansion of a slender flux tube (dashed lines).

(thick, dashed lines), together with the two smooth curves from Fig. 1 (solid lines).

In an absolute sense small and large flux tubes expand at rates that differ by orders of magnitude (over 300 km in height small flux tubes expand by a few hundred km, while large sunspots expand by tens of thousands of km). However, when plotted on the relative radial scale, r/R_0, the expansion of the two extreme types of solar flux tubes is remarkably similar. Note that it is valid to use z without further scaling, since the natural vertical scale for both types of flux tubes, the pressure scale-height, is almost the same. In particular, if a thin tube had a much lower temperature, similar to that of an umbra or penumbra, it would not expand significantly differently. For example, it would reach $z_{\rm c} = 300$ km at an r/R_0 that does not differ by more than 20% from a hot flux tube model, which is far less than the uncertainty in the sunspot curves.

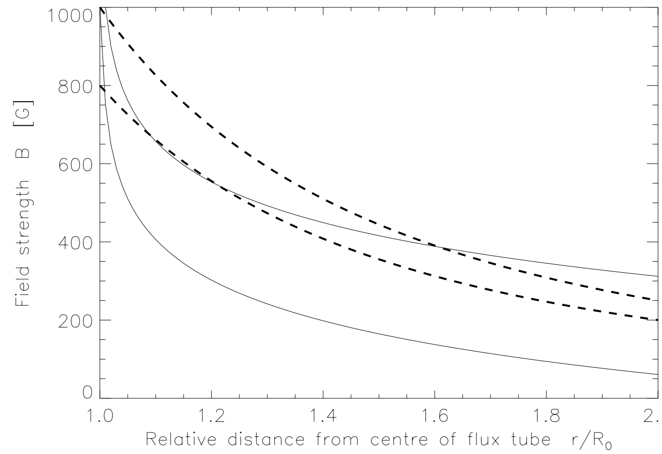

Fig. 4. The same as Fig. 3, but now for $B_{\rm c}$ vs. r/R_0.

Fig. 4 exhibits $B_{\rm c}(r/R_0)$ for both types of flux tubes. Note again the surprising similarity between the smallest and largest flux tubes.

5. Conclusions

The two main results of our analysis, the lower canopy base height $z_{\rm c}(r)$ of the sunspot and the similarity of relative expansion of sunspots and magnetic elements, have consequences that we now briefly explore. The lower $z_{\rm c}$ values agree with those of Giovanelli & Jones (1982) and confirm that in active regions large areas are filled with magnetic field already in the mid-photosphere. In addition to having almost the same field strength, when averaged over their cross-sections, it is now clear that the smallest and largest photospheric flux tubes expand in an almost identical manner, although they carry 6 orders of magnitude different amounts of flux. This result provides a new constraint on models of the sunspot magnetic field. It also shows that at least in some important aspects sunspots behave like thin flux tubes, although they definitely do *not* fulfill the conditions under which the thin tube approximation is valid, since their diameter is far larger than a scale height. The thin-tube approximation even predicts that $dR/dz \sim R$, as follows from flux (Φ) conservation: $\Phi = B\pi R^2 = {\rm const.}$

Solanki et al. (1994) have argued that due to the order of magnitude lower gas density above $z_{\rm c}$ than in the penumbra (since the density scale height ≈ 100 km) only 6–20% of the mass transported by the Evershed flow within the penumbra continues in the canopy.

We find that due to the lower z_c the gas density above z_c is larger by a factor of 2–2.5 compared to that used by Solanki et al. (1994). The density jump at z_c, caused by the imposed pressure balance, only partially compensates for the 100–150 km lower z_c. Taking the new numbers we find that 15–50% of the mass transported by the Evershed effect in the penumbra also passes through the canopy. Inspite of this higher fraction over half of the matter flowing through the penumbra must return to the solar interior at the penumbral edge (cf. Westendorp Plaza et al. 1997).

The expansion of large and small tubes differs clearly only rather close to the flux-tube boundary, between $r/R_0 = 1$ and 1.2. This implies that whatever the distribution of electric currents within these sunspots, they influence the expansion of the field only in a minor manner. Unfortunately, close to the sunspot boundary stray light can be a problem, complicating the issue.

It follows from Figs. 3 and 4 that in the low photosphere slender tubes expand more rapidly with height than sunspots, whereas their field strength decreases less rapidly with r/R_0. The difference in behaviour between the sunspot field and that of thin flux tubes may be due to the presence of some return flux at the outer penumbral boundary, as proposed by Westendorp Plaza et al. (1997). Such a disappearance of the penumbral horizontal field component into the solar interior is expected to cause the field strength to drop rapidly as r becomes greater than R_0, since less flux can now fill the available space. Also a less rapid relative expansion of sunspot fields with height (relative to slender flux tubes) is expected to be produced because only the more vertical magnetic component is left at $r > R_0$, so that z_c increases rapidly with r/R_0 near the spot boundary.

To test this conjecture it is necessary, however, to compare MHD models of the magnetic structure of sunspots with observations of the type presented in Figs. 1 and 2.

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