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LETTER

Letter to the Editor

Mean electromotive force and dynamo action in a turbulent flow

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Abstract. We derive the expression for the mean electromotive force in a turbulent flow taking into account stretching of turbulent magnetic field lines and streamlines by the mean flow. Shear stresses break the symmetry of turbulence in such a way that turbulent motions become suitable for the dynamo action. The contribution of this effect to the mean electromotive force is proportional to the production of spatial derivatives of the mean magnetic field and the mean velocity thus it cannot be described, in principle, in terms of the conventional alpha-effect.

Key words: Magnetohydrodynamics (MHD) – turbulence – stars: magnetic fields – accretion, accretion disks

1. Introduction

The origin of large-scale magnetic fields in various astrophysical bodies still remains a challenging problem. These fields can arise, in principle, by turbulent dynamo generation, from a weak seed field (see, e.g., Moffat 1978, Parker 1979). Turbulent motions showing lack of the reflection symmetry seem to be most suitable for the dynamo action (Krause & Rädler 1980). The conventional alpha-dynamo is an example of such a mechanism where the reflection symmetry of a rotating turbulence is broken by the Coriolis force. Due to rotation, the mean electromotive force, $\boldsymbol{\mathcal{E}} = \langle \boldsymbol{v} \times \boldsymbol{b} \rangle$, has a component proportional to the mean magnetic field, and this component may be responsible for the dynamo action. Note, however, that the capacity of this mechanism to produce the observed magnetic fields, at least in some astrophysical objects, has been debated (Cattaneo & Vainstein 1991, Vainstein & Rosner 1991, Kulsrud & Anderson 1992).

Recently, Urpin and Brandenburg (1999) have suggested the qualitatively different turbulent dynamo mechanism which can operate in a differentially rotating fluid. If rotation is non-uniform, the mean electromotive force contains additional terms proportional to the production of shear stresses and derivatives of the mean magnetic field. These additional terms may generally lead to a rapid amplification of the mean field. Obviously this effect cannot be interpreted in terms of the alpha-dynamo. Contrary to the conventional alpha-effect, this mechanism can amplify a large scale magnetic field even if turbulence is homogeneous. It has also been argued that the similar mechanism can operate in a turbulent shear flow (Urpin 1999). Shear breaks the symmetry of turbulence stretching turbulent magnetic field lines and streamlines. This stretching produces a non-zero contribution to the mean electromotive force even in the simplest case of a plane Couette flow.

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In the present paper we consider the mean electromotive force induced in a turbulent fluid by shear stresses. The main goal of this paper is to show that a non-uniform mean flow may lead to a large scale dynamo action for a wide class of flows.

2. The basic equations

The behaviour of the mean magnetic field is governed by the mean electromotive force, $\boldsymbol{\mathcal{E}} = \langle \boldsymbol{v} \times \boldsymbol{b} \rangle$, where $\langle ... \rangle$ denotes ensemble averaging. We derive the expression for $\boldsymbol{\mathcal{E}}$ by making use of a quasilinear approximation. In this approximation mean quantities are governed by equations including non-linear effects in fluctuating terms, whilst the linearized equation are used for the fluctuating quantities (see, e.g., Krause & Rädler 1980). A quasilinear approximation is sufficiently accurate, for example, to describe various turbulent kinetic processes in plasma when the amplitude of turbulent fluctuations is relatively small (see, e.g., Lifshitz & Pitaevskii 1979). Generally, this approximation applies if the ensemble of turbulent motions is characterized by relatively high frequencies and small amplitudes thus the Strouhal number, $S = v\tau/\ell$, is small; τ and ℓ are the correlation time and the length-scale of turbulence, respectively. Our analysis is based on a double-scale model in which attention is concentrated on the development of the magnetic field on a scale L much greater than the scale ℓ.

Split the magnetic field $\boldsymbol{\mathcal{B}}$ and the velocity $\boldsymbol{u}$ into the mean and fluctuating parts, $\boldsymbol{\mathcal{B}} = \boldsymbol{B} + \boldsymbol{b}$ and $\boldsymbol{u} = \boldsymbol{V} + \boldsymbol{v}$, where $\boldsymbol{B}$ and $\boldsymbol{V}$ are the mean field and velocity, respectively. The linearized induction equation for the fluctuating magnetic field reads

$$\frac{\partial \boldsymbol{b}}{\partial t} = \nabla \times (\boldsymbol{V} \times \boldsymbol{b}) + \nabla \times (\boldsymbol{v} \times \boldsymbol{B}). \qquad (1)$$

We assume that the magnetic Reynolds number is large for fluctuating motions and neglect dissipative effects.

Consider the incompressible flow with $\boldsymbol{V} = \boldsymbol{V}(x, y, z)$ where x, y and z are the Cartesian coordinates; the corresponding unit vectors are $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, and $\boldsymbol{e}_z$. Substituting this expression into Eq. (1), we have

$$\frac{\partial \boldsymbol{b}}{\partial t} + (\boldsymbol{V} \cdot \nabla)\boldsymbol{b} - (\boldsymbol{b} \cdot \nabla)\boldsymbol{V} = \boldsymbol{Q}, \tag{2}$$

where

$$\boldsymbol{Q} = (\boldsymbol{B} \cdot \nabla)\boldsymbol{v} - (\boldsymbol{v} \cdot \nabla)\boldsymbol{B}. \tag{3}$$

The second term on the l.h.s. of Eq. (2) describes the advection of turbulent magnetic field lines by the mean flow and leads only to a small shift of frequences in the spectral integrals if $\ell/\tau \gg V$ (see Urpin 1999). We neglect this term in what follows.

By making use of Fourier transformation,

$$\boldsymbol{b}(\boldsymbol{r}, t) = \int_{-\infty}^{+\infty} d\omega d\boldsymbol{k} e^{i\omega t - i\boldsymbol{k}\cdot\boldsymbol{r}}\ \hat{\boldsymbol{b}}(\boldsymbol{k}, \omega), \tag{4}$$

where the hat labels the Fourier amplitude and neglecting terms of the order of ℓ/L, we obtain from Eq. (2)

$$i\omega \hat{b}_i - \hat{b}_k \frac{\partial V_i}{\partial x_k} = \hat{Q}_i, \tag{5}$$

where

$$\hat{\boldsymbol{Q}} = \frac{1}{(2\pi)^4} \int_{-\infty}^{\infty} d\boldsymbol{r} dt e^{i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t}[(\boldsymbol{B} \cdot \nabla)\boldsymbol{v} - (\boldsymbol{v} \cdot \nabla)\boldsymbol{B}]. \tag{6}$$

In what follows, we assume that inhomogeneity of the mean flow is relatively small and restrict ouselves to linear terms in $\partial V_i/\partial x_k \ll 1/\tau$. Therefore, the solution of Eq. (5) can be represented as

$$\hat{\boldsymbol{b}} = -\frac{i}{\omega}\hat{\boldsymbol{Q}} - \frac{1}{\omega^2}\ (\hat{\boldsymbol{Q}} \cdot \nabla)\boldsymbol{V}. \tag{7}$$

Then, we have

$$\boldsymbol{b}(\boldsymbol{r}, t) = -i \int_{-\infty}^{+\infty} \frac{d\omega d\boldsymbol{k}}{\omega} e^{i\omega t - i\boldsymbol{k}\cdot\boldsymbol{r}} \left[\hat{\boldsymbol{Q}} - \frac{i}{\omega}\ (\hat{\boldsymbol{Q}} \cdot \nabla)\boldsymbol{V}\right]. \tag{8}$$

Substitution of $\boldsymbol{b}(\boldsymbol{r}, t)$ into the definition of $\boldsymbol{\mathcal{E}}$ yields

$$\boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{E}}' + \boldsymbol{\mathcal{E}}'', \tag{9}$$

where

$$\boldsymbol{\mathcal{E}}' = -\int_{-\infty}^{+\infty} \frac{d\omega d\boldsymbol{k}}{\omega^2} d\omega' d\boldsymbol{k'} e^{i(\omega+\omega')t - i(\boldsymbol{k}+\boldsymbol{k'})\cdot\boldsymbol{r}} \langle \hat{\boldsymbol{v}}(\boldsymbol{k'}, \omega') \times [\hat{\boldsymbol{Q}}(\boldsymbol{k}, \omega) \cdot \nabla]\boldsymbol{V} \rangle\ , \tag{10}$$

$$\boldsymbol{\mathcal{E}}'' = -i \int_{-\infty}^{+\infty} \frac{d\omega d\boldsymbol{k}}{\omega} d\omega' d\boldsymbol{k'} e^{i(\omega+\omega')t - i(\boldsymbol{k}+\boldsymbol{k'})\cdot\boldsymbol{r}} \langle \hat{\boldsymbol{v}}(\boldsymbol{k'}, \omega') \times \hat{\boldsymbol{Q}}(\boldsymbol{k}, \omega) \rangle\ . \tag{11}$$

Eqs. (9)-(11) represent the general expression for $\boldsymbol{\mathcal{E}}$ in the presence of a large scale flow.

3. The mean electromotive force in a turbulent flow

Consider the case of incompressible turbulence. Since inhomogeneity of the mean flow is small, we can split Fourier amplitudes into two components as in

$$\hat{\boldsymbol{v}} = \hat{\boldsymbol{v}}_0 + \hat{\boldsymbol{v}}_1, \tag{12}$$

where $\hat{\boldsymbol{v}}_0$ represents the fluctuating velocity at $\boldsymbol{V}(\boldsymbol{r}) = 0$, and $\hat{\boldsymbol{v}}_1$ is a small departure to $\hat{\boldsymbol{v}}_0$ caused by $\boldsymbol{V}(\boldsymbol{r})$. We assume that, in the absence of a mean flow, turbulence is isotropic and homogeneous with the correlation tensor given by

$$\langle \hat{v}_{0i}(\boldsymbol{k}, \omega) \hat{v}_{0j}(\boldsymbol{k'}, \omega') \rangle = \frac{1}{3} v^2(\boldsymbol{k}, \omega) \left(\delta_{ij} - k_i k_j / k^2\right) \delta(\boldsymbol{k} + \boldsymbol{k'}) \delta(\omega + \omega') \tag{13}$$

(see, e.g., Rüdiger 1989).

The velocity $\hat{\boldsymbol{v}}_1$ can be calculated from the momentum equation. In the presence of a mean flow, the fluctuating velocity obeys the equation

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{V} \cdot \nabla)\boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{V} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{\nabla p}{\rho}, \tag{14}$$

where p is the fluctuating pressure and ρ is the density. Following the spirit of a quasilinear approximation, we will neglect the non-linear advective term $(\boldsymbol{v} \cdot \nabla)\boldsymbol{v}$. Making Fourier transformation in t and $\boldsymbol{r}$, we obtain

$$i\omega_1 \hat{\boldsymbol{v}} + (\hat{\boldsymbol{v}} \cdot \nabla)\boldsymbol{V} = i\boldsymbol{k}\ \frac{\hat{p}}{\rho}, \tag{15}$$

where $\omega_1 = \omega - \boldsymbol{k} \cdot \boldsymbol{V}$. If the phase velocity of turbulent fluctuations is larger then V, we have $\omega_1 \approx \omega$. With the accuracy in linear terms in $\partial V_i/\partial x_k$, calculations of $\hat{\boldsymbol{v}}$ can be done by making use of a standard perturbation procedure. The unperturbed velocity, $\boldsymbol{v}_0$ is governed by Eq. (15) with $(\boldsymbol{v} \cdot \nabla)\boldsymbol{V} = 0$. Then, the equation for $\hat{\boldsymbol{v}}_1$ reads

$$i\omega_1 \hat{\boldsymbol{v}}_1 = i\boldsymbol{k}\frac{\hat{p}_1}{\rho} - (\hat{\boldsymbol{v}}_0 \cdot \nabla)\boldsymbol{V}. \tag{16}$$

Taking into account the continuity condition for incompressible fluid, $\boldsymbol{k} \cdot \hat{\boldsymbol{v}}_1 = 0$, we obtain

$$\hat{\boldsymbol{v}}_1 \approx \frac{i}{\omega} \left\{ (\hat{\boldsymbol{v}}_0 \cdot \nabla)\boldsymbol{V} - \frac{\boldsymbol{k}}{k^2}\boldsymbol{k} \cdot [(\hat{\boldsymbol{v}}_0 \cdot \nabla)\boldsymbol{V}] \right\}. \tag{17}$$

Calculating $\boldsymbol{\mathcal{E}}'$, we can neglect small corrections in $\hat{\boldsymbol{v}}$ and $\hat{\boldsymbol{Q}}$ because this part of the electromotive force is already proportional to shear stresses. Therefore,

$$\mathcal{E}'_i = -\varepsilon_{ijk} \frac{\partial V_k}{\partial x_m} J_{jm}, \tag{18}$$

$$J_{jm} = \int_{-\infty}^{+\infty} \frac{d\omega d\boldsymbol{k}}{\omega^2} d\omega' d\boldsymbol{k'} e^{i(\omega+\omega')t - i(\boldsymbol{k}+\boldsymbol{k'})\cdot\boldsymbol{r}} \times \langle \hat{v}_{0j}(\boldsymbol{k'}, \omega') \hat{Q}_{0m}(\boldsymbol{k}, \omega) \rangle; \tag{19}$$

summation is over repeated indexes. Substituting $\hat{\boldsymbol{Q}}$ from Eq. (6) and taking into account the correlation properties of turbulence (Eq. (13)), we have

$$J_{jm} = -\frac{1}{3(2\pi)^4} \int_{-\infty}^{+\infty} \frac{d\omega d\boldsymbol{k}}{\omega^2} d\omega' d\boldsymbol{k}' d\boldsymbol{r}' dt' v^2(\boldsymbol{k}', \omega') \times \left(\delta_{jp} - \frac{k'_j k'_p}{k'^2} \right) e^{i(\omega+\omega')(t-t') - i(\boldsymbol{k}+\boldsymbol{k}')\cdot(\boldsymbol{r}-\boldsymbol{r}')} \frac{\partial B_m}{\partial x'_p}. \quad (20)$$

The integrals over $d\omega$ and $d\boldsymbol{k}$ can be calculated as in

$$\frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{d\omega d\boldsymbol{k}}{\omega^2} e^{i(\omega+\omega')(t-t') - i(\boldsymbol{k}+\boldsymbol{k}')\cdot(\boldsymbol{r}-\boldsymbol{r}')} = \delta[\boldsymbol{r}' - \boldsymbol{r} + \boldsymbol{V}(t-t')] \int_{-\infty}^{+\infty} \frac{d\omega}{(\omega-\omega')^2} e^{i\omega(t-t')}. \quad (21)$$

The last integral in this expression can be taken from the table (see, e.g., Gradshtein & Ryzhik 1965), then

$$J_{jm} = -\frac{1}{3} \int_{-\infty}^{+\infty} d\omega' d\boldsymbol{k}' v^2(\boldsymbol{k}', \omega') \left(\delta_{jp} - \frac{k'_j k'_p}{k'^2} \right) \times \int_{-\infty}^{t} dt'(t'-t) \times e^{-i\omega'(t'-t)} \left. \frac{\partial B_m}{\partial x'_n} \right|_{\boldsymbol{r}'=\boldsymbol{r}-\boldsymbol{V}(t-t')}. \quad (22)$$

Since $\partial B_m / \partial x'_n$ is slowly varying function and does not change significantly on a time scale of the order of τ, integration over dt' gives $(1/\omega'^2)(\partial B_m / \partial x_n)$ with the accuracy in the lowest order in ℓ/L and τ/t_0 (t_0 is the characteristic time scale of mean quantities). Finally,

$$\mathcal{E}'_i = 2\ell^2 \varepsilon_{ijk} \frac{\partial V_k}{\partial x_m} \frac{\partial B_m}{\partial x_j}, \quad (23)$$

where

$$\ell^2 = \frac{1}{9} \int_{-\infty}^{+\infty} \frac{d\omega d\boldsymbol{k}}{\omega^2} v^2(\boldsymbol{k}, \omega). \quad (24)$$

We assume that the ensemble of fluctuating motions does not contain waves with $\omega = 0$ thus the spectral power of turbulence goes to zero at $\omega \to 0$ and there is no singularity in the integral (24).

Contrary to $\boldsymbol{\mathcal{E}}'$, the component $\boldsymbol{\mathcal{E}}''$ should be calculated with taking into account small corrections in the fluctuating velocity and magnetic field. With the accuracy in linear terms in shear stresses, we have

$$\boldsymbol{\mathcal{E}}'' = -i \int_{-\infty}^{+\infty} \frac{d\omega d\boldsymbol{k}}{\omega} d\omega' d\boldsymbol{k}' e^{i(\omega+\omega')t - i(\boldsymbol{k}+\boldsymbol{k}')\cdot\boldsymbol{r}} \langle \hat{\boldsymbol{v}}_0(\boldsymbol{k}', \omega') \times \hat{\boldsymbol{Q}}_1(\boldsymbol{k}, \omega) + \hat{\boldsymbol{v}}_1(\boldsymbol{k}', \omega') \times \hat{\boldsymbol{Q}}_0(\boldsymbol{k}, \omega) \rangle. \quad (25)$$

Substituting $\hat{\boldsymbol{v}}_1$ into this equation and making the same transformations as deriving the expression (23), we obtain

$$\mathcal{E}''_i = \ell^2 \varepsilon_{ijk} \frac{\partial B_k}{\partial x_m} \left(\frac{\partial V_j}{\partial x_m} - \frac{\partial V_m}{\partial x_j} \right). \quad (26)$$

Note that $(\partial V_j / \partial x_m - \partial V_m / \partial x_j)$ determines a large scale vorticity of the flow thus the component $\boldsymbol{\mathcal{E}}''$ of the mean electromotive force is non-vanishing only for the flow with a non-zero vorticity.

Finally, the expression for the mean electromotive force is

$$\mathcal{E}_i = \ell^2 \varepsilon_{ijk} \left[2 \frac{\partial V_k}{\partial x_m} \frac{\partial B_m}{\partial x_j} + \left(\frac{\partial V_j}{\partial x_m} - \frac{\partial V_m}{\partial x_j} \right) \frac{\partial B_k}{\partial x_m} \right]. \quad (27)$$

The terms on the r.h.s. of Eq. (27) contain spatial derivatives of the mean velocity and, generally, the electromotive force is non-vanishing for any non-uniform flow. Note that $\boldsymbol{\mathcal{E}}$ does not contain the component proportional to $\boldsymbol{B}$ that is typical for the alpha-effect. Therefore the effect of shear stresses cannot be described, in principle, in terms of the alpha-effect. The coefficient α of the conventional dynamo theory is proportional to the kinetic helicity, $h = \langle \boldsymbol{v} \cdot (\nabla \times \boldsymbol{v}) \rangle$, which is non-vanishing, for example, in a rotating fluid. However, apart from rotation the condition $\alpha \neq 0$ requires also the presence of a large scale inhomogeneity of the fluid (see, e.g., Krause & Rädler 1980). For example, it can be the density stratification but the presence of inhomogeneity is absolutely necessary because the pseudoscalar α can be formed from the axial vector of the angular velocity, $\boldsymbol{\Omega}$, only as a scalar production of $\boldsymbol{\Omega}$ and some polar vector (e.g., the density gradient). In the present paper, we consider the case of homogeneous unperturbed turbulence where the alpha-effect should be vanishing because of this reasonning in the lowest order in ℓ/L and τ/t_0, at least. This point can easily be checked by direct calculations. Indeed, calculating helicity in terms of Fourier transforms, we obtain that h is proportional to $\langle \hat{\boldsymbol{v}}_0 \cdot (\boldsymbol{k} \times \hat{\boldsymbol{v}}_1) + \hat{\boldsymbol{v}}_1 \cdot (\boldsymbol{k} \times \hat{\boldsymbol{v}}_0) \rangle$ with the accuracy in linear terms in $\hat{\boldsymbol{v}}_1$. Eq. (17) for $\hat{\boldsymbol{v}}_1$ contains only a production of $\hat{\boldsymbol{v}}_0$ and even powers of $\boldsymbol{k}$, therefore the expression for h should contain only odd powers of $\boldsymbol{k}$ after ensemble averaging. If the unperturbed turbulence is assumed to be homogeneous and isotropic (see (13)), integration over $d\boldsymbol{k}$ will give a vanishing contribution, and $h \propto \alpha = 0$ in our model. Thus, the suggested mechanism works under the condition when the alpha-dynamo is unoperative.

4. Discussion

For illustration, consider the behaviour of the mean field in a turbulent flow between two planes, $x = \pm L$. Assume that the mean velocity of the flow is given by $\boldsymbol{V} = \boldsymbol{e}_y V(x)$. In the induction equation, the electromotive force (27) has to be complemented by the standard term representing turbulent magnetic diffusivity and the advective term, $\boldsymbol{V} \times \boldsymbol{B}$. Then, the induction equation reads

$$\frac{\partial \boldsymbol{B}}{\partial t} + V(x) \frac{\partial \boldsymbol{B}}{\partial y} - \boldsymbol{e}_y B_x V'(x) = \nabla \times \boldsymbol{\mathcal{E}} + \eta \Delta \boldsymbol{B}, \quad (28)$$

where

$$\eta = \frac{1}{3} \int_{-\infty}^{+\infty} \frac{\omega_m k^2 R(\omega, \boldsymbol{k})}{\omega^2 + \omega_m^2} d\omega d\boldsymbol{k}, \quad (29)$$

is the scallar turbulent magnetic diffusivity (see, e.g., Krause & Rädler 1980); $\omega_m = \nu_m k^2$, and ν_m is the molecular magnetic diffusivity. Note that the dissipative term does not appear in the expression (27) because we neglect the magnetic diffusivity from the very beginning for the sake of simplicity.

The evolution of the mean field is determined by the behaviour of its x-component which turns out to be not influenced by B_y and B_z. However, these two components are generated from B_x due to either the stretching effect or the turbulent electromotive force thus B_y and B_z are coupled to B_x. The equation for B_x is

$$\frac{\partial B_x}{\partial t} + V(x)\frac{\partial B_x}{\partial y} = 2\ell^2 V'(x)\frac{\partial^2 B_x}{\partial x \partial y} + \eta \Delta B_x. \quad (30)$$

Consider as an example the shear flow with $V(x) = V_0(1 - x^2/L^2)$. The solution of Eq. (30) can be represented in the form

$$B_x = e^{\gamma t - iq_y y - iq_z z - i\ell^2 q_y V_0 x^2/\eta L^2} f(x), \quad (31)$$

where q_y and q_z are the wavevectors in the y- and z-directions, respectively. Then, the equation for f reads

$$\frac{d^2 f}{d\xi^2} + (a + b\xi^2) f = 0 \quad (32)$$

where $\xi = x/L$, $a = -L^2[\gamma - iq_y V_0 + \eta(q_y^2 +_z^2)]/\eta$, and $b = (q_y V_0 L^2/\eta)(-i + 4q_y V_0 \ell^4/\eta L^2)$. The solution of Eq. (32) can easily be obtained by making use of the WKB-approximation. Substituting $f = \exp[-i\int K(\xi')d\xi']$ and assuming $K(\xi)L \gg 1$, we have

$$K_{1,2}(\xi) = \pm\sqrt{a + b\xi^2}. \quad (33)$$

If B_x is vanishing on the surfaces $x = \pm L$, then the eigenvalues are given by the condition

$$\int_{-1}^{+1} \sqrt{a + b\xi^2} d\xi = n\pi \quad (34)$$

where n is integer. We can rewrite this equation as

$$\sqrt{a + b\xi_*^2} = n\pi/2, \quad (35)$$

where ξ_* is some average value of ξ, $1 > \xi_* > 0$. Then, the eigenvalues are given by

$$\gamma = iq_y V_0(1 - \xi_*^2) + \gamma_0 - \eta\lambda^2, \quad (36)$$

where $\gamma_0 = 4\xi_*^2 q_y^2 V_0^2 \ell^4/\eta L^2$, and $\lambda^2 = (n\pi/2L)^2 + q_y^2 + q_z^2$. The first term on the r.h.s. of Eq. (36) describes oscillations of the magnetic field caused by advection of field lines by the mean flow, the second terms represents the generating effect of the electromotive force $\boldsymbol{\mathcal{E}}$, and the third term describes turbulent dissipation. Note that $\boldsymbol{\mathcal{E}}$ always gives a positive contribution to γ resulting in a generation of the magnetic field for any wavevectors. The generating effect becomes stronger for a lower magnetic diffusivity (or, in other words, for a higher conductivity). For a relatively large magnetic Reynolds number, we have $\gamma_0 > \eta\lambda^2$ and the flow becomes unstable to a generation of the mean field. Note that only the field components which depend on the y-coordinate can be unstable.

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