Astron. Astrophys. 347, 1046–1054 (1999)

ASTRONOMY
AND
ASTROPHYSICS

The combination of ground-based astrometric compilation catalogues with the HIPPARCOS Catalogue

I. Single-star mode

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Received 19 January 1999 / Accepted 20 April 1999

Abstract. The combination of ground-based astrometric compilation catalogues, such as the FK5 or the GC, with the results of the ESA Astrometric Satellite HIPPARCOS produces for many thousands of stars proper motions which are significantly more accurate than the proper motions derived from the HIPPARCOS observations alone. In the combination of the basic FK5 with the HIPPARCOS Catalogue (i.e., in the FK6), the gain in accuracy is about a factor of two for the proper motions of single stars. The use of the GC still improves the accuracy of the proper motions by a factor of about 1.2 . We derive and describe in detail how to combine a ground-based compilation catalogue with HIPPARCOS. Our analytic approach is helpful for understanding the principles of the combination method. In real applications we use a numerical approach which avoids some (minor) approximations made in the analytic approach. We give a numerical example of our combination method and present an overall error budget for the combination of the ground-based data for the basic FK5 stars and for the GC stars with the HIPPARCOS observations.

In the present paper we describe the 'single-star mode' of our combination method. This mode is appropriate for truly single stars or for stars which can be treated like single stars. The specific handling of binaries will be discussed in subsequent papers.

Key words: catalogs – astrometry

1. Introduction

The HIPPARCOS astrometric satellite has provided very accurate positions, proper motions, and parallaxes for more than 118 000 stars (ESA 1997). Does this mean that the long series of ground-based astrometric observations of all these stars are now merely of historical value ? This is certainly not the case: It can be shown that the combination of the HIPPARCOS data with ground-based results is providing for many thousands of stars individual proper motions which are significantly more accurate than the HIPPARCOS proper motions themselves (Wielen 1988, Wielen et al. 1998, 1999).

There are two main reasons why ground-based observations can improve the HIPPARCOS results: (1) For many of the brighter stars, the ground-based observations cover a period of time of more than two centuries. The measuring accuracy of these ground-based positions is such that they allow, especially in combination with the HIPPARCOS positions, to derive proper motions which have significantly smaller measuring errors than the HIPPARCOS proper motions. It is well-known that ground-based observations suffer from considerable systematic errors. However, it is just the HIPPARCOS Catalogue itself which allows us to remove these systematic errors from the ground-based data to such a high degree that we can use with much confidence the corrected ground-based results for the direct combination with HIPPARCOS. (2) The HIPPARCOS proper motions are derived from observations which have been carried out within about three years only. Since many of the HIPPARCOS stars are undetected astrometric binaries, these nearly 'instantaneously' measured HIPPARCOS proper motions can differ significantly from 'time-averaged', 'mean' proper motions. We have called the difference between the instantaneous HIPPARCOS proper motion and the mean motion of a star the 'cosmic error' of the HIPPARCOS proper motion (Wielen 1997). In many cases these cosmic errors are significantly larger than the measuring errors of the instantaneous HIPPARCOS proper motions (Wielen 1995a, b, Wielen et al. 1997, 1998, 1999). The ground-based observations, with their long observational history, are often already providing 'mean' proper motions. The ground-based data allow us therefore to identify and to correct partially the cosmic errors in the HIPPARCOS proper motions.

Having argued that a combination of ground-based observations with the HIPPARCOS data is useful for many stars, we now provide a method for carrying out such a combination. The best way would be to combine the individual observational catalogues of ground-based positions directly with HIPPARCOS. However, there exists more than 2000 of such observational catalogues. While we have collected most of these catalogues in our astrometric data base ARIGFH (described in Wielen (1998)) and hope to use them individually in the future, we have to rely for the moment on a few compilation catalogues. Such a compilation catalogue is usually based on a large number of observational catalogues and provides as a 'summary' of these

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individual catalogues a derived proper motion and a mean position at a central epoch.

The most accurate of the ground-based compilation catalogues is the first part of the FK5 (Fricke et al. 1988). It contains the 1535 basic fundamental stars. The second part of the FK5 (Fricke et al. 1991) lists 3117 additional stars in the bright and faint extension. A compilation of the remaining FK4Sup stars (Schwan et al. 1993) provides data for further 995 stars with a good observational history. The combination of the basic FK5 stars with HIPPARCOS data produces the FK6. The proper motions of the FK6 are of unprecedented accuracy (Wielen et al. 1998,1999, and Sect. 6). The accuracy of these FK6 proper motions rests to a high degree on the older observations. This is not only to be expected on theoretical grounds, but can be proved empirically also (Wielen et al. 1998) by combining HIPPARCOS with an older fundamental catalogue such as the FK3 (Kopff 1937, 1938). Because the old observations carry most of the weight in a combination with HIPPARCOS, the GC (Boss et al. 1937) is also relevant for our purpose. The GC is a careful compilation of ground-based observations from the 18th century until about 1930. The main advantage of the GC is its large number of stars. About 29 700 of the 33 342 GC stars have been observed by HIPPARCOS and can therefore be used in our method. The typical accuracy obtained for a GC star is, of course, lower than for an FK6 star (Wielen et al. 1998, 1999).

Instead of using a *compilation* catalogue, we may rely on one old *observational* catalogue only, such as the Astrographic Catalogue (AC). However, the formal accuracy of the proper motions obtained by a combination of the AC with the HIPPARCOS data is in most cases only marginally better than that of the HIPPARCOS proper motions alone. On the other hand, a comparison of a proper motion derived from an AC position and a HIPPARCOS position with the quasi-instantaneous HIPPARCOS proper motion μ_H can reveal the occurence of a large value of the cosmic error in μ_H for the star under consideration.

Before we can combine the ground-based data with HIPPARCOS, the systematic errors of the ground-based positions and proper motions have to be determined and removed. For the determination of these systematic errors we use methods developed for the construction of the FK5 (Bien et al. 1978). These methods do not only provide the systematic errors themselves but also the local uncertainty of the correction, i.e. the mean 'measuring' error of the systematic correction for a given star. As the reference system, we use the HIPPARCOS system. The combined catalogue is therefore always on the HIPPARCOS system, i.e. on the ICRS. For our purpose, we can consider the HIPPARCOS data as free from systematic errors. A possible slight rotation of the HIPPARCOS system with respect to an inertial, extragalactic system does not affect our method (but of course our results).

In the following sections, we assume that the angular coordinates of a star, $\alpha_* = \alpha \cos \delta$ and δ, change linearly in time t. In practice, the non-linear motion of a star in α_* and δ because of spherical effects and the foreshortening effect has to be taken into account. We do this already in the determination of the systematic errors between the ground-based data and HIPPARCOS. Later we do not use the full values of ground-based positions and proper motions, but only their differential values with respect to the HIPPARCOS solution at the proper epoch. If we use these differential values only, the linear approximation is then fully sufficient for our purpose.

We shall now present the mathematical details of our combination method. We offer two approaches: an approximate analytic one and a rigorous numerical one. The analytic approach has the advantage to show more clearly the basic principles. It neglects, however, some of the correlations which occur among the HIPPARCOS results. In practical applications, we use the numerical approach which takes all the correlation among the HIPPARCOS results into account. In all cases, we assume that the HIPPARCOS data are completely uncorrelated with the ground-based data. This is certainly true for all of our stars, although the HIPPARCOS Input Catalogue, used as a first approximation in the HIPPARCOS data reduction procedure, had to rely on the ground-based observations.

2. Analytic approach

We assume that two astrometric catalogues are available. They are identified by the indices 1 and 2. Examples are the FK5 and HIPPARCOS. For the combined catalogue, e.g. the FK6, we use the index C. Each of the two basic catalogues (i = 1, 2) provides for all the stars under consideration a position $x_i(T_i)$ and a proper motion μ_i at a central epoch T_i for two uncorrelated coordinate components, e.g. for α_* and δ. In addition, both catalogues provide mean errors of $x_i(T_i)$ and μ_i which we denote by $\varepsilon_{x,i}$ and $\varepsilon_{\mu,i}$.

In the case of a catalogue which has been reduced to the HIPPARCOS system, the errors $\varepsilon_{x,i}$ and $\varepsilon_{\mu,i}$ have to take into account also the local uncertainty of the systematic corrections, e.g. by using $\varepsilon^2_{x,i,tot} = \varepsilon^2_{x,i,ind} + \varepsilon^2_{x,i,sys}$, where the 'individual' error $\varepsilon_{x,i,ind}$ is the random measuring error given in the catalogue. We should emphasize that $\varepsilon_{x,i,sys}$ is not the systematic correction itself but only its uncertainty.

The central epoch T_i is chosen such that the correlation between $x_i(T_i)$ and μ_i is zero at $t = T_i$. Most of the ground-based catalogues give the central epoch T_i explicitely for each star, separately for α and δ. For HIPPARCOS (index 2) we have to derive $T_2 = T_{H,ind}$ (separately for α_* and δ) from the data given in the catalogue:

$$T_2 = T_{H,ind} = T_H - \frac{\rho_{x\mu,H}\,(T_H)\,\varepsilon_{x,H}\,(T_H)}{\varepsilon_{\mu,H}\,(T_H)}\,, \tag{1}$$

where $T_H = 1991.25$ is the overall reference epoch of the HIPPARCOS Catalogue, $\varepsilon_{x,H}\,(T_H)$ the mean error of $x_H\,(T_H)$, $\varepsilon_{\mu,H}$ the mean error of μ_H, and $\rho_{x\mu,H}\,(T_H)$ the correlation coefficient between $x_H(T_H)$ and μ_H. The position $x_2(T_2)$ is derived from

$$x_2(T_2) = x_H(T_H) + \mu_H(T_2 - T_H)\,. \tag{2}$$

The mean error of $x_2(T_2)$ is given by

$$\begin{aligned}\varepsilon^2_{x,2}(T_2) \;=\; & \varepsilon^2_{x,H}(T_H) + \varepsilon^2_{\mu,H}(T_2 - T_H)^2 \\ & +\; 2\,\rho_{x\mu,H}(T_H)\,\varepsilon_{x,H}(T_H)\,\varepsilon_{\mu,H}(T_2 - T_H)\,.\end{aligned} \tag{3}$$

The proper motion has not to be recalculated, i.e. $\mu_2 = \mu_H$ and $\varepsilon_{\mu,2} = \varepsilon_{\mu,H}$. The HIPPARCOS position and proper motion, $x_2(T_2)$ and μ_2, are now uncorrelated, i.e. $\rho_{x\mu,2}(T_2) = 0$. In the analytic approach, we assume that the values of $x_2(T_2)$ and μ_2 between α_* and δ are also uncorrelated. This is not strictly true because our choice of T_2 (separately for α_* and δ) forces only one correlation coefficient to vanish. All the other correlation coefficients of the HIPPARCOS data remain finite (albeit small) in general. Practical experience shows that this approximation is quite good for most stars (see Sect. 5).

We are now prepared to carry out the combination of the two catalogues. The basic idea of the combination is to reconstruct the normal equations of the least-square solutions from which the two catalogues have been derived. This can be done by using only the data actually given in the two catalogues. These two sets of normal equations are then added together to provide the normal equations for the combined catalogue C. Our method is a generalisation of methods described by Kopff et al. (1964), Eichhorn (1974), and other authors. First partial results were presented by Wielen (1988).

The method of least squares determines the 'best' solution $x_{sol}(T_{ref})$ and μ_{sol} for the position of a star at a reference epoch T_{ref} and for its proper motion from a time series of observed positions,

$$x_{obs}(t) = x_{sol}(T_{ref}) + \mu_{sol}(t - T_{ref}) + v_{obs}(t)\,, \tag{4}$$

by the condition that the weighted sum of the squared residuals $v_{obs}(t)$ should be a minimum:

$$\left[p\, v_{obs}^2\right] = min\,. \tag{5}$$

We use here the classical convention that the brackets [...] imply a summation over the observations. p is the individual weight of $x_{obs}(t)$. (We hope that the reader is not confused by the fact that we use the same letter p for two different quantities in order not to change familiar notations: In this Sect. 2, we denote by p the weights of observations. In all the other sections, p is the stellar parallax.) The condition (5) is fulfilled if we solve the normal equations for $x_{sol}(T_{ref})$ and μ_{sol}:

$$[p]\; x_{sol}(T_{ref}) \;+\; [p(t - T_{ref})]\; \mu_{sol} = [p\, x_{obs}(t)]\;, \tag{6}$$

$$[p(t - T_{ref})]\; x_{sol}(T_{ref}) \;+\; \left[p(t - T_{ref})^2\right]\; \mu_{sol} = [p\, x_{obs}(t)(t - T_{ref})]\;. \tag{7}$$

We now specialize the Eqs. (6) and (7) for each of the basic catalogues (i = 1, 2). For this purpose, we choose individual reference epochs T_{ref} for each catalogue $(T_{ref} = T_i)$ such that

$$[p(t - T_i)]_i = 0\,. \tag{8}$$

The index i at the right bracket indicates that the sum should be extended over all the observations which have been used in catalogue i. The normal equations of catalogue i are then

$$[p]_i\; x_i(T_i) \;=\; [p\, x_{obs}(t)]_i \tag{9}$$

$$\left[p(t - T_i)^2\right]\; \mu_i \;=\; [p\, x_{obs}(t)(t - T_i)]_i\;. \tag{10}$$

The choice (8) of the central epoch T_i for T_{ref} has the consequence that $x_i(T_i)$ and μ_i are not correlated, as indicated by the vanishing diagonal terms which leads to decoupled equations for $x_i(T_i)$ and μ_i.

The mean errors $\varepsilon_{x,i}$ of $x_i(T_i)$ and $\varepsilon_{\mu,i}$ of μ_i are given by:

$$\varepsilon_{x,i}^2 \;=\; \frac{\varepsilon_0^2}{[p]_i}\,, \tag{11}$$

$$\varepsilon_{\mu,i}^2 \;=\; \frac{\varepsilon_0^2}{[p(t - T_i)^2]_i}\,, \tag{12}$$

where ε_0 is the error of unit weight. It will turn out that we do not have to know ε_0 or the adopted system of weights in each of the basic catalogues. We rely only on the fact that mean errors were correctly calculated for the final results given in each of the catalogues.

The four quantities in the brackets of Eqs. (9) and (10) are now redetermined in the following. From Eqs. (11) and (12), we obtain

$$[p]_i \;=\; \frac{\varepsilon_0^2}{\varepsilon_{x,i}^2}\,, \tag{13}$$

$$\left[p(t - T_i)^2\right]_i \;=\; \frac{\varepsilon_0^2}{\varepsilon_{\mu,i}^2}\,. \tag{14}$$

Since the catalogue values of $x_i(T_i)$ and μ_i solve the Eqs. (9) and (10), we can reconstruct the right-hand sides of the normal equations with the help of Eqs. (13) and (14):

$$[p\, x_{obs}(t)]_i \;=\; \frac{\varepsilon_0^2}{\varepsilon_{x,i}^2}\, x_i(T_i)\,, \tag{15}$$

$$[p\, x_{obs}(t)(t - T_i)]_i \;=\; \frac{\varepsilon_0^2}{\varepsilon_{\mu,i}^2}\, \mu_i\,. \tag{16}$$

The normal equations for the combined catalogue C have the same form as Eqs. (6) and (7). In order to simplify these normal equations, we replace T_{ref} by the central epoch T_C of the combined catalogue C. T_C is given by the condition:

$$[p(t - T_C)]_C = 0\,. \tag{17}$$

T_C can be determined from known quantities by using Eqs. (8):

$$\begin{aligned} [p(t - T_C)]_C &= [p(t - T_C)]_1 + [p(t - T_C)]_2 \\ &= [p(t - T_1)]_1 + [p(T_1 - T_C)]_1 \\ &\quad + [p(t - T_2)]_2 + [p(T_2 - T_C)]_2 \\ &= [p]_1\,(T_1 - T_C) + [p]_2\,(T_2 - T_C) = 0\,. \end{aligned} \tag{18}$$

Solving for T_C and inserting Eqs. (13), we find

$$\begin{aligned} T_C \;&=\; \frac{[p]_1\; T_1 + [p]_2\; T_2}{[p]_1 + [p]_2} = \frac{\varepsilon_{x,1}^{-2}\, T_1 + \varepsilon_{x,2}^{-2}\, T_2}{\varepsilon_{x,1}^{-2} + \varepsilon_{x,2}^{-2}} \\ &=\; \frac{w_{x,1}\, T_1 + w_{x,2}\, T_2}{w_{x,1} + w_{x,2}}\,, \end{aligned} \tag{19}$$

The central epoch T_C is therefore the weighted mean of the epochs T_1 and T_2. As weights for T_i, we have to use the weight

$$w_{x,i} = \frac{1}{\varepsilon_{x,1}^2} \tag{20}$$

of the position $x_i(T_i)$. Using T_C as the reference time, the normal equations for the combined catalogue C are reduced to two independent equations for the unknowns $x_C(T_C)$ and μ_C, the position and proper motion of the star in C:

$$[p]_C \; x_C(T_C) \;=\; [p\, x_{obs}(t)]_C \;, \tag{21}$$

$$\left[p(t-T_C)^2\right]_C \; \mu_C \;=\; [p\, x_{obs}(t)(t-T_C)]_C \;. \tag{22}$$

If we solve Eq. (21) for $x_C(T_C)$ and use Eqs. (9), (13), and (15), we find

$$\begin{aligned} x_C(T_C) \;&=\; \frac{[p\, x_{obs}(t)]_C}{[p]_C} = \frac{[p\, x_{obs}(t)]_1 + [p\, x_{obs}(t)]_2}{[p]_1 + [p]_2} \\ &=\; \frac{[p]_1 \; x_1(T_1) + [p]_2 \; x_2(T_2)}{[p]_1 + [p]_2} \\ &=\; \frac{w_{x,1}\, x_1(T_1) + w_{x,2}\, x_2(T_2)}{w_{x,1} + w_{x,2}} \,, \end{aligned} \tag{23}$$

with $w_{x,i}$ given by Eq. (20). As for T_C, the position $x_C(T_C)$ is the weighted mean of $x_1(T_1)$ and $x_2(T_2)$ with $w_{x,i}$ as weights.

In order to write μ_C also as a weighted average, it is helpful to introduce a third proper motion, μ_0, as an auxiliary tool (Wielen 1988):

$$\mu_0 = \frac{x_2(T_2) - x_1(T_1)}{T_2 - T_1} \,. \tag{24}$$

The proper motion μ_0 is based only on the central positions of the two basic catalogues (see also Fig. 1). Since $x_1(T_1)$ and $x_2(T_2)$ are not correlated with the catalogue proper motions μ_1 and μ_2, the new proper motion μ_0 is also not correlated with μ_1 and μ_2. The mean error $\varepsilon_{\mu,0}$ of μ_0 is given by

$$\varepsilon^2_{\mu,0} = \frac{\varepsilon^2_{x,1} + \varepsilon^2_{x,2}}{(T_2 - T_1)^2} \,. \tag{25}$$

The weights $w_{\mu,i}$ of the proper motions are

$$w_{\mu,i} = \frac{1}{\varepsilon^2_{\mu,i}} \,, \tag{26}$$

where the index i now runs from 0 to 2. We now solve Eq. (22) for μ_C:

$$\mu_C = \frac{[p\, x_{obs}(t)(t-T_C)]_C}{[p(t-T_C)^2]_C} \,. \tag{27}$$

For the numerator of Eq. (27) we obtain by using Eqs. (15), (16), (19), (20), (24), (25), and (26):

$$\begin{aligned} &[p\, x_{obs}(t)(t-T_C)]_C \\ &\quad = [p\, x_{obs}(t)(t-T_C)]_1 + [p\, x_{obs}(t)(t-T_C)]_2 \\ &\quad = [p\, x_{obs}(t)(t-T_1)]_1 + [p\, x_{obs}(t)(t-T_2)]_2 \\ &\qquad\quad + (T_1 - T_C)\, [p\, x_{obs}(t)]_1 + (T_2 - T_C)\, [p\, x_{obs}(t)]_2 \\ &\quad = \varepsilon_0^2 \Big(w_{\mu,1}\, \mu_1 + w_{\mu,2}\, \mu_2 + (T_1 - T_C)\, w_{x,1}\, x_1(T_1) \\ &\qquad\quad + (T_2 - T_C)\, w_{x,2}\, x_2(T_2) \Big) \\ &\quad = \varepsilon_0^2 \Big(w_{\mu,1}\, \mu_1 + w_{\mu,2}\, \mu_2 \\ &\qquad\quad + \frac{w_{x,2}}{w_{x,1} + w_{x,2}}\, (T_1 - T_2)\, w_{x,1}\, x_1(T_1) \\ &\qquad\quad + \frac{w_{x,1}}{w_{x,1} + w_{x,2}}\, (T_2 - T_1)\, w_{x,2}\, x_2(T_2) \Big) \\ &\quad = \varepsilon_0^2 \Big(w_{\mu,1}\, \mu_1 + w_{\mu,2}\, \mu_2 \\ &\qquad\quad + \frac{w_{x,1}\, w_{x,2}\, (T_2 - T_1)^2}{w_{x,1} + w_{x,2}} \; \frac{x_2(T_2) - x_1(T_1)}{T_2 - T_1} \Big) \\ &\quad = \varepsilon_0^2 \Big(w_{\mu,1}\, \mu_1 + w_{\mu,2}\, \mu_2 + \frac{(T_2 - T_1)^2}{\varepsilon^2_{x,1} + \varepsilon^2_{x,2}}\, \mu_0 \Big) \\ &\quad = \varepsilon_0^2 \left(w_{\mu,1}\, \mu_1 + w_{\mu,2}\, \mu_2 + w_{\mu,0}\, \mu_0 \right) . \end{aligned} \tag{28}$$

Similar to Eq. (28) we obtain for the denominator of Eq. (27) after some algebra

$$\begin{aligned} &\left[p(t-T_C)^2\right]_C = \left[p(t-T_C)^2\right]_1 + \left[p(t-T_C)^2\right]_2 \\ &\quad = \left[p\Big((t-T_1) + (T_1 - T_C)\Big)^2\right]_1 \\ &\qquad\quad + \left[p\Big((t-T_2) + (T_2 - T_C)\Big)^2\right]_2 \\ &\quad = \left[p(t-T_1)^2\right]_1 + \left[p(t-T_2)^2\right]_2 \\ &\qquad\quad + (T_1 - T_C)^2 \; [p]_1 + (T_2 - T_C)^2 \; [p]_2 \\ &\quad = \varepsilon_0^2 \left(w_{\mu,1} + w_{\mu,2} + w_{\mu,0} \right) . \end{aligned} \tag{29}$$

Our final result for μ_C is therefore, using Eqs. (27) - (29),

$$\mu_C = \frac{w_{\mu,1}\, \mu_1 + w_{\mu,2}\, \mu_2 + w_{\mu,0}\, \mu_0}{w_{\mu,1} + w_{\mu,2} + w_{\mu,0}} \,. \tag{30}$$

Eq. (30) means that the combined proper motion μ_C is the weighted average of the three proper motions μ_1, μ_2, μ_0, where we have to use Eq. (26) for the weights of the proper motions.

The mean errors $\varepsilon_{x,C}$ and $\varepsilon_{\mu,C}$ of the combined position $x_C(T_C)$ and combined proper motion μ_C are derived from Eqs. (11), (12), and (29):

$$\varepsilon^2_{x,C} \;=\; \frac{\varepsilon_0^2}{[p]_C} = \frac{\varepsilon_0^2}{[p]_1 + [p_2]} = \frac{1}{\varepsilon^{-2}_{x,1} + \varepsilon^{-2}_{x,2}} \,, \tag{31}$$

$$\begin{aligned} \varepsilon^2_{\mu,C} \;&=\; \frac{\varepsilon_0^2}{[p(t-T_C)^2]_C} \\ &=\; \frac{1}{w_{\mu,1} + w_{\mu,2} + w_{\mu,0}} = \frac{1}{\varepsilon^{-2}_{\mu,1} + \varepsilon^{-2}_{\mu,2} + \varepsilon^{-2}_{\mu,0}} \,. \end{aligned} \tag{32}$$

The weights of $x_C(T_C)$ and μ_C are therefore just the sum of the weights of the two positions,

$$w_{x,C} = w_{x,1} + w_{x,2} \,, \tag{33}$$

and the sum of the weights of the three proper motions,

$$w_{\mu,C} = w_{\mu,1} + w_{\mu,2} + w_{\mu,0} \,. \tag{34}$$

Due to the separation of the normal Eqs. (21) and (22) by our choice of T_C, the values of $x_C(T_C)$ and μ_C are not correlated.

In summary, our analytic approach to the combination of the two basic catalogues provides a very simple rule: the resulting combined data are weighted averages of the data in the two catalogues, described by Eqs. (19), (23), and (30). For the proper motions we have to include the proper motion μ_0, based on the two positions, into this average. The mean errors of the combined position and proper motion are given by Eqs. (31) and (32). A schematic illustration of the method is shown in Fig. 1. While our method is pleasing because of its conceptional simplicity, it is also fully justified by the use of the normal equations. As mentioned earlier, the only approximation used in our analytic approach is that we have neglected the cross-correlations between the coordinate components of the positions and proper motions in α_* and δ introduced by HIPPARCOS.

As a preparation for Sect. 3, we remark (without giving the proof here) that our analytic result is identical to the solution of another least-square problem in which we treat the catalogue data $x_1(T_1),\ \mu_1,\ x_2(T_2),\ \mu_2$ as 'observations'. The new condition is

$$w_{x,1}\left(x_1(T_1)-x_C(T_1)\right)^2+w_{x,2}\left(x_2(T_2)-x_C(T_2)\right)^2 +w_{\mu,1}\left(\mu_1-\mu_C\right)^2+w_{\mu,2}\left(\mu_2-\mu_C\right)^2=min\,. \quad (35)$$

The weights w are given by Eqs. (20) and (26), and $x_C(T_i)$ follows from

$$x_C(T_i)=x_C(T_C)+\mu_C(T_i-T_C)\,. \quad (36)$$

Finally we would like to point out that our analytic method can be easily used for a 'decomposition' of a combined catalogue: If the catalogues C and 1 are available, then the catalogue 2 can be fully reconstructed. In subsequent papers we shall present results of this 'inverse combination' method by decomposing earlier fundamental catalogues. For example, we have decomposed the FK5 into two parts: the published FK4 and the 'summary of subsequent data' which we denote by FK5-4. The FK5-4 has also the character of a compilation catalogue. The decomposition of the FK5 into a few 'subcatalogues' has the following purpose: If we would like to test the motion of a fundamental star for linearity in time or to derive a non-linear motion for the star similar to the G solutions of HIPPARCOS, then it is helpful to have as many catalogues with different, well-spaced epochs as possible at hand.

3. Numerical approach

We now present our numerical approach to the problem of combining two catalogues. In this numerical approach, the correlations between the five astrometric parameters derived by HIPPARCOS in a standard solution for an apparently single star are fully taken into account.

We change our notation slightly. As a ground-based catalogue, we envisage the FK5, and use the index F (instead of 1). The HIPPARCOS data receive the index H (instead of 2). The index C for the combined catalogue remains. Furthermore we will use now the vector and matrix notation in most cases. **U** is the unit matrix with the elements $U_{kl}=\delta_{kl}\ (k,l=1,2,3,...)$; **0** is a submatrix filled with zeros. $\mathbf{A}^T$ denotes the transpose of the matrix $\mathbf{A}$, and $\mathbf{A}^{-1}$ its inverse, $\mathbf{A}\mathbf{A}^{-1}=\mathbf{U}$. Instead of the right ascension α and the corresponding proper motion μ_α, we

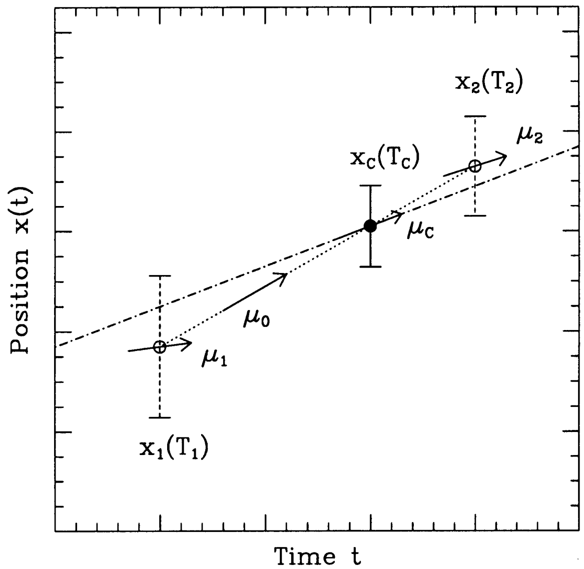

Fig. 1. Schematic illustration of the analytic approach to the combination of two catalogues. The basic data are the positions of a star, x_1 and x_2 at the two epochs T_1 and T_2, and the corresponding proper motions μ_1 and μ_2. The dotted line connects the two positions (open circles). The combined position x_C (filled circle) at the combined central epoch T_C is the weighted average of x_1 and x_2 and is located therefore on this connecting line. For the positions, error bars are shown. The dotted line provides also the direction of the additional proper motion μ_0, derived from the two positions. The combined proper motion μ_C is the weighted average of the three proper motions $\mu_1,\ \mu_2,\ \mu_0$. The dash-dotted line illustrates the combined solution $x_C(t)$ as a function of time t.

use $\alpha_*=\alpha\cos\delta$ and $\mu_{\alpha*}=\mu_\alpha\cos\delta$. The parallax of a star is denoted by p.

As already mentioned in Sect. 1, we use in real applications only quantities which are differential with respect to HIPPARCOS, e.g. $\alpha_{*,F}(t)-\alpha_{*,H}(t)$ instead of $\alpha_{*,F}(t)$. This means, of course, that most of the quantities with the index H are zero in our scheme. Here we keep, however, the full quantities in order not to confuse the reader.

The HIPPARCOS Catalogue provides the 5 astrometric parameters

$$\mathbf{b}_H\,(T_H)=\begin{pmatrix}\alpha_{*,H}(T_H)\\ \delta_H(T_H)\\ \mu_{\alpha*,H}\\ \mu_{\delta,H}\\ p_H\end{pmatrix} \quad (37)$$

for the overall epoch $T_H=1991.25$. Furthermore, the HIPPARCOS Catalogue gives all the data required to derive the corresponding variance-covariance matrix $\mathbf{D}_H(T_H)$. The diagonal line of $\mathbf{D}_H$ is occupied by the squares of the mean errors of the components of $\mathbf{b}_H(T_H)$, e.g. by

$$D_{H,11}=\varepsilon^2_{\alpha*,H}\,(T_H)\,. \quad (38)$$

The non-diagonal elements of $\mathbf{D}_H$ can be derived from the correlation coefficients $\rho_{kl,H}$ $(k,l = 1, ..., 5)$ and the mean errors, e.g. by

$$D_{H,12} = D_{H,21} = \varepsilon_{\alpha*,H}(T_H)\,\varepsilon_{\delta,H}(T_H)\,\rho_{\alpha\delta,H}(T_H)\,. \quad (39)$$

The FK5 provides only 4 astrometric parameters,

$$\mathbf{b}_F = \begin{pmatrix} \alpha_{*,F}(T_{\alpha,F}) \\ \delta_F(T_{\delta,F}) \\ \mu_{\alpha*,F} \\ \mu_{\delta,F} \end{pmatrix}, \quad (40)$$

since no parallax is given. In contrast to HIPPARCOS, the positions are given at the central epochs, $T_{\alpha,F}$ and $T_{\delta,F}$. Hence the four FK5 astrometric parameters are not correlated. This means that all the non-diagonal elements of the variance-covariance matrix $\mathbf{D}_F$ are zero. The diagonal elements of $\mathbf{D}_F$ are given by the squares of the mean errors, e.g. by

$$D_{F,11} = \varepsilon^2_{\alpha*,F}\,(T_{\alpha,F})\,. \quad (41)$$

The total vector **b** of 'observations' (9 elements) is given by

$$\mathbf{b} = \begin{pmatrix} \mathbf{b}_H \\ \mathbf{b}_F \end{pmatrix}, \quad (42)$$

and the total matrix $\mathbf{D}$ $(9 \times 9$ elements) by

$$\mathbf{D} = \begin{pmatrix} \mathbf{D}_H & \mathbf{O} \\ \mathbf{O} & \mathbf{D}_F \end{pmatrix}, \quad (43)$$

since the HIPPARCOS and FK5 data have no cross-correlations. The total matrix $\mathbf{P}$ of the weights is the inverse of $\mathbf{D}$:

$$\mathbf{P} = \mathbf{D}^{-1}\,. \quad (44)$$

As the unknowns to be determined, we use the two positions at an arbitrary reference epoch, T_{ref}, the two proper motions, and the parallax:

$$\mathbf{c} = \begin{pmatrix} \alpha_{*,C}(T_{ref}) \\ \delta_C(T_{ref}) \\ \mu_{\alpha*,C} \\ \mu_{\delta,C} \\ p_C \end{pmatrix}. \quad (45)$$

We now write down the equations of conditions for each 'observation', similar to Eq. (4) of our analytic approach. In matrix form the equations of conditions are

$$\mathbf{b} = \mathbf{A}\,\mathbf{c} + \mathbf{v}\,. \quad (46)$$

The design matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & T_H - T_{ref} & 0 & 0 \\ 0 & 1 & 0 & T_H - T_{ref} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & T_{\alpha,F} - T_{ref} & 0 & 0 \\ 0 & 1 & 0 & T_{\delta,F} - T_{ref} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}. \quad (47)$$

The vector **v** contains the residuals of the 9 observational parameters. The least-square solution of Eq. (46) requires that the sum of the weighted squares of the residuals is minimized:

$$\mathbf{v}^T\,\mathbf{P}\,\mathbf{v} = min\,. \quad (48)$$

From this condition the normal equations follow:

$$\mathbf{A}^T\,\mathbf{P}\,\mathbf{A}\,\mathbf{c} = \mathbf{A}^T\,\mathbf{P}\,\mathbf{b}\,. \quad (49)$$

The solution **c** of this equation gives the five astrometric parameters of the combined catalogue C for the epoch T_{ref}. The mean errors and the correlation coefficients of the elements of **c** are derived from the variance-covariance matrix of **c** which is given by

$$\mathbf{D}_C = (\mathbf{A}^T\,\mathbf{P}\,\mathbf{A})^{-1}\,. \quad (50)$$

For example,

$$\varepsilon^2_{\alpha*,C}(T_{ref}) = D_{C,11}\,, \quad (51)$$

or

$$\rho_{\alpha\delta,C}(T_{ref}) = D_{C,12}\,/\,(D_{C,11}\,D_{C,22})^{1/2}\,. \quad (52)$$

The positions, their mean errors and their correlation coefficients can, of course, be calculated for other epochs than T_{ref}, say for an epoch T, by using

$$\alpha_{*,C}(T) = \alpha_{*,C}(T_{ref}) + \mu_{\alpha*,C}(T - T_{ref}) \quad (53)$$

and a corresponding equation for $\delta_C(T)$. A specially interesting epoch is $T_{\alpha,C,min}$ at which the mean error $\varepsilon_{\alpha*,C}$ of $\alpha_{*,C}$ reaches its minimum. This epoch corresponds to T_C for α_* in our analytic approach. In general, $T_{\alpha,C,min}$ and $T_{\delta,C,min}$ are not the same epochs.

The numerical approach is well adapted for many variations. Firstly, we can easily add further compilation catalogues or individual observational catalogues. Secondly, we can remove unknowns or add further unknowns. For example, the 'long-term prediction mode' of the FK6 requires that we drop the parallax p as an unknown. This can be done either explicitly in our equations or by giving p_H a weight zero, corresponding formally to $\varepsilon_{p,H} \to \infty$. Additional unknowns, e.g. acceleration terms similar to the G solutions of HIPPARCOS, are required if we would like to determine non-linear solutions for some astrometric binaries. The use of the 'decomposed' FK5 mentioned at the end of Sect. 2, demands an increase in both the number of catalogues and the number of unknowns.

4. The effect of cosmic errors

The combination methods presented in Sects. 2 and 3 are valid for single stars only, which move linearly in time on straight lines in space. We call this version of our combination method the 'single-star mode'. Some binaries can be handled without any significant change in this mode of our combination method if we use appropriate 'reference points' for the binary, e.g. the center-of-mass of the double star. The specific treatment of known binaries will be presented elsewhere.

Table 1. Results of the combination method for the star HIP 9884 = FK 74 = GC 2538 = α Ari

	FK5 + HIPPARCOS				GC + HIPPARCOS			
Quantity	α_*	mean error	δ	mean error	α_*	mean error	δ	mean error
Input data:								
$\Delta x_F(T_F)$ or $\Delta x_{GC}(T_{GC})$	– 7.83	± 12.51	+ 96.94	± 14.78	– 75.60	± 39.62	+ 237.68	± 27.01
$\Delta\mu_F$ or $\Delta\mu_{GC}$	+ 0.49	± 0.40	– 1.20	± 0.38	+ 1.70	± 0.99	– 2.83	± 0.95
T_F or T_{GC}	1947.84		1929.73		1892.60		1890.30	
$\Delta x_H(T_{H,ind})$	0.00	± 0.77	0.00	± 0.54	0.00	± 0.77	0.00	± 0.54
$\Delta\mu_H$	0.00	± 1.01	0.00	± 0.77	0.00	± 1.01	0.00	± 0.77
$T_{H,ind}$	1991.26		1991.51		1991.26		1991.51	
p_H	49.48	± 0.99			49.48	± 0.99		
Results from analytic approach:								
$\Delta\mu_0$	+ 0.18	± 0.29	– 1.57	± 0.25	+ 0.77	± 0.40	– 2.35	± 0.27
$\Delta x_C(T_C)$	– 0.03	± 0.77	+ 0.12	± 0.54	– 0.03	± 0.77	+ 0.09	± 0.54
$\Delta\mu_C$	+ 0.27	± 0.23	– 1.36	± 0.20	+ 0.79	± 0.35	– 2.14	± 0.24
T_C	1991.10		1991.44		1991.22		1991.47	
Results from numerical approach:								
$\Delta x_C(T_C)$	– 0.18	± 0.76	+ 0.14	± 0.54	– 0.27	± 0.77	+ 0.15	± 0.54
$\Delta\mu_C$	+ 0.23	± 0.23	– 1.34	± 0.20	+ 0.66	± 0.35	– 2.10	± 0.24
T_C	1991.12		1991.47		1991.25		1991.50	
Δp_C	– 0.57	± 0.95			– 0.92	± 0.95		

Units: mas, mas/year, or years

As mentioned in Sect. 1, undetected binaries introduce cosmic errors into the positions and proper motions, especially into the 'instantaneous' HIPPARCOS data. In a subsequent paper we shall discuss the modifications of our combination method which are required if we take such cosmic errors into account. In many applications, it is only necessary to replace the actual measuring errors of some quantities by formal errors which include both the measuring and the cosmic errors of these quantities.

5. An example: α Ari

In order to illustrate our combination method we give in Table 1 the results for one individual star. All the positions and proper motions are given differentially to the HIPPARCOS results for the given epoch. Since this is only a numerical convenience, the mean errors of the quantities given in Table 1 are those of the full quantities themselves. The mean errors of the ground-based quantities, e.g. of x_F, μ_F, μ_0, are 'total' values which contain both the individual measuring errors and the uncertainty in the systematic corrections. The ground-based data given are already reduced to the HIPPARCOS system. Hence the differences in Table 1 contain only the 'individual' differences, not anymore a 'systematic' part. In order to obtain the final result of our combination method, one should add the results with the index C to the full HIPPARCOS data. e.g. $x_C(T_C) = \Delta x_C(T_C) + x_H(T_C)$ or $\mu_C = \Delta\mu_C + \mu_H$.

The comparison of the results of the analytic approach with those of the numerical approach shows a very good agreement. The numerical approach has made use of the correlation coefficients between the HIPPARCOS data for α_*(1991.25), δ(1991.25), $\mu_{\alpha*}$, μ_δ, p (+0.20, -0.01, +0.10, +0.26, +0.00, -0.35, -0.25, +0.33, +0.02, +0.29). The effect of the correlations is therefore small. This holds for most stars. The correlation coefficients for the combined data for α_*(1991.12), δ(1991.47), $\mu_{\alpha*}$, μ_δ, p (+0.26, 0.00, +0.04, +0.24, +0.04, 0.00, -0.16, +0.02, -0.01, +0.08) are smaller than for HIPPARCOS, because the ground-based data are not correlated at all if we use the concept of central epochs.

A comparison of the mean errors of the combined quantities with those of HIPPARCOS shows the following: (1) There is a significant gain in the accuracy of the proper motion which justifies the combination of HIPPARCOS results with ground-based data. In our example, the mean errors of the combined proper motions are smaller than those of the HIPPARCOS proper motion by a factor of 1/4.4 in $\mu_{\alpha*}$ and of 1/3.9 in μ_δ. (2) Since the mean measuring errors of x_H are extremely small, there is practically no gain in the accuracy of the central positions, and we have $T_C \sim T_H$. However, for predicting positions at epochs which differ from T_H by more than a few years, the accuracy of the proper motion is governing the error of these predicted positions. (3) The improvement of the HIPPARCOS parallax p_H is here and in most other cases not very significant, since the correlation of p_H with the other HIPPARCOS data (x_H, μ_H) is usually small.

The comparison of the combinations FK5+HIPP and GC+HIPP shows the importance of the old observations. The FK5 contains essentially all the observations compiled in the GC, and in addition many, more accurate, modern observations. Nevertheless, the mean errors of the combined data for the GC+HIPP are only slightly larger than for FK5+HIPP. The analytic approach shows the reason: The accuracy of the proper motion μ_0 is already very high for the GC+HIPP, mainly because of the large epoch difference of the central positions.

6. Error budget

In order to show the overall improvement in the accuracy of the proper motions obtained by using our combination method, we present here the statistics of the appropriate mean errors for a few samples of stars.

The mean errors ε_μ of the proper motions μ, given in Tables 2 and 3, refer to one 'mean' coordinate component of μ. For each star, the 'mean' value $\varepsilon_{\mu,1\mathrm{D}}$ is obtained as a root-mean-square (rms) average over $\varepsilon_{\mu,\alpha*}$ and $\varepsilon_{\mu,\delta}$. Then these individual values of $\varepsilon_{\mu,1\mathrm{D}}$ are averaged over the sample of stars under consideration, either by taking an rms average over all the $\varepsilon_{\mu,1\mathrm{D}}$ or by selecting the median value of $\varepsilon_{\mu,1\mathrm{D}}$ in the sample.

In Table 2, we present the error budget for the combination of the FK5 with HIPPARCOS for the basic FK5 stars. This corresponds to the results of the 'single-star mode' of the FK6 (Wielen et al. 1998). We consider two samples of stars: (a) all the 1535 basic FK5 stars, and (b) 1202 'apparently single' basic FK5 stars. The error budget of all the basic FK5 stars is slightly fictious in so far as this sample also contains binaries which cannot be safely treated by a direct combination method. Nevertheless, the error budget for the 1535 stars is still a valid indicator for the overall accuracy of our combination method. The error budget for the 1202 stars is slightly biased in favour of the HIPPARCOS Catalogue, since most of the stars for which especially HIPPARCOS is less accurate than usually (e.g., C solutions for visual binaries) are removed from this sample. Table 2 shows that the FK6 proper motions, obtained from the combination of the FK5 data with the HIPPARCOS observations, are typically by a factor of about two more accurate than the proper motions in either the FK5 alone or in HIPPARCOS Catalogue alone. Already μ_0 is typically slightly more accurate than μ_H.

In Table 3, the error budget for the combination GC+HIP of the GC data with the HIPPARCOS observations are shown. The samples of stars are the following: (a) all the 29 717 GC stars which we have identified in the HIPPARCOS Catalogue; (b) 11 773 stars in the GC which have standard solutions by HIPPARCOS (i.e., they do not occur in the DMS Annex of the HIPPARCOS Catalogue, which would indicate a double or multiple object) and which are not known to be double from ground-based observations and which have accurate values of $\mu_{0(GC)}$ (mean errors smaller than 2.0 mas/year in each coordinate); (c) all the 1534 basic FK5 stars in the GC (one basic FK5 star is not in the GC); (d) 1201 'apparently single' basic FK5 stars in the GC. Table 3 shows that for the 11 773 apparently single GC stars the combination GC+HIP produces proper motions which are more accurate than the HIPPARCOS proper motions by about 20 %. The overwhelming part of this improvement stems from the proper motion $\mu_{0(GC)}$. For the brighter GC stars, represented by the FK stars in Table 3, the gain in accuracy is much larger, typically by a factor 1.5 to 2 . A comparison of the results for GC+HIP given in Table 3 with those for FK6 = FK5+HIP in Table 2 for the basic FK5 stars confirms that the older ground-based observations carry a very large weight in the combination with the HIPPARCOS data: For the basic FK5 stars, the mean errors of the proper motions of the combination GC+HIP are only slightly larger (by about 20 %) than those of the FK6 = FK5+HIP.

Table 2. Error budget for FK6 proper motions in the 'single-star mode'

Typical mean errors of proper motions (in one component, averaged over $\mu_{\alpha*}$ and μ_δ; units: mas/year)				
Sample of stars:	1535 FK		1202 FK	
	rms aver.	median	rms aver.	median
HIPPARCOS	0.82	0.63	0.68	0.61
FK5				
random	0.76	0.64	0.77	0.67
system	0.28	0.25	0.28	0.25
total	0.81	0.70	0.83	0.72
μ_0				
random	0.53	0.43	0.54	0.45
system	0.24	0.23	0.25	0.23
total	0.58	0.49	0.59	0.51
FK6 = FK5+HIP	0.35	0.33	0.35	0.34
ratio of HIPPARCOS to FK6 errors	2.3	1.9	1.9	1.8

The error budgets for the stars in the bright and faint extension of the FK5 and for the remaining FK4Sup stars are not given here. For these stars the gain in accuracy of their proper motions by using our combination method lies inbetween the gain for the basic FK5 stars and that for the GC stars.

We would like to warn those readers who would like to test our combination method with the help of the numbers given in Tables 2 and 3, using the equations given in Sect. 2. The quantities given in Tables 2 and 3 are ensemble averages of the mean errors ε_μ, while our combination method works individually star by star. Since $< 1/\varepsilon_\mu^2 >$ differs in general from $1/ < \varepsilon_\mu^2 >$, such a test is not meaningful. A numerical check of our method can, of course, be carried out by using the numbers given in Table 1 for the individual star α Ari.

It should be emphasized that the gain in accuracy is significantly larger in the 'long-term prediction mode' (Wielen et al. 1998) in which we take into account the 'cosmic errors' in the HIPPARCOS results (see Sect. 4). The error budget for the long-term predictions will be discussed in detail in a subsequent paper.

7. Conclusions and outlook

We have shown that the combination of ground-based compilation catalogues with the HIPPARCOS data produces for many

Table 3. Error budget for GC+HIP proper motions in the 'single-star mode'

Typical mean errors of proper motions (in one component, averaged over $\mu_{\alpha*}$ and μ_δ; units: mas/year)								
Sample of stars:	29 717 GC		11 773 GC		1534 FK from GC		1201 FK from GC	
Median of m_V:	$7^m_.0$		$6^m_.8$		$4^m_.8$		$4^m_.9$	
	rms average	median	rms average	median	rms average	median	rms average	median
HIPPARCOS	1.47	0.73	0.75	0.69	0.82	0.63	0.68	0.61
GC								
random	10.57	9.38	8.59	7.55	3.33	2.31	3.44	2.45
system	0.38	0.34	0.38	0.33	0.47	0.40	0.47	0.40
total	10.58	9.39	8.60	7.56	3.37	2.38	3.47	2.48
μ_0								
random	1.78	1.75	1.43	1.45	0.66	0.55	0.67	0.57
system	0.15	0.11	0.12	0.11	0.19	0.14	0.18	0.14
total	1.78	1.75	1.43	1.45	0.66	0.55	0.67	0.57
GC+HIP	0.72	0.63	0.62	0.58	0.42	0.39	0.41	0.39
ratio of HIPPARCOS to GC+HIP errors	2.0	1.16	1.21	1.19	2.0	1.6	1.7	1.6

stars proper motions which are significantly more accurate than the HIPPARCOS proper motions themselves. For stars which are bright ($\sim 5^m$), well-observed and truly single, the gain in accuracy is about a factor of two. For stars of intermediate magnitudes ($\sim 7^m$), the smaller gain of about 20 % on average is still valuable for many applications.

The combination method described above has been used intensively and successfully in the construction of the FK6, in which the basic FK5 is combined with HIPPARCOS. The method described here provides the FK6 results in the 'single-star mode' (see Wielen et al. 1998). The first part of the FK6 will be published in the near future. The second part of the FK6 deals with double stars and requires a slightly modified version of our combination method.

In order to take into account the cosmic errors in the HIPPARCOS results, which are caused by undetected astrometric binaries, our combination method has to be changed slightly. The corresponding procedures, which lead to the 'long-term' and 'short-term' prediction modes (Wielen et al. 1998, 1999), will be presented in a subsequent paper.

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