Astron. Astrophys. 348, 249–250 (1999)

ASTRONOMY AND ASTROPHYSICS

Research Note

A new approximation of electron-cyclotron-maser frequencies

A. Stupp

School of Physics and Astronomy, Tel-Aviv University, Israel (stupp@post.tau.ac.il)

Received 4 November 1998 / Accepted 11 February 1999

Abstract. We present a new approximation for the frequencies where electron-cyclotron-maser is produced by a loss-cone distribution. This approximation is fully relativistic, and is good also for masering frequencies far from the cyclotron harmonics.

Key words: methods: analytical – methods: numerical – Sun: activity – Sun: flares – Sun: radio radiation

1. Introduction

The observation of millisecond microwave spikes from the Sun has been interpreted as gyrosynchrotron maser emission since the late 1970‘s (Wu & Lee 1979; Holman et al. 1980). Melrose and Dulk (1982) have used the semi-relativistic approximation to derive approximate formulae for the frequency of the largest growth rate, and for the growth rate. Aschwanden (1990a,1990b) followed the evolution of the ECM numerically, and gives frequencies and angles of largest growth. All previous studies concentrated on the *growth rate* of the amplified wave, while we use the *absorption coefficient* approach of Ramaty (1969), and calculate the frequencies at which the absorption coefficient is negative.

2. Semi-analytical formulae

An electron in a magnetic field can emit radiation only by satisfying the resonance condition:

$$\frac{s\omega_{\rm B}}{\gamma} = \omega(1 - n_{\pm}\beta\cos(\theta)\cos(\phi)) \quad (1)$$

where $\omega_{\rm B} = {\rm e}B/{\rm m_e}/{\rm c}$ is the cyclotron frequency, ω is the emission frequency, s is an integer, θ is the angle of the wave vector **k** to the magnetic field, ϕ is the angle between the velocity of the electron and the magnetic field (the pitch angle), $n_{\pm}$ is the refraction index, and γ and β are the usual Lorentz factor and velocity.

The important factor in Eq. 1 is $n_{\pm}$ the refraction index for the Ordinary Mode (OM or ‘+’), and the eXtra-Ordinary mode (XO or ‘-’). The refraction index is a function of the ambient density, of the cyclotron frequency, of the emission frequency, and of the angle to the magnetic field, and in general is quite complicated.

The resonance condition defines an ellipse on the electron velocity plane $(v_{||}, v_{\perp})$ for a given s,θ, and frequency of emission. In the weakly relativistic case $1/\gamma = 1 - \beta^2/2$ this ellipse becomes a circle (Melrose & Dulk 1982), and Melrose and Dulk use this to derive the frequency of largest growth rate. The frequency and angle for which the largest (in absolute value) negative absorption is expected are those for which the resonance circle is tangent to the outer edge of the loss-cone at α the loss-cone opening angle (Melrose & Dulk 1982). This definition results in an equation for the frequency where the largest absorption is expected which is:

$$\frac{\nu}{\nu_{\rm B}} = s\left(1 + \frac{n_{\pm}^2\cos^2(\theta)\cos^2(\alpha)}{2}\right) \quad (2)$$

For the fully relativistic case it is more convenient to consider the plane $(\gamma, \cos(\phi))$, and the resonance condition in the form:

$$\cos(\phi_s) = \frac{1 - \frac{s\nu_{\rm B}}{\gamma\nu}}{n_{\pm}\cos(\theta)\beta} \quad (3)$$

This equation has two possible forms of a curve. One for $s\nu_{\rm B} > \nu$, where as γ approaches 1 from above, the $\cos(\phi_s)$ solution goes to minus infinity. The second form is for $s\nu_{\rm B} < \nu$, where the $\cos(\phi_s)$ solution goes to positive infinity as the kinetic energy approaches zero. Both curves go asymptotically to $[n_{\pm}\cos(\theta)]^{-1}$ as γ goes to infinity.

For the fully relativistic case the equivalent curve to the special resonance circle is a curve of the second form which is tangent to the line $\cos(\phi) = \cos(\alpha)$.

The equation which results for the fully relativistic case, giving the frequency at which ECM occurs is:

$$\frac{\nu}{\nu_{\rm B}} = \frac{s}{\sqrt{1 - n_{\pm}^2\cos^2(\theta)\cos^2(\alpha)}} \quad (4)$$

This new approximation is identical with the Melrose and Dulk approximation (Eq. 2) for the case $n_{\pm}^2\cos^2(\theta)\cos^2(\alpha) \ll 1$. It is, however, more general and is a good approximation for the frequencies of negative absorption, even when $n_{\pm}^2\cos^2(\theta)\cos^2(\alpha)$ is large.

It is important to note that even though Eq. 4 is a better approximation, it still assumes a single harmonic s while the absorption coefficient includes a sum over some harmonics (Ramaty 1969). As a result, the frequency with largest negative absorption is usually not exactly the solution of Eq. 4, but a frequency close to it.

Our equation includes the refraction index $n_{\pm}$ which is a function of the frequency ν, and also of the ratio between the plasma frequency $\nu_{\rm p}$ and the cyclotron frequency $\nu_{\rm B}$. Our approximation is therefore only semi-analytical, and some numerical computation must be made. The computation, however, is very simple and can be performed very quickly.

Some general properties of the solution can be deduced, and in summary they are for the OM and XO mode:

- For the OM the frequency should be derived from a harmonic s which has $s > \nu_{\rm p}/\nu_{\rm B}$.
- For the XO mode the frequency should be derived from a harmonic s which has $s^2 - s > (\nu_{\rm p}/\nu_{\rm B})^2$.
- For both modes the frequency is smaller than a limiting frequency

$$\nu_{\max} = \frac{s\nu_{\rm B}}{\sqrt{1-\cos^2(\theta)\cos^2(\alpha)}} \quad (5)$$

The harmonic number s of a frequency can be derived from the condition 5 and should be the smallest integer which fits the inequality

$$s \geq \frac{\nu}{\nu_{\rm B}}\sqrt{1-\cos^2(\theta)\cos^2(\alpha)} \quad (6)$$

Since it is reasonable to assume that the smallest harmonic has the largest contribution, it is sufficient to consider it alone for the purposes of the approximation.

There is another magneto-ionic mode, the Z-mode. The Z-mode is the lower branch of the XO mode, for frequencies smaller than $\nu_{\rm z}$

$$2\nu_{\rm z}^2 = \nu_{\rm B}^2 + \nu_{\rm p}^2 + \sqrt{(\nu_{\rm B}^2+\nu_{\rm p}^2)^2 - 4\nu_{\rm B}^2\nu_{\rm p}^2\cos^2(\theta)} \quad (7)$$

Emission in the Z-mode can not emerge from the plasma, and therefore can not be observed. Since our purpose is to derive estimates for the observable frequencies, approximate solutions for the Z-mode are not of great interest. However, the Z-mode may be the dominant mode and quench the maser before the other modes are amplified. It is therefore of interest to determine where possible negative frequencies can appear.

The condition for negative absorption in the Z-mode is either $\nu_{\rm p} > s\nu_{\rm B}$, or $\cos(\theta) \ll 1$ and $\nu_{\rm p}/\nu_{\rm B}$ not too small. Empirically “not too small” translates as $\nu_{\rm p}/\nu_{\rm B} \geq 0.25$.

We compared the Dulk-Melrose approximation, our new approximation, and a numerical calculation of the absorption coefficients from which the frequencies where the absorption is negative was extracted. The results are that for small $\cos(\theta)$ the Dulk-Melrose approximation(Eq. 2) is very similar to our new approximation. However, for larger $\cos(\theta)$, the new approximation is much better. The new approximation is identical with the result of the full numerical computation for most of the $\cos(\theta)$ range where there is negative absorption.

3. Discussion

We developed a new approximation for the frequencies at which the absorption coefficient is negative, and therefore the electron-cyclotron-maser mechanism operates. This new approximation is given by Eq. 4, and is easy to compute. Our approximation gives results which are much more accurate than previous approximations, and are practically the same as the results of a full numerical calculation. The frequencies derived with the approximation are within 0.01–0.02 $\nu_{\rm B}$ of the numerically calculated frequencies.

The parameters entering the approximation are the angle of emission to the magnetic field θ, the loss-cone opening angle α, the ratio $\nu_{\rm p}/\nu_{\rm B}$, and the ratio $\nu/\nu_{\rm B}$.

The new approximation can be used to define a range of possible frequencies of millisecond spike emission, given the ratio $\nu_{\rm p}/\nu_{\rm B}$. Or, when spike emission is detected, the approximation can be used to give the range of physical parameters in the emission region.

Acknowledgements. I would like to thank Dr. R. Ramaty for his time and comments.

References

Aschwanden M.J, 1990a, A&AS 85, 1141
Aschwanden M.J., 1990b, A&A 237, 512
Holman G.D., Eichler D., Kundu M.R., 1980, An Interpretation of Solar Flare Microwave Spikes as Gyrosynchrotron Masering. In: Kundu M.R., Gergely T.E. (eds.) 86th IAU Symposium, Radio Physics of the Sun. Reidel, Dordrecht, p. 457
Melrose D.B, Dulk G.A., 1982, ApJ 259, 844
Ramaty R., 1969, ApJ 158, 753
Wu C.S., Lee L.C, 1979, ApJ 230, 621