Astron. Astrophys. 348, L13–L16 (1999)

ASTRONOMY AND ASTROPHYSICS

LETTER

Letter to the Editor

A broadband theory of Langmuir-whistler events in the solar wind

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Received 7 June 1999 / Accepted 19 June 1999

Abstract. Recent interplanetary observations presented evidence of simultaneous excitation of high-frequency Langmuir waves and low-frequency electromagnetic whistler waves in the magnetic holes of the solar wind. In this letter, we formulate a broadband theory of the interplanetary Langmuir-whistler events. We show that whistler waves can grow exponentially as a result of nonlinear interactions of Langmuir waves and left-hand circularly polarized radio waves. In particular, the growth of whistler waves favors propagation parallel to the Langmuir waves.

Key words: plasmas – Sun: solar wind – turbulence

1. Introduction

Ulysses observation of high-frequency electrostatic Langmuir waves in close temporal correlation with low-frequency electromagnetic whistler waves suggests that nonlinear coupling between these two types of plasma waves may occur in the solar wind (Kellogg et al. 1992; Lin et al. 1995; Stone et al. 1995). These interplanetary Langmuir-whistler events were seen within the local depressions of solar wind magnetic field known as magnetic holes. A theoretical model was proposed by Chian & Abalde (1999) to describe the nonlinear interaction between Langmuir waves and whistler waves in the solar wind. In their model the pump is the Langmuir wave, whereas whistler waves and circularly polarized radio waves are produced through three-wave interactions. Their model was based on the fixed-phase formalism, which is applicable only to narrowband waves with the associated bandwidth less than the growth rate. Fixed-phase analysis of nonlinear Langmuir-whistler mode coupling has also been considered by Hasegawa (1974) and Novikov et al. (1976). In addition, parametric interactions of Langmuir waves and circularly polarized electromagnetic waves have been studied by Stenflo (1970), Larsson & Stenflo (1973), Shukla & Sharma (1982), Chian, Lopes & Alves (1994), Stenflo & Shukla (1995) and Chian et al. (1997). In the solar wind, nonlinear wave-wave interactions often occur in the broadband regime whereby the bandwidth of growing waves is greater than the growth rate. The aim of this paper is to perform a random-phase analysis of nonlinear coupling of Langmuir waves with whistler waves in order to extend the validity of the interplanetary Langmuir-whistler model of Chian and Abalde (1999) to the regime of broadband waves.

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2. Theory

We consider three-wave interactions among Langmuir wave (L), right- and left-hand circularly polarized radio waves (r, l), and electromagnetic whistler wave (W) in the weak turbulence approximation, in which the growth rate of any wave is much less than the reciprocal of the lowest frequency of the three waves. We discuss specifically the following two three-wave processes: $L \rightleftarrows l + W$, and $L + W \rightleftarrows r$, with the frequencies and wave numbers satisfying the conditions, $\omega = \omega_1 + \omega_2$, $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$ (l-waves); $\omega_1 = \omega + \omega_2$, $\mathbf{k}_1 = \mathbf{k} + \mathbf{k}_2$ (r-waves), where $(\mathbf{k}, \omega)$, $(\mathbf{k}_1, \omega_1)$, $(\mathbf{k}_2, \omega_2)$ are the wave vectors and frequencies of the L, r (or l) and W waves. Examples of three-wave interactions (forward-scattering) are shown in Fig. 1.

In the random-phase approximation, waves are regarded as a collection of quanta with occupation number represented by $N(\mathbf{k})$, and wave-wave interaction is then described by a set of kinetic equations for $N(\mathbf{k})$ for various waves involved [for a detailed discussion see Davidson (1972), Tsytovich (1972) and Melrose (1986a,b)]. To estimate the growth rate of whistler waves, it is convenient to consider the kinetic equations in the form similar to radiative transfer equations (Luo & Melrose 1995, 1997; Luo & Chian 1997),

$$\frac{dN_W(\mathbf{k}_2)}{dt} = -\Gamma_W^{(\pm)} N_W(\mathbf{k}_2) + S_W^{(\pm)}(\mathbf{k}_2), \quad (1)$$

where $\pm$ correspond respectively to r- and l-polarization, and

$$\Gamma_W^{(\pm)} = -\int \frac{d^3k}{(2\pi)^3}\, u_{LW}^{(\pm)} \left[\pm N_\pm(\mathbf{k}_1) \mp N_L(\mathbf{k})\right], \quad (2)$$

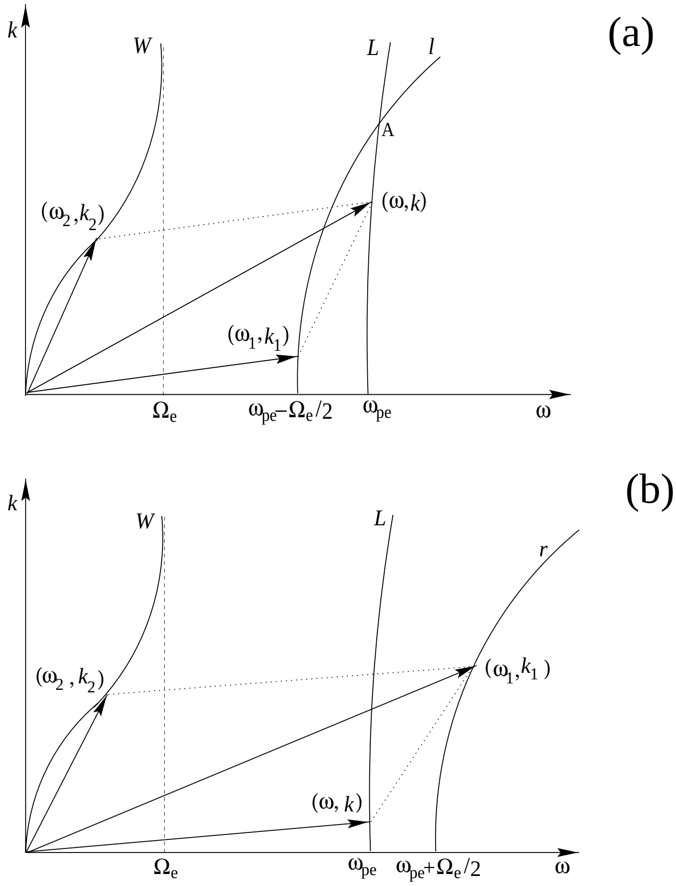


Fig. 1a and b. Three-wave conditions. **a**: $\omega = \omega_1 + \omega_2$, $k = k_1 + k_2$ with $\omega \approx \omega_1 \gg \omega_2 \approx \Omega_e/2$, $k \approx k_2 \gg k_1$. **b**: $\omega_1 = \omega + \omega_2$, $k_1 = k + k_2$ with $\omega \approx \omega_1 \gg \omega_2 \approx \Omega_e/2$, $k_1 \approx k_2 \gg k$.

is the nonlinear absorption coefficients, and

$$S_W^{(\pm)} = \int \frac{d^3k}{(2\pi)^3}\, u_{LW}^{(\pm)}\, N_L(\mathbf{k}) N_\pm(\mathbf{k}_1), \tag{3}$$

describes emission of whistler waves as a result of induced decay of L into l (or r into L), where $\mathbf{k}_1 = \mathbf{k} \pm \mathbf{k}_2$, $u_{LW}^{(\pm)}$ is the three-wave probability. The absorption coefficient (2) can be negative, corresponding to wave growth. So, $\Gamma_W^{(\pm)}$ is sometimes referred to as the damping (growth) rate.

We assume that Langmuir waves, which act as the pump, are produced mainly through a beam-plasma instability, which can be the case in the solar wind, and that $N_L > N_\pm$. Then, we have nonlinear growth of whistler waves, $\Gamma_W^{(-)} < 0$, as a result of the pump depletion through three-wave interaction $L \rightleftarrows l + W$. Similarly, we have nonlinear damping, $\Gamma_W^{(+)} > 0$, through $L + W \rightleftarrows r$. The two processes compete with each other.

The essential part of the calculation of $\Gamma_W^{(\pm)}$ is the derivation of the three-wave probability. Here we write down the probability for parallel propagation and forward-scattering ($k_{2z} > 0$),

$$u_{LW}^{(\pm)} = r_e c^2 R_W \left(\frac{\pi^2 \hbar \omega_2}{2 m_e c^2}\right) \frac{\omega_1 \omega_{pe}^2}{\omega} \left(\frac{n_\pm}{\omega_2 - \Omega_e} \mp \frac{n_W}{\omega_1 \mp \Omega_e}\right)^2 \times \delta(\omega - \omega_1 \pm \omega_2), \tag{4}$$

where ω_{pe} is the plasma frequency, Ω_e is the cyclotron frequency, R_W is the ratio of electric to magnetic energy denisty in the whistler waves, $r_e = e^2/4\pi\varepsilon_0 m_e c^2$ is the classical radius of electron, $n_\pm$ and n_W are the refraction indices. A similar form can be derived for back-scattering ($k_{2z} < 0$) by replacing $n_W \to -n_W$. We assume $N_L(k)$ is peaked at k_0 with a spread, $\Delta k < k_0$, where k_0 satisfies $\xi(k_0) \equiv \omega(k_0) - \omega_1(k_0) \pm \omega_2(k_0) = 0$. Because $\Gamma_W^{(\pm)} \propto 1/|\xi'|$, the growth rate must peak at $|\xi'| \approx 0$.

For $n_\pm \gg 3\beta_{\rm th}^2 n_L = 2\times 10^{-5} n_L (T_e/4\times 10^4\,{\rm K})$, we have $|\xi'| \approx c n_\pm$. Assuming $n_W^2/(\omega_1 \mp \Omega_e)^2 \gg n_\pm^2/(\omega_2 - \Omega_e)^2$ (when either $\omega_2 \ll \Omega_e$ or $n_\pm \ll 1$) and using (4), we obtain

$$\Gamma_W^{(\pm)} \approx \pm 2\pi^2 r_e c^2 \frac{W_L}{\Delta n_L n_\pm} \frac{\omega_2}{\omega^2} \left(1 - \frac{\omega_2}{\Omega_e}\right), \tag{5}$$

where $W_L \approx (\Delta k/2\pi)\hbar\omega\, N_L(k)/m_e c^2$ is the the energy density in Langmuir waves, $\Delta n_L = \Delta k c/\omega$, we neglect the difference between $u_{LW}^{(+)}$ and $u_{LW}^{(-)}$ which is of the order magnitude Ω_e/ω. In general, $\Gamma_W^{(+)} \neq -\Gamma_W^{(-)}$, i.e. in some parameter regions, one dominates the other.

One can calculate the growth rate for circularly polarized radio waves by writing the relevant kinetic equations into the form similar to Eq. (1), i.e. $N_W \to N_\pm$, $\Gamma_W^{(\pm)} \to \Gamma_\pm$ (the absorption coefficients for r-, l-waves), $S_W^{(\pm)} \to S_\pm$ (stimulated emission of r-waves as a result of induced fusion of L with W, and stimulated emission of l-waves due to induced decay of L into W). Similarly to whistler waves, l-waves can grow exponentially, provided that $N_L \gg N_W$. However, the absorption coefficient $\Gamma_+(\mathbf{k}_1)$ is always positive, and hence r-waves can only be produced through fusion of L with W.

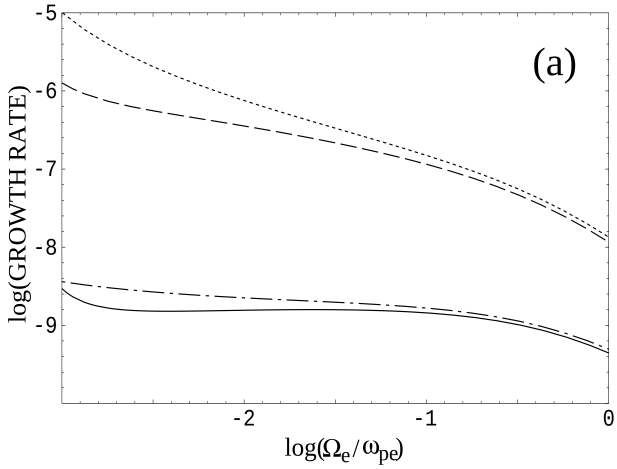
For $N_L \gg N_W$, $\Gamma_\pm$ can be written into the form similar to Eq. (5) but with $|\xi'| = |d\omega/dk \pm d\omega_2/dk|$. When ω_2, Ω_e are not too small, i.e., they satisfy the condition $\omega_2\Omega_e/\omega_{pe}^2 \gg 9\beta_{\rm th}^4 n_L^2$, we have $|\xi'| \approx c n_W [2\omega_2(\omega_2 - \Omega_e)^2/\omega_{pe}^2\Omega_e]$. Then, the damping (growth) rate is given by

$$\Gamma_\pm \approx \pm 2\pi^2 \frac{r_e c^2}{\omega} \frac{W_L}{\Delta n_L} \left(\frac{\omega_2}{\Omega_e}\right)^{1/2}, \tag{6}$$

where we assume $\omega_2 \leq \Omega_e/2$. When we choose $n_L = 1$, the three-wave condition cannot be satisfied for $\Omega_e \to \omega_2$. Note that the damping (growth) rate (6) is different from that for whistler waves (cf. Eq. 5), while fixed-phase analysis predicts the two should be the same (e.g., Chian & Abalde 1999). Unlike the fixed-phase formalism in which the three interacting waves are monochromatic and their frequencies satisfy the three-wave conditions, in the random-phase formalism the relevant waves are broadband and so the frequency matching condition can be satisfied at different part of the frequency spreads. Thus, the damping (growth) rates for the three interacting waves are in general different.

3. Application to solar wind

Simultaneous observations of Langmuir waves and electromagnetic whistler waves have been made by the Ulysses spacecraft



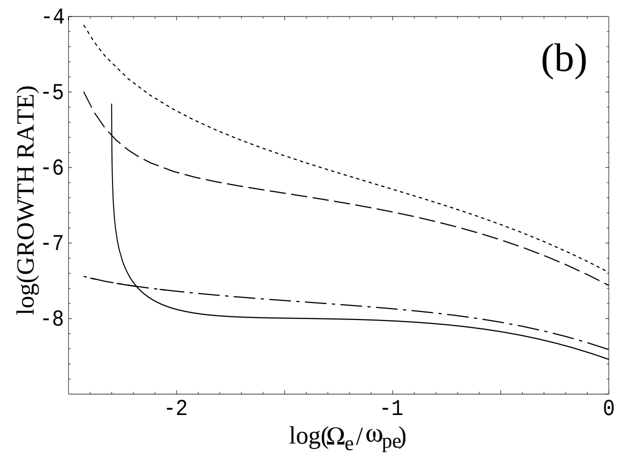


Fig. 2a and b. Growth rates, $\Gamma_W^{(-)}$ and Γ_-, as a function of Ω_e/ω_{pe}. **a**: $\omega_2/\omega_{pe} = 3.5 \times 10^{-4}$ and $n_L = 1$. **b**: $\omega_2/\omega_{pe} = 2.5 \times 10^{-3}$, $n_- = 5 \times 10^{-4}$ at $\Omega_e = 2\omega_2$. The solid, dot-dashed curves correspond respectively to the growth rate of forward-, back-propagating W-waves. The growth rates for the l-waves are plotted as dashed (forward-scattering) and dotted (back-scattering) curves.

in the solar wind (Kellogg et al. 1992; Lin et al. 1995; Stone et al. 1995). These interplanetary Langmuir-whistler events were detected in solar wind magnetic holes (Turner et al. 1977; Tsurutani et al. 1992; Winterhalter et al. 1994). In Sec. 2, we have formulated a three-wave theory for nonlinear coupling of Langmuir waves with whistler waves, valid for broadband plasma waves which are often seen in the solar wind. According to our theory, the nonlinear Langmuir-whistler interactions generate as a by-product circularly polarized radio waves near the solar wind plasma frequency.

From Sec. 2 we see that when L is the pump, both whistler and l-waves can grow exponentially at rates $\Gamma_W^{(-)}$ and Γ_-, respectively, through $L \rightleftarrows l + W$. On the other hand, in the process $L + W \rightleftarrows r$, r-waves are in general damped at rate Γ_+ and can grow only through stimulated emissions (S_+). Assuming $dN_+/dt = 0$, one may find the maximum level of r-waves $N_+ \sim N_L N_W/(N_L + N_W)$, which shows that high level of N_+ can be achieved only when N_W has reached a sufficiently high level.

Two different regimes of whistler growth in $L \rightleftarrows l + W$ can be identified, depending on the ratio ω_2/Ω_e Consider first the case $\omega_2/\Omega_e \ll 1$. In this frequency regime, the three-wave matching conditions can be satisfied only for the triplet (L, l, W), but not for the triplet (L, r, W). As shown in Fig. 1a, the dispersion curves of L and l meet at the point A, i.e., the wave frequencies of L and l can be arbitrarily close to each other; whereas the wave frequencies of r and L differ at least by $\Omega_e/2$ (cf Fig. 1b). From $k = k_1 \pm k_2$, where $+$, $-$ correspond respectively to the forward- and back-scattering, we have $n_L \approx n_- \pm n_W(\omega_2/\omega) \approx n_-$ as $n_W(\omega_2/\omega) \ll 1$. While Chian & Abalde (1999) discussed only the back-scattering case, we see both back- and forward-scattering involving l are allowed in the limit $\omega_2 \ll \Omega_e$. Assuming $n_L \approx n_- \sim 1$, $\Delta n_L = n_L/2$ and using the typical solar wind parameters used by Chian & Abalde (1999), $\omega_2/2\pi = 5.6\,\mathrm{Hz}$, $\Omega_e/2\pi = 82\,\mathrm{Hz}$, $\omega_{pe}/2\pi = 16\,\mathrm{kHz}$, $\omega \sim \omega_{pe}$, $E_0 = 1\,\mathrm{mV\,m^{-1}}$, which gives the energy density ($m_e c^2$ per unit volume) $W_L = 5.4 \times 10^{-5}\,\mathrm{m^{-3}}$, we have $\Gamma_W^{(-)} \approx -2 \times 10^{-9}\,\mathrm{rad\,s^{-1}}$. The growth rate is much less than the fixed-phase growth rate obtained by Chian & Abalde (1999). This is expected since in general the random-phase processes are less efficient than the fixed-phase processes. In principle, one can boost the growth rate by increasing W_L. This is possible, for example, if microstructures in the distribution of electrons (similar to type III events) appear in interplanetary mangetic holes, which may enhance Langmuir wave amplitude to levels much higher than those observed (Melrose & Goldman 1987; Chian & Alves 1988). From Eq. (6), a higher growth rate can be obtained for l-waves. With the same solar wind parameters given above, we have $\Gamma_- \approx -1.5 \times 10^{-6}\,\mathrm{rad\,s^{-1}}$. The relevant growth rates as a function of Ω_e/ω_{pe} for $\omega_2 \ll \Omega_e/2$ are shown in Fig. 2a. In this frequency regime, the growth rate for l-waves is significantly larger than that for whistler waves. In each case, back-scattering gives rise to higher growth rate than forward-scattering.

Consider next the frequency regime $\omega_2/\Omega_e \sim 1/2$ in which one of the two high-frequency waves has very small refraction index. In this case, the growth (damping) rate can increase significantly. For Fig. 1b, we have $n_L \ll 1$ (but $n_\pm$ cannot be small), which will not be considered further as we restrict our discussion only to the case of $n_L \sim 1$. For Fig. 1a, we have $n_+ \ll 1$ (while $n_L \sim 1$). Back-propagating whistler waves are damped at rate $\Gamma_W^{(+)}$ given by Eq. (5). For $\Delta n_L \sim n_L/2$, we estimate the damping rate $\Gamma_W^{(+)} \approx 2\pi^2 r_e c^2 (W_L/n_+)(\omega_2/\omega^2)$. However, for forward-propagating whistler waves, as shown in Fig. 1a, we can have $n_- \ll 1$ (with $n_L \sim 1$) for $\omega_2 \sim \Omega_e/2$. For smaller ω_2, the resonance shifts towards even smaller Ω_e. Forward-propagating waves grow exponentially at rate $\Gamma_W^{(-)} \approx -2\pi^2 r_e c^2 (W_L/n_-)(\omega_2/\omega^2)$ where $\omega_2 = \Omega_e/2$ is used, and from Eq. (6), the growth rate for l waves is $\Gamma_- \approx -2\pi^2 (r_e c^2/\omega)(W_L/\Delta n_L)$. Plots of $-\Gamma_W^{(-)}$, $-\Gamma_-$ as a function of Ω_e/ω_{pe} with a relatively higher ω_2 are shown in Fig. 2b. As we choose $n_L = 1$, the three-wave condition cannot be satisfied for $\Omega_e \to \omega_2$, thus the plot range of Γ_- and $\Gamma_W^{(-)}$ are restricted to $\Omega_e \geq 1.5\omega_2$. Fig. 2b shows that the growth rate for the W waves can be comparable to and exceed that for the l circularly polarized waves. In principle, we can have very large growth rate, $\Gamma_W^{(-)}$, by simply choosing smaller n_- such that $|\xi'| \to 0$. Thus, it is possible that whistler waves with a frequency close $\Omega_e/2$ grow rapidly to the level at which the weak turbulence approximation is no longer valid (cf. Davidson 1972). Therefore, our random-phase result suggests a possibility of generating whistler waves with frequency close to $\Omega_e/2$.

When both waves grow to a level comparable to the Langmuir wave, i.e. $N_- \sim N_W \sim N_L$, the growth stops. Let the energy densities of whistler and l-waves be W_W and W_-, respectively. The saturation levels for these two waves are estimated to be $W_W = (\omega_2/\omega)^2(n_W/n_{_L})W_L$, $W_- = (n_-/n_{_L})W_L$. Then, we have $W_W \approx (\omega_2/n_{_L}\omega)(\omega_2/\Omega_e)^{1/2}W_L$ and hence, $W_W/W_L \ll 1$. For $\omega_2 = \Omega_e/2$, $\omega/2\pi \approx \omega_{pe}/2\pi = 16\,\mathrm{kHz}$, $n_{_L} = 1$, we have $W_W/W_L = 4 \times 10^{-3}$ for $\omega_2/2\pi = 41\,\mathrm{Hz}$. The energy density of the l-waves can reach the level of the pump.

4. Conclusions

In the solar wind, whistler waves can grow exponentially as a result of three-wave interactions involving $L \rightleftarrows l + W$, provided that $N_L > N_-$. Whistler waves are damped exponentially through three-wave interaction involving $L + W \rightleftarrows r$. Growth of whistler waves favors forward-propagating with the frequency $\omega_2 \approx \Omega_e/2$. Thus, in the broadband approximation, for a given ambient mangetic field the W-waves produced most likely have a frequency near $\Omega_e/2$. The saturated energy density of whistler waves is at most a $\omega_2/n_{_L}\omega_{pe}$ fraction of the energy density of the Langmuir pump waves. In general, the saturated energy density of l-waves can reach the level comparable to the pump. l-mode radio waves can grow exponentially, while r-waves are usually damped. Exceptionally, however, r-waves can be produced through stimulated emissions, with the maximum value given by $N_+ \sim N_L N_W/(N_L + N_W)$, when W-waves achieve a sufficiently high level of energy density through exponential growth (via $L \rightleftarrows l + W$). It follows that unless all relevant waves reach saturation so that $N_- \sim N_+$, in general l waves are more likely observed than r-waves. The results of this paper show that in addition to Langmuir-ion acoustic turbulence (Chian & Alves 1988; Robinson, Cairns & Willes 1994; Abalde, Alves & Chian 1998) the solar wind seems to be rich in other types of nonlinear Langmuir phenomena such as Langmuir-whistler turbulence.

Acknowledgements. QL thanks ARC (Australia) for financial support through a fellowship. ACLC and FAB acknowledge the financial support from FAPESP (Brazil).

References

Abalde, J. R., Alves, M. V., & Chian, A. C.-L., 1998, A&A, 331, L21.
Chian, A. C.-L., & Alves, M. V., 1988, ApJ, 330, L77.
Chian, A.C.-L., Lopes, S. R., & Alves, M. V., 1994, A&A, 290, L13.
Chian, A. C.-L., Abalde, J. R., Alves, M. V., & Lopes, S. R., 1997, Solar Phys., 173, 199.
Chian, A. C.-L., & Abalde, J. R. 1999, Solar Phys. 184, 403.
Davidson, R. C., 1972, Methods in Nonlinear Plasma Theory (New York: Academic Press).
Hasegawa, A., 1974, Phys. Rev. Lett., 32, 817.
Kellogg, P. T., Goetz, K., Lin, N., et al. 1992, Geophys. Res. Lett., 19, 1299.
Larsson, J. & Stenflo, L., 1973, Beitr. Plasma-phys., 13, 169.
Lin, N., Kellogg, P. J., MacDowall, R. J., et al. 1995, Geophys. Res. Lett., 22, 3417.
Luo, Q., & Melrose, D. B., 1995, ApJ, 452, 346.
Luo, Q., & Melrose, D. B., 1997, J. Plasma Phys., 58, 345.
Luo, Q., & Chian, A. C.-L., 1997, MNRAS, 289, 52.
Melrose, D. B., 1986a, Instabilities in Space and Laboratory Plasmas (Cambridge: Cambridge University Press).
Melrose, D. B., 1986b, J. Plasma Phys., 36, 269.
Melrose, D. B., & Goldman, M. V., 1987, Solar Phys., 107, 329.
Novikov, A. A., Rabinovich, M. I., & Fainstein, S. M., 1976, Sov. Phys. Tech. Phys., 20, 829.
Robinson, P. A., Cairns, I. H., & Willes, A. J., 1994, ApJ, 422, 870.
Shukla, P. K. & Sharma, R. P., 1982, Phys. Rev., A25, 2816.
Stenflo, L., 1970, J. Plasma Phys., 4, 585.
Stenflo, L., & Shukla, P. K., 1995, J. Geophys. Res. 100, 17261.
Stone, R. G. et al., 1995, Science, 268, 1026.
Tsurutani, B. T., Southwood, J., Smith, E. J., & Balogh, A., 1992, Geophys. Res. Lett., 19, 1267.
Turner, J. M., Burlaga, L. F., Ness, N. F., & Lemaire, J. F., 1977, J. Geophys. Res., 82, 1921.
Tsytovich, V. V., 1972, An Introduction to the Theory of Plasma Turbulence (Oxford: Pergamon Press).
Winterhalter, D., Neugebauer, M., Goldstein, B. E., et al. 1994, J. Geophys. Res., 99, 23371.