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On outflowing viscous disc models for Be stars

John M. Porter

Astrophysics Research Institute, Liverpool John Moores University, Twelve Quays House, Egerton Wharf, Birkenhead L41 1LD, UK (jmp@astro.livjm.ac.uk)

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Abstract. It is assumed that Be star discs are driven by viscosity. Emission from disc models is calculated and is confronted with continuum observations. It is found that the outflowing viscous disc models can reproduce the observed IR continuum emission. However, to exist as *outflowing* discs, either the discs are significantly acted upon by the stellar radiation field and/or there is significant cooling with radius in the disc. The energy generated via viscous dissipation is calculated and shown to play only a minor rôle in the energy balance of the disc.

A scenario whereby a B star may change into a Be star (and vice versa) by generating (reaccreting) the disc is suggested.

Key words: stars: circumstellar matter – stars: emission-line, Be – stars: mass-loss – stars: rotation

1. Introduction

Be stars are fast rotating stars (Slettebak 1982) with rotation rates up to break-up, and a modal value of $\sim 70\%$ of break-up rotation (Porter 1996). The circumstellar matter around Be stars is thought to consist of two distinct regions: a diffuse polar stellar wind and a dense equatorial "disc" (Dachs 1987, Slettebak 1988). The nature of the polar wind seems to be well understood in the context of radiation-driven wind theory (Castor et al. 1975, Kudritzki et al. 1989). However, the mechanism generating the equatorial disc is more elusive: several theories have been proposed including wind compression (Bjorkman & Cassinelli 1993), outflowing viscous discs (Lee et al. 1991), and wind bistability (Lamers & Pauldrach 1991). Recently it has been found that stars with high rotation will generate a metallicity enhancement in their equatorial planes (Porter 1999), which through the metallicity dependence of the radiation driving parameters may change the nature of the wind there.

Wind compression has already been shown to be insufficient on theoretical (Owocki et al. 1996) and observational (Porter 1997) grounds. The major problem with wind bistability is that the disc appears to be rotating in a Keplerian fashion (e.g. Dachs et al. 1986, Hanuschik 1989, 1996), whereas any wind causing the equatorial enhancement is likely to conserve angular momentum, and should not be observed to be rotating *faster* than the star – Hanuschik (1996) has shown that the half-line widths of emission lines created in the disc are indeed larger than $v \sin i$ of the star.

The theory which seems to be most applicable to Be star discs is that of viscously driven outflow (Lee et al. 1991). Here angular momentum is added to the inner edge of the disc increasing its angular velocity to slightly super-Keplerian. The disc interacts with itself via viscosity and angular momentum is transported outwards. The angular momentum source has been suggested by Osaki (1986) to be due to non-radial pulsations dissipating in the atmosphere of the star. The outflowing viscous disc model has several circumstantial corroborations – the variation in the line asymmetry (the V/R variation) can be naturally generated in Keplerian discs (Papaloizou et al. 1992, Okazaki 1991). Excess IR emission generated by the disc is seen to disappear over a timescale of months (Hanuschik et al. 1993) which is similar to the viscous timescale in these discs – this is perhaps indicative of phases of outflow and inflow within the disc.

Although the outflowing viscous disc model appears to be the most likely, it has never been subjected to observational tests – can these discs generate the IR excess? What is the source of viscosity? How and why do they dissipate and regenerate? In this investigation, Be star discs are assumed to be outflowing viscous discs and an attempt is made to answer these questions.

In Sect. 2 aspects of the outflowing viscous disc model are considered along with their application to Be star discs. In Sect. 3 the energy balance within the disc is studied, and the theory is applied to a test case in Sect. 4. Discussions and conclusions are presented in Sect. 5 and Sect. 6.

2. Can outflowing viscous discs exist?

2.1. Viscous discs

The gas dynamics of viscous discs has been analysed and developed by several authors (Shakura & Sunyaev 1973, Pringle 1981, Frank et al. 1992). Here, some comments are made regarding the structure of such discs (following arguments of the references above). First, a disc interacting with itself via viscous stresses transports angular momentum outward. In order for viscous stresses to exist in a disc, there must be (i) a non-zero viscosity and (ii) some differential rotation in the disc. Conservation of angular momentum in a cylindrical co-ordinate system (R, ϕ, z), yields

$$R\frac{\partial}{\partial t}\left(\Sigma R^2\Omega\right)+\frac{\partial}{\partial R}\left(R\Sigma v_R R^2\Omega\right)=\frac{1}{2\pi}\frac{\partial G}{\partial R} \quad (1)$$

where Σ is the surface density of the disc, Ω is the angular velocity, and v_R is the radial "drift" velocity. The viscous torque is $G = 2\pi R\nu\Sigma R^2(\partial\Omega/\partial R)$ where ν is the viscosity. If a ring of matter is placed in a Keplerian orbit and allowed to interact with itself through viscous stresses then part of it will move to smaller radii having $v_R < 0$ and parts of it will move outward $v_R > 0$ (e.g. Pringle's 1981, Fig. 1). This is solely due to the redistribution of angular momentum in the gas. The outflowing (inflowing) parts have increased (decreased) angular momentum with respect to the initial state. Time steady ($\partial/\partial t = 0$) solutions which show no drift velocities $v_R = 0$ occur when G is a constant with radius, i.e. $\partial\Omega/\partial R \propto (\nu\Sigma R^3)^{-1}$, although these solutions have limited physical application in discs around stars. These static discs still transport angular momentum outwards, even though the gas in them is not moving radially.

For an accretion disc, matter is added at the outer part of the disc. The viscous stresses lower the angular momentum of a ring of gas which in response moves inward and increases its angular velocity until it it regains rotational support. This mechanism creates the accretion flow: the gas moves inward $v_R < 0$ throughout the disc. The disc's velocity field is typically rotationally dominated with rotational velocities very close to Keplerian, and a subsonic radial velocity.

Eq. 1 also permits solutions with positive radial drift velocity v_R everywhere i.e. outflowing discs. This sort of disc requires an angular momentum source at the inner boundary – the disc-star interface – as well as a gas source taken to be the atmosphere. The disc recieves angular momentum from the star which will itself spin down. It is exactly this sort of model which has been suggested by Lee et al. (1991) to apply to the discs of Be star and are investigated below. Porter (1998a) has already condisered the angular momentum evolution of Be stars for this sort of disc, and finds that the observational result that there is little or no angular momentum evolution of Be stars (Zorec & Briot 1997, Steele 1999) may be explained if the viscous stresses at the inner edge of the disc are small or that the disc is present intermittently in the lifetime of the star. A small couple applied from the star at the inner edge leads to slow outflow velocities of $v_R \lesssim 0.01$ km s^{-1}at the inner edge of the disc. Note that an outflowing disc cannot exist if there is no couple from the star at the inner edge.

Aside from Lee et al.'s (1991) initial suggestion and modelling of Be star discs, outflowing viscous discs have been discussed by Pringle (1991), Narita et al. (1994) and Okazaki (1997). Narita et al.'s numerical analysis showed that if the central object rotates fast enough, then the disc does indeed become an outflowing one. Let us assume then that Be star discs *are* outflowing viscous discs and subject them to some observational tests.

2.2. Angular momentum transport in a viscous disc

The existence of an outflowing disc is dependent on angular momentum being added at the inner edge of the disc whereas the dynamics of the disc is determined by the viscous stresses: if only this initial couple is present then a ring of gas will move away from the star until it finds its Keplerian orbit, at which point it will stop. The evolution of the disc away from the inner boundary is governed by the viscosity.

The rotational velocity of a outflowing viscous disc is $v_\phi \approx \sqrt{GM_*/R}$, where M_* is the mass of the star, and G is the gravitational constant. Assuming that the disc is steady (so $\partial/\partial t = 0$), the equation for transport of angular momentum becomes

$$\Sigma v_R R^{1/2} = -3\frac{\partial}{\partial R}\left(R^{1/2}\nu\Sigma\right) \quad (2)$$

where Σ is the surface density of the disc, v_R is the radial velocity, and ν is the viscosity.

The surface density is $\Sigma = \rho(R,0)H$, where $\rho(R,0)$ is the density in the equatorial plane at radius R, and $H = R\,c_s\,/\,v_\phi$ is the density scale height (c_s is the sound speed). The disc is isothermal in the z direction and assuming that the density in the equatorial plane $\rho(R,0)$ is a power law, the density field is

$$\rho(R,z)=\rho_0\left(\frac{R}{R_*}\right)^{-n}\exp\left(-\frac{z^2}{2H^2}\right) \quad (3)$$

where ρ_0 is the density at the inner boundary. Note that the numerical simulations of outflowing discs by Narita et al. (1994) find that steady state outflowing discs do indeed have power law surface densities from $R \gtrsim 2R_*$, justifying the power-law assumption in Eq. 3.

Following convention, an alpha prescription is used for the viscosity $\nu = \alpha c_s H$ (Shakura & Sunyaev, 1973). Finally, to ensure the disc is as general as possible, the temperature of the disc is assumed to follow a power law: $T_d = 0.8T_{\rm eff}(R/R_*)^{-m}$, where $T_{\rm eff}$ is the effective temperature of the star.

2.3. Inflow or outflow?

Let us assume that we have a disc which is viscously interacting with itself. Is it possible to tell whether the gas flow is inward, outward or zero? In asking this question, the nature of the addition of torque to the inner boundary of the disc for outflowing discs, or that of the angular momentum removal for inflowing discs has been ignored. An attempt is made to ascertain whether outflowing viscous discs are credible Be star disc candidates.

For $v_R > 0$ then the radial exponent of the term in brackets in Eq. 2 must be less than zero, ensuring the differential is negative and hence the right hand side is positive. Inserting Eq. 3 and the α viscosity into Eq. 2 and collecting powers of radius R together, then the right hand side is proportional to $-\partial/\partial R(R^{3.5-n-1.5m})$. Therefore the disc is an outflowing disc if $2n+3m > 7$, assuming that α is a constant (see later). It is worthy of note that when outflowing viscous discs have been modelled, this limit on n and m has always been obeyed (e.g. see Narita et al. 1994 and Okazaki 1997).

2.4. Confrontation with observations

There does appear to be a problem with the outflowing viscous disc paradigm straight away when confronting it with observations. Cóte & Waters (1987) found that the IR emission (originating in the inner 10s of R_* of the disc) is well fit with an isothermal disc ($m = 0$) having a density power law index of $2 < n < 3.5$. Values of n are larger when fitting the emission in the radio regime, corresponding to larger radii of the disc (Dougherty et al., 1991). According to the above work, the inner parts of the discs *cannot* be outflowing viscous discs, and must actually be accretion discs!

If the outflowing viscous disc model is applicable to Be star discs then this discrepancy *must* be examined. First, consider the emission fitting procedure: typically both a constant opening angle for the disc and an isothermal disc are used. The constant opening angle leads to $H \propto R$ as opposed to $H \propto R^{3/2}$ for viscous discs above. Inserting this slight modification into the angular momentum conservation expression, leads to the new limit $2n + 3m > 6$ for outflow.

However, comparing this with the derived values of n does not really change the conclusion that almost all discs should be accreting! We must be wary of taking the fits at face value – usually the disc was assumed *a priori* to be isothermal, and the parameter m was not allowed to vary. If a non-isothermal disc is used then the fits to the IR excess may change.

What can be gleaned from this apparent paradox that all Be star discs should be inflowing, given that there is a strong belief that they are in fact outflowing? It is unlikely that the fitting procedure produces values of n consistently too small in almost every cases (see Sect. 4). The conclusion, therefore, is that the outflowing viscous disc can not fit the observations, and cannot be applicable to Be stars.

This is too hasty: aspects of the problem have yet to be considered. The first is the radiation field from the star: the total electron-scattering optical depth in the equatorial plane is

$$\begin{aligned}\tau &= \int_{R_*}^{\infty} \sigma_e \rho dR = \frac{\sigma_e \rho_0 R_*}{n} \\ &= \frac{0.24}{n}\left(\frac{\rho_0}{10^{-11}\mathrm{g\ cm^{-3}}}\right)\left(\frac{R_*}{R_\odot}\right)\end{aligned} \quad (4)$$

where $\sigma_e = 0.35\,\mathrm{cm^2 g^{-1}}$ is the electron scattering cross-section. Inserting typical values of $n = 2.5$, $\rho_0 = 10^{-11}\mathrm{g\,cm^{-3}}$ and $R_* = 5R_\odot$ yields an optical depth of $\tau = 0.5$. Therefore the disc is optically thin to electron scattering, and hence the disc may be acted upon by optically thin lines, as suggested by Chen & Marlborough (1994) (note that the optical depth argument derived in Sect. 4.1 of Chen & Marlborough where they rule out radially subsonic discs is misleading due to their choice of wind parameters: viscous discs may have mass-loss rates of $10^{-11}\mathrm{M_\odot\,yr^{-1}}$ with small radial velocities and still be dense enough to produce the IR excess emission e.g. Okazaki 1997).

If regions of the disc are partially supported and driven outwards by radiation, then they will deviate from Keplerian rotation. This is because an insufficient amount of angular momentum is given to a ring of material as it moves outward. Hence the rotation velocity of the disc will lie between Keplerian ($v_\phi \propto r^{-1/2}$) and angular momentum conserving ($v_\phi \propto r^{-1}$). This difference may be very difficult to discriminate observationally, and so radiation-driven discs cannot be ruled out. However, it should also be noted that the inclusion of optically thin radiation driving does not necessarily produce non-Keplerian discs (e.g. see Okazaki 1997).

It is noted that the viscosity parameter α may be dependent on radius. The parameter α represents the difference between the *actual* value of (size×velocity) of the turbulent eddies and the characteristic product of scale and speed in the disc (disc height×sound speed). In fact if the turbulence becomes supersonic then it is possible that $\alpha > 1$. In the situation currently considered, a dense rotationally-dominated disc is adjacent to the fast wind which is dominated by the radial component of velocity. In the interaction region there are large shearing velocities and therefore the interaction region will become unstable to Kelvin-Helmholtz instabilities.

Consequently it might be expected that the typical turbulent velocity will be a function of the Mach number of the fast wind. In fact, if the eddy size is the typical scale height of the disc, then the maximum value that is possible if this is the case will be the fast wind Mach number – the wind typically reaches velocities of >1000 km s^{-1} after a few R_* , and with the typical temperature of the disc of 10 km s^{-1}, yield Mach numbers of $\mathcal{M} \sim 100$. Therefore, it is conceviable that may become larger than unity at quite a small radius, although $\alpha < \mathcal{M} \approx 100$.

As the fast wind's velocity increases with distance from the star, then α might be expected to increase with radius. This aspect of the viscosity is difficult to examine and a full study is left to a future paper. It is expected that if this Kelvin-Helmholtz process is significant in determining α, then it should increase with radius. This, however, makes the gas in the observed discs more likely to be inflowing rather than outflowing.

It is also noted that the disc may be outflowing if there is a sufficient radial temperature gradient in the disc (corresponding to large m). This requires the disc to cool significantly as it drifts away from the star. However, recent work by Millar & Marlborough (1998) casts doubt on a such a large temperature gradient being present.

2.5. Accretion discs?

The above paragraphs attempt to show that outflowing viscous discs are acceptable models for Be star discs. However, to make that discussion the primary one for the rest of this paper, we must answer the question "why are Be star discs not normal accretion discs?"

There are two prerequisites for them to be accretion discs: there must be a supply of gas to make the disc, and the gas must have larger specific angular momentum than Keplerian at the disc surface. The first of these two aspects can be fulfilled at least for some stars with weak radiatively driven winds: Porter & Skouza (1999) have examined models of radiatively driven winds where the gas and radiation field decouple due to ion stripping (e.g. Springmann & Pauldrach 1992), before the flow becomes unbound from the star. The gas then stalls in the star's gravitational potential and reaccretes. This may provide the material to make an accretion disc if the shell then was concentrated in the equatorial plane, but it will not have enough specific angular momentum to *form* an accretion disc. If the gas conserves angular momentum (almost certainly true for the outflowing phase), then when the gas reaccretes, it will have the same angular velocity as it left the star, i.e. sub-Keplerian. Consequently, unless there is some way of adding angular momentum to the wind as it stalls, this does not seem a feasible way to make accretion discs. A second possible reservoir of gas is a companion star, which is losing mass to the Be star – this does not have the angular momentum problem, although it does mean that all Be stars are in binary systems.

On the balance of these points, it seems unlikely that Be stars discs are *accretion* discs because (i) not all Be stars are in interacting binary systems with the secondary star as the mass donor, and (ii) an accretion disc will not form when a star is reaccreting its own wind.

3. Energy balance within the disc

3.1. Liberation of energy via viscous dissipation

The viscous stresses enables the disc to flow outwards and in doing so liberates energy. What happens to this energy? The timescale over which the viscous disc can evolve significantly τ_v is defined:

$$\begin{aligned}\tau_v &\sim \frac{R^2}{\nu} = \frac{(GM_*\,R)^{1/2}}{\alpha c_s^2} \\ &\approx 21\left(\frac{M_*\,R_*}{M_\odot\,R_\odot}\right)^{1/2}\frac{1}{\alpha T_4}\ \text{days}\end{aligned} \tag{5}$$

where T_4 is the disc temperature in 10^4 K. Inserting typical parameters leads to $\tau_v \sim 10^2$s of days. Given that Be star discs are observed to be present for many years (a pre-requisite for them to be able to develop and evolve V/R variations in the lines), then a typical disc is stable over a viscous timescale. Therefore any energy liberated via viscous stresses *must* emerge as radiation.

The luminosity emitted is

$$\begin{aligned}\frac{dL}{dR} &= \frac{9\pi}{2}GM_*\,\nu\Sigma R^{-2} \\ &= \frac{9\pi}{2}\rho_0 c_s^3 R_*\,\alpha\left(\frac{R}{R_*}\right)^{1-n}\end{aligned} \tag{6}$$

(Frank et al. 1992, p73), where the expressions for H, Σ and ν have been inserted to obtain the second equality. This luminosity source is termed the "viscous luminosity" as it is the energy emitted caused solely by the viscous processes leading to outflow. This expression only gives the luminosity as a function of radius, and contains no spectral information. As the disc is optically thin in the z direction, then the standard approach to the disc emission (e.g. Sect. 5.5 of Frank et al. 1992) are inapplicable, and an alternative discussion is needed.

3.2. IR free-free and free-bound emission

The continuum emission of Be star discs is limited to the IR–radio spectral regions (e.g. see Fig. 11 of Poeckert & Marlborough 1978) and was identified as free-free and free-bound emission from an ionized plasma by Gehrz et al. (1974). The most successful empirical model used to calculate the emission is due to Waters (1986), and represents the disc using a density power law, and an opening angle. Three parameters determine the emission: the opening angle, the density at the star-disc boundary, and the density power-law exponent.

The above expressions for a outflowing viscous disc changes the calculation of disc emission and so some of Waters' expressions are now restated. The optical depth τ_ν of the disc may be calculated at a frequency ν. The expression is simpler than Waters' expression as the integral through the disc (equivalent to his $C(\theta, n)$) may be completed analytically. Using Waters' notation

$$\left.\begin{aligned}&\tau_\nu(R) = X_\lambda\,X_{*d}\,R^{-2n+3/2-m/2}\sqrt{\frac{\pi c_{s,0}^2 R_*}{GM_*}} \\ &\quad X_\lambda = \lambda^2\frac{\left(1-\mathrm{e}^{-h\nu/kT_d}\right)}{\left(\dfrac{h\nu}{kT_d}\right)}(g_{ff}+g_{fb}) \\ &\quad X_{*d} = 4.923\times10^{35}\frac{\bar{z^2}}{T_d^{3/2}\mu^2}\gamma\rho_0^2\left(\frac{R_*}{R_\odot}\right)\end{aligned}\right\} \tag{7}$$

where λ is the wavelength in cm, g_{ff} and g_{fb} are the Gaunt factors for free-free and free-bound emission respectively, $\bar{z^2}$ is the mean squared atomic charge, γ is the ratio of the number of electrons to ions, μ is the mean atomic mass, and $c_{s,0}$ is the sound speed at the star.

The intensity $I_\nu(R)$ is calculated in the same way as in Waters (1986), and hence the spectrum of the disc may be calculated. However, here it is the total emission at a given radius which provides a direct comparison with outflowing viscous disc models. The differential energy liberated is

$$\frac{dL_\nu}{dR} = 8\pi^2 R I_\nu(R) \tag{8}$$

(equivalent to Waters' Eq. 12). Finally to obtain an expression to relate to Eq. 6, this is integrated over all frequencies:

$$\frac{dL}{dR} = 8\pi^2 R\int_0^\infty I_\nu(R)d\nu. \tag{9}$$

3.3. How important is the viscous luminosity?

There are now two expressions for differential disc luminosity deriving from the viscous stresses (Eq. 6) and the free-free, free-bound emission of the plasma (Eq. 9). Here the fraction of viscous luminosity to the free-free, free-bound emission of the disc is considered. This is an important check on the model – if the viscous luminosity (Eq. 6) is larger then the disc emission (Eq. 9) then the disc cannot be driven by viscosity. Eq. 8 provides a prescription for the disc emission which may be directly

Table 1. Best fit parameters for the IR excess of χ-Oph. Model 1 is isothermal and so $m = 0$.

	$\log(\rho_0)$ (g cm^{-3})	n	m	$2n + 3m$
Model 1	-11.22	2.20	(0)	4.40
Model 2	-11.24	1.90	0.23	4.49

related to the observations by integrating over radius to produce a spectrum, whereas Eq. 6 provides the theoretical emission due to viscous dissipation.

The density parameter ρ_0 and the power law exponent n, can be constrained by fitting the spectrum of the disc with the spectrum produced by Waters' method. The ratio $\mathcal{F}$ of luminosity emitted from free-free and free-bound transitions to the viscously dissipated energy is

$$\mathcal{F} = \frac{16\pi}{9c_s^3\rho_0\alpha}\left(\frac{R}{R_*}\right)^n \int_0^\infty I_\nu(R)d\nu. \qquad (10)$$

The most poorly constrained parameter is then the viscosity parameter α which is *a priori* limited to the region $0 < \alpha < 1$. However this may an underestimate if the turbulence becomes supersonic (see Sect. 2.4). For the current calculation α is set to unity, although the limit that $\alpha \lesssim 100$ is kept in mind.

4. A case study – χ-Oph

To illustrate the discussion above, a test case is considered – this provides us with a "real" example. As an example, the observations of the B2IV star χ-Oph are used (Waters 1986). The star was assumed to have a mass, radius and temperature of $10.0 M_\odot$, $5.7 R_*$, and 22,500 K.

4.1. IR continuum excess fits

To constrain the power law exponents, the IR excess is first fitted by the outflowing viscous disc outlined in Sect. 2 using the procedure due to Waters (1986). Two fits to the excess were calculated: one assumed an isothermal disc (Model 1) and the other allowed the temperature power law exponent m to vary (Model 2). The disc temperature at the inner edge was assumed to be 18000 K (i.e. 0.8 of the effective temperature). In this latter case, the emission was calculated only for radii which had temperatures larger than 10^4 K to ensure that the region was ionized. In both cases the disc is assumed to have a large radius ($R_{\rm disc} > 50 R_*$). The best fit parameters for the models are presented in Table 1, and are shown in Fig. 1.

Note that the derived parameters are not the same as those of Waters (1986) as the disc model used in that work differs from that used here. Model 2 is formally a better fit to the data – the reduced χ^2 values are 1.1 and 0.7 for Models 1 and 2 respectively. Although Model 2 fits the data better, it's reduced χ^2 value of 0.7 indicates that it is possibly "over fit" and so it does not provide strong evidence for non-isothermality in the disc. The temperature falls below 10^4 K at $R \approx 13 R_*$ providing a natural outer boundary to the emitting part of the disc – this radius is

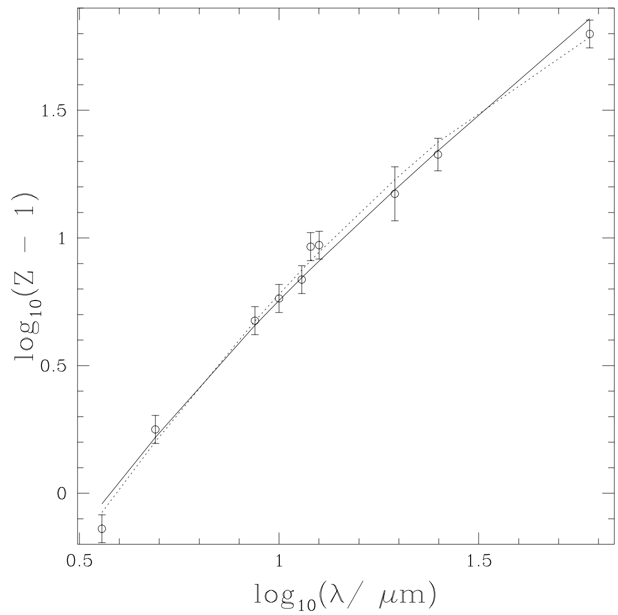

Fig. 1. The IR continuum excess emission as a function of wavelength. $Z - 1$ is the fractional emission excess over the stellar photosphere. All data is taken from Waters (1986). The solid line is the fit from the isothermal Model 1, and the dotted line corresponds to the non-isothermal Model 2.

low when compared to fits including millimeter observations of other stars (Waters et al. 1991). Millimeter–centimeter data provide constraints on the extent of the outer parts of the disc, and hence the temperature exponent m. Column 5 of Table 1 shows the combination of $2n + 3m$ which must be larger than 7 for a non-radiation driven outflowing viscous disc – both models fall significantly short.

4.2. Viscous luminosity versus free-free & free-bound emission

Now the power law exponents have been fitted from the spectrum, the relative contribution to the total emission of the viscously dissipated energy can be calculated. The ratio $\mathcal{F}$ is evaluated from Eq. 10 and is shown in Fig. 2 for both models.

Clearly the free-free and free-bound emission dominates over the viscously dissipated energy, by of order 1000 for both models. This result states that the energy balance within the disc is not dominated by the energy produced by the shear motions within the disc itself, and hence must be dominated by the stellar radiation field. Even with the maximum value of $\alpha \sim 100$, the free-free and free-bound emission still dominates, making the result secure.

5. Discussion

The previous sections have provided a consistent description of Be star discs as outflowing viscous discs. This study links outflowing viscous disc theory with observations and finds that the two are indeed compatible (the first outflowing viscous disc paper by Lee et al. 1991 produces very high disc densities due to their choice of high disc mass-loss rate which can be ruled out

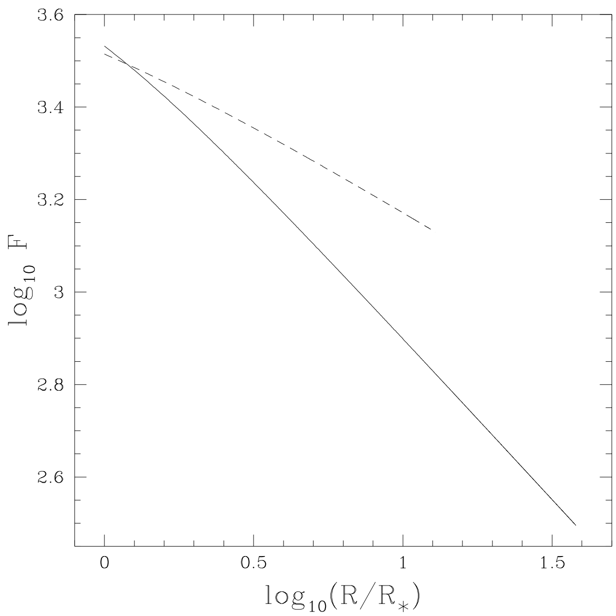

Fig. 2. Ratio of free-free and free-bound disc emission to the viscous emission from Eq. 10. The solid line corresponds to Model 1, the dotted line to Model 2.

by examining the IR continuum emission). This study therefore adds to the growing amount of evidence that Be star discs are in reality viscously driven. There is already substantial work regarding instabilities in Keplerian discs which give rise to density perturbations which produce asymmetric line profiles (e.g. Okazaki 1991). Viscous discs can now explain almost all of the observations of Be star discs.

One major observational aspect of Be stars is that they change phase from Be–Be-shell–normal B star, occurring apparently at random (although the disc may disperse and reappear in a more orderly fashion e.g. μCen, Hanuschik et al. 1993). How may the outflowing viscous disc model explain this? There may be a clue to these changes in the suggestions made in Sect. 2.3 regarding the regimes where the observed discs can be outflowing discs.

Two possibilities have been speculated upon: (i) if the disc is supported by radiation, then it should be noted that the electron scattering optical depth is typically in the range 0.1–1.0 (Eq. 5). With this so close to unity, variations in the density at the star-disc boundary of factors of a few may be enough to decrease the radiative driving through the disc. This loss of radiative support will lead naturally to an accretion phase where the disc falls back on to the star. The density variations at the star-disc boundary could, in principle be created as a by product of disc warping (Porter 1998b), or non-radial pulsations of the star (e.g. Lee & Saio 1990). Alternatively, (ii) the disc may be a outflowing viscous disc due to a possible large radial temperature gradient. However the recent modelling of Millar & Marlborough appears to rule this out.

With the outflowing viscous disc model able to explain the current observations it is pertinent to ask whether future observations could confirm the paradigm. The current major weakness with the theory regards the input of angular momentum at the inner boundary. Although this process has been suggested to be dissipation of non-radial pulsations in the atmosphere (Osaki 1986), it has not been clearly demonstrated. If this is the underlying mechanism, then there should be statistical correlations of disc and pulsation parameters – an observational aspect which is still to be resolved convincingly.

6. Conclusion

This paper has produced several points: first, outflowing viscous discs can only exist around Be stars if the disc is partly driven outwards by the stellar radiation field This has been discussed and shown to be viable for actual Be star discs.

Secondly, it is found (confirming *a priori* expectations) that a viscous disc can account for the observed excess IR emission of Be stars. Also, it has been found that the energy balance in the disc is dominated by the stellar radiation field. The energy liberated in viscous dissipation has been calculated and is shown to be a small fraction of the observed luminosity of the disc ($\sim 10^{-3}$–10^{-1}). These second two points are the first time it had been demonstrated that outflowing viscous discs are energetically allowable models.

The underlying reason why Be stars go through phases where their discs are lost has been speculated upon. This may be due to blocking of radiative support and driving of the disc at the inner regions – causing the disc to change to an accretion disc.

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