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ASTRONOMY AND ASTROPHYSICS

A numerical time ephemeris of the Earth

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Abstract. We present a time ephemeris of the Earth, TE405, which approximates a relativistic time-dilation integral from 1600 to 2200 using numerical quadrature of quantities supplied by the recent JPL ephemeris, DE405. The integral is required to transform between terrestrial time, TT, and the (solar-system) barycentric time scales T_{eph} or TCB. T_{eph} is a linear transformation of TCB that represents the independent variable of a modern ephemeris such as DE405. Our time-ephemeris results have an accuracy of order 0.1 ns, and we distribute them (at ftp://astroftp.phys.uvic.ca/pub/irwin/tephemeris/) in a Chebyshev form that requires much less computer time to evaluate than a detailed time-ephemeris series.

We find angular-frequency and mass-transformation corrections that should be applied to the time-ephemeris series of Fairhead & Bretagnon (1990). These corrections make an extended form of this series with 1705 terms agree with our work to within 15 ns over the epoch range. We find a further correction of two long-term sinusoids that reduces this maximum residual to 5 ns. We suggest the long-term residuals fit by these sinusoids and the remaining short-term residuals are the result of errors in the fit of VSOP82/ELP2000 (the analytical ephemeris upon which the Fairhead & Bretagnon series is based) to the earlier JPL ephemeris, DE200.

Following previous work (Fukushima 1995) we eliminate the linear term from TE405 by comparing with the corrected series results. The result for the linear coefficient of the term that is subtracted is $\Delta L_C^{(\mathrm{TE405})} = 1.480\,826\,855\,94 \times 10^{-8} \pm 1. \times 10^{-17}$. We have not included the *periodic* post-Newtonian and asteroid effects in the time-ephemeris calculation because they are negligible. However, when we add an *average* post-Newtonian and asteroid corrections of $\Delta L_C^{(\mathrm{PN})} = 109.7 \times 10^{-18}$ and $\Delta L_C^{(\mathrm{A})} = (5. \pm 5.) \times 10^{-18}$ to $\Delta L_C^{(\mathrm{TE405})}$ the result is $L_C = 1.480\,826\,867\,41 \times 10^{-8} \pm 2. \times 10^{-17}$. When this result is combined with a recent result for the potential at the geoid (Bursa et al. 1997) corresponding to $L_G = 6.969\,290\,112 \times 10^{-10} \pm 6. \times 10^{-18}$ we obtain

$$K \equiv \frac{d\mathrm{TCB}}{dT_{eph}} \equiv \frac{1}{(1-L_C)(1-L_G)} = 1 + (1.550\,519\,791\,54 \times 10^{-8} \pm 3. \times 10^{-17}).$$

The factor K relates ephemeris units for time and distance and the corresponding SI units for the same quantities.

Key words: ephemerides – time – solar system: general

1. Introduction

The relativistic time-dilation integral, $\Delta T_{\oplus}$ (Eq. [3]), is a quantity that is required to transform between terrestrial time, TT, and the (solar-system) barycentric time scales T_{eph} or TCB. T_{eph} is a linear transformation of TCB that represents (Standish 1998c) the independent variable of a modern ephemeris such as the JPL ephemeris. Thus, an accurate approximation of $\Delta T_{\oplus}$ (we denote such approximations as time ephemerides of the Earth or just as time ephemerides for short) is required to analyze all astronautical or astronomical measurements that are precisely reduced with the aid of a modern ephemeris. From the view point of large scale metrology, time-related measurements made on the Earth suffer the variation of topocentric time scales. Examples which depend on locally realized frequency standards are measurements of spacecraft ranges and pulse-arrival times from pulsars.

The observational errors of spacecraft ranges pose a stringent constraint on the maximum acceptable errors of a time-ephemeris derivative. The typical error of a spacecraft range observed with the Deep Space Network is 1 m (DSN 1999). At the distance of Pluto this translates to a relative range error and corresponding time-ephemeris derivative error of 2×10^{-13}. Systematic errors should ideally be at least two orders of magnitude smaller than random errors in the best single observations (thus making the systematic error equivalent to the random error of the mean of 10^4 such observations). Thus, the time-ephemeris derivative errors should be kept less than 2×10^{-15} for the interpretation of observed ranges of spacecraft. This limit requires some care to achieve because it is 5 orders of magnitude smaller than the maximum absolute value of the time-ephemeris derivative, 3×10^{-10}.

The observational errors of daily mean pulse-arrival epochs of pulsars pose a stringent constraint on the maximum acceptable errors of a time ephemeris. These epoch errors are less than 1 μs and are beginning to approach 0.1 μs (Kaspi et al. 1994).

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We adopt this latter value as the nominal best error of a daily mean pulse-arrival epoch. Using the criterion that systematic errors should be 2 orders of magnitude less than the smallest random errors leads to a maximum acceptable error for a time ephemeris of 1 ns. This error limit requires some care to achieve because it is 6 orders of magnitude smaller than the maximum absolute value of the time-ephemeris, 2 ms.

Efforts to obtain analytical approximations for $\Delta T_\oplus$ (Eq. [3]) or its derivative go back to Aoki (1964) and Clemence & Szebehely (1967). This initial work has been followed by analytical approximations of increasing accuracy and complexity (e.g., Moyer 1981b; Hirayama et al. 1987) culminating in a 1705-term version (Bretagnon 1995) of a time-ephemeris series given by Fairhead & Bretagnon (1990, FB). This series is based on the VSOP82/ELP2000 analytical planetary and lunar ephemeris (Bretagnon 1982; Chapront-Touzé & Chapront 1983) that was fit to DE200 from 1890 to 2000.

As an alternative to the series approach, one may directly calculate a time ephemeris using numerical quadrature of quantities supplied by a planetary and lunar ephemeris (see Backer & Hellings 1986). The mass-corrected series results and numerical results made privately available from the JPL group to Fairhead and Bretagnon agreed within 3 ns over the epoch range from 1900 to 2000 (see Fig. 3 of FB).

The 3-ns level of agreement between the JPL and FB time ephemerides was not obtained by subsequent much more extensive comparisons of numerical and analytical time ephemerides (Fukushima 1995, Paper I). For example, the RMS deviation of TE200 (a numerical time ephemeris based on the JPL ephemeris, DE200) with the FB2 series (an extended form of the FB series containing 791 terms) is 26 ns (Table 7 of Paper I). Subsequently, we found the source of this large disagreement was an inappropriate angular-frequency transformation (see discussion in Sect. 4) that was used for the published FB coefficients. Presumably, this source of error was not present in the published comparison between the FB results and the numerical time ephemeris from the JPL group.

The purpose of the current paper is to follow up Paper I by presenting new results for a numerical time ephemeris, series corrections, and the ratio of ephemeris units to SI units (the K value). We rigorously define in Sect. 2 the relativistic time-dilation integral that is used to transform between Earth-based and solar-system-based time scales. We present in Sect. 3 a numerical approximation of this integral, TE405, which has an unprecedented accuracy of 0.1 ns. We present in Sect. 4 angular-frequency and mass transformations of the FB time-ephemeris series which reduce its maximum errors to 15 ns from 1600 to 2200 and suggest the remaining long-term residuals (which can be fit by two sinusoids) and short-term residuals are due to errors in the fit of VSOP82/ELP2000 to DE200. We use in Sect. 5 the combination of TE405 and the corrected FB series to determine $\Delta L_C^{(\mathrm{TE405})}$, the coefficient of the linear term that is subtracted from TE405. We also determine K which relates ephemeris units for time and distance and the corresponding SI units for the same quantities. We conclude the paper in Sect. 6.

2. Definitions

The purpose of this section is to rigorously define the relativistic time-dilation integral that is used to transform between terrestrial time, TT, or geocentric coordinate time, TCG, and the (solar-system) barycentric coordinate times, T_{eph} and TCB. TT, TCG, and TCB are defined by Recommendations III and IV of IAU Resolution A4 (1992; see also Paper I for further discussion). T_{eph} is a linear transformation (Standish 1998c) of TCB that represents the independent variable of a modern ephemeris such as the JPL ephemeris. Our development follows Seidelmann & Fukushima (1992), Seidelmann et al. (1992), and Paper I, but we have replaced the ambiguous time designations "JD" or "t" that occurred in the previous work with explicit time scales. We prefer the current approach because of its definiteness although the removed ambiguities are only the order of the post-Newtonian corrections.

TCB, TCG, and TT are usually expressed in Julian day numbers with a common epoch (if we ignore the effects of observer location) of $JD = 2\,443\,144.500\,372\,5$. However, for the present paper *we subtract the common epoch from these time scales and also* T_{eph}. This change in zero point simplifies the exposition so that we have been able to use *units of seconds (s) for all time scales in our equations.*

The linear transformation between TCB and T_{eph} is defined by

$$\mathrm{TCB} \equiv K(T_{eph} - T_{eph_0}), \tag{1}$$

where

$$K \equiv \frac{1}{1 - L_B} \approx 1 + (1.55\ldots \times 10^{-8}) \tag{2}$$

is a rate coefficient defined (Eq. [12]) so that the mean rate (in a particular sense, see discussion after Eq. [15]) of T_{eph} and TT are the same. (We discuss the recommended values for K and the equivalent L_B in Sect. 5.) T_{eph_0} is the value of T_{eph} corresponding to the instant when TCB (and TCG and TT if we ignore the effects of observer location) is zero. T_{eph_0} may vary from ephemeris to ephemeris depending on the treatment of the integration constant for the relativistic time-dilation integral (see discussion prior to Eq. [17]).

Eq. (1) implies the ephemeris unit of a second is equivalent to K SI seconds and similarly for meters because the speed of light in vacuum is identical in all coordinate systems. *Thus, all times (s), positions (m), and values of GM (gravitational constant times mass in units of* $\mathrm{m}^3\ \mathrm{s}^{-2}$*) associated with, for example, the JPL ephemeris and header values must be multiplied by K to convert to the SI units appropriate for the coordinate systems associated with TCB, TCG, or TT.* Ephemeris and SI units are the same for velocity and $GM/$(distance).

We define the relativistic time-dilation integral,

$$\Delta T_\oplus(T_{eph}) \equiv \int_0^{T_{eph}} \left[\frac{1}{c^2}\left(U_E + \frac{|v_E|^2}{2}\right) - \Delta L_C\right] dt, \tag{3}$$

where

$$U_E = \sum_{i \neq E} \frac{GM_i}{|r_{Ei}|} \tag{4}$$

and

$$\boldsymbol{r}_{Ei} = \boldsymbol{r}_E - \boldsymbol{r}_i. \tag{5}$$

Here, c is the speed of light in vacuum, $\boldsymbol{r}_E$ and $\boldsymbol{v}_E$ are the position and velocity of the geocenter relative to the solar-system barycenter (SSB), the sum in Eq. (4) is over all solar-system objects excluding the Earth that are more massive than the asteroids, and M_i and $\boldsymbol{r}_i$ are the mass and SSB position vector of the ith object in the solar system. Note that all quantities in Eqs. (3) through (5) are expressed in terms of ephemeris meters and seconds rather than SI meters and seconds.

The current definition of $\Delta T_\oplus$ has the linear trend conveniently removed by subtracting $\Delta L_C \approx 1.48\ldots \times 10^{-8}$ from the integrand. This is an important difference from the previous definition in Paper I which includes the linear trend. ΔL_C is determined (Sect. 5) by finding the zero of the slope of the residuals between the numerical time ephemerides which approximate the integral and a corrected secular + sinusoidal series (with origin at J2000 but no linear term) for the time ephemeris. In this way the separation of the linear trend from the time ephemeris is done similarly to the series results, but the exact value of ΔL_C depends on the underlying planetary and lunar ephemeris used to create a time ephemeris and is partially insulated from the remaining errors in the corrected series results.

Our definition of $\Delta T_\oplus$ formally excludes post-Newtonian and asteroid effects. However, we define

$$L_C \equiv \Delta L_C + \Delta L_C^{(\mathrm{PN})} + \Delta L_C^{(\mathrm{A})} \approx 1.48\ldots \times 10^{-8}. \tag{6}$$

With this definition our calculations implicitly include the mean rate effect of the post-Newtonian and asteroid corrections, $\Delta L_C^{(\mathrm{PN})}$ and $\Delta L_C^{(\mathrm{A})}$ (Sect. 5). Our definition of $\Delta T_\oplus$ is equivalent to one where these corrections are added to the integrand and L_C (rather than ΔL_C) subtracted from the integrand.

The relationship between TCB and TCG is expanded as

$$\begin{aligned}\mathrm{TCB} - \mathrm{TCG} \;=\; & K\frac{(\boldsymbol{r}_o - \boldsymbol{r}_E)\cdot \boldsymbol{v}_E}{c^2} \\ & + K\left[\Delta T_\oplus(T_{eph}) - \Delta T_\oplus(T_{eph_0})\right] \\ & + L_C\mathrm{TCB} + \text{small correction terms}, \end{aligned} \tag{7}$$

where $\boldsymbol{r}_o$ is the barycentric position vector of the observer in the ephemeris coordinate system, the K factor converts from ephemeris units to SI units (see previous discussion), and the recommended value of $L_C \approx 1.48\ldots \times 10^{-8}$ is discussed in Sect. 5. (For consistency in notation we have expressed the position vector of the observer relative to the geocenter, $\boldsymbol{r}_o - \boldsymbol{r}_E$, in ephemeris units, but if this vector is expressed in SI units, as is normally the case, then the K factor should not multiply the first term.)

The first term of Eq. (7) depends on observer location and can be derived from the Principle of Equivalence; in the limit of small fields and accelerations, this term is required by the theory of special relativity to correct for the lack of synchronization of clocks (see discussion on p. 25 of Eisberg 1961) in the moving TCG frame when observed from the TCB frame. More generally, this term is the first term in a Taylor series with both Newtonian and post-Newtonian higher order terms (Thomas 1975; Moyer 1981a). Near the geoid, these higher order terms are smaller than the periodic post-Newtonian and asteroid corrections to the second term of Eq. (7). These latter corrections have a maximum magnitude of 33 ps for the post-Newtonian correction and 15-ps for the asteroid correction. In practice we ignore all these corrections (indicated as "small correction terms" in Eq. [7]) because they are smaller than the maximum errors, of order 0.1 ns, in TE405, our best approximation to $\Delta T_\oplus$ (Sect. 3).

The relationship between TCG and TT is defined by

$$\mathrm{TCG} - \mathrm{TT} \equiv L_G\mathrm{TCG} \equiv \frac{L_G}{1-L_G}\mathrm{TT} \tag{8}$$

or

$$\frac{d\mathrm{TT}}{d\mathrm{TCG}} \equiv 1 - L_G, \tag{9}$$

where L_G is the dimensionless constant,

$$L_G \equiv \frac{W_0}{c^2} \approx 6.96\ldots \times 10^{-10}, \tag{10}$$

and W_0 is the gravitational plus spin potential of the Earth at the geoid. (We discuss the recommended values of W_0 and L_G in Sect. 5.)

If we combine Eqs. (1), (7), and (8), then after some manipulation we obtain

$$\begin{aligned}T_{eph} - \mathrm{TT} = {} & T_{eph_0} + (1-L_G)K\frac{(\boldsymbol{r}_o - \boldsymbol{r}_E)\cdot \boldsymbol{v}_E}{c^2} \\ & + (1-L_G)K\left[\Delta T_\oplus(T_{eph}) - \Delta T_\oplus(T_{eph_0})\right] \\ & + \left[1 - K(1-L_C)(1-L_G)\right](T_{eph} - T_{eph_0}). \end{aligned} \tag{11}$$

To force the mean rate of T_{eph} and TT to be equal in a particular sense (see below) we define

$$K \equiv \frac{1}{(1-L_C)(1-L_G)}, \tag{12}$$

and Eq. (11) reduces to

$$\begin{aligned}T_{eph} - \mathrm{TT} \;=\; & T_{eph_0} + \frac{(\boldsymbol{r}_o - \boldsymbol{r}_E)\cdot \boldsymbol{v}_E}{(1-L_C)c^2} \\ & + \frac{\Delta T_\oplus(T_{eph}) - \Delta T_\oplus(T_{eph_0})}{1-L_C}. \end{aligned} \tag{13}$$

In Eqs. (11) and (13) we have ignored small correction terms (see the discussion after Eq. [7]). We have kept the factor, $(1 - L_C)$, in Eq. (13) to be exactly consistent with our previous equations, but replacing this factor by unity would cause a maximum error of only 30 ps.

From Eqs. (7) and (13) we derive the following mean rates:

$$\left\langle \frac{d\mathrm{TCG}}{d\mathrm{TCB}} \right\rangle = 1 - L_C, \tag{14}$$

and

$$\left\langle \frac{dT_{eph}}{d\mathrm{TT}} \right\rangle = 1, \tag{15}$$

where the angle brackets correspond to taking a specially defined mean. This mean is formally defined by determining a secular plus periodic series with origin at J2000 and ignoring all but the linear term before taking the derivative. By definition no such mean rate exists for $\Delta T_{\oplus}$ (see discussion of Eq. [3]). Thus, in this particularly defined sense *the mean rate of the* T_{eph} *and TT are equal.* We emphasize that definition (12) is necessary to insure this condition.

Aside from a difference in the way the offset is formulated (to be discussed below) our Eq. (13) also differs slightly from Eq. (3) of Standish (1998c) in one other particular. His equation (with $K' \equiv 1/(1 - L_C - L'_G)$ and $T \equiv T_{eph}$) includes a term $K' L'_G T_{eph}$ (where the primes distinguish his K and L_G constants from ours). Our formulation replaces this term by L_GTCG because we prefer the relationship between TCG and TT to be defined independently of T_{eph} (see Eq. [8]). To keep $K' \equiv K$ (and therefore the mean rate of T_{eph} exactly the same in both formulations) we must have

$$L'_G \equiv L_G(1 - L_C). \quad (16)$$

With this identification, the two equations give identical results (once the difference in offset formulation is taken into account) except that the factor $1 - L_C$ appearing in our Eq. (13) is replaced by the factor $1 - L_C - L'_G$ in the Standish equation. This difference is negligible.

We have formulated our equations in such a way that any arbitrary value of the offset, T_{eph_0}, may be used for the construction of future planetary and lunar ephemerides. Nevertheless, when interpreting observations with a presently existing ephemeris, it is necessary to employ the same effective T_{eph_0} value that was used for the construction of that ephemeris. In the case of the recent JPL ephemerides (DE143, DE145, and DE403-DE406), for example, the quantity, $T_{eph_0} + (\Delta T_{\oplus}[T_{eph}] - \Delta T_{\oplus}[T_{eph}]_0)/(1 - L_C)$, in our present notation (see also Eq. [3] from Standish 1998c) was approximated by a series (from Hirayama et al. (1987) as corrected by Paper I). Evaluation of this series at $T_{eph} = T_{eph_0}$ yields $-65.564\,518\ \mu$s (Standish 1999). Therefore, to interpret observations correctly with the indicated recent JPL ephemerides and our formulation it is necessary to adopt

$$T_{eph_0} = -65.564\,518 \mu\text{s}. \quad (17)$$

(For offsets with this small a magnitude, dropping the constant $\Delta T_{\oplus}(T_{eph_0})/(1 - L_C)$ from Eq. [13] would cause a negligible error.)

The right-hand-side of Eq. (13) is a function of T_{eph} so, strictly speaking, iterations should be performed to calculate T_{eph} from TT using this equation. However, because $T_{eph} -$ TT is less than 2 ms, evaluating the right-hand-side of Eq. (13) using the approximation $T_{eph} \approx$ TT introduces a negligible error.

3. Numerical calculation of the time ephemeris

3.1. Procedure

Following Paper I we have used a 10th-order numerical quadrature method (a double-precision [8 bytes total floating-point word length] version of subroutine QROMB from Press et al. 1992) to evaluate Eq. (3). The integrand was evaluated using the JPL ephemerides for the Sun, Moon, and all planets in the solar system. We ignored the 15-ps periodic effect of the asteroids (Paper I) because it is negligible compared to the 0.1-ns accuracy of our final result (see later discussion). We performed the time-ephemeris calculation for both the DE200 and DE405 versions of the JPL ephemeris and we designate the corresponding time-ephemeris results as TE200 and TE405. The integrations were performed with a preliminary ΔL_C value that was subsequently corrected for each time ephemeris (Sect. 5). The granule interpolation boundaries (Standish et al. 1992) that occur every 4th midnight for the JPL ephemeris necessarily have second and higher-order derivative discontinuities which would reduce the effective order and efficiency of our numerical quadrature method if it spanned the interpolation boundaries. To avoid this potential problem the results were calculated on half-day integration intervals with end points at noon and midnight (or nearest midnight to the common epoch of $JD = 2\,443\,144.500\,372\,5$ for the special integration interval that is required to help establish the zero point).

We have interpolated the time-ephemeris results using Chebyshev polynomials on 4-day granules. These polynomials fit the time ephemeris and its derivative exactly at the end points of the granule (thus providing overall continuity in these quantities) and fit these quantities by least squares at the 7 interior points. The Chebyshev coefficients consistent with these requirement have been calculated with the CHEBFIT routine (described by Newhall 1989 and communicated by Standish 1998a) that is used to calculate the Chebyshev coefficients of the JPL ephemeris. We have used 7 polynomial coefficients per 4-day granule to represent the TE200 and TE405 versions of the time ephemeris. This choice results in a gzip-compressed ASCII file size of 5 Mbytes and binary file size of 3 Mbytes for the full 600-yr epoch range of either version of the time ephemeris. These disk-space requirements are modest.

Tests of the quadrature error and significance-loss error were obtained by comparing complete time ephemerides from 1600 to 2200 based on half-day and one-day integrations and based on a numerical quadrature of an integrand of the form, (sinusoid + ΔL_C) - ΔL_C, where the sinusoid is the derivative of the principal term of the series for the time ephemeris. These tests indicated the maximum numerical errors from the quadrature and significance loss were negligible compared to the maximum numerical errors of 0.3 ps in the time ephemeris and 3×10^{-17} in its derivative caused by interpolating the results with 7 polynomial coefficients per 4-day granule. This high degree of numerical precision is straightforward to achieve, requires modest disk space, interferes negligibly with the estimated level of accuracy (see below), and provides a numerically clean benchmark to compare with other work. These results supersede the time-ephemeris results described in Paper I which were based on DE200 and DE245 and which were presented with an interpolation precision of order 1 ns. (Detailed results from the Paper I had been lost in a disk crash, but we checked that our new TE200 does reproduce residual plots made with the old TE200 consistently with the 1 ns interpolation errors of the older calculation.)

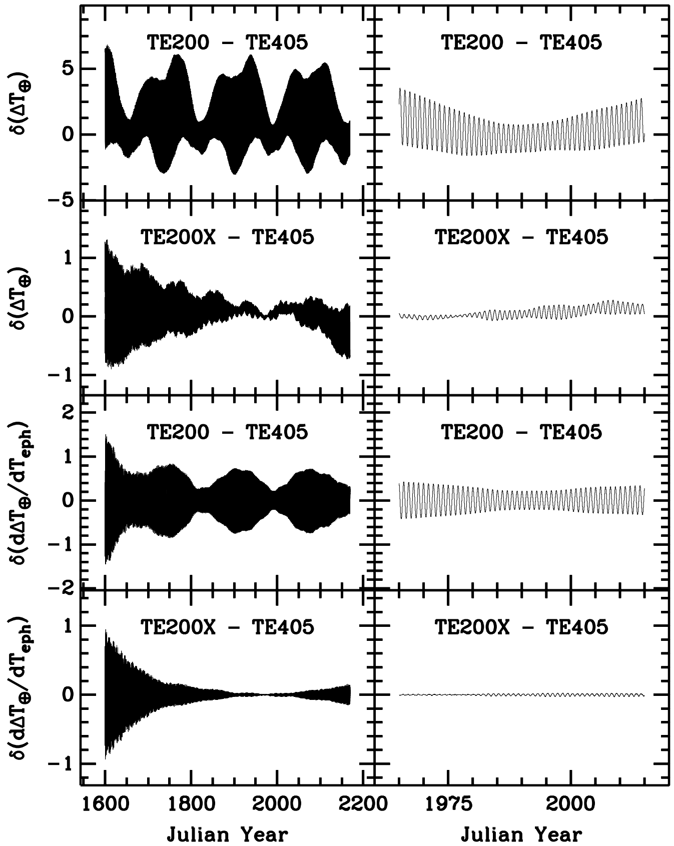

Fig. 1. Differences of numerical time ephemerides (with units of ns) and their derivatives (with units of 10^{-15}) over their common epoch range and over a modern epoch range. TE200 and TE405 correspond to the DE200 and DE405 versions of the JPL ephemeris. The TE200X ephemeris has been mass-corrected to the DE405 mass system using the procedure given in Sect. 4.

3.2. Accuracy

Comparisons between TE200 and TE405 (Fig. 1) help us estimate the accuracy (maximum value of the actual errors rather than the numerical precision) of our results. Because of the good numerical precision of our calculations, the accuracy of the time ephemerides is essentially determined only by the accuracy of the JPL mass parameters and ephemerides. The comparisons between TE200 and TE405 show there are both mass-dependent and mass-independent components to the time-ephemeris errors. The mass-independent errors still remain after mass correction according to the method of Sect. 4 and are presumably the result of different models and starting conditions for the JPL ephemerides extrapolating in different ways to epochs far removed from the modern era where astrometry constrains the solutions. DE405 is a recent JPL ephemeris (Standish 1998b; see also Standish et al. 1995) that fits a substantial amount of additional precise astrometry compared to DE200 which was prepared in 1981. Therefore, the errors of TE200 should be substantially larger than the errors of TE405 so that Fig. 1 essentially illustrates both the mass-dependent and mass-independent errors of TE200 and its derivative.

Table 1. Propagation of mass-parameter errors to the TE405 time ephemeris and its derivative

Mass parameter[a]	Mass error[b] (10^{-6})	$(\sum A)^{c}$ (μs)	$\Delta T_{\oplus}$ error[d] (ps)	$(\sum A\omega)^{e}$ (10^{-12})	$\frac{d\Delta T_{\oplus}}{dT_{eph}}$ error[f] (10^{-18})
m_4	2.9	3.8	10.	0.21	1.
m_5	0.76	33.	30.	5.0	4.
m_6	5.1	8.3	40.	1.1	6.
m_L	0.30	1.9	0.	4.6	1.
Total	...	...	80.	...	12.

[a] Mass parameter. m_4, m_5,m_6, and m_L are the respective ratios of the Mars, Jupiter, Saturn, and Moon masses to the solar mass. We have not included in this table the calculated error budget associated with other mass parameters because their contributions are insignificant.
[b] Relative error in the given mass parameter derived from Standish (1995).
[c] Sum over amplitudes that are affected by the given mass parameter (see text).
[d] Estimated maximum error in $\Delta T_{\oplus}$ (rounded to 10 ps) from the product of the relative error in the given mass parameter and $\sum A$.
[e] Sum over amplitudes $\times$ angular frequencies that are affected by the given mass parameter (see text). [f] Estimated maximum error in $d\Delta T_{\oplus}/dT_{eph}$ (rounded to 1×10^{-18}) from the product of the relative error in the given mass parameter and $\sum A\omega$.

The maximum mass-dependent errors of TE200 are 6 ns for the time ephemeris and 1×10^{-15} for its derivative. Of all the mass parameters of the JPL ephemerides, the Uranus and Neptune masses have the largest relative change from DE200 to DE405. Consequently, the largest terms in the mass-dependent errors of TE200 are 4 sinusoids with the Uranus and Neptune synodic periods (relative to Earth) and sidereal periods. Differentiation amplifies short-term changes so the mass-dependent errors of the TE200 derivative are dominated by the Uranus and Neptune synodic periods near a year. These short periods beat together with the synodic period (170 yr) of Uranus relative to Neptune.

As expected, the envelope of the mass-independent errors of TE200 grows substantially larger as we extrapolate from the modern era where DE200 was calibrated by the astrometry that was available when it was created. The maximum values of these errors within the epoch range are 1 ns for the time ephemeris and 1×10^{-15} for its derivative.

For TE405, the mass-dependent and mass-independent error components are more difficult to estimate since we have no time ephemeris with higher accuracy for comparison.

We estimate the maximum mass-dependent errors (see Table 1) for TE405 by propagating the relative mass-parameter errors taken from Standish (1995) to the relevant amplitudes of the series results using the same model that is used in Sect. 4 to correct for changes in mass. For each mass parameter we have summed amplitudes that are affected by that mass parameter using the extended form of the FB series. Summing amplitudes ignores phase information, but this approximation should be

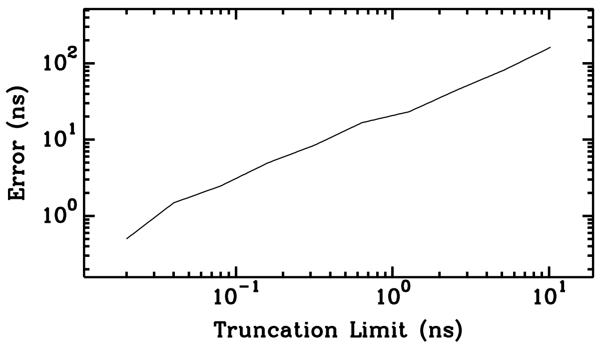

Fig. 2. Maximum difference over the epoch range from 1600 to 2200 of results from truncated FB3 series compared with results from the original FB3 series (which includes terms with amplitudes of 10 ps or more). The truncation limit is the minimum amplitude term included in the truncated series.

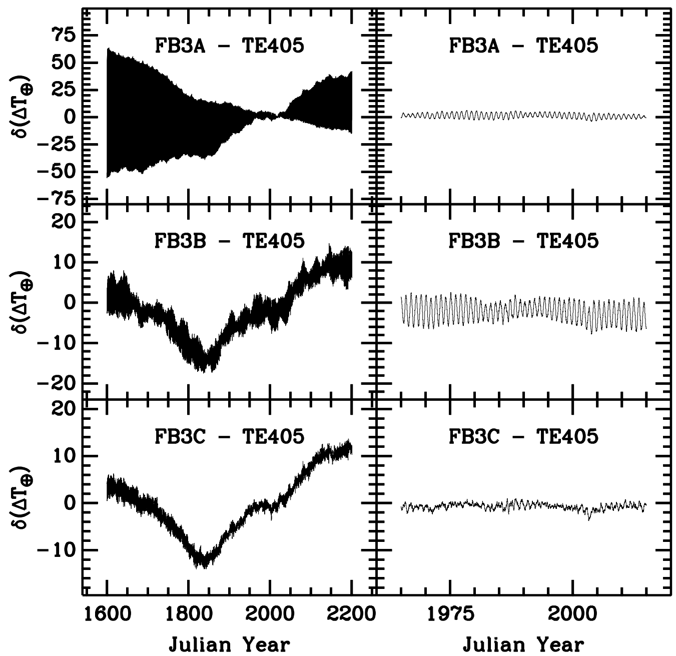

Fig. 3. Differences of time ephemerides (with units of ns) over the complete TE405 epoch range and over a modern epoch range. Three versions of the FB3 series are compared with TE405 results. The FB3A series is the original FB3 series transformed to the published angular frequencies (see text). The FB3B series is the original FB3 series. The FB3C series is the original FB3 series transformed to the mass parameters of DE405 (see text).

excellent because usually the sums are dominated by just one component, and even in the case of two dominant components the two error sinusoids should be in phase for a number of epochs within the large range we are considering here. We evaluate the mixed secular-periodic amplitudes at 1600 to maximize their contribution to the sums and error estimates. The total mass-dependent error estimates from Table 1 are obtained by simply adding the individual results (again ignoring phase information). The maximum mass-dependent error estimates are 80 ps for TE405 and 1.2×10^{-17} for its derivative.

We estimate the maximum mass-independent errors are 0.1 ns for TE405 and 1×10^{-16} for its derivative, i.e., an order of magnitude less than the corresponding errors for TE200. These order-of-magnitude estimates of the mass-independent TE405 errors are based on the observation that consistency between modern JPL ephemerides is roughly an order of magnitude better than the consistency between DE200 and the modern ephemerides (see Standish et al. 1995; Standish 1998b).

Combining the mass-independent and mass-dependent errors estimates together we conclude that the accuracy is the order of 0.1 ns for TE405 and 1×10^{-16} for its derivative. This level of accuracy is directly attributable to the great care that has gone into the preparation of the JPL ephemerides and is an order of magnitude better than the accuracy currently required for reducing pulse-arrival times of pulsars and spacecraft ranging. (Sect. 1). This level of accuracy is also at least an order of magnitude better on all time scales than the accuracy of the best frequency standards (linear ion traps, see Tjoelker et al. 1996) now being installed (see http://horology.jpl.nasa.gov/research.html) at the ground stations of the Deep Space Network.

Our estimate of the accuracy of the time ephemeris ignores the effect of the error in the ΔL_C rate adjustment that is subtracted from the time ephemeris. We estimate the ΔL_C value and error in Sect. 5 for each version of our time ephemeris.

4. Corrections to the FB time ephemeris series

Analytical time ephemerides are the result of integration of Eq. (3) using an integrand determined from analytical planetary and lunar ephemerides such as VSOP82/ELP2000 (Bretagnon 1982; Chapront-Touzé & Chapront 1983). Analytical and numerical time ephemerides complement each other. The analytical approach is essential for identifying the important terms of the numerical results, for error analysis and mass correction of the numerical results (Sect. 3), and for calculating the ΔL_C and K values (Sect. 5). On the other hand, the current numerical results are almost 3 orders of magnitude more accurate than the published series results. In this section we present corrections to the analytic results that reduce their errors by an order of magnitude. In addition we suggest a plausible explanation for the remaining errors of the analytical results.

We believe a 1705-term version (Bretagnon 1995) of the FB series has the best potential for correction so we have concentrated exclusively on it in the present investigation. We designate this series "FB3" to distinguish it from the (shorter) series that were investigated in Paper I.

The FB3 series includes all known terms whose amplitudes are 10 ps or more. Fig. 2 shows the maximum truncation errors over the epoch range from 1600 to 2200 as a function of truncation limit. From this figure, the truncation limit of 10 ps for the FB3 series should correspond approximately to 0.5 ns error which is an acceptable result. However, truncating the series at larger limits than the FB3 series cutoff (as occurs, for example, for many of the series which were investigated in Paper I) would produce substantial truncation errors.

Fig. 3 illustrates how removing the angular-frequency transformation that was incorrectly applied to the published results can improve the FB series. The original FB3 series as received from Bretagnon is expressed in terms of the independent time variable of the VSOP82/ELP2000 ephemeris. The most-important angular frequencies (ω_{a1}, ω_{a2}, and ω_{a3}) published in FB are actually the original FB3 series angular frequency values multiplied by the factor $1/(1 - L_C) \approx 1 + 1.48\ldots \times 10^{-8}$. Apparently this frequency transformation was done to compensate for a mean rate difference that was *thought* to occur between the independent variable of the VSOP82/ELP2000 ephemeris and TT. However, the independent variable of the VSOP82/ELP2000 ephemeris (like the independent variable of numerical ephemerides, see Sect. 2) has *no* mean rate difference with TT. The results in Fig. 3 serve as numerical proof of this because we obtain much better agreement with TE405 if we *do not* apply the angular frequency transformation to the FB3 series. The invalid angular frequency transform was also present for the FB (127 terms truncated at 10 ns) and FB2 (791 terms truncated at 0.1 ns) forms of the series investigated in Paper I. The invalid angular frequency transformation contributes to the errors found for the FB series (although the FB series truncation errors should be larger, see Fig. 2) and should provide the principal source of the errors found for the FB2 series (see Table 7 of Paper I).

Fig. 3 also illustrates the modest but important improvements possible with mass transformation of series. For this transformation (and also for the mass transformations and error analyses in Sects. 3 and 5) we use parameters defined as the ratio of various planetary masses or the lunar mass to the solar mass. We also use the mass parameter, S^2, the square of the number of meters per AU. We use S^2 rather than S^3 (which is proportional to $GM_\odot$) as a mass parameter because with the planetary and lunar mass ratios fixed, the other quantities that appear in Eqs. (3) and (4) are $GM_\odot/(\text{distance})$ and $|v_E|^2$ which are both proportional to S^2. The VSOP82/ELP2000 ephemeris and the resulting time-ephemeris series are mostly based on the 1976 IAU system of masses. (Compare Bretagnon 1982, Table 5 with Seidelmann 1992. See also FB.) Through the fit to DE200, the DE200 masses also indirectly affect the VSOP82/ELP2000 ephemeris and the resulting time-ephemeris series, but this inconsistency does not seem to introduce appreciable errors to the mass-correction procedure (see Fig. 1).

To transform from the IAU 1976 system to the DE405 system, we multiply all important amplitudes that are affected by a particular planet or the Moon by the ratio of the associated mass-ratio parameters. Similarly, all important amplitudes are multiplied by the ratio of the S^2 parameters. We transform TE200 (Sect. 3) to the DE405 mass system using similar procedures. The various forms of the Fairhead & Bretagnon series have convenient identifications of the solar, planetary (excluding the Earth) or lunar source of each term. The mass-transformation procedure necessarily excludes Pluto because no analytical theory of Pluto is yet available. However, this exclusion should not make much difference since from Eq. (13) of Paper I the time-ephemeris amplitude of the principal term of the Pluto effect should be only 0.6 ns.

Table 2. Coefficients of the correction to the FB3 series

i	ΔA_i (ns)	ω_{ai} (rd/10^3 yr)	ϕ_{ai} (rd)	Period (yr)
1	9.289	9.726519522	−3.157958164	645.9850
2	2.184	29.022384716	0.110947587	216.4945

These coefficients are defined in the same way as the coefficients of the original FB3 series (aside from the difference of the amplitude unit). The correction to be added to the FB3 series is $\sum_i \Delta A_i \sin(\omega_{ai}[T_{eph} - 2451545.]/365250. + \phi_{ai})$.

Work on an improved analytical theory is in progress (Bretagnon & Moisson 1998), but until that is completed it is difficult to identify the exact source of the remaining discrepancies between the mass-corrected FB3 series and TE405. However, since the 1-ns discrepancies (Fig. 1) between the mass-corrected TE200 and TE405 results are substantially smaller than the 15-ns discrepancies between the mass-corrected analytical results and TE405 (Fig. 3), we suggest that a possible cause of these latter discrepancies could be the deviations between VSOP82/ELP2000 results and DE200 (and DE405) results. The analytical ephemerides were only fit to the DE200 positions over a limited time interval (1890 to 2000) with masses that are inconsistent with the DE200 masses. The maximum residuals in the fitting interval of the Earth's latitude, longitude, and heliocentric distance (Bretagnon 1982, Table 6) correspond respectively to 114, 240, and 160×10^{-10} AU. If we propagate these maximum position errors to the term proportional to the square of the earth speed in Eq. (3) and corresponding time-ephemeris error ignoring phase effects we derive an upper limit of 2.6 ns independent of assumed period for the error perturbation. If we propagate the maximum error of the heliocentric distance of the Earth to the gravitational potential term in Eq. (3) and corresponding time-ephemeris error ignoring phase effects we derive an upper limit of 0.79 ns $\times P$ where P is the assumed period (in years) of the perturbation. Inside the fitting interval the VSOP82/ELP2000 model and mass inconsistencies with DE200 are partially compensated by the fit. Outside the fitting interval the position discrepancies and associated mass-corrected time-ephemeris errors should grow substantially. When this additional factor is combined with the calculated upper limits for the time-ephemeris error due to the speed and gravitational potential terms it seems plausible that VSOP82/ELP200 fitting errors are responsible for the errors of the mass-corrected series illustrated in Fig. 3.

The final correction we add to the mass-corrected FB3 series is two long-period sinusoids with amplitudes significantly above the FB3 truncation limit. Table 2 presents sinusoidal coefficients which were determined by the method of non-linear least squares using a fit to the residuals between the mass-corrected series and TE405 results. Fig. 4 presents the residuals which are smaller (when compared over the same limited epoch ranges) than the best residuals between numerical and analytical work

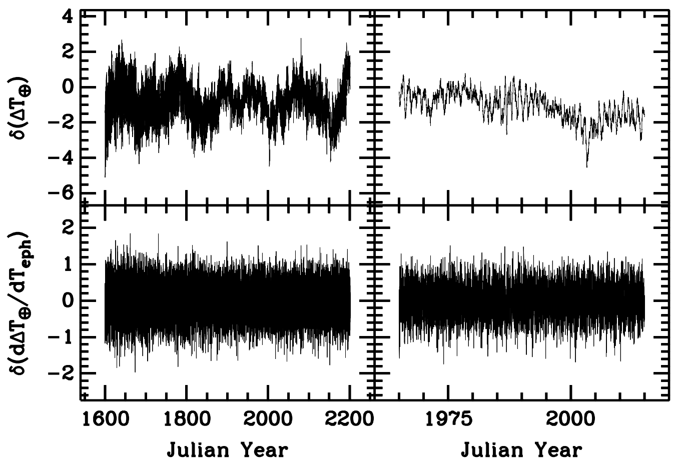

Fig. 4. Differences of a corrected and supplemented FB3 series and TE405 results (with units of ns) and their derivatives (with units of 10^{-15}) over the complete TE405 epoch range and over a modern epoch range. The original FB3 series has been corrected to the DE405 masses (see text) and also supplemented by two long-term sinusoids. The coefficients of these sinusoids are given in Table 2.

that have been previously published (see Fig. 3 of FB and Fig. 6 of Paper I).

Comparison of Figs. 3 and 4 shows how the long-term residuals are improved by adding the sinusoidal corrections to the series. (Similar sinusoidal corrections and residuals were obtained from a fit to the residuals between mass-corrected series and TE200 results.) From these results it should be worthwhile to look for additional terms in the analytical results with periods near 200 and 600 years. However, we caution that although these sinusoids give a good representation of the long-term residuals in Fig. 3 the actual deviations may be caused by a correction of different functional form (e.g., a sum of mixed secular and sinusoidal terms). Because of this uncertainty and because of general extrapolation uncertainties, the correction sinusoids should not be used outside the epoch range of TE405. Even within the epoch range the correction sinusoids only reduce the maximum residuals to 5 ns. Thus, for the most accurate results we recommend using the numerical time-ephemeris rather than the corrected series results. Nevertheless, the corrected series results are useful for determining ΔL_C with the hybrid technique (see next section).

5. ΔL_C, L_C, L_G, L_B, and K

5.1. $\Delta L_C^{(TE200)}$ *and* $\Delta L_C^{(TE405)}$

The determination of ΔL_C from numerical quadrature results *alone* is not recommended. One could adjust ΔL_C so the mean of the integrand of Eq. (3) was zero, but then the definition of ΔL_C would be ambiguous because it would depend strongly on the epoch range chosen for the integration. A much better approach is to define ΔL_C as the constant term (at J2000) in a series approximation for $(U_E + |v_E|^2/2)/c^2$ or the corresponding linear term in a series approximation to the integral of this same quantity. (This integral is the same as the time ephemeris defined by Eq. [3] except that the ΔL_C term is not subtracted from the integrand.) A series approach inevitably has some degree of ambiguity about how the periodic and secular terms are separated since a compromise must be made between the accuracy of the result and the longest period allowed in the analysis (FB). Beyond this period limit the periodic terms are approximated as secular perturbations which in turn affect the derived ΔL_C value. However, the degree of ambiguity is much smaller than in the strictly numerical approach because the period limit is usually quite long. For example, FB treat a sinusoidal term with a $93\,000$-yr period (and by implication all terms of longer period) as a secular perturbation in their work.

A drawback of the analytical approach for determining ΔL_C is the current series are not as accurate as the numerical time ephemeris results (see previous section). Thus, the best current method of determining ΔL_C is with a hybrid approach (Paper I) that combines results from the numerical and series methods. The hybrid method uses a linear least-squares fit of the residuals between series results (with the ΔL_C term removed) and a numerical time ephemeris that is calculated with a preliminary value of ΔL_C. The fit gives a correction to the numerical ΔL_C value that is completely independent of the the series ΔL_C value. The hybrid method removes the slope in the residuals of the fit to a high degree of precision (see next subsection) by averaging out the errors in the series which have time scales less than the epoch range of the numerical ephemeris.

The ΔL_C results calculated with the hybrid technique for the TE200 and TE405 versions of the time ephemeris and for the corrected and supplemented series results of the last section are

$$\Delta L_C^{(\text{TE200})} = 1.480\,826\,857\,03 \times 10^{-8} \qquad (18)$$

and

$$\Delta L_C^{(\text{TE405})} = 1.480\,826\,855\,94 \times 10^{-8}. \qquad (19)$$

We have presented rounded values here that are consistent (aside from two guard digits) with the accuracy discussed in the next subsection. For highest numerical precision of the time ephemeris we use machine-precision values of ΔL_C to remove the slope. These precise ΔL_C values are also stored in the ephemeris header to facilitate future rate adjustments to the time ephemeris when more accurate values of ΔL_C become available.

The present results supersede the $\Delta L_C^{(\text{TE200})}$ and $\Delta L_C^{(\text{TE245})}$ results presented in Table 7 of Paper I. (We have changed our notation from Paper I; currently we prepend a "Δ" to the L_C symbol to indicate whenever post-Newtonian and asteroid effects are not corrected.) Our present $\Delta L_C^{(\text{TE200})}$ value is 3×10^{-19} larger than the previous hybrid value derived from the FB2 series. We ascribe this difference to the previously discussed errors in the FB2 series. Our present $\Delta L_C^{(\text{TE405})}$ value is 13×10^{-19} smaller than the previous hybrid value of $\Delta L_C^{(\text{TE245})}$ derived from the FB2 series. We ascribe most of this difference to differences between the DE245 and DE405 ephemerides. For comparison, note that our present $\Delta L_C^{(\text{TE405})}$ value is 109×10^{-19} smaller than the present value of $\Delta L_C^{(\text{TE200})}$.

5.2. Errors in $\Delta L_C^{(TE200)}$ and $\Delta L_C^{(TE405)}$

The ΔL_C values derived in the last subsection are used for both adjusting the rate of time-ephemeris results and calculating the ratio of ephemeris units to SI units. These two different uses have different error definitions which require separate error discussions.

5.2.1. Error in ΔL_C associated with the rate adjustment of the time ephemeris

We have discussed in Sect. 3 the time-ephemeris errors caused by the JPL ephemeris errors (assuming a perfect rate adjustment), and here we discuss errors in the rate adjustment (assuming a perfect JPL ephemeris) and the associated time-ephemeris errors. Assuming there are no important missing terms in the analytical integration of the VSOP82/ELP200 ephemeris to form the series for the time ephemeris, the errors in the rate adjustment must be ultimately caused by inconsistencies between the VSOP82/ELP200 ephemeris and the particular JPL ephemeris that forms the basis for a numerical time ephemeris. These inconsistencies propagate to differences (e.g., Fig. 3) between the mass-corrected series results for the time ephemeris and the numerical time ephemeris. Furthermore, even when the present analytical time ephemeris is supplemented by additional terms, there are remaining differences (e.g., Fig. 4) which are propagated via the hybrid technique to errors in the derived ΔL_C value and rate adjustment for the numerical time ephemeris.

One measure of the rate-adjustment errors is the formal least-squares errors of the ΔL_C values determined by the hybrid technique. These errors are respectively 1×10^{-21} and 2×10^{-21} for $\Delta L_C^{(\mathrm{TE200})}$ and $\Delta L_C^{(\mathrm{TE405})}$. These fitting errors greatly underestimate the actual errors in the rate adjustment because they only account for residuals with time scales shorter than the epoch range of the time ephemeris.

For future work it should be straightforward to calculate the time ephemeris associated with the long JPL ephemeris, DE406. This ephemeris has 10 times the epoch range of DE405 and the associated time ephemeris could be used to evaluate the long-term errors in the series results. For the present we have no such long-term comparison so a discussion of the long-term errors in the analytical time ephemeris must be speculative. However, it is not unreasonable to suppose that sinusoidal error terms with 10 times the amplitude and period of the 600-yr sinusoid from Table 2 exists in the present series results. The maximum rate adjustment corresponding to such an error term is 3×10^{-18}, and we take this as a speculative estimate of the rate error in the present series results (without ΔL_C term) for the time ephemeris caused by inconsistencies between the VSOP82/ELP200 and JPL ephemerides. The present numerical results for the time ephemeris also share this same rate error because the hybrid technique forces the average rates to be the same for the analytical (without ΔL_C term) and numerical time ephemerides. The speculative estimate of the rate error corresponds to ± 30 ns over the 600-yr epoch range of the present time ephemeris.

Table 3. Propagation of mass-parameter differences and errors to ΔL_C

Mass para.[a]	ΔL_C comp.[b] (10^{-8})	Mass diff.[c] (10^{-3})	ΔL_C diff.[d] (10^{-18})	Mass error[e] (10^{-6})	ΔL_C error[f] (10^{-18})
S^2	1.5	−0.00000041	−6.	0.00040	6.
m_5	0.00018	−0.0013	−2.	0.76	1.
m_6	0.000030	−0.029	−9.	5.1	2.
m_7	0.0000022	−2.5	−55.	1.3	0.
m_8	0.0000017	5.1	87.	2.1	0.
Total	...	...	15.	...	9.

[a] Mass parameter. S^2 is the square of the number of meters per AU, and m_5, m_6, m_7, and m_8 are the respective ratios of the Jupiter, Saturn, Uranus, and Neptune masses to the solar mass. We have not included in this table the calculated ΔL_C differences and errors associated with other mass parameters because their contributions are insignificant.
[b] Component of ΔL_C from Table 8 of Paper I that corresponds to the given mass parameter (see text).
[c] Relative difference (in the sense of DE200 − DE405) in the given mass parameter derived from data in the JPL ephemeris headers.
[d] Calculated difference $\Delta L_C^{(\mathrm{TE200})} - \Delta L_C^{(\mathrm{TE405})}$ from the product of the given ΔL_C component and associated relative mass difference.
[e] Relative error in the given mass parameter derived from Standish (1995).
[f] Estimated maximum error in $\Delta L_C^{(\mathrm{TE405})}$ from the product of the given ΔL_C component and the associated relative error in the mass parameter.

This source of error is much larger than the 0.1 ns estimated accuracy (aside from rate-adjustment errors) of the TE405 time ephemeris (Sect. 3). However, a rate adjustment is a simple correction that can be made whenever improved ΔL_C values become available without having to recalculate the numerical quadrature and Chebyshev interpolation coefficients of the present time ephemeris.

5.2.2. Total error in ΔL_C associated with the conversion from ephemeris to SI units

The total errors of $\Delta L_C^{(\mathrm{TE200})}$ and $\Delta L_C^{(\mathrm{TE2405})}$ can be split into the (just discussed) rate-adjustment component due to inconsistencies between VSOP82/ELP200 and JPL ephemerides and the component due to the errors in the JPL ephemerides themselves.

Because the DE405 ephemeris errors should be substantially smaller than the DE200 ephemeris errors, the ephemeris component of the $\Delta L_C^{(\mathrm{TE200})}$ error should be close to $\Delta L_C^{(\mathrm{TE200})} - \Delta L_C^{(\mathrm{TE405})} = 11 \times 10^{-18}$. The mass-dependent part of this difference can be estimated using the components of ΔL_C from Paper I and the known differences in mass parameters between DE200 and DE405 (Table 3). For this calculation we have assumed that all ΔL_C components are proportional to S^2 (following an argument made in Sect. 4) while the individual components other than the solar component and the component proportional to the square of the Earth speed are proportional to the appropriate ratio of the lunar or planetary mass to the solar mass. The result is the mass-dependent part of $\Delta L_C^{\rm (TE200)} - \Delta L_C^{\rm (TE405)} = 15 \times 10^{-18}$. Thus, by subtraction the mass-independent part of the ephemeris component of the $\Delta L_C^{\rm (TE200)}$ error should be close to -4×10^{-18}.

The ephemeris component of the $\Delta L_C^{\rm (TE405)}$ error is difficult to estimate because we have no better value for comparison. If we propagate the estimated mass parameter errors for DE405 using the same mass model that was used to predict the mass-dependent part of $\Delta L_C^{\rm (TE200)} - \Delta L_C^{\rm (TE405)}$ the resulting mass-dependent error is 9×10^{-18} (Table 3). This error estimate is quite uncertain so for simplicity we have simply added the various error components. The mass-independent error estimate for $\Delta L_C^{\rm (TE405)}$ is even more uncertain; following an argument in Sect. 3 we estimate it as 0.4×10^{-18}, i.e., an order of magnitude less than the mass-independent error for $\Delta L_C^{\rm (TE200)}$.

If we add the speculative rate-adjustment error (previous sub-section) to the present results and round to one significant digit we find the total error in $\Delta L_C^{\rm (TE405)}$ associated with the conversion from ephemeris to SI units is 1×10^{-17}.

5.3. *Recommended values and errors of L_C, L_G, L_B, and K*

Our recommended L_C value is

$$\begin{aligned} L_C &= \Delta L_C^{\rm (TE405)} + \Delta L_C^{\rm (PN)} + \Delta L_C^{\rm (A)} \\ &= 1.480\,826\,867\,41 \times 10^{-8} \pm 2. \times 10^{-17}, \end{aligned} \tag{20}$$

where the post-Newtonian correction (Paper I) is

$$\Delta L_C^{\rm (PN)} = 109.7 \times 10^{-18} \tag{21}$$

and the asteroid correction (Paper I) is

$$\Delta L_C^{\rm (A)} = (5. \pm 5.) \times 10^{-18}. \tag{22}$$

(For Eq. [21] we have corrected a sign error that occurred in front of the integral in Eq. [24] of Paper I that propagated to Eqs. [26], [30], [31], and [38] of that paper.) Even though the value of the asteroid correction is the same as its uncertainty we add this correction anyway because it is known to be positive.

A recent determination of the potential at the geoid yields (Bursa et al. 1997),

$$W_0 = 62\,636\,855.8 \pm 0.5 {\rm m}^2\ {\rm s}^{-2}. \tag{23}$$

(Note there is a typographical error in the value of the W_0 error stated in the abstract of the Bursa et al. paper. We have taken the error from Eq. [12] of that paper which is 10 times the abstract value but consistent with the other error values given in that paper.) We derive from this value and Eq. (10) our recommended value of

$$L_G = 6.969\,290\,112 \times 10^{-10} \pm 6. \times 10^{-18}. \tag{24}$$

From Eqs. (12), (20), and (24) we derive our recommended value of

$$K = 1 + (1.550\,519\,791\,54 \times 10^{-8} \pm 3. \times 10^{-17}), \tag{25}$$

which from Eq. (2) is equivalent to

$$L_B = 1.550\,519\,767\,49 \times 10^{-8} \pm 3. \times 10^{-17}. \tag{26}$$

The uncertainties in $\Delta L_C^{\rm (TE405)}$ and $\Delta L_C^{\rm (A)}$, are probably only reliable in order of magnitude and similarly for the resultant uncertainties given for L_C, K, and L_B which we have estimated by simply adding the component uncertainties and rounding.

6. Conclusion

We present a numerical time ephemeris, TE405, which approximates a relativistic time-dilation integral ($\Delta T_\oplus$, see Eq. [3]) that is required to transform (see Eqs. [1] and [13]) between terrestrial time, TT, and either the independent variable of a modern ephemerides, T_{eph}, or the (solar-system) barycentric coordinate time, TCB. We calculate TE405 using numerical quadrature of quantities supplied by the JPL ephemeris, DE405, and the results are presented (at ftp://astroftp.phys.uvic.ca/pub/irwin/tephemeris/) as a file of Chebyshev interpolation coefficients and associated software similar to the JPL ephemeris itself. The results are meant to be a companion to DE405 and can be used to quickly interpolate values of the time ephemeris or its derivative for any epoch within the range of DE405 (1600 to 2200). The interpolation errors are 0.3 ps in the time ephemeris and 3×10^{-17} in its derivative which provides a numerically clean result for comparison with future time ephemerides. The accuracy (maximum total error from all sources aside from mean rate adjustments) is estimated to be the order of 0.1 ns in the time ephemeris and 1×10^{-16} in its derivative. These levels of accuracy should satisfy all current needs.

We have investigated errors in the the time-ephemeris series of FB. The published coefficients were given with an invalid angular frequency transformation, and this affected the comparisons with both the FB and FB2 series given in Paper I (although the FB3 form of this series as privately communicated from Bretagnon for the present paper does not have this transformation applied). Truncation of the series at a finite number of terms can be another substantial source of error (Fig. 2), and it is essential to use an amplitude limit of at most 10 ps (the limit of the 1705-term FB3 series) in order to obtain precise series results. A mass transformation reduces the discrepancies of short time-scale between the FB3 series and TE405 but still leaves a long-term residual which still deviates by 15 ns from TE405 (Fig. 3). We suggest that the long-term residuals (which can be fit by two sinusoids [Table 2 and Fig. 4]) and somewhat smaller short-term residuals reflect known differences between the analytical VSOP82/ELP2000 ephemerides upon which the FB3 series is based and the numerical JPL ephemeris, DE200.

We use the hybrid technique of Paper I in combination with TE405 and the sinusoid- and mass-corrected form of the FB3 series to determine the $\Delta L_C^{\rm (TE405)}$ value which we use to adjust the rate of TE405 and ultimately determine the K value (Eq. [25]). The K value relates ephemeris units for time and distance and the corresponding SI units for the same quantities.

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