

Non-local bias and the problem of large-scale power in the *Standard Cold Dark Matter* model

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Abstract. We study the effect of non-radial motions, originating from the gravitational interaction of the quadrupole moment of a protogalaxy with the tidal field of the matter of the neighboring protostructures, on the angular correlation function of galaxies. We calculate the angular correlation function using a *Standard Cold Dark Matter* (hereafter SCDM) model ($\Omega = 1$, $h=0.5$, $n = 1$) and we compare it with the angular correlation function of the APM galaxy survey (Maddox et al. 1990; Maddox et al. 1996). We find that taking account of non-radial motions in the calculation of the angular correlation function gives a better agreement of the theoretical prediction of the SCDM model to the observed estimates of large-scale power in the galaxy distribution.

Key words: cosmology: theory – cosmology: large-scale structure of Universe – galaxies: formation

1. Introduction

The galaxy two-point correlation function $\xi_g(r)$ is a powerful discriminant between distinct models of structure formation in the universe. On scales $\geq 10h^{-1}\text{Mpc}$ correlations between galaxies are weak, namely $\xi_g(r) \ll 1$, so one may reasonably expect that $\xi_g(r)$ can be related to the fluctuations in the early universe by linear perturbation theory. The role of the two-points correlation function is particularly important in models that predict fluctuations that obey Gaussian statistics (see Bardeen et al. 1986) because in this case the field is completely specified, in a statistical sense, by a single function: the power spectrum $P(k)$ or by its Fourier transform, the autocorrelation function, $\xi(r)$. This means that a knowledge of $\xi_g(r)$ on large scales would give a powerful constraint on models of the early Universe, and if the fluctuations are Gaussian, we would obtain a complete description of large-scale structure. Analyses of galaxy surveys (APM, QDOT) have shown that excess correlations are found on scales larger than $10h^{-1}\text{Mpc}$ (Maddox et al. 1990; Efstathiou et al.

1990b; Saunders et al. 1991; Maddox et al. 1996) when observations are compared with the predictions of the standard CDM model and also radio galaxies are also strongly clustered on large scales (Peacock 1991; Peacock & Nicholson 1991). These data have hence been widely interpreted as ruling out SCDM model and new alternative models have been introduced in an effort to solve this and other problems of the model.

Several authors (Peebles 1984; Efstathiou et al. 1990a; Turner 1991) have lowered the matter density under the critical value ($\Omega_m < 1$) and have added a cosmological constant in order to retain a flat Universe ($\Omega_m + \Omega_\Lambda = 1$). Mixed dark matter models (MDM) (Shafi & Stecker 1984; Valdarnini & Bonometto 1985; Schaefer et al. 1989; Holtzman 1989; Schaefer 1991; Schaefer & Shafi 1993; Holtzman & Primack 1993) increase the large-scale power because neutrinos free-streaming damps the power on small scales. Alternatively, changing the primeval spectrum solves several problems of CDM (Cen et al. 1992). Finally it is possible to assume that the threshold for galaxy formation is not spatially invariant but weakly modulated (2% ÷ 3% on scales $r > 10h^{-1}\text{Mpc}$) by large scale density fluctuations, with the result that the clustering on large-scale is significantly increased (Bower et al. 1993). This last alternative is part of those models attempting to rescue SCDM by invoking a form of bias that has different effects on clustering on different scales (Babul & White 1991; Bower et al. 1993). In fact as previously reported, large-scale clustering studies such as APM (Maddox et al. 1990; Maddox et al. 1996) and QDOT (Efstathiou et al. 1990b; Saunders et al. 1991; Peacock 1991) suggest a clustering amplitude which is larger than one would expect on the basis of the SCDM model. Moreover the level of temperature fluctuations seen by COBE are consistent with no large-scale bias, but when the CDM model is normalized to COBE results it has problems in accounting for small-scale structure. An obvious way to rescue the model, then, is a scale-dependent bias which can modify the slope of the correlation function so as to make it decay less steeply than the mass autocovariance function on large scales. As shown by Coles (1993) for Gaussian fields, a change of slope of this kind can be achieved by non-local biasing effects such as cooperative galaxy formation (Bower et al. 1993). In this last model Bower et al. (1993)

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adopt the assumption that the threshold level, δ_c , depends on the mean mass density in the domain of influence, rather than being spatially invariant. Galaxy formation is assumed to occur according to the prescriptions of the standard biased galaxy formation theory (Kaiser 1984; Bardeen et al. 1986) but is enhanced by the presence of nearby galaxies. This approach is able to produce enough additional clustering to fit the $\xi_g(r)$ of the APM galaxy survey. The main problem of this and of any theory of galaxy formation involving a bias of any kind is that they are not acceptable until the physical mechanisms producing the bias are elucidated. In some recent papers (Del Popolo & Gambera 1998a,b) we introduced a model that is able to reduce several of the SCDM model problems and also includes (differently from Bower et al. 1993) a clear explanation for the physical mechanisms that produce the bias.

As shown by Barrow & Silk (1981) and Szalay & Silk (1983) the gravitational interaction of the irregular mass distribution of a test proto-structure with the neighbouring ones gives rise to non-radial motions, within the test proto-structure, which are expected to slow the rate of growth of the density contrast and to delay or suppress the collapse. According to Davis & Peebles (1977), Villumsen & Davis (1986) and Peebles (1990) the kinetic energy of the resulting non-radial motions at the epoch of maximum expansion increases so much as to oppose the recollapse of the proto-structure. As shown by Del Popolo & Gambera (1998a,b), within high-density environments, such as rich clusters of galaxies, non-radial motions slow down the collapse of low- ν peaks thus producing an observable variation in the time of collapse of the shell and, as a consequence, a reduction in the mass bound to the collapsed perturbation. Moreover, the delay of the collapse produces a tendency for less dense regions to accrete less mass, with respect to a classical spherical model, inducing a *biasing* of over-dense regions toward higher mass. Non-radial motions change the energetics of the collapse model by introducing another potential energy term in the equation of collapse, leading to a change of the *turn around* epoch, t_m , and consequently the critical threshold, δ_c , for collapse. The change of δ_c is in the same sense as that described by Bower et al. (1993).

In this paper we apply the quoted model to galaxies showing how non-radial motions modify the galaxies correlations. The plan of the paper is the following: in Sect. 2 we introduce the model; in Sect. 3 show the results and in Sect. 4 we draw our conclusions.

2. The model

In the standard high-peak galaxy model, galaxies form from mass located near high peaks of the linear density field. The density contrast at early times, $\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \rho_b}{\rho_b}$ is assumed to be Gaussian, and the field $\delta(\mathbf{x})$ is smoothed by convolving it with a spherical symmetric window function $W(r, R_g)$, where the characteristic scale R_g is chosen so that the enclosed mass matches the halo of a bright galaxy. Galaxy formation sites are identified with peaks rising above a threshold, $\delta > \delta_c$. The value of δ_c is quite dependent on the choice of smoothing window

used to obtain the dispersion (Lacey & Cole 1994). Using a top-hat window function $\delta_c = 1.7 \pm 0.1$, while for a Gaussian window the threshold is significantly lower. In non-spherical situation things are more complicated (Monaco 1995). In any case in the standard biased galaxy formation the threshold is not scale-dependent and is taken to be universal.

Several studies have shown that there is no convincing justification for this choice (Cen & Ostriker 1992; Bower et al. 1993; Coles 1993; Del Popolo & Gambera 1998a,b,c; Kauffmann et al. 1998; Willmer et al. 1998; Governato et al. 1998; Peacock 1998).

Some authors (see Barrow & Silk 1981; Szalay & Silk 1983 and Peebles 1990) have proposed that non-radial motions would be expected within a developing proto-galaxy due to the tidal interaction of the irregular mass distribution around them, typical of hierarchical clustering models, with the neighbouring proto-galaxies. The kinetic energy of these non-radial motions prevents the collapse of the proto-structure, enabling the same to reach statistical equilibrium before the final collapse (the so-called previrialization conjecture by Davis & Peebles 1977, Peebles 1990). In other words one expects that non-radial motions change the characteristics of the collapse and in particular the *turn around* epoch, t_m , and consequently the critical threshold, δ_c , for collapse.

As shown by Del Popolo & Gambera (1998a,b,c), if non-radial motions are taken into account, the threshold δ_c is not constant but is function of mass, M , (Del Popolo & Gambera 1998a,b):

$$\delta_c(\nu) = \delta_{co} \left[1 + \frac{8G^2}{\Omega_0^3 H_0^6 r_i^{10} \bar{\delta} (1 + \bar{\delta})^3} \int_{a_{min}}^{a_{max}} \frac{L^2 \cdot da}{a^3} \right] \quad (1)$$

where $\delta_{co} = 1.68$ is the critical threshold for a spherical model, r_i is the initial radius, L the angular momentum, H_0 and Ω_0 the Hubble constant and the density parameter at the current epoch, respectively, a the expansion parameter and $\bar{\delta}$ the mean fractional density excess inside a shell of given radius. The mass dependence of the threshold parameter, $\delta_c(\nu)$, and the total specific angular momentum, $h(r, \nu) = L(r, \nu)/M_{sh}$, acquired during expansion, was obtained in the same way as described in Del Popolo & Gambera (1998b) and is displayed in Fig. 1.

In order to find the galaxy correlation function, we combine Kaiser's (1984) analysis with the theory of gravitational clustering (Press & Schechter 1974). In this last theory non-linear clumps are identified as overdensities in the filtered linear density field. When these overdensities exceed a critical threshold, δ_c , they will be incorporated in a non-linear object of mass $M \propto R_g^3$ or greater. Since the linear density field is assumed to be Gaussian, the probability that on scale M one would find a density contrast between δ and $\delta + d\delta$ would be:

$$p(\delta)d\delta = \frac{1}{\sqrt{2\pi}\sigma(M)} \exp \left[-\frac{\delta^2}{2\sigma(M)^2} \right] d\delta \quad (2)$$

and the Press-Schechter ansatz leads to the following fraction of mass incorporated in objects of mass $> M$ (or the probability

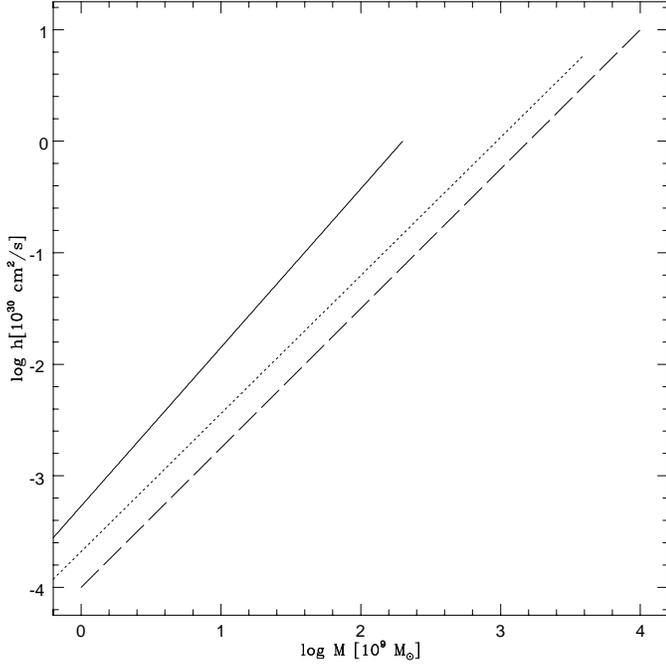


Fig. 1. The specific angular momentum for three values of the parameter ν ($\nu = 2$ solid line, $\nu = 3$ dotted line, $\nu = 4$ dashed line).

that an object of mass M has turned around at any time in the past):

$$P_M = \int_{\delta_c}^{\infty} p(\delta) d\delta = \frac{1}{2} \operatorname{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma}\right) \quad (3)$$

And in a similar way, the probability density for finding simultaneously, on scale M , the density contrast $\delta_1 = \delta(\mathbf{x}_1)$ and $\delta_2 = \delta(\mathbf{x}_2)$ of two field points separated by $r = |\mathbf{x}_1 - \mathbf{x}_2|$ is:

$$p(\delta_1, \delta_2, \xi/\sigma^2) = \frac{1}{2\pi\sqrt{\xi^2(0) - \xi^2}} \times \exp\left[-\frac{\xi(0)\delta_1^2 + \xi(0)\delta_2^2 - 2\xi\delta_1\delta_2}{2(\xi^2(0) - \xi^2)}\right] \quad (4)$$

where

$$\xi(r) = \frac{1}{2\pi^2} \int_0^{\infty} P(k) k^2 \frac{\sin(kr)}{kr} \exp(-k^2 R_g^2) \quad (5)$$

is the mass autocorrelation function of the primeval density distribution, δ_1 and δ_2 are used to denote the density contrast at the positions \mathbf{x}_1 and \mathbf{x}_2 of the two field points and σ denotes the r.m.s. of δ . The power spectrum used has the form given by Bardeen et al. (1986):

$$P(k) = Ak^{-1} [\ln(1 + 4.164k)]^2 \cdot (192.9 + 1340k + 1.599 \cdot 10^5 k^2 + 1.78 \cdot 10^5 k^3 + 3.995 \cdot 10^6 k^4)^{-1/2} \quad (6)$$

and A is the normalizing constant, which gives the amplitude of the power spectrum. Similarly to Bower et al. (1993), since our model, like the original high-peak model, calculates ξ_g from ξ/σ^2 , the amplitude of the power spectrum drops out of our analysis. The probability of finding two objects of masses M

separated by r that have turned around in any time between the actual time and $t = 0$ is:

$$P_{MM} = \int_{\delta_c}^{\infty} \int_{\delta_c}^{\infty} p(\delta_1, \delta_2, \xi/\sigma^2) d\delta_1 d\delta_2 = \frac{1}{2\sigma\sqrt{2\pi}} \int_{\delta_c}^{\infty} \exp\left(-\frac{\delta_1^2}{2\sigma^2}\right) \operatorname{erfc}\left[\frac{\nu - \frac{\xi}{\xi(0)} \frac{\delta_1}{\sigma}}{\sqrt{2(1 - \xi^2/\xi^2(0))}}\right] d\delta_1 \quad (7)$$

Following Kaiser (1984), the correlation function of peaks above a given threshold, on scales larger than R_g , can be approximated by that of points above the same threshold. According to Kaiser's (1984) definition we have:

$$\xi_g(r) = \frac{P_{MM}}{P_M^2} - 1 \quad (8)$$

giving the fractional excess probability that two points at separation r are both above the threshold. Eq. (8) shows that the $\xi_g(r)$ is a function of $\xi(r)/\sigma^2$ and of δ_c . In the limit $\delta_c \rightarrow \infty$, $\xi \rightarrow 0$ we have:

$$\xi_g(r) \simeq \left(\frac{\delta_c}{\sigma}\right)^2 \xi(r) \quad (9)$$

(Kaiser 1984). This approximation is not so accurate on the scales we are considering, we thus prefer to evaluate $\xi_g(r)$ numerically after having reduced the dimensionality of the integrals involved in the calculation, as Bower et al. (1993) have done, by means of:

$$\xi_g(r) = \xi(r) \int_0^1 [\xi(0)^2 - s^2 \xi(r)^2]^{-1/2} \times \exp\left[-\frac{\delta_c^2 \xi(0)}{\xi(0) + s\xi(r)}\right] ds \left[\int_{\delta_c}^{\infty} \exp\left(-\frac{u^2}{2}\right) du\right]^{-2} \quad (10)$$

In order to compare our model predictions of large-scale power in galaxy distribution with its estimates from the APM survey (Maddox et al. 1990) we have to calculate the angular two points autocorrelation function, $w(\vartheta)$. This last is related to the spatial correlation function, $\xi(r)$, through Limber's (1954) equation (see also Peebles 1980, Peacock 1991):

$$w(\vartheta) = \int_0^{\infty} y^4 \phi^2 dy \int_{-\infty}^{\infty} \xi(\sqrt{x^2 + y^2 \theta^2}) \times dx / \left[\int_0^{\infty} y^2 \phi dy\right] \quad (11)$$

where the luminosity function, $\phi(y)$, is that recommended by Maddox et al. (1990) and where:

$$\phi(y) dy = \phi^* y^\alpha \exp(-y) dy \quad (12)$$

being

$$y = 10^{0.4(M_b^*(z) - M)} \quad (13)$$

and

$$\phi^* = 1.3 \times 10^{-2} h^3 \text{Mpc}^{-3} \quad (14)$$

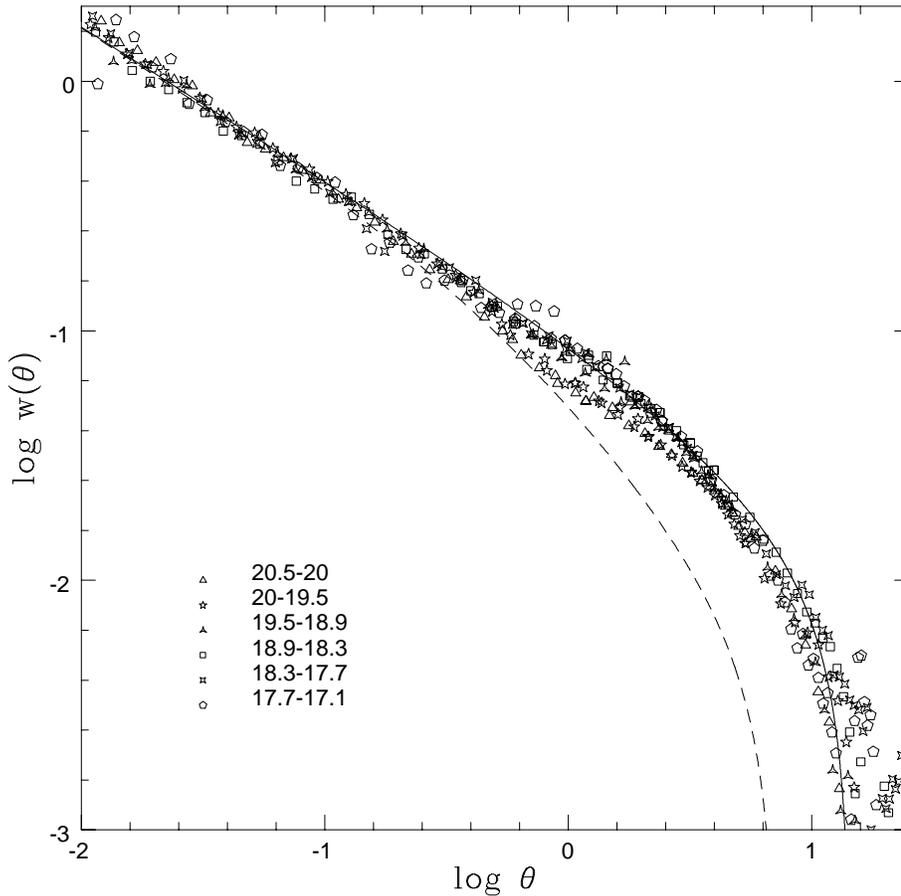


Fig. 2. Angular correlation function for galaxies for the APM survey and in CDM models. The data (kindly provided by W. Sutherland) represents estimates of $w(\theta)$ for six disjoint magnitude slices in the range $17 \leq b_j \leq 20.5$ scaled to the magnitude limit of the Lick catalogue, $b_j = 18.4$, published in Maddox et al. 1996. The dashed line shows correlations in the SCDM model while the solid line shows correlations in our model.

with

$$\begin{aligned}
 M_b^*(z) &= M_0^* + M_1^* z \\
 \alpha(z) &= \alpha_0 + \alpha_1 z \\
 M_0^* &= -19.8 \\
 M_1^* &= 1 \\
 \alpha_0 &= -1 \\
 \alpha_1 &= -2
 \end{aligned}$$

In order to calculate Eq.(11) we need the spatial counterpart $\xi(r)$ for all r . We have used an approach similar to that by Maddox et al. (1990) and Bower et al. (1993), namely we calculated correlation functions according to what we have previously described in this section but on small scales ($r \leq 5.7h^{-1}\text{Mpc}$) we extrapolated our model correlation functions by using $\xi(r) = \left(\frac{5.7h^{-1}\text{Mpc}}{r}\right)^{1.7}$.

3. Results and discussion

The result of our model is directly compared (see Fig. 2) with the angular correlation function estimate by Maddox et al. (1990) from the APM survey, in order to find whether it can match observed estimates of large-scale power in the galaxy distribution. The data (kindly provided by W. Sutherland) plot the angular correlation function, $w(\theta)$, for six disjoint apparent magnitude slices between $17 \leq b_j \leq 20.5$, all scaled to the magnitude

limit of the Lick catalogue (Groth & Peebles 1977), $b_j = 18.4$. At small angles the angular correlation function is a power law:

$$w(\theta) = B\theta^{1-\gamma} \quad (15)$$

For $0.01^\circ < \theta < 1^\circ$ the values of γ and B are:

$$\begin{aligned}
 \gamma &= 1.699 \pm 0.032 \\
 B &= 0.0284 \pm 0.0029
 \end{aligned}$$

(Maddox et al. 1996). At larger angles the angular correlation function steepens and lies below the extrapolation of the power law. At magnitude $b_j = 20$ the steepening occurs at $\simeq 2^\circ$.

The dotted line shows the angular correlation function calculated using the SCDM model, then assuming a uniform biasing threshold. Fig. 2 clearly shows the well known SCDM problem of lack of large-scale power, namely the two-point angular correlation function, when fit to the observations on the 0.03–0.3 degree scale, is significantly below the observations on scales greater than 1 degree, when these are scaled to the depth of the Lick survey. This provides strong evidence for large-scale power in the galaxy distribution that cannot be reconciled with the SCDM model. The same conclusion is achieved by analysis of three-dimensional data (Vogeley et al. 1992) and from an independent redshift survey of a subset of the APM-Stromlo galaxies (Loveday et al. 1992). An important point to stress is that this problem is normalization independent, it cannot be solved by changing the bias strength, (Maddox et al. 1990; Bartlett & Silk

1993; Ostriker 1993) because the theoretical $w(\theta)$ has the wrong shape.

The solid line shows the angular correlation function obtained in our model (CDM with $\Omega = 1$, $h = 0.5$ and taking account of non-radial motions). Our $w(\vartheta)$ is less steep, at large angles, than that expected from the SCDM model and is in better agreement with observations. The relatively enhanced power on large scales, with respect to smaller scales, is insensitive to the amplitude of the power spectrum: the better agreement with observations is due to the non uniform biasing threshold in our model as given by Eq. (1).

Similarly to what is shown in Del Popolo & Gambera (1998a,b), Fig. 3 shows that the threshold, δ_c , is a decreasing function of the mass, M . This means that peaks in more dense regions must have a lower value of the threshold, δ_c , with respect to those of under-dense regions, in order to form structure. In fact, as clearly shown in Fig. 1, the angular momentum acquired by a shell centered on a peak in the CDM density distribution is anti-correlated with density: high-density peaks acquire less angular momentum than low-density peaks (Hoffman 1986; Ryden 1988). A greater amount of angular momentum acquired by low-density peaks (with respect to the high-density ones) implies that these peaks can more easily resist gravitational collapse and consequently it is more difficult for them to form structure. This results in a tendency for less dense regions to accrete less mass, with respect to a classical spherical model, inducing a *biasing* of over-dense regions toward higher mass. This also explains why the value of δ_c , that a peak must rise above in order to form a structure, is larger for low-mass peaks than high density ones. The space dependence of the threshold implies also a scale dependence of the bias parameter, b , because the two parameters are connected (see Borgani 1990; Mo & White 1996; Del Popolo & Gambera 1998a,b,c).

Another way of describing the differences between our model and the standard biased galaxy formation is the following. In order to describe the distribution of objects we consider the biased field:

$$\rho_{\delta_c, R_g}(x) = t[\delta_R(x) - \delta_c] \quad (16)$$

where $t(y)$, the threshold function, relates the biased field $\rho_{\delta_c, R_g}(x)$ to the background field, $\rho_{R_g}(x)$. The threshold function, $0 \leq t(y) \leq 1$, gives the probability that a fluctuation of a given amplitude turns out to be revealed as an object, while the threshold level δ_c is defined as the value of the density contrast at which a fluctuation has a probability of 1/2 of giving rise to an object. The simplest case is that of the standard biased galaxy formation in which the selection function, is a Heaviside function $t(y) = \theta(y)$ and δ_c is the limiting height of the selected fluctuations. In this scheme, fluctuations below δ_c have zero probability of developing an observable object and fluctuations above δ_c have zero probability not to develop an object. As we showed in a previous paper (Del Popolo & Gambera 1998a) one of the effects of non-radial motions is that the threshold function differs from a Heaviside function (sharp threshold), (see Fig. 7 by Del Popolo & Gambera 1998a). This means that objects can

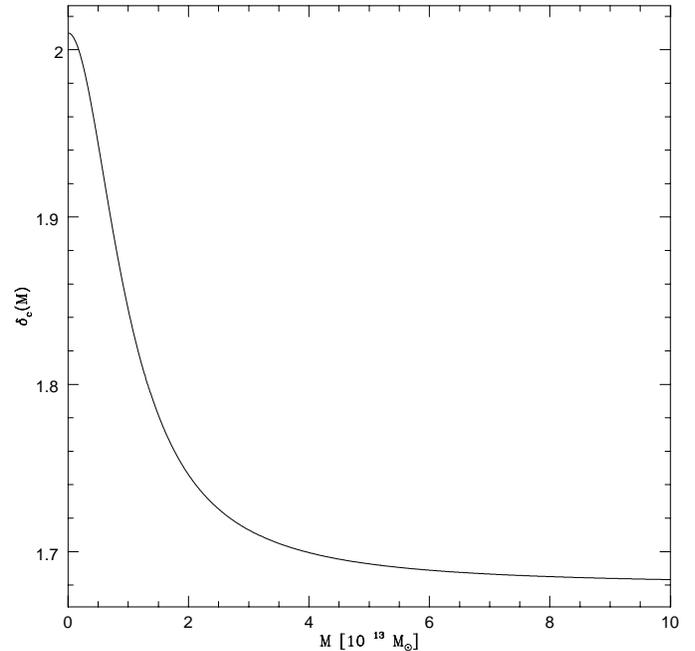


Fig. 3. The threshold δ_c as a function of the mass M , for a CDM spectrum ($\Omega_0 = 1$, $h = 1/2$), taking account of non-radial motions.

also be formed from fluctuations below δ_c and that there is a non-zero probability for fluctuations above δ_c to be sterile.

The model is similar to the cooperative galaxy formation theory but now there is a simple explanation for the mass dependence of the threshold, δ_c : it is due to non-radial motions. As we said previously non-radial motions arise from the gravitational interaction of the quadrupole moment of the system with the tidal field of the matter of the neighbouring proto-galaxies (Barrow & Silk 1981; Szalay & Silk 1983; Peebles 1990). The energy connected to these motions enters the equation of spherical collapse thus changing the turnaround epoch and δ_c . Being Eq. (9) dependent on the threshold, δ_c , the galaxy correlation function is changed as well. The final result is an increase of the galaxy correlations, ξ_g , predicted on large scales.

4. Conclusion

The galaxy two-point correlation function $\xi_g(r)$ has a special place amongst statistics of the galaxy distribution and is a powerful discriminant between distinct models of structure formation in the universe. Several studies (Maddox et al. 1990, Efstathiou et al. 1990a; Saunders et al. 1991; Loveday et al. 1992; Maddox et al. 1996) have shown that the $\xi_g(r)$ obtained from an SCDM model, independently from normalization, is difficult to reconcile with the observed $\xi_g(r)$ if the bias is scale independent. In this paper we showed how a scale dependent bias, due to non-radial motions, can reduce the problem of large-scale lack of power in the SCDM model. We calculated the mass dependence of the threshold parameter, δ_c , (due to non-radial motions) and then used it to find the two-points correlation function following Press & Schechter (1974) and Kaiser (1984). This was used to find the angular correlation function, $w(\theta)$, through Limber's

(1954) equation. The $w(\theta)$ found in this way was compared with data of the APM survey (Maddox et al. 1990; Maddox et al. 1996). We found a less steep $w(\theta)$ in good agreement with estimates of large-scale power in the galaxy distribution.

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References

- Babul A., White S.D.M., 1991, MNRAS 253, 31P
 Bardeen J.M., Bond J.R., Kaiser N., Szalay A.S., 1986, ApJ 304, 15
 Barrow J.D., Silk J., 1981, ApJ 250, 432
 Bartlett J.G., Silk J., 1993, ApJ 407, L45
 Borgani S., 1990, A&A 240, 223
 Bower R.G., Coles P., Frenk C.S., White S.D.M., 1993, ApJ 405, 403
 Cen R., Ostriker J.P., 1992, ApJ 399, L113
 Cen R.Y., Gnedin N.Y., Kofman L.A., Ostriker J.P., 1992, preprint
 Coles P., 1993, MNRAS 262, 1065
 Davis M., Peebles P.J.E., 1977, ApJS 34, 425
 Del Popolo A., Gambera M., 1998a, A&A 337, 96
 Del Popolo A., Gambera M., 1998b, A&A in print (see also SISSA preprint, astro-ph/9806044)
 Del Popolo A., Gambera M., 1998c, Proceedings of the VIII Conference on Theoretical Physics: General Relativity and Gravitation, Bistritza, Rumania, June 15–18, 1998
 Efstathiou G., Sutherland W.J., Maddox S.J., 1990a, Nat 348, 705
 Efstathiou G., Kaiser N., Saunders W., et al., 1990b, MNRAS 247, 10p
 Governato F., Babul, A., Quinn T., et al., 1998, SISSA preprint astro-ph/9810189
 Groth E.J., Peebles P.J.E., 1977, ApJ 217, 385
 Hoffman Y., 1986, ApJ 301, 65
 Holtzman J., 1989, ApJS 71, 1
 Holtzman J., Primack J., 1993, Phys. Rev. D43, 3155
 Kaiser N., 1984, ApJ 284, L9
 Kauffmann G., Colberg J.M., Diaferio A., White S.D.M., 1998, SISSA preprint astro-ph/9805283
 Lacey C., Cole S., 1994, MNRAS 271, 676
 Limber, D.N., 1954, ApJ 119, 655
 Loveday J., Efstathiou G., Peterson B.A., Maddox S.J., 1992, ApJ 400, L43
 Maddox S.J., Efstathiou G., Sutherland W.J., Loveday J., 1990, MNRAS 242, 43p
 Maddox S.J., Efstathiou G., Sutherland W.J., 1996, MNRAS 283, 1227
 Mo H.J., White S.D.M., 1996, MNRAS 282, 347
 Monaco P., 1995, ApJ 447, 23
 Ostriker J.P., 1993, ARA&A 31, 689
 Peacock J.A., 1991, MNRAS 253, 1p
 Peacock J.A., 1998, SISSA preprint astro-ph/9805208
 Peacock J.A., Nicholson D., 1991, MNRAS 253, 307
 Peebles P.J.E., 1980, The large scale structure of the Universe. Princeton University Press
 Peebles P.J.E., 1984, ApJ 284, 439
 Peebles P.J.E., 1990, ApJ 365, 27
 Press W.H., Schechter P., 1974, ApJ 187, 425
 Ryden B.S., 1988, ApJ 329, 589
 Saunders W., Frenk C., Rowan-Robinson M., et al., 1991, Nat 349, 32
 Schaefer R.K., 1991, Int. J. Mod. Phys. A6, 2075
 Schaefer R.K., Shafi Q., 1993, Phys. Rev. D47, 1333
 Schaefer R.K., Shafi Q., Stecker F., 1989, ApJ 347, 575
 Shafi Q., Stecker F.W., 1984, Phys. Rev. D29, 187
 Szalay A.S., Silk J., 1983, ApJ 264, L31
 Turner M.S., 1991, Phys. Scr. 36, 167
 Valdarnini R., Bonometto S.A., 1985, A&A 146, 235
 Villumsen J.V., Davis M., 1986, ApJ 308, 499
 Vogeley M.S., Park C., Geller M.J., Huchra J.P., 1992, ApJ 391, L5
 Willmer C.N.A., Nicolaci da Costa L., Pellegrini P.S., 1998, SISSA preprint astro-ph/9803118