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A dynamic solar core model: on the activity-related changes of the neutrino fluxes

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Abstract. I point out that the energy sources of the Sun may actually involve runaway nuclear reactions as well, developed by the fundamental thermonuclear instability present in stellar energy producing regions. In this paper I consider the conjectures of the derived model for the solar neutrino fluxes in case of a solar core allowed to vary in relation to the surface activity cycle. The observed neutrino flux data suggest a solar core possibly varying in time. In the dynamic solar model the solar core involves a " quasi-static" energy source produced by the quiet core with a lower than standard temperature which may vary in time. Moreover, the solar core involves another, dynamic energy source, which also changes in time. The sum of the two different energy sources may produce quasi-constant flux in the SuperKamiokande because it is sensitive to neutral currents, anti-neutrinos (and axions), therefore it observes the sum of the neutrino flux of two sources which together produce the solar luminosity. A dynamic solar core model is developed to calculate the contributions of the runaway source to the individual neutrino detectors. The results of the dynamic solar core model suggest that since the HOMESTAKE detects mostly the high energy electron neutrinos, therefore the HOMESTAKE data may anticorrelate with the activity cycle. Activity correlated changes are expected to be present only marginally in the GALLEX and GNO data. The gallium detectors are sensitive mostly to the pp neutrinos, and the changes of the pp neutrinos arising from the SSM-like core is mostly compensated by the high-energy electron neutrinos produced by the hot bubbles of the dynamic energy source. The dynamic solar model suggests that the GALLEX data may show an anti-correlation, while the SuperKamiokande data may show a correlation with the activity cycle. Predictions of the dynamic solar model are presented for the SNO and Borexino experiments which can distinguish between the effects of the MSW mechanism and the consequences of the dynamic solar model. The results of the dynamic solar model are consistent with the present helioseismic measurements and can be checked with future helioseismic measurements as well. In the Appendix, the physical parameters of the bubbles are derived, including their temperatures, energy contents, sizes, velocities, lifetimes, and the possible lengths of path they can travel in the Sun.

Key words: nuclear reactions, nucleosynthesis, abundances – Sun: activity – elementary particles – stars: interiors

1. Introduction

The presence of the neutrino problems extends not only to the field of astrophysics (Bahcall et al. 1998a), but to the anomalies of the atmospheric neutrinos. Apparently, the possible solutions to these unexpected anomalies are not compatible allowing three neutrino flavours (Kayser, 1998).

Nevertheless, the apparent paradox may be resolved when taking into account the effects of the thermonuclear runaways present in stellar cores (Grandpierre, 1996). The neutrinos produced by the runaways may contribute to the events detected in the different neutrino detectors. Estimating the terms arising from the runaways, the results obtained here are at present compatible even with standard neutrinos. The objections raised against a possible astrophysical solutions to the solar neutrino problems (see e.g. Hata et al. 1994, Heeger & Robertson, 1996, Berezinsky et al., 1996, Hata & Langacker, 1997) are valid only if the assumption, that the solar luminosity is supplied exclusively by pp and CNO chains, is fulfilled. Therefore, when this assumption is not fulfilled, i.e. a runaway energy source supplies in part the total solar energy production, a more general case has to be considered. In this manuscript I attempt to show that the presence of the runaway energy source, indicated already by first physical principles (Grandpierre, 1977, 1984, 1990, 1996; Zeldovich & Novikov, 1971, Zeldovich et al. 1981; see also the Appendix) could be described in a mathematically and physically consistent way. The contributions of the runaway source to the neutrino detector data may be determined, allowing also solar cycle changes in the neutrino production. The results of Fourier analysis (Haubold, 1997) and wavelet analysis (Haubold, 1998) of the new solar neutrino capture rate data for the Homestake experiment revealed periodicities close to 10 and 4.76 years.

2. Basic equations and the SSM-like approach

The basic equations of the neutrino fluxes in the standard solar models are the followings (see e.g. Heeger & Robertson, 1996):

$$S_K = a_{K8}\Phi_8 \quad (1)$$

$$S_C = a_{C1}\Phi_1 + a_{C7}\Phi_7 + a_{C8}\Phi_8 \quad (2)$$

$$S_G = a_{G1}\Phi_1 + a_{G7}\Phi_7 + a_{G9}\Phi_8, \quad (3)$$

with a notation similar to that of Heeger & Robertson (1996): the subscripts i = 1, 7 and 8 refer to $pp + pep$, $Be + CNO$ and B reactions. The S_j-s are the observed neutrino fluxes at the different neutrino detectors, in dimensionless units, j = K, C, G to the SuperKamiokande, chlorine, and gallium detectors. The observed averaged values are $S_K = 2.44$ (Fukuda et al., 1998), $S_C = 2.56$ (Cleveland et al., 1988) and $S_G = 76$ (Cleveland et al., 1998). Φ_i are measured in $10^{10} \nu cm^{-2} s^{-1}$. Similar equations are presented by Castellani et al. (1994), Calabresu et al. (1996), and Dar & Shaviv (1998) with slightly different parameter values. Using these three detector-equations to determine the individual neutrino fluxes Φ_i, I derived that

$$\Phi_8 = S_K / a_{K8} \tag{4}$$

$$\Phi_1 = (a_{G7} S_C - a_{C7} S_G + S_K / a_{K8} (a_{C7} a_{G8} - a_{G7} a_{C8})) / a_{G7} a_{C1} - a_{C7} a_{G1} \tag{5}$$

and

$$\Phi_7 = (a_{G1} S_C - a_{C1} S_G + S_K / a_{K8} (a_{C1} a_{G8} - a_{G1} a_{C8})) / a_{G1} a_{C7} - a_{C1} a_{G7}. \tag{6}$$

Now let us see how these equations may serve to solve the solar neutrino problems. There are three solar neutrino problems distinguished by Bahcall (1994, 1996, 1997): the first is related to the lower than expected neutrino fluxes, the second to the problem of missing beryllium neutrinos as relative to the boron neutrinos, and the third to the gallium detector data which do not allow a positive flux for the beryllium neutrinos in the frame of the standard solar model. It is possible to find a solution to all of the three neutrino problems if we are able to find positive values for all of the neutrino fluxes in the above presented equations. I point out, that the condition of this requirement can be formulated with the following inequality:

$$S_K < (a_{G1} S_C - a_{C1} S_G) / (a_{C1} a_{G8} / a_{K8} - a_{G1} a_{C8} / a_{K8}) \tag{7}$$

Numerically,

$$\Phi_7 = 0.4647 S_C - 0.0014 S_G - 0.5125 S_K. \tag{8}$$

If we require a physical $\Phi_7 > 0$, with the numerical values of the detector sensitivity coefficients, this constraint will take the following form:

$$S_K < 0.9024 S_C - 0.0027 S_G \simeq 2.115. \tag{9}$$

In the obtained solutions the total neutrino flux is compatible with the observed solar luminosity L_{Sun}, but the reactions involved in the SSM (the pp and CNO chains) do not produce the total solar luminosity. The detector rate inequalities (7) or (9) can be fulfilled only if we separate a term from $S_K(0)$, $S_K(x)$ which represents the contribution of non-pp,CNO neutrinos to the SuperKamiokande measurements (Fukuda et al. 1998)(and, possibly, also allow the existence of $S_C(x)$ and $S_G(x)$). The presence of a non-electron neutrino term in the SuperKamiokande is interpreted until now as indication to neutrino oscillations. Nevertheless, thermal runaways are indicated to be present in the solar core that may produce high-energy electron neutrinos, as well as muon and tau neutrinos, since $T > 10^{11} K$ is predicted for the hot bubbles (Grandpierre, 1996). Moreover, the explosive reactions have to produce high-energy axions to which also only the SuperKamiokande is sensitive (Raffelt, 1997, Engel et al., 1990). Also, the SuperKamiokande may detect electron antineutrinos arising from the hot bubbles. This indication suggests a possibility to interpret the neutrino data with standard neutrinos as well.

To determine the $S_i(x)$ terms I introduced the "a priori" knowledge on the pp,CNO chains, namely, their temperature dependence. This is a necessary step to subtract more detailed information from the neutrino detector data. In this way one can derive the temperature in the solar core as seen by the different type of neutrino detectors. I note that finding the temperature of the solar core as deduced from the observed neutrino fluxes does not involve the introduction of any solar model dependency, since the neutrino fluxes of the SSM pp,CNO reactions depend on temperature only through nuclear physics. Instead, it points out the still remaining solar model dependencies of the previous SSM calculations and allowing other types of chains, it removes a hypothetical limitation, and accepting the presence of explosive chains as well, it probably presents a better approach to the actual Sun.

The calculations of the previous section suggested to complete the SuperKamiokande neutrino-equation with a new term

$$S_K = T^{24.5} \Phi_8(SSM) a_{K8} + S_K(x), \tag{10}$$

where T is the dimensionless temperature $T = T(actual)/T(SSM)$. The one-parameter allowance describes a quiet solar core with a temperature distribution similar to the SSM, therefore it leads to an SSM-like solution of the standard neutrino flux equations (see Grandpierre, 1999). An essential point in my calculations is that I have to use the temperature dependence proper in the case when the luminosity is not constrained by the SSM luminosity constraint, because another type of energy source is also present. The SSM luminosity constraint and the resulting composition and density readjustments, together with the radial extension of the different sources of neutrinos, modify this temperature dependence. The largest effect arises in the temperature dependence of the pp flux: $\Phi_1 \propto T^{-1/2}$ for the SSM luminosity constraint (see the results of the Monte-Carlo simulations of Bahcall & Ulrich, 1988), but $\Phi_1 \propto T^4$ without the SSM luminosity constraint. Inserting the temperature-dependence of the individual neutrino fluxes for the case when the solar luminosity is not constrained by the usual assumption behind the SSM (Turck-Chieze & Lopes, 1993) into the chlorine-equation, we got the temperature dependent chlorine neutrino-equation

$$S_C(T) = a_{C1} T_{C,0}^4 \Phi_1(SSM) + a_{C7} T_{C,0}^{11.5} \Phi_7(SSM) + a_{C8} T_{C,0}^{24.5} \Phi_8(SSM) + S_C(x) \tag{11}$$

Similarly, the temperature-dependent gallium-equation will take the form:

$$S_G(T) = a_{G1}T^4_{G,0}\Phi_1(SSM) + a_{G7}T^{11.5}_{G,0}\Phi_7(SSM) + a_{G8}T^{24.5}_{G,0}\Phi_8(SSM) + S_G(x). \quad (12)$$

Now let us first determine the solutions of these equations in the case $S_i(x) = 0$. The obtained solutions T_i will be relevant to the SSM-like solar core. Now we know that the Sun can have only one central temperature T. Therefore, the smallest T_i-s will be the closer to the actual T of the SSM-like solar core, and the larger T_i-s will indicate the terms arising from the runaways. In this way, it is possible to determine the desired quantities $S_i(x)$.

From the observed S_i values, it is easy to obtain $\Phi_1(SSM) = 5.95 \times 10^{10}\, cm^{-2}s^{-1}$, $\Phi_7(SSM) = 0.594 \times 10^{10}\, cm^{-2}s^{-1}$ and $\Phi_8(SSM) = 0.000515 \times 10^{10} cm^{-2}s^{-1}$. With these values, the chlorine neutrino-temperature from (11) $T_{Cl} \simeq 0.949T(SSM)$, the gallium neutrino-temperature is from (12) $T_{Ga} \simeq 0.922T(SSM)$ and the SuperKamiokande neutrino-temperature is from (10) $T_{SK} \simeq 0.970T(SSM)$. The neutrino flux equations are highly sensitive to the value of the temperature. Assuming that the actual Sun follows a standard solar model but with a different central temperature, the above result shows that the different neutrino detectors see different temperatures. This result suggest that the different neutrino detectors show sensitivities different from the one expected from the standard solar model, i.e. some reactions produce neutrinos which is not taken into account into the standard solar model, and/or that they are sensitive to different types of non-pp,CNO runaway reactions. Let us explore the consequences of this conjecture.

3. Dynamic models of the solar core

3.1. Static core

Obtaining a Ga neutrino-temperature is $T_{Ga} \simeq 0.922T(SSM)$, this leads to a pp luminosity of the Sun around $L_{pp} \simeq 72\%L(SSM)$. The remaining part of the solar luminosity should be produced by the hot bubbles, $L_b \simeq 28\%L(SSM)$. The runaway nuclear reactions proceeding in the bubbles (and possibly in the microinstabilities) should also produce neutrinos, and this additional neutrino-production, Φ_b should generate the surplus terms in the chlorine and water Cherenkov detectors as well. At present, I was not able to determine directly, which reactions would proceed in the bubbles, and so it is not possible to determine directly the accompanying neutrino production as well. Nevertheless, it is plausible that at high temperatures $10^{10-11}K$ (Grandpierre, 1996), such nuclear reactions occur as at nova-explosions or other types of stellar explosions. Admittedly, these could be rapid hydrogen-burning reactions, explosive CNO cycle, and also nuclear reactions producing heat but not neutrinos, like e.g. the explosive triple-alpha reaction (Audouze et al. 1973, Dearborn et al. 1978). At present, I note that the calculated bubble luminosity ($\simeq 28\%$) may be easily consistent with the calculated non-pp,CNO neutrino fluxes $\Delta S_{Cl} = S_C(T_{Cl} = 0.949) - S_C(T_{Ga} = 0.922) \simeq 1.04$, $\Delta S_{Cl}/S_{Cl} \simeq 41\%$, and $\Delta S_{SK} = S_{SK}(T_{SK} = 0.970) - S_{SK}(T_{Ga} = 0.922) \simeq 1.74$, $\Delta S_{SK}/S_{SK} \simeq 71\%$.

The above results are in complete agreement with the conclusion of Hata et al. (1994), namely: "We conclude that at least one of our original assumptions are wrong, either (1) Some mechanism other than the pp and the CNO chains generates the solar luminosity, or the Sun is not in quasi-static equilibrium, (2) The neutrino energy spectrum is distorted by some mechanism such as the MSW effect; (3) Either the Kamiokande or Homestake result is grossly wrong." These conclusions are concretised here to the following statements: (1) a runaway energy source is present in the solar core, and the Sun is not in a thermodynamic equilibrium, (2) this runaway source distorts the standard neutrino energy spectrum, and perhaps the MSW effect also contributes to the spectrum distortion (3) The Homestake, Gallex and SuperKamiokande results contains a term arising from the non-pp,CNO source, which has the largest contribution to the SuperKamiokande, less to the Homestake, and the smallest to the Gallex.

The helioseismic measurements are regarded as being in very good agreement with the SSM. However, the interpretation of these measurements depends on the inversion process, which uses the SSM as its basis. Moreover, the different helioseismic measurements at present are contradicting below $0.2R_{Sun}$ (Corbard et al., 1998).

We can pay attention to the fact that the energy produced in the solar core do not necessarily pours into thermal energy, as other, non-thermal forms of energy may also be produced, like e.g. energy of magnetic fields. The production of magnetic fields can significantly compensate the change in the sound speed related to the lower temperature, as the presence of magnetic fields may accelerate the propagation of sound waves with the inclusion of magnetosonic and Alfven magnetohydrodynamical waves.

The continuously present microinstabilities should produce a temperature distribution with a double character, as part of ions may posses higher energies. Their densities may be much lower than the respective ions closer to the standard thermodynamic equilibrium, and so they may affect and compensate the sound speed in a subtle way. Recent calculations of the non-maxwellian character of the energy distribution of particles in the solar core (Degl'Innocenti et al., 1998) indicate that the non-maxwellian character leads to lowering the SSM neutrino fluxes and, at the same time, produces higher central temperatures. This effect may also compensate for the lowering of the sound speed by lowering of central temperature.

At the same time, an approach specially developed using helioseismic data input instead of the luminosity constraint, the seismic solar model indicates a most likely solar luminosity around $0.8L_{Sun}$(Shibahashi & Takata, 1996, Figs. 7–10), which leads to a seismological temperature lower than its SSM counterpart, $\Delta T \simeq 6\%$. On the other hand, as Bludman et al. (1993) pointed out, the production of high energy ^8B neutrinos and intermediate energy ^7Be neutrinos depends very sensitively on the solar temperature in the innermost 5% of the Sun's radius.

Accepting the average value of $R_K = S_{Kam}(obs.)/S_{Kam}(SSM) = 0.474$, this value gives $T_K \simeq 0.97$. With $S_G = 73.4SNU$, the derived gallium-temperature will be $T_G \simeq$

0.93. With $S_C = 2.56$ (Cleveland et al. 1998), $T_C \simeq 0.95$. The result that $S_G(x) < S_C(x) < S_K(x)$ can arise from the circumstance that the gallium detectors are less sensitive to intermediate and high-energy neutrinos than the chlorine one, which detects less runaway neutrino than the SuperKamiokande. Therefore, if thermonuclear runaways produce intermediate- and/or high-energy neutrino flux in the Sun, it results a relatively smaller contribution in the gallium detectors than in the chlorine one. Moreover, the SuperKamiokande can detect also runaway muon and tau neutrinos besides the high-energy electron neutrinos, therefore they can contribute with an extra term which would give account why the Kamiokande observes a larger neutrino flux than the Homestake. Therefore, the deduced three temperatures actually indicate that the solar core is actually cooler than the standard one by an amount around 7%. Therefore, the beryllium neutrino flux in the dynamical solar model is estimated as 43% of its SSM expected value. The luminosity of the SSM-like core is around 75% of L_{Sun}, therefore the bubble luminosity has to be $25\% L_{Sun}$. The boron neutrino flux of the SSM like core will be 16.9% of the SSM value. Therefore, the bubbles has to produce the remaining $\Phi_b = 30.5\% \Phi_K(SSM)$ of the high energy neutrinos observed by the SuperKamiokande. This requirement may be easily satisfied and it may be consistent with the result obtained that the bubble luminosity is 25% of the solar luminosity, too.

The dynamic solar model predicts a beryllium neutrino flux $\leq 43\%$ of the SSM value, corresponding to a temperature of $T(DSM) \leq 93\%$. This estimation offers a prediction for the Borexino neutrino detector $\Phi(Borexino) = \Phi_{Be}(SSM) \times T(DSM)^{11.5} + \Phi(bubbles) \leq \mathbf{0.43\Phi_{Be}(SSM)} + \Phi(bubbles)$. Regarding the SNO detector, I can assume that the neutral currents are produced by the electron neutrinos of the SSM-like core plus all kinds of neutrinos produced by the hot bubbles. Therefore, the prediction of the DSM is $\Phi(SNO) = \Phi(SSM) \times T(DSM)^{24.5} + \Phi(bubbles) \simeq \mathbf{0.17\Phi(SSM)} + \Phi(bubbles)$. These predictions differ significantly from the MSW SSM-values. Therefore, the future observations may definitively decide which model describes better the actual Sun, the SSM-based MSW effect or the dynamic solar model. In the interpretation of the future measurements it will be important also to take into account the possible dependence of the neutrino fluxes on the solar cycle.

3.2. Around activity maximum

Similarly, we can apply the equations given to derive the temperatures as seen by the different neutrino detectors in relation to the phases of solar activity. Around solar activity maximum the Kamiokande reported no significant deviancies from the averaged neutrino flux, therefore I can take $R_K(max) = 0.474$ which leads to $T_K(max) = 0.97$ again. With the data of Cleveland et al. (1998), neutrino fluxes were measured in two solar activity maximum period, in 1980 the result was 17.2% and in 1989 around 42.5%, which compares to the reported average value of 47.8%. Since the average absolute flux is $2.56 SNU$, this refers to an expected flux of $5.36 SNU$. These values lead to $S_C(max) \simeq 1.60 SNU$. Also, the Gallex collaboration did not report about activity related changes in their observed neutrino data, therefore $S_G(max) = 76 SNU$ can be used. Solving the neutrino flux equations for an assumed SSM-like solar core, the resulting temperatures will be $T_C(max) \simeq \mathbf{0.93}$ and $T_G(max) \simeq 0.922$.

The obtained results, $T_K(max) \simeq 0.97$, $T_C(max) \simeq 0.93$, $T_G(max) \simeq 0.92$, are consistent with the assumption that in the solar maximum the Gallex and the Homestake detect only the neutrinos from the SSM-like solar core, which has a temperature around 8% lower than in the SSM, or that in the solar maximum the neutrinos produced by the hot bubbles contribute mostly to the SuperKamiokande data. This results may be regarded as well fitting to the main point of the paper, namely that the neutrino flux produced by the hot bubbles produce muon and tau neutrinos, axions and anti-neutrinos, to which only the SuperKamiokande is sensitive.

3.3. Around solar activity minimum

Using a value $R_K(min) = 0.474$ for the minimum of the solar activity, the Kamiokande temperature will be $T_K(min) \simeq 0.97$. With the data presented in Cleveland et al. (1998), in the periods around solar minimum the Homestake measured $0.823\, counts\ day^{-1}$ around 1977, $0.636\, counts\ day^{-1}$ around 1987, and $0.634\, counts\ day^{-1}$ around 1997. These values average to $0.299\, counts\ day^{-1}$, suggesting an $S_C(min) \simeq 3.737$ SNU. With this $S_C(min)$ the neutrino flux equations leads to $T_C(min) \simeq 0.97$. Now the Gallex results marginally indicates larger than average counts around 1995–1997, as reported by the Gallex-IV measurements of $117 \pm 20\, SNU$. This value leads to a $T_G(min) \simeq 0.99$, i.e. an anti-correlation with the solar cycle. For a temperature of $T_G(min) \simeq 0.97$ the $S_G(min)$ would be $\simeq 96\, SNU$.

The results obtained above suggest that around solar minimum all the neutrino detector data are consistent with a uniform temperature $T_K(min) = T_C(min) = T_G(min) = 0.97$. In this case the results would suggest that all the neutrino detectors observe only the SSM-like solar core and not the neutrinos arising from the hot bubbles of the thermonuclear runaways. In the dynamic solar model there is a quick and direct contact between the solar surface and the solar core. In the dynamic solar model the transit time scale of the hot blobs from he solar core to the surface is estimated to be around one day (Grandpierre, 1996), therefore, the absence of the surface sunspots may indicate the simultaneous absence (or negligible role) of runaways in the core. Therefore, the result that in solar minimum no bubble neutrino flux are observed in each of the neutrino detectors, is consistent with the fact that in solar minimum there are no (or very few) sunspots observed at the solar surface.

4. Discussion and conclusions

The calculated solutions of the neutrino flux equations are consistent with the data of the neutrino detectors. I have shown that introducing the runaway energy source, it is possible to resolve the apparent contradiction between the different neutrino detectors even assuming standard neutrinos. Moreover, the results presented here suggest that the physical neutrino problems of the atmospheric neutrinos may be consistent with the solution of the solar neutrino problems even without introducing sterile neutrinos.

Considering the hypothetical activity-related changes of the solar neutrino fluxes, I found that the twofold energy source of the Sun produces different contributions in the different neutrino detectors. Apparently, it is the SuperKamiokande that is the most sensitive to the runaway processes. The contribution of the runaway neutrinos and the neutrinos of the standard-like quiet solar core runs in anti-correlation to each other. Therefore, their effects may largely compensate each other in the SuperKamiokande data. Nevertheless, it is indicated that intermediate and high-energy neutrinos may produce a slight correlation with the solar activity in the SuperKamiokande data since they correlate more closely with the runaway neutrino fluxes than with the neutrinos of the SSM-like solar core. They give a 64% of the total counts observed in the SuperKamiokande, therefore, the total flux may slightly correlate with the solar cycle. On the contrary, since the Homestake do not see the runaways, except the intermediate and high-energy electron neutrinos produced by the hot bubbles, its data may anti-correlate with the solar activity. Moreover, the dynamic solar model suggests that GALLEX data may anti-correlate with the solar cycle as well since it is more sensitive to the low-energy neutrinos arising from the proton-proton cycle, although it is also sensitive to the intermediate and high-energy electron neutrinos produced by the hot bubbles.

Now, obtaining indications of possible correlations between the solar neutrino fluxes and activity parameters, I can have a short look to the data whether they show or not such changes in their finer details. Such a marginal change may be indicated in the Fig. 3 of Fukuda et al. (1996). In this figure, the maximum value is detected just in 1991, at solar maximum, consistently with the results obtained here. Moreover, its value as read from that figure seems to be 68% of the value expected from the SSM. In 1995, in solar minimum, the lowest value, 34% is detected, again consistently with the interpretation we reached. Later on, the SuperKamiokande started to work and measured a value of $2.44 \times 10^6\, cm^{-2}s^{-1}$ for the boron neutrino flux. Assuming that the values in 1995 and 1996–1997 did not differ significantly, as it is a period of the solar minimum, the two observation can be taken as equal, i.e. the 34% is equal with $2.44 \times 10^6\, cm^{-2}s^{-1}$. This method gives for the 68% value a boron flux of $4.88 \times 10^6\, cm^{-2}s^{-1}$. Now Bahcall et al. (1998b) developed an improved standard solar model, with significantly lower $^7Be(p,\gamma)^8B$ cross sections, $5.15\, cm^{-2}s^{-1}$ instead of the previous $6.6 \times 10^6\, cm^{-2}s^{-1}$ of Bahcall & Pinsonneault (1995). With this improved value the $4.88 \times 10^6\, cm^{-2}s^{-1}$ leads to a $\Phi_k(min) \simeq 95\%\ \Phi_K(SSM)$! This means that actually even the SuperKamiokande data may contain some, yet not noticed correlation tendency with the solar cycle. These indications make the future neutrino detector data more interesting to a possible solar cycle relation analysis.

The dynamic solar model has a definite suggestion that below 0.10 solar radius the standard solar model is to be replaced by a significantly cooler and possibly varying core. These predictions can be checked with future helioseismic observations. Helioseismology is not able to tell us the temperature in this deepermost central region. On the other hand, the presence of the thermonuclear micro-instabilities causes a significant departure from the thermal equilibrium and changes the Maxwell-Boltzmann distribution of the plasma particles. It is shown that such modification leads to increase the temperature of the solar core, which can compensate the non-standard cooling (Kaniadakis et al. 1996) and so the simple dynamic solar model can be easily consistent with the helioseismic results as well.

The indicated presence of a runaway energy source in the solar core - if it will be confirmed - will have a huge significance in our understanding of the Sun, the stars, and the neutrinos. This subtle and compact phenomena turns the Sun from a simple gaseous mass being in hydrostatic balance to a complex and dynamic system being far from the thermodynamic equilibrium. This complex, dynamic Sun ceases to be a closed system, because its energy production is partly regulated by tiny outer influences like planetary tides. This subtle dynamics is possibly related to stellar activity and variability. Modifying the participation of the MSW effect in the solar neutrino problem, the dynamic energy source has a role in the physics of neutrino mass and oscillation. An achievement of the suggested dynamic solar model is that it may help to solve the physical and astrophysical neutrino problems without the introduction of sterile neutrinos, and, possibly, it may improve the bad fit of the MSW effect (Bahcall et al. 1998a).

Appendix A: the travel of the hot bubbles towards the solar surface

A.1. The velocity of the bubbles

Accepting that a hot bubble forms in the solar core, we can ask how can such a bubble survive when it starts to rise, when accelerated by the buoyant force. At first sight, it may seem that such a bubble can easily loose its surplus inner energy on its pathway by turbulent viscosity and other forms of dissipation. Moreover, heat expansion may quickly cool the bubble and so it may dissolve with its surroundings. Nevertheless, quantitative estimates show that the bubbles may easily keep their identity on travelling towards the solar surface and that if their energy surplus is significant enough, they can even reach the surface layers, where they are certainly disintegrate due to a developing shock wave ("sonic boom", Grandpierre 1981).

Let us see some equations determining the conditions of the bubbles on their way towards the surface. Following Gorbatsky (1964), if the hot bubble's surplus energy is dominated by radiation, and their initial energy surplus when they start to rise is Q_0, then

$$4/3\pi R^3 a T_2^4 = Q_0 \tag{A1}$$

where T_2 is the temperature in the bubble and R is the radius of the bubble, and

$$a/3 \times T_2^4 = p_1, \tag{A2}$$

where p_1 is the pressure outside the bubble.

$$R = 1/(4\pi)^{1/3}(Q_0/p_1)^{1/3} \approx 0.43(Q_0/p_1)^{1/3} \tag{A3}$$

The more exact calculation modifies the value of this coefficient to 0.40. Using a value for $Q_0 = 10^{33} - 10^{35} ergs$, and $p_1 \approx 10^{17} dyn$, one can get for $T_2 \approx 8 \times 10^7\,K$ and $R \approx 8.6 \times 10^4\,cm - -4 \times 10^5\,cm$. Of course, a hotter bubble can contain the same amount of internal energy with a smaller size.

Now let us calculate the velocity of a bubble! Following Gorbatsky (1964), let us assume the following plausible assumptions:

1. the bubble keeps its spherical form
2. the region between the bubble and its environment is thin (very narrow turbulent wake)
3. the density and temperature may be regarded uniform within the bubble
4. the pressure within the bubble equals with the pressure outside it.

On its travel the bubble meets with resistance, therefore it is necessary to put a term in the equation of motion of the bubble describing it. Since the molecular viscosity is extremely low, it is enough to take into account only the turbulent drag here:

$$\begin{aligned}(4/3\pi R^3\rho_2 + 2/3\pi R^3\rho_1)d^2r/dt^2 \\ = -4/3\pi R^3(dp_1/dr + \rho_2 GM_r/r^2) - c_x\rho_1 v^2/2\pi r^2\end{aligned} \tag{A4}$$

where c_x is the coefficient of the turbulent drag, v is the velocity of the bubble. The quantity $2/3(\pi R^3\rho_1)$ represents the so-called "induced-mass" term occuring for a body moving in a hydrodynamic medium (see Landau, Lifshitz, 1959, Sect. 11) in case of a spherical bubble. Using the equality $\rho_1 T_1 = \rho_2 T_2$, a consequence of the assumption 3, Eq. (16) can be arranged into a more suitable form:

$$\begin{aligned}d^2r/dt^2 = -2(1 - T_1/T_2)/(1 + 2T_1/T_2)\ 1/\rho_1\ dp_1/dr \\ -3/4 \times c_x/R \times 1/(1 + 2T_1/T_2)\ v^2\end{aligned} \tag{A5}$$

The initial conditions for these equation are: at $t = 0$ $r = r_0$, and $dr/dt = 0$. As the bubble becomes accelerated to a large enough velocity, the turbulent drag and the gravitational force will balance the buoyant force and the velocity becomes steady,

$$v = (8R/3c_x|(1/\rho_1 dp_1/dr)|(1 - T_1/T_2))^{1/2}. \tag{A6}$$

Now accepting that $c_x = 1$ (since for a movement with a constant speed the potential flow v and its potential ϕ does not depend on time, and therefore the pressure distribution becomes $p = p_0 - 1/2\rho v^2$, see Problem 2, Sect. 10, Landau & Lifshitz 1959, p. 25), and substituting dp_1/dr from the condition of hydrostatic equilibrium $dp_1/dr = -\rho_1 GM_r/r^2$,

$$v \approx 1.6(Rg)^{1/2}(1 - T_1/T_2)^{1/2}. \tag{A7}$$

The acceleration "a" can be estimated with $T_1 \approx 10^8\,K$, $T_2 \approx 10^7\,K$, $\rho_1 \approx 43 g/cm^3$, $v \approx 3 \times 10^5\,cm/s$, $g \approx 10^5\,cm/s^2$, so $a = d^2r/dt^2 \approx 2.1 \times 10^5\,cm/s^2$ and the bubble reach the constant speed during $\tau = 1.4\,s$. The obtained velocity of the hot bubbles 10^5–$10^6\,cm/s$ is much larger then the average speed of convective cells in the solar convective zone.

It has to note here that recent experimental and computational results suggest that for extremely high Rayleigh numbers $Ra > 10^7$ the turbulent convection turns to a thready flow. "The flow is driven entirely (in the limit of infinite Ra) by these threads. The heat flux is carried by flows that maintain their identity...and can cross a convecting layer with little mixing between them. The width of the threads, in spite of entrainment, decreases with Rayleigh number instead of increasing as one might have expected on the basis of the simple 'higher Ra means more turbulence means more mixing' line of argument" (Spruit, 1997).

A.2. What amount of temperature surplus is necessary for the bubble to reach the solar envelope from the core?

Calculating the adiabatic temperature as

$$(dT/dr)_{ad} = -\rho/C_p \tag{A8}$$

(see e.g. Lang, 1980, formula 3–296); here g is the local gravitational acceleration, and C_p is the specific heat at constant pressure. Estimating its value at $r = 0.4R_{Sun}$, $(dT/dr)_{ad}(r = 0.4R_{Sun}) \simeq 4.1 \times 10^{-4} K/cm$. At $r = 0.3R_{Sun}$ the adiabatic temperature gradient is $(dT/dr)_{ad}(r = 0.3R_{Sun}) \simeq 5.5 \times 10^{-4} K/cm$. This means that the temperature difference between the adiabatic gradient and the actual one in the solar core, when going from $0.4R_{Sun}$ to $0.3R_{Sun}$ will be $\Delta T_{ad-act} \simeq 3 \times 10^6\,K$. If one estimate the temperature difference when going downwards from the bottom of the convective zone to $0.4R_{Sun}$, this will be $\Delta T_{ad-act}(0.7R_{Sun} - 0.4R_{Sun}) \simeq 2.5 \times 10^6\,K$. The estimation shows that it is necessary a surplus local heating 2–$5 \times 10^6\,K$ for the bubbles to be able to reach the bottom of the convective from 0.4–$0.3R_{Sun}$. This amount of heating (and more) is easily produced with electric heating induced by the interaction of tidal waves and the local magnetic field (Grandpierre, 1990). Therefore, a small amount of heating may be able to trigger the movement of the bubbles upwards and they may arrive to the bottom of the connectivezone, from where their movement is again facilitated.

A.3. What is the characteristic life-time of a hot bubble?

The bubble may get a significant energy input from electric heating (Grandpierre, 1990). Moreover, its temperature may grow exponentially when the timescale of the volume expansion is smaller than that of the local thermal instability. Now, $\tau_{expansion}$ is significantly larger than $\tau_{freeexpansion} \simeq 446/\rho^{1/2} \simeq 45\,s$ (Arnett & Clayton 1970) for $\rho = 100 g/cm^3$, since the expanding bubble has to work against the large hydrostatic pressure of the star. The timescale of the thermal runaway is estimated to be $\tau_{runaway} \simeq 10^{-5}\,s$ (Grandpierre, 1990) for

$\epsilon \simeq 5 \times 10^{17}\, ergs/g/s$ (Edwards, 1969). Even for smaller rate of energy production like $\epsilon \simeq 10^{13}\, ergs/g/s$ the runaway may be faster than volume expansion. Now the nuclear depletion lifetime of a bubble, in the absence of entrainment, can be estimated as

$$\tau_d = QA/\epsilon_b \tag{A9}$$

where Q is the heat energy produced in the dominant reaction per nucleon, and A is the number of nuclei participating in the reaction per gram, ϵ_b is the average energy production in the bubble. With $Q \simeq 7MeV$ amd $\epsilon_b \simeq 10^{13}\, ergs/g/s$ the lifetime of the bubble is $\tau_b \simeq 7 \times 10^5\, s$. During its nuclear lifetime the bubble may travel the solar radius $7 \times 10^{10}\, cm$ if its velocity is $v_b \simeq 10^5\, cm/s$. Therefore, even when the energy production goes largely from the triple alpha reaction, with a smaller Q, the bubble may travel from the core to the solar convective zone, especially since the triple alpha reaction prevails above $10^8\, K$, therefore in that case the bubble has a larger temperature surplus over its environment and so it is accelerated to a larger speed.

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