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Research Note

A new method for calculating a special class of self-consistent three-dimensional magnetohydrostatic equilibria

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Abstract. A new mathematical formulation for calculating a special class of self-consistent three-dimensional magnetohydrostatic equilibria in Cartesian and spherical coordinates is presented. The method uses a representation of the magnetic field in terms of poloidal and toroidal field components which automatically guarantees the solenoidal character of the magnetic field. This representation is commonly used in the theory of linear force-free magnetic fields. One advantage of this representation of the magnetic field is that the calculation involves only one scalar function whereas in previous treatments one was forced to operate with all three components of the magnetic field independently.

Key words: Sun: magnetic fields – magnetic fields – methods: analytical

1. Introduction

Three-dimensional magnetohydrostatic equilibria are very difficult to calculate as compared to systems with helical symmetry (including the limiting cases of rotational and translational symmetry). In toroidal geometry the very existence of non-symmetric equilibria is very much in doubt (e.g. Grad 1985; Kaiser et al. 1995; Salat 1995) unless simplifying symmetry assumptions are made.

Nevertheless, simple non-toroidal three-dimensional magnetohydrostatic equilibria have been found by Woolley (1976, 1977), Shivamoggi (1986), Salat & Kaiser (1995) and Kaiser & Salat (1996, 1997). In these papers no gravitational force has been included. The equilibria have relatively little relevance for astrophysical applications (but see Petrie & Neukirch 1999) and will therefore not be discussed further.

For astrophysical applications, Parker (1972, 1979) has shown that no three-dimensional MHD equilibria exist which are small perturbations of two-dimensional equilibria (having translational, rotational or helical symmetry), are bounded and extend throughout the complete $\mathbf{R}^3$. However, Arendt & Schindler (1988) have shown that as soon as finite boundaries are present Parker's result does no longer hold true (see also Hu et al. 1983).

If the gravitational force (and/or the centrifugal force) is included the calculation of special classes of three-dimensional magnetohydrostatic equilibria is somewhat easier, though far from simple, as has been shown in a series of papers by Low (1982, 1984, 1985, 1991, 1992, 1993a,b) and Bogdan & Low (1986). There has also been work along similar lines by Osherovich (1985a,b) although, as we show in Appendix C, Osherovich's approach is far less general than Low's method. Neukirch (1995, 1997b) has reformulated and slightly generalized Low's approach and has in this way been able to find nonlinear solutions (Neukirch 1997a).

The value of Low's class of MHS solutions lies in their potential astrophysical applications. Solutions in spherical coordinates have been used to construct global models of the solar corona (Bagenal & Gibson 1991; Gibson & Bagenal 1995; Zhao & Hoeksema 1993, 1994) and solutions in Cartesian coordinates have been applied to model prominences and pre-flare magnetic fields (Aulanier et al. 1998, 1999).

In the present paper, we present a new and different mathematical formulation of Low's method for Cartesian and for spherical coordinates. As we will show this new method has several advantages compared to previous methods. Firstly, it uses a well-known representation for solenoidal vector fields commonly used for linear force-free magnetic fields which allows the calculation of the complete magnetic field by solving a single scalar equation. Secondly, this formulation also shows the intrinsic connection between linear force-free magnetic fields and this special class of magnetohydrostatic fields. Finally, as a by-product it is possible to show that there is only a limited number of coordinate systems for which such a simple formulation of the problem is possible. Although it is not the topic of this paper, we also mention that the new formulation allows the straightforward development of the Green's function method for

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the magnetohydrostatic problem in very much the same way as for the linear force-free case (see e.g. Chiu & Hilton 1977). Work along these lines is presently in progress and will be reported in a separate publication (Petrie & Neukirch, in preparation).

2. The new formulation

Including a gravitational potential ψ the magnetohydrostatic equations have the following form:

$$\boldsymbol{j}\times\boldsymbol{B}-\nabla p-\rho\nabla\psi \;=\; 0, \tag{1}$$
$$\nabla\times\boldsymbol{B}=\mu_0\boldsymbol{j}, \tag{2}$$
$$\nabla\cdot\boldsymbol{B}=0. \tag{3}$$

It is well-known that in Cartesian coordinates any solenoidal vector field can be represented in terms of two scalar functions P and T in the following way

$$\boldsymbol{B}=\nabla\times\nabla\times(P\hat{\boldsymbol{a}})+\nabla\times(T\hat{\boldsymbol{a}}), \tag{4}$$

where $\hat{\boldsymbol{a}}$ is an arbitrary constant vector (e.g. Nakagawa & Raadu 1972). The vector $\hat{\boldsymbol{a}}$ is usually chosen to be perpendicular to one of the boundaries of the physical system under consideration. Such a choice is convenient because the term $\nabla\times\nabla\times(P\hat{\boldsymbol{a}})=-\Delta P\hat{\boldsymbol{a}}$ is then perpendicular to the boundary (parallel to $\hat{\boldsymbol{a}}$) whereas the second term $\nabla\times(T\hat{\boldsymbol{a}})=\nabla T\times\hat{\boldsymbol{a}}$ is tangential to the boundary. As discussed by Morse & Feshbach (1953) for the case of the vector Helmholtz equation, this allows for a much more convenient way of incorporating boundary conditions when solving the equation. Throughout this paper we will assume that $\hat{\mathbf{a}}=\hat{\boldsymbol{z}}$, where $\hat{\boldsymbol{z}}$ is the unit normal vector in the z-direction.

A similarly useful representation exists in spherical coordinates (r, θ, ϕ) in the form

$$\boldsymbol{B}=\nabla\times\nabla\times(\Phi\boldsymbol{r})+\nabla\times(\Psi\boldsymbol{r}) \tag{5}$$

(e.g. Chandrasekhar & Kendall 1957; Chandrasekhar 1961), with $\boldsymbol{r}$ being the coordinate vector ($\boldsymbol{r}=r\hat{\boldsymbol{r}}$). This expression for $\mathbf{B}$ is especially useful if the boundaries are concentric spherical surfaces centred at the origin. We will restrict our treatment in the main part of the manuscript to the Cartesian case and refer the reader to Appendix B for the case of spherical coordinates. For a discussion of similar representations in other coordinate system we refer the reader to Sect. 4.

Progress can now be made if we make assumptions about the structure of the current density. In the well-studied linear force free case we have $\mu_0\boldsymbol{j}=\alpha\boldsymbol{B}$ and $\alpha=$ constant. Then it can be shown that for a special gauge (see e.g. Nakagawa & Raadu 1972)

$$T \;=\; \alpha P, \tag{6}$$
$$\Delta P+\alpha^2 P \;=\; 0. \tag{7}$$

In Low (1991, 1992) and Neukirch (1995, 1997a,b) it is assumed that the current density has the structure

$$\mu_0\boldsymbol{j}=\alpha\boldsymbol{B}+\nabla\times(F\hat{\boldsymbol{z}}) \tag{8}$$

where α is constant. The only condition we impose on the function F is that it should not depend on z alone. This would lead to the trivial case that the second term on the right hand side of Eq. (8) vanishes because ∇F is then parallel to $\hat{\boldsymbol{z}}$. So basically we assume here that without loss of generality the vector fields ∇F and $\hat{\boldsymbol{z}}$ are linearly independent.

Then, as in the linear force-free case (see Appendix A) one can show that

$$T \;=\; \alpha P, \tag{9}$$
$$\Delta P+\alpha^2 P+F \;=\; 0. \tag{10}$$

To be able to solve Eq. (10) we have to specify F. There are two obvious conditions that F should satisfy if we want to make any analytical progress: the choice for F should be such that a) Eq. (10) can be solved analytically and b) the force balance equation should be easily integrated.

We look at condition b) first and substitute Eq. (8) into Eq. (1) and get

$$\frac{1}{\mu_0}(\boldsymbol{B}\cdot\nabla F)\hat{\boldsymbol{z}}-\frac{1}{\mu_0}(\boldsymbol{B}\cdot\hat{\boldsymbol{z}})\nabla F-\nabla p-\rho\nabla\psi=\mathbf{0}. \tag{11}$$

In Cartesian coordinates $\psi=gz$ and therefore, because ∇F and $\hat{\boldsymbol{z}}$ are linearly independent by assumption, we can split Eq. (11) into two components along ∇F and along $\hat{\boldsymbol{z}}$ giving

$$\left(\frac{\partial p}{\partial F}\right)_z \;=\; -\frac{1}{\mu_0}\boldsymbol{B}\cdot\hat{\boldsymbol{z}}, \tag{12}$$
$$\left(\frac{\partial p}{\partial z}\right)_F \;=\; \frac{1}{\mu_0}\boldsymbol{B}\cdot\nabla F-g\rho. \tag{13}$$

The equations show that the plasma pressure p can only depend on F and z. We remark that this dependence does not necessarily have to be single-valued as long as p is a single-valued function of the coordinates. Since p does only depend on F and z, so does $(\partial p/\partial F)_z$ and therefore, by Eq. (12) also does $\boldsymbol{B}\cdot\hat{\boldsymbol{z}}=B_z$. Reverting this relation (which is always possible, at least locally), we find that F can depend only on B_z and z (Low 1991). A similar line of reasoning leads to the conclusion that in the case of spherical coordinates F can only depend on B_r and r (see Appendix B). The fact that F can only depend on B_z (B_r) and z (r) does have direct consequences for other approaches to the same problem (Osherovich 1985a,b). We discuss these consequences in Appendix C.

Eq. (13) is then used to calculate the density ρ in the form

$$\rho=-\frac{1}{g}\left[\left(\frac{\partial p}{\partial z}\right)_F-\frac{1}{\mu_0}\boldsymbol{B}\cdot\nabla F\right]. \tag{14}$$

Assuming the plasma to be an ideal gas it is possible to define the plasma temperature by

$$T=\frac{\mu p}{\mathrm{k_B}\rho}, \tag{15}$$

where μ is the average mass of the plasma particles and $\mathrm{k_B}$ the Boltzmann constant. The temperature will usually also be varying in all three directions since the pressure (p) and the density (ρ) vary.

3. Relation to previous work

We recover the treatment given by Low (1991, 1992) and by Neukirch (1995, 1997b) for Cartesian coordinates (for spherical coordinates see Appendix B) if we let

$$F = \xi(z)B_z. \tag{16}$$

Using Eq. (4), we find

$$B_z = -\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) P. \tag{17}$$

Substituting Eqs. (16) and (17) into Eq. (10) we obtain the linear partial differential equation

$$\Delta P + \alpha^2 P - \xi(z)\left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\right) = 0. \tag{18}$$

A similar equation can be derived for the case of spherical coordinates (see Appendix B).

The mathematical structure of Eq. (18) matches exactly the structure of the equation for B_z derived by Neukirch (1995, 1997b)

$$\Delta B_z + \alpha^2 B_z - \xi(z)\left(\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2}\right) = 0. \tag{19}$$

It is easy to see that we obtain this equation for B_z from Eq. (18) if we simply apply the operator

$$\hat{L}_C^2 = -\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \tag{20}$$

from the left to that equation. Since all coefficients of Eq. (18) are independent of x and y, the operator operates on P only and using Eq. (17) we obtain Eq. (19) for B_z. In this sense the present and the previous mathematical formulation are completely equivalent.

But whereas in the previous work of Low (1991, 1992) and Neukirch (1995) one needed to ensure that $\boldsymbol{B}$ is solenoidal by imposing this condition explicitely when calculating the other magnetic field components, here we are basically finished once we have calculated P. The solenoidal character of the magnetic field is automatically ensured and the other components of $\boldsymbol{B}$ can be calculated from the appropriate derivatives of P. This disposes of the tedious task to calculate the correct combinations of field components and their derivatives necessary in the previous approach.

The pressure can be found by integrating Eq. (12) giving

$$p = p_0(z) - \xi(z)\frac{B_z^2}{2\mu_0}. \tag{21}$$

The density can be found from Eq. (14)

$$\rho = \frac{1}{g}\left(-\frac{\mathrm{d}p_0}{\mathrm{d}z} + \frac{\mathrm{d}\xi}{\mathrm{d}z}\frac{B_z^2}{2\mu_0} + \frac{1}{\mu_0}\xi\,\boldsymbol{B}\cdot\nabla B_z\right) \tag{22}$$

and the temperature is then found from Eq. (15).

Similarly the treatment of the nonlinear cases discussed by Neukirch (1997a) is much easier when the representation (4) is used. Neukirch (1997a) discussed cases in which F depends nonlinearly on B_z. Independently of F, it is immediately clear that for any F the mathematical structure of the equation for P will be the same as the mathematical structure of the corresponding equation for B_z. As in the linear case discussed above, one can derive the B_z equation from the P equation by applying the $\hat{L}_C^2$ operator. That means that the equation for P and the equation for B_z have exactly the same mathemathical structure. However, the other magnetic field components follow immediately by differentiation of P without the tedious task to integrate the other equations further as in the previous treatment.

4. Conclusions

In this paper we have presented a new alternative way of deriving a special class of three-dimensional solutions of the magnetohydrostatic equations. The new method is completely equivalent to the methods which have been used before but it has the advantage that the solenoidal nature of the magnetic field is automatically guaranteed. This facilitates the calculations considerably because it is not necessary to calculate each magnetic field component separately. Only one scalar function (and its derivatives) is needed to gain full knowledge of the complete magnetic field. This is very similar to the well-known case of linear force-free magnetic fields.

Another advantage of the new method is that we can use textbook knowledge to show that such a simple treatment of the problem is only possible in a limited number of coordinate systems. As is shown in Morse & Feshbach (1953) apart from the possibility of choosing different coordinate systems in the plane perpendicular to $\hat{\boldsymbol{a}}$ (e.g. a polar coordinate system in this plane leads to cylindrical coordinates in $\mathbf{R}^3$) or conical instead of spherical coordinates, there are no other coordinate systems for which the representation of a solenoidal vector field in this form leads to a splitting into components perpendicular and tangential to one of the coordinate lines (which we could then identify with one of the boundaries) and at the same time satifies the vector Helmholtz equation. Since our method for the magnetohydrostatic case is based on exactly this property of the representation for $\boldsymbol{B}$, it cannot work either for other coordinate systems. This implies that the method presented here can only work for these (basically two) coordinate systems. One case which would have been relevant for space plasma or astrophysical applications is the case of cylindrical coordinates (ϖ, ϕ, z), but with $\hat{\varpi}$ instead of $\hat{\mathbf{z}}$ in Eq. (4). Therefore the interesting case of rigidly rotating systems is excluded from the simple treatment derived in this paper and one has to use other, more complicated methods to calculate solutions for this and other cases.

Finally we mention that the similarity of the equations to those of the linear force-free case can be used to develop the Green's function method for the MHS case with a linear F. This is currently under study (Petrie & Neukirch, in preparation) and will be published elsewhere.

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Appendix A: solution of gauge condition

Substituting Eq. (4) into Eq. (2) and using Eq. (8) we get

$$\nabla \times (-\Delta P \hat{\boldsymbol{z}}) + \nabla \times [\nabla \times (T\hat{\boldsymbol{z}})] = \nabla \times [\nabla \times (\alpha P \hat{\boldsymbol{z}})] + \nabla \times [(\alpha T + F)\, \hat{\boldsymbol{z}}] \,. \tag{A1}$$

By straightforward rearrangement of this equation we obtain

$$\nabla \times [(-\Delta P - \alpha T - F)\, \hat{\boldsymbol{z}}] + \nabla \times \{\nabla \times [(T - \alpha P)\, \hat{\boldsymbol{z}}]\} = \boldsymbol{0}. \tag{A2}$$

It can be shown (see e.g. Nakagawa & Raadu 1972) that this equation is solved without loss of generality by

$$\Delta P + \alpha T + F \;=\; 0, \tag{A3}$$

$$T - \alpha P \;=\; 0. \tag{A4}$$

Eq. (10) is identical with Eq. (A3) if we use T from Eq. (A4). We also recover the linear force free case for $F = 0$.

Appendix B: spherical coordinates

In spherical coordinates we have the following general representation for solenoidal vector fields

$$\boldsymbol{B} = \nabla \times \nabla \times (\Phi \boldsymbol{r}) + \nabla \times (\Psi \boldsymbol{r}) \tag{B1}$$

(e.g. Chandrasekhar & Kendall 1957; Chandrasekhar 1961), where $\boldsymbol{r}$ is the coordinate vector expressed in spherical coordinates. We remark that we have changed the definitions of Φ and Ψ from those used by Chandrasekhar (1961) by a factor of r for mathematical convenience. Substituting this expression for $\boldsymbol{B}$ into Eq. (2) and using the equivalent of Eq. (8) in spherical coordinates for the current density,

$$\mu_0 \boldsymbol{j} = \alpha \boldsymbol{B} + \nabla \times (F\boldsymbol{r}), \tag{B2}$$

we obtain

$$\nabla \times [(-\Delta\Phi)\boldsymbol{r}] + \nabla \times [\nabla \times (\Psi \boldsymbol{r})] = \nabla \times [\nabla \times (\alpha \Phi \boldsymbol{r})] + \nabla \times [(\alpha\Psi + F)\, \boldsymbol{r}] \,. \tag{B3}$$

Eq. (B3) can be rearranged into the form

$$\nabla \times [(-\Delta\Phi - \alpha\Psi - F)\, \boldsymbol{r}] + \nabla \times \{\nabla \times [(\Psi - \alpha\Phi)\, \boldsymbol{r}]\} = \boldsymbol{0}, \tag{B4}$$

which without loss of generality is solved by

$$\Delta\Phi + \alpha\Psi + F \;=\; 0 \tag{B5}$$

$$\Psi - \alpha\Phi \;=\; 0. \tag{B6}$$

The ultimate equation for Φ then is

$$\Delta\Phi + \alpha^2\Phi + F = 0. \tag{B7}$$

Substituting our expression (B2) into the force balance equation we get

$$\frac{1}{\mu_0}(\boldsymbol{B} \cdot \nabla F)\boldsymbol{r} - \frac{1}{\mu_0}(\boldsymbol{B} \cdot \boldsymbol{r})\nabla F - \nabla p - \rho \nabla \psi = \boldsymbol{0}. \tag{B8}$$

In spherical coordinates and assuming a spherical central body of mass M we have $\psi = -\mathrm{GM}/r$ and we can therefore split Eq. (B8) into two components along ∇F and $\boldsymbol{r}$ giving

$$\left(\frac{\partial p}{\partial F}\right)_r \;=\; -\frac{1}{\mu_0}\boldsymbol{B} \cdot \boldsymbol{r} = -\frac{1}{\mu_0} r B_r, \tag{B9}$$

$$\left(\frac{\partial p}{\partial r}\right)_F \;=\; \frac{1}{\mu_0}\boldsymbol{B} \cdot \nabla F - \frac{\mathrm{GM}}{r^3}\rho. \tag{B10}$$

We see that the plasma pressure depends only on F and r in spherical coordinates. The same is of course true for $(\partial p/\partial F)_r$ and by Eq. (B9) for B_r itself. Inversion of this relation shows that F can depend only on B_r and r.

We recover the discussion of Neukirch (1995) if we assume that

$$F = \xi(r) r B_r. \tag{B11}$$

Using Eq. (B1) we obtain

$$r B_r = \hat{L}^2 \Phi \tag{B12}$$

with the differential operator $\hat{L}^2$ defined by

$$\hat{L}^2 = -\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) - \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}. \tag{B13}$$

Substituting Eqs. (B11) and (B12) into Eq. (B7) we obtain the linear partial differential equation

$$\Delta\Phi + \alpha^2\Phi + \xi(r)\hat{L}^2\Phi = 0. \tag{B14}$$

We recover the equation for B_r derived in Neukirch (1995)

$$\Delta(rB_r) + \alpha^2(rB_r) + \xi(r)\hat{L}^2(rB_r) = 0, \tag{B15}$$

by applying $\hat{L}^2$ to Eq. (B14) from the left. Since all coefficients of Eq. (B14) depend only on r (or are constant), $\hat{L}^2$ commutes with the differential operator of Eq. (B14). By using Eq. (B12), we obtain Eq. (B15) for rB_r.

Appendix C: Osherovich's method

In this Appendix we discuss the implications of the fact that F can only depend on B_z and z in the Cartesian coordinate case, respectively on B_r and r in the spherical coordinate case for the method given by Osherovich (1985a,b). He investigates only cases in spherical coordinates with $\alpha = 0$. Then from the structure of the current density (8) and Ampère's law we get

$$\boldsymbol{B} = \nabla\Phi + F\hat{\mathbf{r}}. \tag{C1}$$

Instead of imposing F to be a function of B_r and r as discussed above, Osherovich uses

$$F = Q(r)\Phi. \tag{C2}$$

Since this is in general not consistent with the constraint that F *must* be a function of B_r and r only, one can only get solutions if Φ satisfies an additional equation as we will show now. Assuming the linear F of Eq. (B11) we find that

$$Q(r)\Phi = \xi(r)B_r. \tag{C3}$$

But from Eq. (C1) we get

$$B_r = \frac{\partial\Phi}{\partial r} + Q(r)\Phi. \tag{C4}$$

Substituting this relation into Eq. (C3) we arrive at

$$\xi(r)\frac{\partial\Phi}{\partial r} = Q(r)(1-\xi(r))\Phi. \tag{C5}$$

This is exactly the relation that Osherovich (1985a) *assumes* to obtain a tractable equation. Here we have shown that it is actually the *only* possible way to get solutions from his approach. There are two immediate conclusions we can draw: a) Osherovich's approach can only give separable solutions and b) any other way to treat Osherovich's approach as for example proposed in Neukirch (1995) cannot work.

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