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Damping and frequency reduction of the *f*-mode due to turbulent motion in the solar convection zone

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Abstract. Solar *f*-mode properties were observed recently with high accuracy using high-resolution Michelson Doppler Imager (MDI) data from SOHO (Duvall et al. 1998). According to these observations, linewidths increase with wavenumber k and the *f*-mode frequency ω is significantly lower than the frequency ω_0 given by the simple dispersion relation $\omega_0^2 = gk$. This paper provides a possible explanation of these observations on the basis of the turbulent flow that is in the convection zone. The *f*-mode spends more time propagating against the flow than with the flow. As a result, its effective speed and consequently frequency are reduced. This reduction is revealed by the real part of ω. A negative imaginary part of the frequency ω represents the damping of the coherent *f*-mode field due to scattering by the turbulent flow. The *f*-mode damping is a result of the generation of the turbulent field at the expense of the coherent field.

Key words: convection – Sun: atmosphere – Sun: granulation – Sun: oscillations – turbulence

1. Introduction

The fundamental solar *f*-mode is recognized as a surface gravity wave with dispersion relation

$$\omega_0^2 = gk, \tag{1}$$

where ω_0 and $k = \sqrt{l(l+1)}/R$ are the frequency and wavenumber, g is the surface gravity, R is the solar radius, and l is the spherical degree. This dispersion relation shows that the *f*-mode frequency is independent of the internal structure of the Sun.

Detailed observations of the high-degree *f*-mode by Libbrecht et al. (1990), Rhodes et al. (1991), Fernandes et al. (1992), Bachmann et al. (1995) and Duvall et al. (1998), have shown that the frequency of this mode is substantially lower than that implied by the parabolic dispersion relation (1). An even stronger frequency decrease is observed for solar *p*-modes in the high-frequency range. Kosovichev (1995) has argued that the *p*-mode frequencies may be decreased due to turbulent pressure in the upper convection zone. Eq. (1) is obtained making two kinds of approximation. Firstly, the oscillations are assumed adiabatic. Secondly, even if the adiabatic approximation is accepted, parabolic relation (1) is only a first order result and a more accurate dispersion relation is model dependent (Gabriel 1991). In view of this second approximation, Kosovichev suggestion might be right. However, we rather consider here the consequences of the first approximation and among all the dissipative terms we choose to discuss the influence of the scattering terms.

Brown (1984) first suggested that the frequencies of global oscillations can be decreased by fluctuating velocity fields in the upper convection zone. This dynamical effect of solar convection applies for both *f*- and *p*-modes. Theoretical models of the effect for the *f*-mode have been proposed by several authors. Murawski & Roberts (1993a, 1993b), Murawski et al. (1998) and Gruzinov (1998) showed that the *f*-mode frequency is reduced by granulation, modeled as a turbulent velocity field located in the convection zone. Gruzinov (1998) has used a perturbative theory to show that turbulent convection produces an unobservably small blue shift for low spherical degree l, while the fractional frequency shift for $l > 500$ is $-M^2$, where M is the Mach number of convection near the solar surface. But the *f*-mode is affected by other factors too. For example, the *f*-mode is affected by chromosphere magnetism which causes an increase in its frequency (e.g., Campbell & Roberts 1989, Evans & Roberts 1990, Jain & Roberts 1994). The combined influences of flow and magnetism were considered by Murawski & Goossens (1993). Also Rosenthal & Gough (1994) suggested that a surface mode propagating along the chromosphere-corona transition region gives a close quantitative agreement with the observed frequency reduction. The effects of a hot chromosphere and corona on the acoustic-gravity waves in the Sun have been examined by Hindman & Zweibel (1994) who considered a simple solar model consisting of a neutrally stable polytrope matched to an isothermal chromosphere. This model shows that *f*-mode frequencies lie below those given by the static dispersion relation (1) while frequency shifts of *p*-modes can be positive or negative.

Numerical simulations of the external parts of the solar convection zone and atmosphere have been performed by Rosenthal et al. (1999) to quantify the influence of turbulent convection on frequencies of solar *p*-modes. It has been found that the main

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model effects are due to the turbulent pressure that provides additional support against gravity. As a consequence, the turning points of high-frequency *p*-modes are raised and frequencies are reduced.

It is noteworthy that by collecting data in a magnetically quiet Sun at disc center, Kiefer & Balthasar (1998) have shown that both *f*- and *p*-modes exhibit a tendency to increase their amplitudes in the darkest features of the granulation structure. The recent analysis of the *f*-mode low frequencies obtained from SOHO/MDI suggest that the value of the solar radius has to be reduced by about 300 km in order to match model calculations and observed frequencies (Brown & Christiansen-Dalsgaard 1998; Schou et al. 1998). Rudiger (1997) has shown that while the fluctuations of temperature and density produce a redshift of the *p*-mode frequencies at low l, the pressure part of the Reynolds stress causes frequency shifts of *p*-modes in the opposite direction. In a periodic model of the convection in homogeneous atmosphere, it has been shown by Zhughzda (1998) that a dispersion relation possesses an infinite number of solutions but just one of them tends to the solution for the static atmosphere when the convection vanishes. As a consequence, the *f*-mode oscillations are split into waves which propagates with slightly different speeds. This effect broadens line-widths and shifts the frequency of the *f*-mode.

This paper aims to display some of the features that go into a theoretical description of the turbulence and its effect on the spectrum of the *f*-mode oscillations. In particular, we employ the dispersion relation developed by Murawski & Roberts (1993a, 1993b) to determine the turbulent velocity field corrections to the solar *f*-mode. Our discussion is stimulated by recent observations (Duvall et al. 1998) that indicate that the line width Γ increases with wavenumber. A turbulent flow makes its appearance through a negative imaginary part of frequency, ν_i. This negative component represents the damping of the mean field, *i. e.* the generation of the turbulent field at the expense of the mean field energy. The line width Γ is proportional to the magnitude of the imaginary part ν_i of the frequency, $\Gamma = -2\nu_i$. We apply an analytical perturbation technique to estimate the line width and the frequency shift, and show that the results of our model are consistent with the properties of the *f*-mode obtained from the high-resolution MDI data (Duvall et al. 1998).

The paper is organized as follows. In the next section we present a theoretical model to study the influence of turbulence on the frequency and line width of the solar *f*-mode. Sect. 3 contains the results of a numerical study of the dispersion relation. The paper concludes with a summary of the main results and discussion.

2. Theoretical model

The solar plasma below the visible layers is a dynamic environment, supporting convection which reveals itself principally on two scales of motion: a large scale supergranulation with a horizontal scale of $3 \cdot 10^4$ km and flows of $0.1 - 0.4\,\mathrm{km\,s^{-1}}$ and much smaller scale granulation with a horizontal scale of $0.2–2 \cdot 10^3$ km and flows of $1–3\,\mathrm{km\,s^{-1}}$. In such a dynamic medium waves are generated by turbulent convection (Goldreich & Kumar 1990, Bogdan et al. 1993, Goldreich et al. 1994).

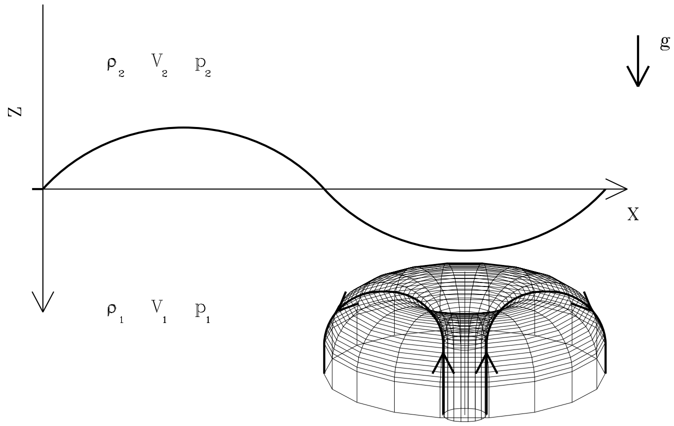

Fig. 1. The region close to the interface between the convection zone (lower half plane) and the chromosphere-photosphere (upper half plane) containing the *f*-mode (sinusoidal line) and the convective random flow.

In this paper we employ the turbulent *f*-mode model of Murawski & Roberts (1993b). In this model the region $z < 0$ is taken to correspond to the photosphere-chromosphere of constant mass density ρ_2. Below this region lies the convection zone, taken to have a mass density ρ_1 ($> \rho_2$). The atmosphere is assumed free of magnetic field. A weak turbulent velocity field is located in the convection zone (Fig. 1). It is assumed that the two regions are inviscid and incompressible, and that motions are described by the incompressible hydrodynamic equations. These equations are matched across the interface $z = 0$ by requiring continuity of the vertical velocity V_z and the gas pressure p. Application of these boundary conditions and the use of the binary collision approximation in a perturbative method (Howe 1971) yields the turbulent dispersion relation (Murawski & Roberts 1993b)

$$\omega^2 - gk\frac{1-\kappa}{1+\kappa} = \varphi(\sigma^2, k, l_x, \kappa, g), \qquad (2)$$

where the function $\varphi(\sigma^2, k, l_x, \kappa, g)$ depends on the parameters of the turbulent flow. This function is too complex to be presented here. Its exact form is given by Murawski & Roberts (1993b), Eq. (4.19) with $\alpha = 1/l_x$, $\sigma_2 = \sigma/l_x^2$ and $\sigma_{02} = \sigma/l_x$. In formula (2) σ and l_x denote respectively the variance and correlation length of the turbulent flow, κ is a density ratio ($\kappa \equiv \rho_2/\rho_1$) and k is the horizontal wavenumber; $\sigma = 1\,\mathrm{km\,s^{-1}}$ and $l_x = 10^3$ km correspond to a typical granule flow speed and size (Title et al. 1989).

To choose an appropriate value for the density ratio κ we either select arbitrarily a small value (as in Murawski & Roberts 1993b) or instead we attempt to allow for solar stratification, as follows. Consider a simple model of the solar atmosphere (Fig. 1) in which the density profile in the convection zone $\rho_{conv}(z)$, $z > 0$, and in the chromosphere $\rho_{chrom}(z)$, $z < 0$, is described by:

$$\rho_{chrom}(z) = \rho_0 \exp(\frac{z}{\Lambda_0}), \qquad z < 0, \qquad (3)$$

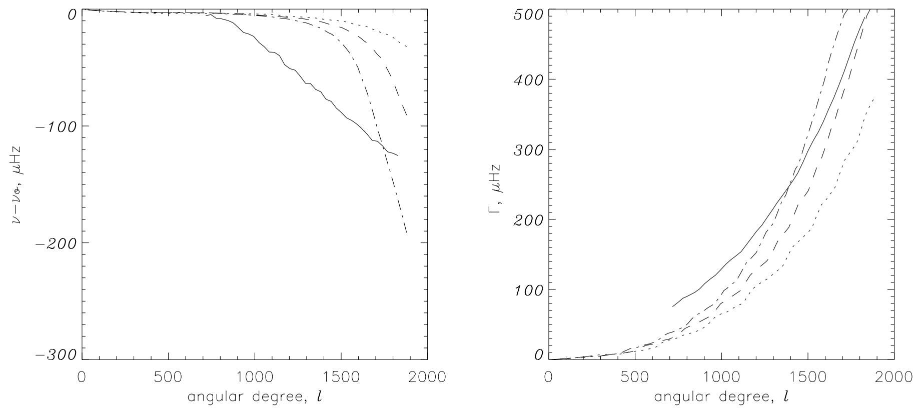

Fig. 2. The cyclic frequency difference $\nu - \nu_0$ and linewidth Γ as a function of angular degree l. The frequency $\nu_0\ (= \omega_0/2\pi)$ is determined by Eq. (1). For the broken curves, the frequency $\nu\ (= \omega/2\pi)$ and linewidth Γ are determined by Eq. (2) for density ratio $\kappa = 10^{-3}$ and convective flow with variance $\sigma = 1.0\,\mathrm{km\,s^{-1}}$ and correlation lengths $l_x = 0.9 \cdot 10^3$ km (dotted curve), $l_x = 1.1 \cdot 10^3$ km (broken curve) and $l_x = 1.3 \cdot 10^3$ km (dash-dotted curve). The solid curves represent SOHO/MDI data (Duvall et al. 1998).

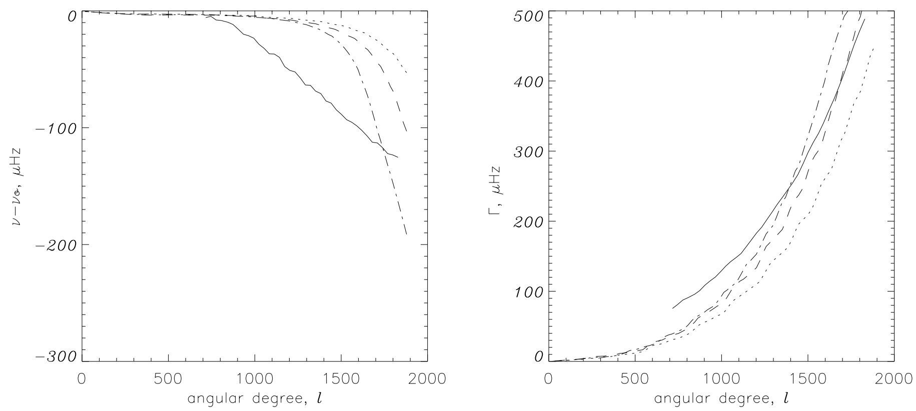

Fig. 3. The frequency difference $\nu - \nu_0$ and the linewidth Γ as a function of angular degree l, for density ratio $\kappa = 10^{-3}$ and various convective flows. The flow is characterized by a correlation length $l_x = 1.0 \cdot 10^3$ km, and variances $\sigma = 1.0\,\mathrm{km\,s^{-1}}$ (dotted curve), $\sigma = 1.1\,\mathrm{km\,s^{-1}}$ (broken curve) and $\sigma = 1.2\,\mathrm{km\,s^{-1}}$ (dash-dotted curve). The solid lines represent SOHO/MDI data.

$$\rho_{conv}(z) = \rho_0 (1 + \frac{z}{z_{0t}})^{m-1}, \qquad z > 0, \tag{4}$$

where $z_{0t} = 200$ km is the temperature scale height at $z = 0$, $c_0 = (\gamma p_0/\rho_0)^{1/2} = 6.6829\,\mathrm{km\,s^{-1}}$ is the sound speed and $\Lambda_0 = c_0^2/(\gamma g)$ is the pressure scale-height. Also, we take $g = 0.274\,\mathrm{km\,s^{-2}}$, $\gamma = 5/3$, and atmospheric base pressure $p_0 = 86.82$ Pa; the polytropic index is $m = \gamma g z_{0t}/c_0^2$. The density profiles (3) and (4) correspond to a temperature which grows linearly with depth in the convection zone and is isothermal in the chromosphere-photosphere.

We assume that the *f*-mode penetrates the convection zone and the photosphere-chromosphere to a depth which is comparable to the wavelength λ. The density ratio $\kappa = \rho_2/\rho_1$ can then be approximated by averaging the densities over the wavelength λ, *viz.*

$$\kappa(l) = \frac{\frac{1}{\lambda_{up}} \int_{-\lambda_{up}}^{0} \rho_{chrom}(z) dz}{\frac{1}{\lambda_{dn}} \int_{0}^{\lambda_{dn}} \rho_{conv}(z) dz}, \tag{5}$$

where λ_{up} and λ_{dn} are the upper and lower levels to which the surface wave penetrates.

The dispersion relation (2) has some physical consequences. The dependence $\omega(k)$ differs from the static (non-turbulent) dispersion relation, Eq. (1). The turbulent velocity field introduces a correction to the wave frequency. This correction is described by the real part of ω. Due to scattering by the turbulent flow, the energy of the coherent field that is associated with the *f*-mode is partially transformed into the turbulent field. The mathematical manifestation of this phenomenon corresponds to the appearance of ν_i, the imaginary part of ν. The linewidth Γ is expressed through the imaginary part of the frequency, $\Gamma = -2Im(\nu)$ (e.g. Osaki, 1990).

3. Numerical solutions of the turbulent dispersion relation

In this section we consider the numerical solution of dispersion relation (2) for various values of the velocity variance σ, correlation length l_x and density ratio κ. Our results are compared to MDI data (Duvall et al. 1998).

We split the integration function φ into real and imaginary parts. For integration we used the 8th point Gauss-Hermite method, and the transcendental Eq. (2) has been solved by use of an analytic function solver (the ZANLYT routine from the IMSL library).

In the following figures, the frequency difference $\Delta\nu \equiv \nu - \nu_0$ is displayed as a function of various parameters. The quantity $\Delta\nu$ is obtained by determining the cyclic frequency $\nu\,(= \omega/2\pi)$ numerically from the dispersion relation (2) and then subtracting the cyclic frequency $\nu_0 = \omega_0/2\pi$ that pertains in a static medium with $\kappa = 0$. The MDI data (Duvall et al. 1998) is shown by solid curves for comparison purposes.

Fig. 2 illustrates the difference $\Delta\nu$ and linewidth Γ of the *f*-mode as a function of the angular degree l. This figure compares numerical results for a fixed σ and κ and various values of l_x with the observed frequency depression. From this comparison we conclude that the theoretical results (broken curves) are close to

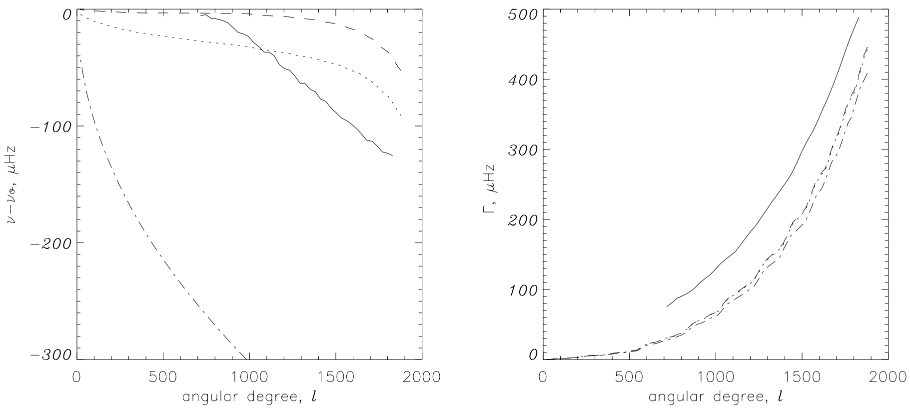

Fig. 4. The frequency difference $\nu - \nu_0$ and the linewidth Γ as a function of degree l, for flow correlation length $l_x = 1.0 \cdot 10^3$ km and variance $\sigma = 1.0\,\mathrm{km\,s^{-1}}$, with density ratios $\kappa = 10^{-3}$ (broken curve), $\kappa = 10^{-2}$ (dotted curve) and $\kappa = 10^{-1}$ (dash-dotted curve).

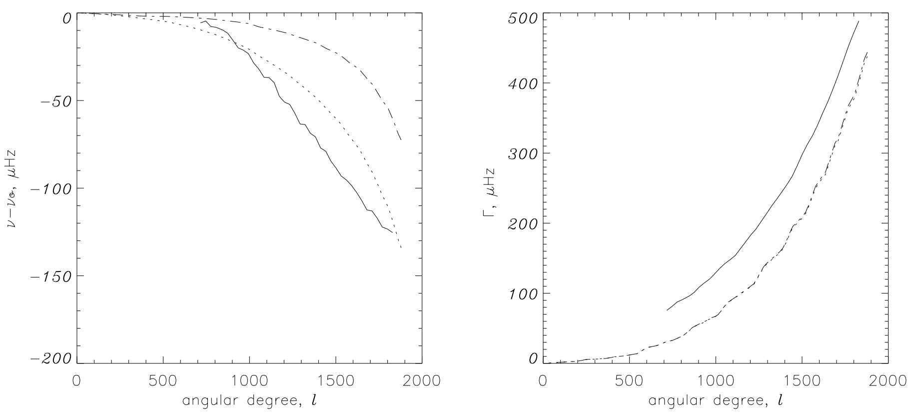

Fig. 5. The frequency difference $\nu - \nu_0$ and the linewidth Γ as a function of angular degree l. The observational data is represented by the solid curve. The parameters of the flow are: correlation length $l_x = 1.0 \cdot 10^3$ km and variance $\sigma = 1.0\,\mathrm{km\,s^{-1}}$. The density ratio $\kappa(\lambda)$ is computed according to Eq. (5) with $\lambda_{up} = \lambda_{dn} = 2\pi/k$ (dash-dotted curve) and with $\lambda_{up} = \lambda_{dn} = \pi/k$ (dotted curve), for wavenumber k.

the observational data (solid curve). A good fit is obtained for a linewidth Γ which grows with the correlation length. However, the frequency difference $\nu - \nu_0$ that is obtained numerically depends weakly on the angular degree up to $l = 1200$. For $l > 1400$ higher values of the correlation length (l_x) lead to a reduction in the frequency difference $\nu - \nu_0$.

The frequency difference and linewidth for different values of the variance σ are shown in Fig. 3. The effect of the stronger turbulent flow is to reduce the real part of the *f*-mode frequency and to increase the linewidth. This effect increases with angular degree, l. The results which are drawn for typical values of the convective flow ($\sigma = 1\,\mathrm{km\,s^{-1}}$ and $l_x = 10^3$ km) follow the trend that is revealed by the observational data, for both the frequency reduction and the linewidth (Figs. 2 and 3).

The numerical results show that the magnitude of the frequency decrease depends both on the velocity variance σ (see Fig. 3), the correlation length l_x (Fig. 2), and the density ratio κ (Fig. 4). In particular, at very small correlation lengths (0.2 Mm) the frequency reduction is much smaller than seen in the observational data. A good fit to the observational data is obtained for small values of κ. The linewidth depends weakly on κ.

Fig. 5 presents the numerically obtained results in the case when the density ratio κ is computed using Eq. (5). As a result of the averaging, the density ratio depends on the angular degree l. See Fig. 6. The dash-dotted and dotted curves are drawn for the penetration length equal to $2\pi/k$ and π/k. A better agreement between the theoretical and observational data corresponds to a

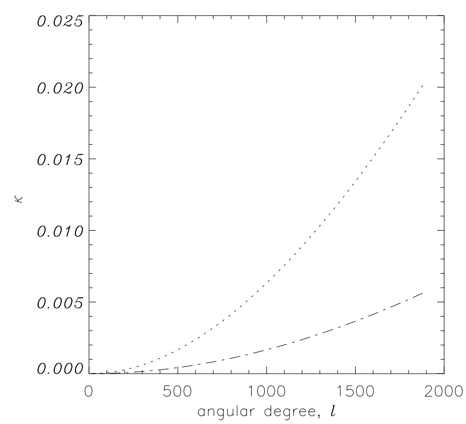

Fig. 6. Density ratio κ as a function of angular degree l according to Eq. (5) for $\lambda_{up} = \lambda_{dn} = 2\pi/k$ (dash-dotted curve) and $\lambda_{up} = \lambda_{dn} = \pi/k$ (dotted curve).

penetration length π/k (dotted curve). The linewidth Γ is hardly dependent on the density ratio (Fig. 5), in agreement with Fig. 4. The use of Eq. (5) significantly improves our results; compare Fig. 2 and Fig. 5.

4. Summary and discussion

The observed frequencies of the solar *f*-mode show deviations from the standard dispersion relation $\omega_0^2 = gk$. We have modelled the *f*-mode as the surface gravity wave that propagates along the interface between the photosphere-chromosphere and the convection zone. Such a wave is influenced by the turbulent field in the lower, denser medium. Numerical computations of the dispersion relation (2) for the *f*-mode in the model atmosphere containing a turbulent velocity field were performed for various parameters of the flow and wavevector. The effect of the turbulent flow is to decrease the frequency of the *f*-mode and broaden lines profiles. This reduction is revealed by the real part of ω. The negative imaginary part of the frequency represents damping of the coherent *f*-mode field due to scattering by the turbulent flow. The *f*-mode damping is a result of the generation of the turbulent field at the expense of the coherent field.

The *f*-mode damping, which corresponds to the negative imaginary part of ν, enhances the speed reduction and vice versa. As a consequence, a moderately low flow amplitude ($\sigma = 1\,\mathrm{km\,s^{-1}}$) is sufficient to fit the observational data while for the case when ν_i was neglected (see Murawski & Roberts 1993a) a flow of $\sigma = 3\,\mathrm{km\,s^{-1}}$ was required to match with observational data. We have obtained improved agreement between our theoretical results and the observational data for the density ratio being averaged over the *f*-mode penetration length (Eq. 5). The theoretical curve which correspond to the penetration length π/k lies close to the MDI observational curve.

Our current model, based on a single correlation length l_x of convection, remains not fully adequate because the spectrum of the solar granules is in fact continuous, and the solar stratification is more complex than considered here. A statistical model of turbulent convection based on a Kolmogorov spectrum, for instance, requires consideration, as it introduces a continuous range of spatial scales which correspond to various sizes of granules. There are also other effects which can influence both the frequency ν and the line-width Γ of the *f*-mode, including perturbations of the Reynolds tensor, radiative and convective fluxes, a dissipation rate of turbulent kinetic energy and power associated with the buoyancy force (Gabriel 1998), and magnetism. We neglected all these effects in our model although there inclusion may lead to a better agreement between theoretical and observational data.

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