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Accretion-disk dynamo models with dynamo-induced alpha-effect

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Abstract. A nonlinear 1D dynamo model with a heuristic α-ansatz for magnetic field-induced turbulence is considered to be growing for weak fields and suppressed by strong fields. The *amplification of initial magnetic fields* is regarded in accretion disks like both Kepler disks and galaxies. The solutions for the magnetic field are symmetric with respect to the equator, they are steady for positive α-effect and oscillatory for negative α-effect (in the northern hemisphere). Positive α-effect is assumed for galaxies while there are also reasons to assume negative α-effect in accretion disks (northern hemisphere). For galaxies, the question is discussed whether the amplification of initial fields by differential rotation and α-effect is strong enough to reach the observed field strengths of order μG. The pitch angle of the field lines, describing the spiral structure of the field, is determined. A model with pitch angles of (say) 10° as observed in galaxies reaches a magnetic amplification factor of only 300. Only a few rotation times are necessary to establish the final configuration. Once established, the highly nonlinear dynamo never decays.

Key words: accretion, accretion disks – instabilities – magnetic fields – turbulence – galaxies: magnetic fields

1. Introduction

Mean-field dynamo simulations usually required hydrodynamic turbulence as a prerequisite for dynamo action, that is, no magnetic field is needed to drive turbulence initially, and therefore infinitely small initial fields deliver us with non-trivial solutions, simply by the action of the electromotive forces of the small-scale fluid motion.

Investigations of the origin of turbulence proved its magnetohydrodynamic background in astrophysical disks. Weak magnetic fields can turn differentially rotating matter into turbulent flows (Balbus & Hawley 1991). Such a generation of turbulence cannot be directly included in a mean-field dynamo simulation. We need expressions for the dynamo-α representing both the generation of turbulence by weak fields and the suppression of turbulence by strong fields. Recently Brandenburg & Schmitt (1998) simulated an α-effect due to magnetic buoyancy which grows with growing fields. The 3D simulations of Brandenburg et al. (1995) have shown that large-scale magnetic fields can be generated by magnetic-induced turbulence. Kitchatinov & Rüdiger (1997) demonstrated the global character of the magnetic shear instability in spheres.

We used a heuristic function for the α-effect, which grows with the magnetic energy density B^2 for weak fields and falls with $1/B^2$ for strong fields. Unlike a standard dynamo simulation where hydrodynamic turbulence is a prerequisite, *we now need a finite minimum magnetic field* to start up a dynamo. The relevance of such an approach will be validated if the ratio of seed field to final large-scale field is of the same order of magnitude as suggested for astrophysical objects.

The question of how strong initial magnetic fields had to be during galaxy formation to explain typical galactic magnetic fields of 1–10 μG (see e.g. Berkhuijsen et al. (1997), for M51) has been discussed for the last couple of decades. Mechanisms for the creation of primordial seed fields like that of Harrison (1973) lead to a maximum of 10^{-16} G before dynamo and differential rotation act as amplifiers. Intergalactic magnetic field strengths are assumed of order 10^{-10} G according to the analysis of Vallée (1983), which differ by more than four orders of magnitude from the galactic fields. A detailed review on the models of galactic magnetic fields is given in Beck et al. (1996). Accretion disk dynamos are considered by Stepinski & Levy (1990), Campbell (1992), Torkelsson & Brandenburg (1994a,b), Rüdiger et al. (1995).

2. The dynamo model

The dynamo equation in terms of mean-field theory consists of the action of the turbulent electromotive force $\boldsymbol{\mathcal{E}}$ caused by fluctuations in the velocity field and magnetic field, $\boldsymbol{\mathcal{E}} = \langle \boldsymbol{u}' \times \boldsymbol{B}' \rangle$, and the action of large-scale velocity gradients like differential rotation

$$\frac{\partial \langle \boldsymbol{B} \rangle}{\partial t} = \mathrm{rot}\left(\langle \boldsymbol{u} \rangle \times \langle \boldsymbol{B} \rangle + \boldsymbol{\mathcal{E}} \right). \tag{1}$$

The turbulent EMF can be developed in a series

$$\mathcal{E}_i = \alpha_{ij} \langle B_j \rangle + \eta_{ijk} \langle B_j \rangle_{,k} + \ldots, \tag{2}$$

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where α_{ij} represents the dynamo effect, and the η-tensor gives the eddy diffusivity. We will assume isotropic properties in our model by $\alpha_{ij} = \alpha\,\delta_{ij}$ and $\eta_{ijk} = \eta_{\rm T}\,\epsilon_{ijk}$. Both tensors α and η are suppressed by strong magnetic fields, because the motion will be more and more tightly bound to the large-scale magnetic field; this study only considers the suppression of α, whereas $\eta_{\rm T}$ is constant.

The dynamo equation is studied with a 1D model, representing an infinite disk extending in the xy-plane with height H. The integration region extends only in the z-direction hence the remaining radial and azimuthal components of the magnetic field depend on z only. With these restrictions, the dynamo equation leads to

$$\frac{\partial B_x}{\partial t} = -\frac{\partial}{\partial z}\Big(\alpha B_y\Big) + \eta_{\rm T}\frac{\partial^2 B_x}{\partial z^2}, \tag{3}$$

$$\frac{\partial B_y}{\partial t} = \frac{\partial}{\partial z}\Big(\alpha B_x\Big) + \frac{\partial u_y}{\partial x}B_x + \eta_{\rm T}\frac{\partial^2 B_y}{\partial z^2} \tag{4}$$

for the x- and y-component of the field. Normalization of times with the diffusion time $\tau_{\rm diff} = H^2/\eta_{\rm T}$ and vertical distances z with H yield for the poloidal potential A and the toroidal field B the equations

$$\frac{\partial A}{\partial t} = C_\alpha \hat\alpha(z)\,\psi(B_{\rm tot})B + \frac{\partial^2 A}{\partial z^2}, \tag{5}$$

$$\frac{\partial B}{\partial t} = -C_\alpha \frac{\partial}{\partial z}\left(\hat\alpha(z)\,\psi(B_{\rm tot})\frac{\partial A}{\partial z}\right) - C_\Omega\frac{\partial A}{\partial z} + \frac{\partial^2 B}{\partial z^2} \tag{6}$$

with $B_{\rm tot}^2 = B^2 + (\partial A/\partial z)^2$. The dimensionless parameters

$$C_\alpha = \frac{\alpha_0 H}{\eta_{\rm T}}, \qquad C_\Omega = \frac{\partial u_y}{\partial x}\frac{H^2}{\eta_{\rm T}} < 0 \tag{7}$$

represent the strength of the α-effect and the shear flow, resp. Without loss of generality for galaxies and Kepler disks the dimensionless quantity (7) can be assumed as negative. The outer parts of a galactic disk can be represented by $C_\Omega = -\Omega H^2/\eta_{\rm T}$; a Keplerian disk has a somewhat steeper rotation profile giving $C_\Omega = -1.5\Omega H^2/\eta_{\rm T}$. This slight difference will not be relevant for the results so we shall use the definition in (7) throughout this paper. The parameter becomes, with the ansatz $\nu_{\rm T} = \alpha_{\rm SS}H^2\Omega$ for the eddy viscosity,

$$C_\Omega = -{\rm Pm}/\alpha_{\rm SS}. \tag{8}$$

With ${\rm Pm} \simeq 1$ and $\alpha_{\rm SS} \geq 0.01\ldots 1$ the C_Ω amplitude should vary between 1 and 100, $C_\Omega \simeq -10$ Pm might be a good estimate.[1] For galaxies with $H\Omega \approx c_{\rm ac}$ ($c_{\rm ac}$ speed of sound), $\Omega^* = 2\tau_{\rm corr}\,\Omega$ and the Mach number $\mathcal{M} = \sqrt{\langle u'^2\rangle}/c_{\rm ac}$ follows

$$C_\Omega \simeq -\frac{\rm Pm}{c_\nu \mathcal{M}^2\Omega^*}. \tag{9}$$

With $\mathcal{M} \lesssim 1, \Omega^* \simeq 0.1$ and $c_\nu \simeq 0.33$ we again arrive at an amplitude $|C_\Omega|$ of about 10 Pm.

The α depends on the magnetic field as well as on the location in the object. Hence we split the dependencies into two factors, i.e. $\alpha = \alpha_0\psi(B)\hat\alpha(z)$. Both the suppression of α by large magnetic fields and the non-existence of turbulence for zero magnetic fields is expressed by the function

$$\psi(\beta) = \frac{\beta^2}{1+\beta^4}, \tag{10}$$

where $\beta = |\langle B\rangle|/B_{\rm max}$ is the magnetic field normalized by the cut-off field strength $B_{\rm max} = \sqrt{\mu_0 p}$. The magnetic field is thus considered to be weak in the sense that the Alfvèn speed is small compared with the sound speed (Papaloizou & Szuszkiewicz 1992; Brandenburg & Campbell 1997; Rüdiger et al. 1999).

Also the eddy diffusivity can be considered as magnetic-induced and as magnetic-quenched. Even the magnetic diffusivity-quenching in standard dynamo theory has been considered only seldom (see, however, Rüdiger et al. 1994). Studies are missing about the generation of eddy diffusivity by MHD instability. On the other hand, dissipation is such a wide-spread phenomenon that certainly part of it does not depend on the magnetic field. The problem of the dynamo theory is always to discuss the effects which are inducing magnetic fields rather than dissipating. Hence we have to follow the procedure of concentration to modeling the magnetic features of the α-effect.

Concerning the sign of the α-effect we have to distinguish two cases. Simulations of SN explosions in the interstellar matter always yield positive α (more exactly: positive $\alpha_{\phi\phi}$-component of the α-tensor) in the northern hemisphere (Ferrière 1996; Ziegler 1996; Korpi et al. 1998). In strong shear flows such as Kepler disks the resulting MHD-turbulence seems to produce *negative* α-effect in the northern hemisphere and positive in the southern hemisphere (Brandenburg & Donner 1997). There is thus reason to work here with both possible signs. Recent second-order correlation approximation computations of fluctuating field and momentum in MHD shear flows yield

$$\alpha = -\frac{\tau_{\rm corr}^2(\boldsymbol{g\Omega})}{c_{\rm ac}^2}\left(\frac{4}{15}+\frac{1}{5}\frac{\partial\log\Omega}{\partial\log R}\right)\frac{\langle \boldsymbol{B}'^2\rangle}{\mu_0\rho} \tag{11}$$

for the α-effect. The flow is driven by fluctuating magnetic fields and the basic modifications are due to buoyancy. With $\boldsymbol{g}$ as the gravitation, in Kepler flows, one obtains

$$\alpha = -\frac{z\tau_{\rm corr}^2\Omega^3}{30}\,\frac{\langle \boldsymbol{B}'^2\rangle}{\mu_0\rho c_{\rm ac}^2} \tag{12}$$

(Pipin et al. 1999) and $\langle \boldsymbol{B}'^2\rangle \propto \langle \boldsymbol{B}\rangle^2$. Indeed, magnetically driven turbulence under the simultaneous influence of buoyancy and rotation can produce negative values of the α-effect in shear flows (Brandenburg 1998).

The α-quenching part of ψ for high field amplitudes was thoroughly analyzed by Rüdiger & Kitchatinov (1993) and is approximated here by a β^{-2}-dependence. Such a strong-field dependence was also used by Stix (1972), Brandenburg et al. (1989) and Schmitt & Schüssler (1989).

Both pressure and velocity fluctuations depend on the vertical position z and thus lead to a z-dependence of α which decreases exponentially with the height above and below the equatorial plane. The strong decrease of α allows for boundaries at finite height H, and its z-dependence is approximated

[1] $C_\Omega \simeq -100$ are also used by v. Rekowski et al. (1999)

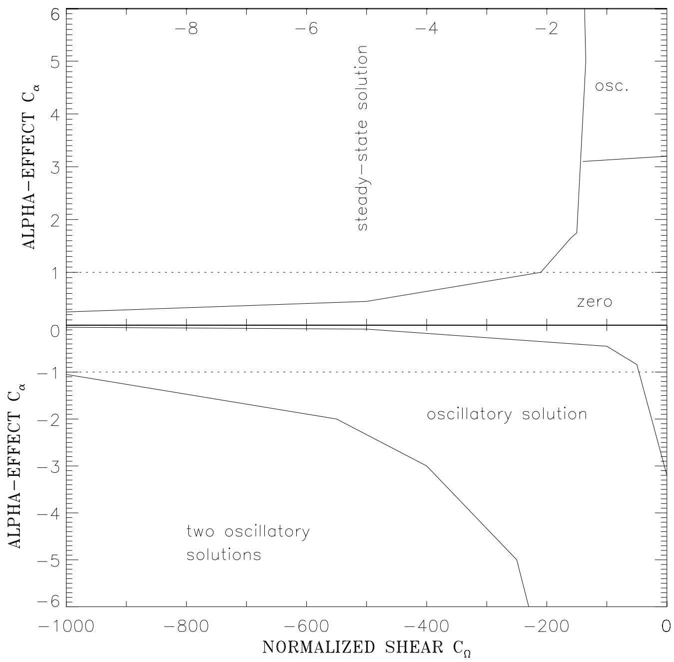

Fig. 1. Solutions of a 1D $\alpha^2\Omega$-dynamo in the C_α-C_Ω-plane. Positive C_α yield stationary solutions and negative C_α yield oscillatory solutions

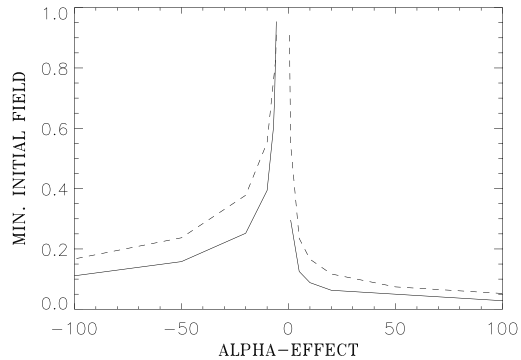

Fig. 2. Minimum initial field for the $\alpha^2\Omega$-dynamo with magnetic-field-induced turbulence. The dotted line represents the roots of Eq. (13). $C_\Omega = -10$

by $\hat{\alpha} \sim \sin 2z$, where z runs from 0 to π in dimensionless coordinates. We limit the magnetic field to the disk by the boundary conditions $B = \partial A/\partial z = 0$ at $z = 0$ and $z = \pi$.

The spatial domain is discretized by 201 grid points, and the time step must be smaller than $10^{-4}\tau_{\rm diff}$ as required for parabolic differential equations.

3. Results

The 1D $\alpha^2\Omega$-dynamo provides oscillatory and steady-state solutions. It has been widely believed that such a dynamo should become a simpler $\alpha\Omega$-dynamo for strong differential rotation neglecting the C_α-term in (6). There are, however, two possible solutions (see Fig. 1) for sufficiently supercritical dynamo parameters C_α and C_Ω. Unlike $\alpha\Omega$-dynamos, in an $\alpha^2\Omega$-dynamo, the dynamo action cannot be described by a single dynamo number. Both the coexisting solutions can be excited by different initial field strengths.
The dotted horizontal lines in Fig. 1 give a typical parameter space of an $\alpha\Omega$-dynamo which may be directly compared with Rüdiger et al. (1994) delivering the critical $|C_\Omega^{\rm crit}| = 2.08$ and 44.1 for steady-state and oscillatory quadrupolar solutions respectively The particular case of pure α^2-dynamo is given by the right-hand ordinate yielding the critical $C_\alpha^{\rm crit} = 3.14$ as determined analytically by Meinel (1990).

The new α-function which implies the excitation of turbulence by weak magnetic fields *and* the suppression of velocity fluctuations by strong magnetic fields, requires *a minimum initial field to start the dynamo* operation. The existence of a minimum initial field is in great contrast to the kinematic dynamo theory. The critical initial field amplitudes are shown in Fig. 2. The solid line is derived from simulations with $C_\Omega = -10$, that is, with a differential rotation comparable to the strength of C_α. The dashed line is the result of an upper-limit estimate of $B_{\rm min}$ as the solution of the over-all equation

$$C_\alpha^{\rm crit} = C_\alpha \frac{B_{\rm min}^2}{1 + B_{\rm min}^4}. \tag{13}$$

A characteristic number for a dynamo excited with such a growth-and-quenching function is the amplification factor of the initial field $B_{\rm min}$. We concentrate on stationary solutions. Initial and final field strength are shown in Fig. 4, in dependence on C_α. The maximum amplification is presented, being achieved with the smallest initial magnetic field $B_{\rm min}$ that excites a non-trivial dynamo solution. The amplification $\mathcal{A} = B_{\rm max}/B_{\rm min}$ grows linearly with C_α. On the positive side is $\mathcal{A} \propto 30 C_\alpha$ and on the negative side it is only

$$\mathcal{A} \propto 3|C_\alpha|. \tag{14}$$

For a fixed C_α the amplification $\mathcal{A}$ increases with the differential rotation like $\mathcal{A} \propto C_\Omega^{1.7}$.

3.1. Kepler disks: oscillating solutions

For negative C_α all the solutions oscillate (cf. Torkelsson & Brandenburg 1994b). The cycle times of such dynamos have been studied earlier in Rüdiger & Arlt (1996). One or two diffusion times proved to be the typical cycle time. As the diffusion time here equals $C_\Omega/2\pi$ rotation periods (see below), we find only a few rotation periods as characteristic cycle time. The number of 30 rotation periods as a characteristic cycle time found by Brandenburg & Campbell (1997) fits here as a direct consequence of their small eddy diffusivity. With their $\eta_{\rm T} \simeq 0.008\Omega H^2$, one finds $C_\Omega \simeq 125$ whence $\tau_{\rm cyc} \simeq 20\ldots 40\ P_{\rm rot}$ in perfect agreement with their numerical result.

Fig. 3 shows the temporal behavior of the magnetic-field distribution over the integration domain for the two possible oscillatory solutions. In only one of the solutions, the fields are migrating (in opposite direction to the solar activity). All stable solutions of the model *are symmetric* with respect to the

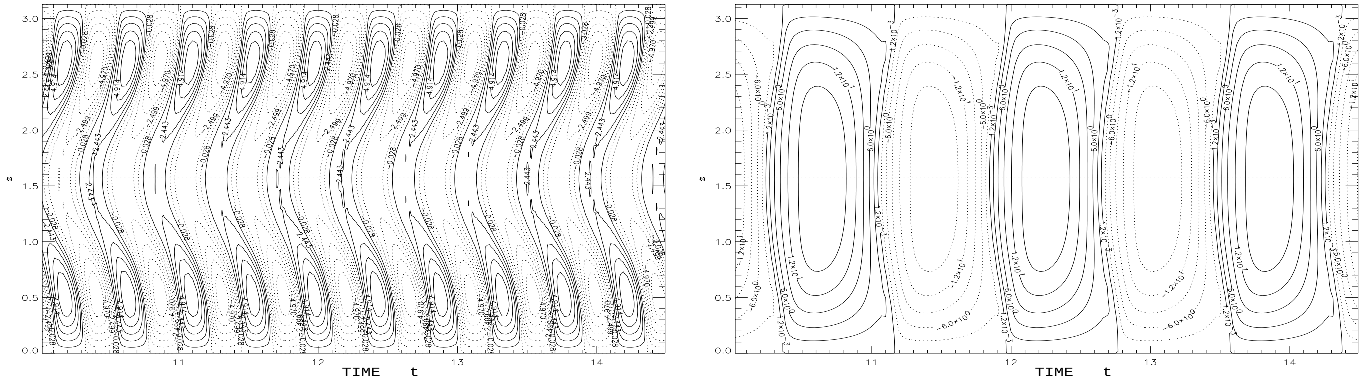

Fig. 3. 'Butterfly' diagram of the two oscillatory solutions of a 1D $\alpha^2\Omega$-dynamo showing both migrating and non-migrating magnetic fields. The time is measured in diffusion times, z is the integration domain

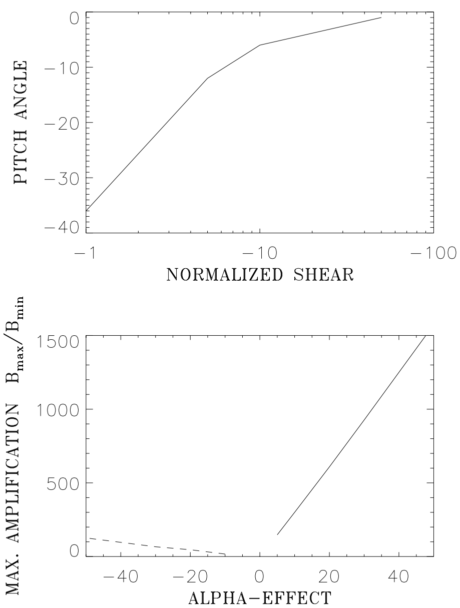

Fig. 4. TOP: The pitch angle for models with $C_\alpha = 10$ versus C_Ω. Large C_Ω yield very small pitch angles ("$\alpha\Omega$-dynamos"), small C_Ω yield large pitch angles ("α^2-dynamos"). BOTTOM: The amplification factor $\mathcal{A}$ of the $\alpha^2\Omega$-dynamo with magnetic field-induced turbulence versus C_α ($C_\Omega = -10$)

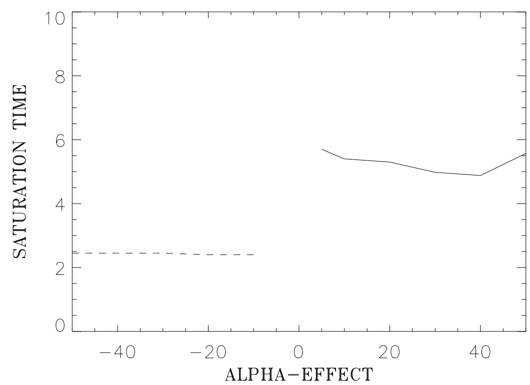

Fig. 5. The growth time $\delta\tau$ of the $\alpha^2\Omega$-dynamo does *not* depend on the normalized α-effect. Times are given in units of diffusion time, $C_\Omega = -10$

equator ('quadrupolar'). The direction of the migration towards the poles cannot be changed in this simple geometry.

3.2. Galaxies: steady solutions

Here the relation of the azimuthal component to the radial field component is of particular interest, as the inclination of the field lines against circles can be derived from polarization measurements in galaxies. The pitch angles found are mostly between $15°$ and $35°$ in spiral galaxies (Beck et al. 1996). One of the smallest values is $11°$ in M51 (Berkhuijsen et al. 1997).

The pitch angle can easily be determined in our models by $\arctan(B_x/B_y)$, the ratio of radial and azimuthal field component. The pitch angle depends only on the differential rotation – expressed by C_Ω – rather than the α-effect, since an α^2-dynamo transforms the azimuthal field into a radial field and vice versa with the same growth rate. The variation of the pitch angle with the strength of the differential rotation is shown in Fig. 4. For $C_\alpha = -C_\Omega = 10$, the plot gives a pitch angle of about $10°$ corresponding to the observed galactic fields. The amplification $\mathcal{A}$, however, is then only of order 300.

3.3. Growth rates

We have also checked how fast the amplification occurs. In Fig. 5, the growth times $\delta\tau$ are given in units of diffusion time, so that for the physical growth rate, $\delta t = \delta\tau H^2/\eta$, follows

$$\frac{\delta t}{P_{\rm rot}} = \frac{C_\Omega}{2\pi}\,\delta\tau \qquad (15)$$

in units of rotation periods. This time only depends on the sign of the α-effect, not on its amplitude. Always after only a few rotation periods, the solutions are saturated.

4. Conclusions

After Fig. 1 shear flows with negative shear like Kepler disks and galaxies produce stationary solutions for positive C_α and cyclic solutions for negative C_α, both with quadrupolar symmetry. In any case, the 'dynamo' with dynamo-induced α-effect needs a minimum magnetic field to start its operation which then consists of a fast but finite amplification of the field strength. For dynamos like galaxies with remarkable magnetic pitch angles the amplification is only of order $10^{2\ldots3}$.

From our experience, once established, the nonlinear dynamo never decays. This finding even remains true if the α-effect is forced to exist only in a narrow range between two magnetic field amplitudes of the same order.

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