Astron. Astrophys. 349, L17–L20 (1999)

ASTRONOMY AND ASTROPHYSICS

LETTER

Letter to the Editor

Stability and energetics of mass transfer in double white dwarfs

Zhanwen Han[1,2,3] **and Ronald F. Webbink**[4]

[1] Yunnan Observatory, Academia Sinica, Kunming, 650011, P.R. China
[2] National Astronomical Observatories, Chinese Academy of Sciences
[3] Institute of Astronomy, Madingley Road, Cambridge, CB3 0HA, UK (zhanwen_han@163.net or zhanwen@ast.cam.ac.uk)
[4] Department of Astronomy, University of Illinois at Urbana-Champaign, 1002 West Green Street, Urbana, IL 61801, USA (webbink@astro.uiuc.edu)

Received 15 July 1999 / Accepted 12 August 1999

Abstract. The donor star in an interacting double white dwarf lies deep within the potential well of the accretor, a condition which both alters the minimum mass transfer rate needed to generate an Eddington luminosity, and limits the fraction of matter which can be driven from the binary in a super-Eddington wind. That mass loss itself alters the threshold condition for the onset of dynamical instability of the donor star. We show that, unless the mass of the accreting star is extremely near the Chandrasekhar limit, a significant fraction (exceeding 0.5 for accretors of mass $< 1M_{\odot}$) must be retained, even when the nominal accretion luminosity far exceeds the Eddington limit. In super-Eddington accretion, the accreted envelope almost certainly extends beyond the orbit of the donor, and will trigger white dwarf merger in a common envelope.

Key words: instabilities – stars: binaries: close – stars: white dwarfs

1. Introduction

Close double white dwarfs (DWDs) play important roles in a number of astrophysical problems. Not only do their physical and statistical properties hold important information about common envelope evolution (e.g. Han 1998), but they are expected to dominate the galactic gravitational wave background between 0.1 mHz and 0.1 Hz (Evans, Iben, & Smarr 1987; Hils, Bender, & Webbink 1990). Orbital energy and angular momentum losses to gravitational radiation can drive close DWDs to mass transfer, an evolutionary path which may lead to merger and the production of Type Ia supernovae, R Coronae Borealis (R CrB) stars, and sdO stars (Webbink 1984; Iben & Tutukov 1984), depending on the underlying compositions of the two white dwarfs. Some DWDs clearly avoid merger, as we see their survivors, in the form of AM Canum Venaticorum (AM CVn) binaries (Warner 1995) — interacting DWDs with helium white dwarf donors.

Send offprint requests to: Z. Han

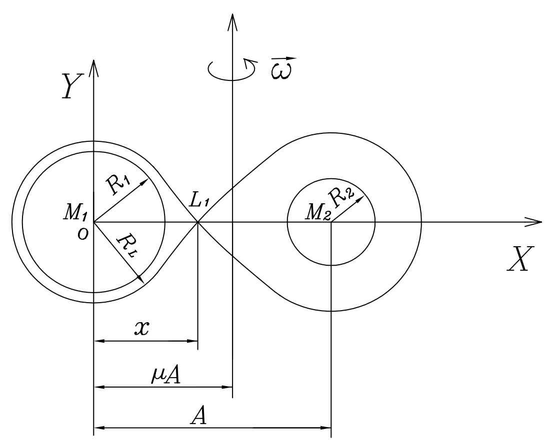

Fig. 1. Schematic model of a DWD system.

Which evolutionary channel a given DWD will follow depends sensitively on the stability and rate of mass transfer when it occurs. The stability of Roche lobe overflow (RLOF) in close DWDs has been the subject of many past investigations (e.g., Pringle & Webbink 1975, Tutukov & Yungelson 1979, Webbink 1984, Cameron & Iben 1986, Webbink & Iben 1992), usually on the assumption that total mass and orbital angular momentum of the binary are conserved by mass transfer. Here, we explore issues raised in the last of these papers, concerning super-Eddington accretion and the energetics of mass loss, as they pertain to the stability of RLOF.

2. The model

Consider a DWD system (Fig. 1), consisting of components WD1 and WD2 in circular orbit, with $M_1 < M_2$. As the orbit decays due to gravitational radiation, the less-massive, larger component, WD1, will be the first to fill its Roche lobe and will become the mass donor, with WD2 the mass accretor.

The rate of closure of the Roche lobe about the donor radius can be written

$$\frac{\mathrm{d}\ln(R_1/R_\mathrm{L})}{\mathrm{d}t} = \left(\frac{\partial \ln R_1}{\partial t}\right)_{M_1} - \left(\frac{\partial \ln R_\mathrm{L}}{\partial t}\right)_{M_1} + (\xi_\mathrm{ad} - \xi_\mathrm{L})\frac{\dot{M}_1}{M_1} , \quad (1)$$

where

$$\xi_\mathrm{ad} \equiv \left(\frac{\partial \ln R_1}{\partial \ln M_1}\right)_\mathrm{ad} , \quad (2)$$

and

$$\xi_\mathrm{L} \equiv \left(\frac{\partial \ln R_\mathrm{L}}{\partial \ln M_1}\right)_\mathrm{RLOF} . \quad (3)$$

For DWD's, we may neglect any residual thermal contraction at constant mass, and set

$$\left(\frac{\partial \ln R_1}{\partial t}\right)_{M_1} = 0 . \quad (4)$$

In the absence of mass transfer (M_1 constant), R_L scales simply as the square of the orbital angular momentum, J, and we have:

$$\left(\frac{\partial \ln R_\mathrm{L}}{\partial t}\right)_{M_1} = \frac{2\dot{J}_\mathrm{GR}}{J} \equiv -\frac{2}{\tau_\mathrm{GR}} , \quad (5)$$

where

$$\tau_\mathrm{GR} = \frac{5}{32}\frac{c^5}{G^3}\frac{A^4}{M_1M_2(M_1+M_2)} \quad (6)$$

is the time scale for orbital angular momentum loss due to gravitational radiation (Landau & Lifshitz 1962), and A is the orbital separation.

When the contracting Roche lobe reaches the surface of WD1 ($R_1 = R_\mathrm{L}$), mass loss from WD1 begins. Since $\dot{M}_1 < 0$, if $\xi_\mathrm{ad} > \xi_\mathrm{L}$, then any increase in the *magnitude* of $\dot{M}_1$ tends to draw WD1 inside its Roche lobe, cutting off mass transfer. Mass transfer is then dynamically stable, and proceeds at a rate for which $\dot{R}_1 = \dot{R}_\mathrm{L}$. If $\xi_\mathrm{ad} < \xi_\mathrm{L}$, on the other hand, perturbations in $\dot{M}_1$ are self-amplifying, and mass transfer is dynamically unstable.

To evaluate ξ_ad, we note that in the limit of complete degeneracy, white dwarfs have zero specific entropy. Their adiabatic mass-radius relation is therefore well-approximated by the zero-temperature mass-radius relation for white dwarfs. Finite-temperature white dwarfs depart significantly from zero-temperature white dwarfs (at least insofar as the run of density, or mass, with radius is concerned) only in their outer envelopes, which may become only partially degenerate or even non-degenerate. Once these partially-degenerate or non-degenerate layers are stripped away (and they generally constitute a *very* small fraction of the white dwarf's mass), the interiors behave practically indistinguishably from zero-temperature white dwarfs. For our purposes, the analytic mass-radius relation for zero-temperature white dwarfs obtained by Nauenberg (1972) suffices for both WD1 and WD2:

$$R_{1,2} = 0.01125\, R_\odot\, (m_{1,2}^{-2/3} - m_{1,2}^{2/3})^{1/2} \quad (7)$$

where $m_{1,2} \equiv M_{1,2}/1.433 M_\odot$, the mass of the respective donor (M_1) or accretor (M_2) in terms of the Chandrasekhar mass limit. The adiabatic mass-radius exponent for the donor star therefore becomes

$$\xi_\mathrm{ad} = -\frac{1}{3}\left(\frac{m_1^{-2/3} + m_1^{2/3}}{m_1^{-2/3} - m_1^{2/3}}\right) . \quad (8)$$

To evaluate ξ_L, we assume that a fraction β of the mass lost by WD1 is accreted by WD2. The remaining fraction, $(1-\beta)$, is lost from the system, carrying away the orbital angular momentum per unit mass of WD2. Employing Eggleton's formula for Roche lobe radius (Eggleton 1983), we find

$$\xi_\mathrm{L} = C_1 + C_2\beta \quad (9)$$

where

$$C_1 = \frac{2}{3} - \frac{0.4q^{2/3} + \frac{1}{3}q^{1/3}/(1+q^{1/3})}{0.6q^{2/3} + \ln(1+q^{1/3})} + \frac{2q^2 - q - 2}{1+q} \quad (10)$$

and

$$C_2 = \left(\frac{2}{3} - \frac{0.4q^{2/3} + \frac{1}{3}q^{1/3}/(1+q^{1/3})}{0.6q^{2/3} + \ln(1+q^{1/3})}\right) q + \frac{q}{1+q} , \quad (11)$$

and q is the mass ratio, $q \equiv M_1/M_2$.

Unlike nondegenerate donor stars in other compact binaries, e.g. cataclysmic variables, the WD donors in DWDs lie deeper within the potential of the accreting star. This has two important consequences for mass transfer. First, the accretion stream does not sample the full depth of the accreting star potential, so that a higher accretion rate is needed to generate an Eddington luminosity,

$$L_\mathrm{Edd} = \frac{4\pi R_2^2 cg}{\kappa} , \quad (12)$$

at which radiation pressure balances gravity. In this expression, $\kappa = 0.2(1+X)\,\mathrm{cm^2 g^{-1}}$ is the opacity of the accreted gas (with X, the hydrogen mass fraction, assumed zero here), and g is the gravitational acceleration at the surface of WD2. Second, and more importantly, the energy needed to remove matter from the inner Lagrangian point, L_1, to infinity is no longer negligible compared with the depth of the accreting star potential, a factor which we shall see below severely limits the fraction of matter a mass-transferring DWD can expel.

The accretion luminosity can be expressed straightforwardly in terms of the Roche potential at the inner Lagrangian point, ϕ_L1, and at the surface of the accreting white dwarf, ϕ_R2:

$$L_\mathrm{acc} = \dot{M}_2(\phi_\mathrm{L1} - \phi_\mathrm{R2}) , \quad (13)$$

where $\dot{M}_2 = -\beta\dot{M}_1$ is of course the accretion rate. The potential ϕ_L1 is

$$\phi_\mathrm{L1} = -\frac{GM_1}{x} - \frac{GM_2}{A-x} - \frac{G(M_1+M_2)}{2A^3}(x-\mu A)^2 , \quad (14)$$

where $\mu = M_2/(M_1+M_2) = 1/(1+q)$, and x is the distance between the centre of the WD1 and the inner Lagrangian point (see Fig. 1). A useful approximation to x, accurate to within a relative error of less than 0.35% over the range $0 < q \leq 1$, is

$$\frac{x}{A} = \frac{(0.696q^{1/3} - 0.189q^{2/3})}{(1 + 0.014q)} \; . \tag{15}$$

The potential $\phi_{\rm R2}$ is, to lowest order in tidal and rotational terms, (see, however, Chapter III of Kopal 1959)

$$\phi_{\rm R2} = -\frac{GM_1}{A} - \frac{GM_2}{R_2} - \frac{G(M_1 + M_2)}{2A^3}\left(\frac{2}{3}R_2^2 + (A - \mu A)^2\right) , \tag{16}$$

where R_2 is the (volume-equivalent) radius of WD2.

We assume below that, if the accretion luminosity (Eq. 13) is less than the Eddington limit (Eq. 12), mass transfer is conservative ($\dot{M}_1 + \dot{M}_2 = 0$, i.e., $\beta = 1$). Eqs. (1)-(11) then give

$$\dot{M}_1 = -\frac{2M_1/\tau_{\rm GR}}{\xi_{\rm ad} - C_1 - C_2} \; . \tag{17}$$

The condition that $L_{\rm acc} < L_{\rm Edd}$ is fulfilled under these assumptions only so long as

$$-\dot{M}_1 < \dot{M}_{\rm Edd} = \frac{4\pi R_2^2 c\bar{g}}{\kappa(\phi_{\rm L1} - \phi_{\rm R2})} , \tag{18}$$

where $\bar{g}$ is the mean surface gravity on WD2. From Eq. (16), we find

$$\bar{g} = \frac{GM_2}{R_2^2} - \frac{2}{3}R_2\frac{G(M_1 + M_2)}{A^3} \; . \tag{19}$$

When $-\dot{M}_1 > M_{\rm Edd}$, it becomes possible for radiation pressure alone to drive matter from the binary system. However, in the absence of non-gravitational energy sources, the energy to do so must also come from accretion.

It is important to recognize that the Eddington accretion limit is physically meaningful only if the photons released in the accretion flow can diffuse outward faster than they are swept inward by the accretion flow itself. Otherwise, the accretion energy is buried (absorbed) in the accreted envelope and an accretion rate exceeding Eddington's limit is quite physical. Assuming a free-fall, spherically-symmetric accretion flow, we obtain a ratio of photon diffusion velocity $v_{\rm diff}$ to mass flow velocity $v_{\rm infall}$

$$\frac{v_{\rm diff}}{v_{\rm infall}} = \frac{10\pi c R_2}{3\kappa\dot{M}} \; . \tag{20}$$

Departure from spherical symmetry or free fall will tend to aid diffusion, but comparison of Eqs. (18) and (20) suggests that self-absorption of energy within the accretion flow severely limits the emergent radiant flux when the mass accretion rate surpasses the nominal Eddington-limited rate.

We assume, therefore, that any accretion luminosity in excess of the Eddington limit is absorbed in the accretion flow. In terms of energetics, radiative losses represent accretion energy not used to power a mass outflow (other possibilities are beyond the scope of this paper). Energy conservation then gives

$$-\dot{M}_1(\phi_{\rm L1} - \phi_{\rm R2}) = (1 - \beta)\dot{M}_1\phi_{\rm R2} + L_{\rm Edd} \tag{21}$$

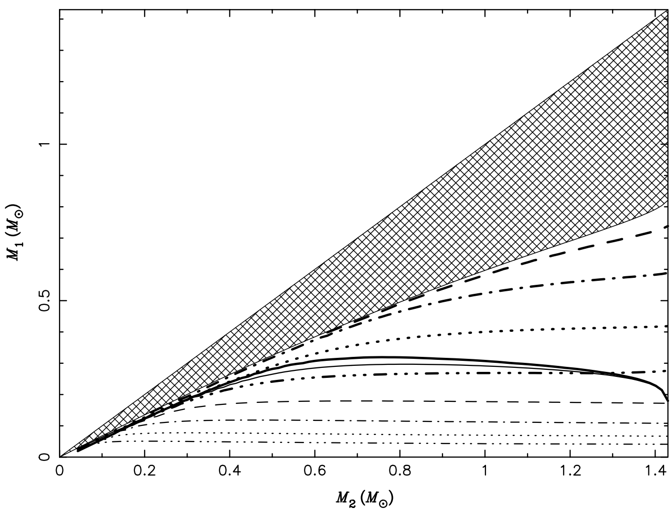

Fig. 2. RLOF stability and mass transfer rates for DWDs. The donor is WD1, the less massive white dwarf. The crosshatched area indicates dynamically unstable mass transfer. Above the thick solid line, mass transfer rates exceed the Eddington limit $\dot{M}_{\rm Edd}$ (see Eq. [18]). The thin solid line marks the corresponding Eddington mass transfer rate which would pertain to a donor star at infinity. The remaining dashed and dotted lines trace contours of constant mass transfer rate, from top to bottom, $10^{-2},\ 10^{-3}, \ldots,\ 10^{-9} M_\odot {\rm yr}^{-1}$, respectively.

for $-\dot{M}_1 > \dot{M}_{\rm Edd}$. The fraction of matter which must be accreted by WD2 to power the outflow is thus

$$\beta = \frac{L_{\rm Edd}}{\phi_{\rm R2}\dot{M}_1} + \frac{\phi_{\rm L1}}{\phi_{\rm R2}} \; . \tag{22}$$

From Eq. (22), we see that the larger the radiated luminosity at a given mass transfer rate, the *higher* in fact must be the fraction β of material retained by the accretor. Substituting Eq. (22) into Eqs. (9) and (1) we find (instead of Eq. [17]) an equilibrium mass transfer rate

$$\dot{M}_1 = \frac{C_2 L_{\rm Edd}/\phi_{\rm R2} - 2M_1/\tau_{\rm GR}}{\xi_{\rm ad} - C_1 - C_2\phi_{\rm L1}/\phi_{\rm R2}} , \tag{23}$$

provided that the denominator in this expression is positive. This is the condition for dynamical stability in the presence of radiatively-driven mass loss. It differs from Eq. (17) above because of the dependence of β on $\dot{M}_1$ through Eq. (9).

3. Results

The limiting masses for dynamically stable mass transfer (from the denominator of Eq. [23]), threshold for Eddington-limited mass transfer (from Eq. [18]), and equilibrium mass transfer rates (from Eqs. [17] and [23], as appropriate) are all plotted in Fig. 2. For comparison, the threshold for Eddington-limited mass transfer is also plotted for the case in which one neglects $\phi_{\rm L1}$, the finite depth of the potential at the source of the accretion stream. One sees immediately that the introduction of a more appropriate potential has scarcely any effect on the position of this threshold, which is nearly constant at $\sim 0.3M_\odot$. It is also apparent that the equilibrium mass transfer rates are virtually

LETTER

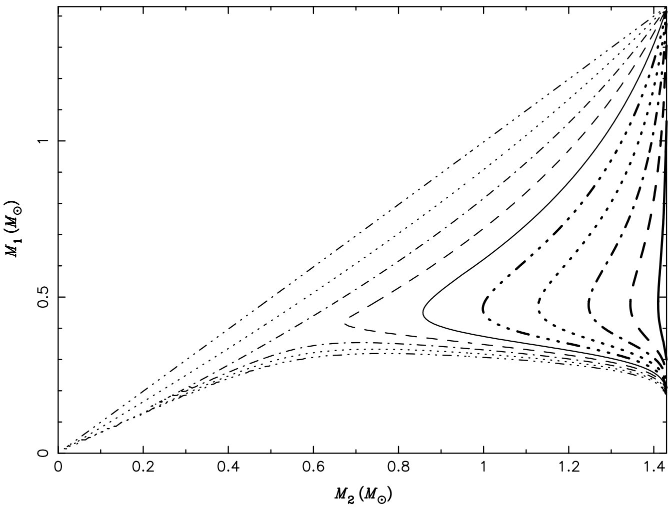

Fig. 3. Ejected mass fraction $(1-\beta)$ of an interacting DWD (Eq. [22]). Contours mark, from left to right, $1-\beta = 0.0, 0.1, \ldots,\ 0.9$, respectively.

a function of donor mass alone, except very near the limit for dynamical instability.

Fig. 2 reveals a broad region of stable donor masses between the threshold for super-Eddington mass transfer and that for dynamically-unstable mass transfer. This region is expanded by the stabilizing effects of super-Eddington mass transfer (compare Eq. [23] with Eq. [17]), which lead to a modest increase in the maximum stable donor mass from $0.66 M_\odot$ to $0.82 M_\odot$ for accreting white dwarfs at the Chandrasekhar limit. To a good approximation, the threshold for dynamically unstable mass transfer in the presence of super-Eddington mass loss becomes

$$q \gtrsim 0.7\text{–}0.1(M_2/M_\odot)\ . \qquad (24)$$

The importance of energetic limitations on the extent of mass loss can be seen in Fig. 3. The ejected mass fractions have been calculated from Eq. (22), but with the assumption that radiative losses are negligible in dynamically unstable mass transfer (i.e., with the first right-hand term in Eq. [22] omitted in this case). The fraction of the mass transfer stream which can be ejected approaches unity only for accreting white dwarf masses extremely close to the Chandrasekhar limit. In more typical cases, a large fraction of the transferred mass remains bound to the system (most of the mass, for $M_2 < 1.0 M_\odot$), even when mass transfer is dynamically stable at super-Eddington rates.

4. Discussion

The preceding analysis has focussed on the initial stability of a donor white dwarf in a DWD, and the question of what fraction of the mass transfer stream can be expelled from the binary while conserving energy, if no energy sources other than gravitational potential energy are at play. The retention fraction derived above is strictly a lower limit in this case, because the entire accretion energy has been assumed lost from the binary, either in radiation, or in material just marginally unbound. It is likely that a large fraction of the accretion energy is retained as thermal energy in the accreted material in super-Eddington mass transfer, in which case the hot accreted envelope must extend beyond the orbit of the donor. We expect, therefore, that even should nuclear burning at the base of the accreted envelope not immediately come into play (for example, in the stable transfer of material from one CO white dwarf to another), a common envelope will be created. The dissipative loss of orbital energy in such a common envelope is certain to lead to runaway mass transfer and merger, because the donor star must in fact evolve to orbits of higher, not lower, total energy if it is to survive.

Acknowledgements. ZH thanks Dr. J.C. Wang at the Yunnan Observatory for a discussion. This work is supported in part by the Pandeng Scheme of China, NAOCAS and a Royal Society Fellowship and by US National Science Foundation grant AST96-18462.

References

Cameron A.G.W., Iben I.Jr., 1986, ApJ 305, 228
Eggleton P.P., 1983, ApJ 268, 368
Evans C.R., Iben I.Jr., Smarr L., 1987, ApJ 323, 129
Han Z., 1998 MNRAS 296, 1019
Hils D., Bender P.L., Webbink R.F., 1990, ApJ 360, 75; 369, 271
Iben I. Jr., Tutukov A.V., 1984, ApJS 54, 335
Kopal Z., 1959, Close Binary Systems, Chapman & Hall, London
Landau L.D., Lifshitz E.M., 1962, The Classical Theory of Fields. Pergamon, Oxford
Nauenberg M., 1972, ApJ 175, 417
Pringle J.E., Webbink R.F., 1975, MNRAS 172, 493
Tutukov A.V., Yungelson L.R., 1979, Acta Astron. 29, 665
Warner B., 1995, Ap&SS 225, 249
Webbink R.F., 1984, ApJ 277, 355
Webbink R.F., Iben I.Jr., 1992, in A.G. Davis Philip, D.S. Hayes and J.W. Liebert, eds, IAU Colloq. No. 95, The Second Conference on Faint Blue Stars, L. Davis Press, p.445