

# Orbital period modulation and quadrupole moment changes in magnetically active close binaries

A.F. Lanza<sup>1</sup> and M. Rodonò<sup>1,2</sup>

<sup>1</sup> Osservatorio Astrofisico di Catania, Città Universitaria, Viale A. Doria, 6, I-95125 Catania, Italy (nlanza@alpha4.ct.astro.it)

<sup>2</sup> Città Universitaria, Istituto di Astronomia dell'Università degli Studi, Viale A. Doria, 6, I-95125 Catania, Italy (mrodono@alpha4.ct.astro.it)

Received 8 April 1999 / Accepted 27 July 1999

**Abstract.** We discuss the main characteristics of the orbital period modulation in close binaries with late-type components. We focus on the various physical scenarios proposed to explain this phenomenon and, in particular, Hall's (1989) suggestion that it may be connected with magnetic activity. Starting from the work of Applegate (1992) and Lanza et al. (1998a), we develop an integral approach to evaluate the gravitational quadrupole moment of an active star and its variations, which we consider to be an important driver of the observed orbital period changes. The method applies the tensor virial theorem after Chandrasekhar (1961) and directly relates the variation of the quadrupole moment with the changes of kinetic and magnetic energy of the stellar hydromagnetic dynamo. Particular effort has been applied in minimizing the number of free parameters entering the problem.

A sample of 46 close binaries with period changes of alternate signs has been studied by our method. The amplitude of the quadrupole moment change appears to decrease with increasing angular velocity, implying that the time-variable part of the kinetic energy of rotation varies as  $\delta\mathcal{T}/\mathcal{T} \propto \Omega^{-0.93 \pm 0.10}$ , with a correlation coefficient of 0.83. The length of the cycle of the orbital period modulation seems to be correlated with the angular velocity as  $P_{mod} \propto \Omega^{-0.36 \pm 0.10}$ , but with a smaller correlation coefficient of 0.62. These results support the suggestion that a distributed non-linear dynamo is at work in the convective envelopes of very active stars and that it strongly affects the differential rotation. We also discuss the energy budget of the process responsible for the quadrupole moment variation and find that, on average, only  $\sim 10\%$  of the energy required to maintain the differential rotation may be lost by dissipation in the turbulent convective envelope during a cycle of the orbital period change. The problems of the magnetic field geometry and stability and the relationship between the length of the activity cycle, as determined by the change of the area of the starspots and the orbital period modulation, respectively, are also addressed.

**Key words:** stars: binaries: close – stars: activity – stars: late-type – magnetic fields

## 1. Introduction

Long-term timings of eclipses have revealed cyclic orbital period variations in several members of the Algol, RS Canum Venaticorum and W Ursae Maioris close binaries, as well as in some Cataclismic Variables (hereinafter CVs).

The modulation time scale for Algols and RS CVn goes from a few to several decades with a median value around 40–50 yrs. The amplitude of the modulation is usually of the order of  $\Delta P/P \sim 10^{-5}$  (Hall 1989, 1990). W UMa binaries show typical variations with  $\Delta P/P \sim 10^{-6}$  over time scales of some decades. In CVs the characteristic time scales go from a few years up to a few decades with amplitudes of the order of  $\Delta P/P \sim 10^{-7} - 10^{-6}$  and the phenomenon is observed in systems both above and below the orbital period gap (cf. Warner 1988, 1995).

Apsidal motion is not a viable explanation for such period changes because the orbital eccentricity for active close binaries is negligible, and both primary and secondary minima show rather identical  $O - C$  residuals.

The presence of a third body in the system, inducing a motion of the binary around a common centre of mass, with the consequent light-time effect and the apparent orbital period change, has often been invoked as an alternative explanation. Frieboes-Conde & Herczeg (1973), Mayer (1990), Chambliss (1992) and Borkovits & Hegedüs (1996) discussed the orbital period modulation of several close binaries in the framework of such a hypothesis. However, independent evidence for the presence of a third body is still limited and for only a few systems. In Algols and RS CVn binaries, the mass derived for the third body is usually  $1-5 M_{\odot}$  and, therefore, it is very difficult to explain how it has systematically escaped detection. Donati (1999) found evidence for a variation of the orbital period of HR 1099, by comparing radial velocity measurements obtained over a time span of six years, but, despite the sufficiently high precision of his measurements, the variation of the radial velocity of the binary baricentre, as expected under the hypothesis of a third body, was not detected. A similar conclusion was reached by Patterson et al. (1978) for the cataclismic variable DQ Her, because the observed 71-sec light oscillations did not show the residuals expected in the hypothesis of a light-time effect.

Moreover, a light-time effect implies a strictly periodic variation of the  $O - C$  residuals, which is usually not observed when data covering several cycles of the modulation are available (see, e.g., the case of RW Tri discussed by Robinson et al. 1991). In the case of the best studied systems, several variations with different periodicities and amplitudes seem to be present. For instance, in Algol there is clear evidence for a modulation with a period of  $\sim 32$  yr superposed upon a much longer, possibly cyclic change with a period of  $\sim 180$  yr (Söderhjelm 1980). Neither of these variations can be explained by a light-time effect due to the third component of the system, the orbital period of which is only 1.86 yr (Fekel 1981, Chambliss 1992).

Multiply periodic variations seem to be present also in all the other Algols and RS CVn systems with a sufficiently long  $O - C$  sequences, i.e., U Cep (Hall 1975), RS CVn (Rodonò et al. 1995), AR Lac (Jetsu et al. 1997, Lanza et al. 1998b), RT Lac (Keskin et al. 1994).

In view of the difficulties with the third-body hypothesis, other possible causes of orbital period modulation have been suggested. In particular, Hall (1989) has shown that orbital period changes of alternate sign, in a sample of 101 Algols, are observed only in systems having a secondary of spectral type F5 or later, i.e., those systems with components possessing sizeable outer convective envelopes. Systems with an early secondary exhibit monotone period changes. Such a result has recently been confirmed by Simon (1999), who analysed a sample of 16 binaries with early-type components. He found only one case of alternate period changes, that could be attributed to a light-time effect induced by a third body. Therefore, the observations suggest to investigate a possible connection between orbital period modulation and magnetic activity, because sizeable convective envelope and rapid rotation are sufficient conditions for the development of a strong dynamo action in stars (e.g., Rosner & Weiss 1992, Weiss 1994).

The possibility that a time-variable angular momentum loss, due to magnetic activity, be responsible for alternate orbital period changes was investigated by DeCampi & Baliunas (1979). They concluded that this explanation was implausible because of the large mass loss required (about 3–4 orders of magnitude larger than allowed by observations) and also because of the long time scale needed to couple the variation of the stellar rotation to the orbital motion through tidal effects. Actually, the characteristic time-scale for spin-orbit coupling in RS CVn's turns out to be of the order of  $10^3$  years, i.e., about two orders of magnitude longer than the periods of the observed short-term modulations (cf. Zahn 1989).

In view of such difficulties, Matese & Whitmire (1983) proposed a new mechanism, based on a direct effect related to a cyclic change of the gravitational quadrupole moment of active stars. Several physical scenarios have been suggested to produce such a change, by, e.g., Applegate & Patterson (1987), Warner (1988), Applegate (1992), Lanza et al. (1998a, hereinafter Paper I), and they are discussed in Sect. 2 below.

In the present study, we shall discuss the orbital period modulation under the hypothesis that a change of the gravitational quadrupole moment of the active star is responsible for the phe-

nomenon, and develop a general approach, based on the tensor virial theorem of Chandrasekhar (1961). Its main advantage is the possibility of relating the variations of the quadrupole moment to the rotational and magnetic energy variations in active stars.

The method is applied to a sample of close binaries for which orbital period changes of alternate sign have been observed, from which information on the energy changes connected with the operation of hydromagnetic dynamo are derived. The discussion based on the available data gives further support to the connection between orbital period modulation and magnetic activity in close binaries, suggesting a new approach to the study of the hydromagnetic dynamo action in very active stars.

## 2. Gravitational quadrupole moment and orbital period variation in close binaries

The equilibrium figures of the components of a close binary are perturbed away from a spherical shape by the centrifugal and tidal potentials, therefore the external gravitational potential of each star  $\Phi$  can be expressed by a multipole expansion. For our purposes an expansion up to the quadrupole moment term is adequate (cf. Paper I):

$$\Phi(\mathbf{x}) = \frac{GM}{r} + \frac{3}{2}GQ_{ik} \frac{x_i x_k}{r^5} \quad (1)$$

where  $\mathbf{x}$  is the position vector,  $r$  the radial distance from the barycentre of the star,  $M$  its mass,  $G$  the gravitational constant,  $Q$  is the quadrupole moment tensor which is defined as the traceless part of the inertia tensor  $I_{ik}$ :

$$Q_{ik} = I_{ik} - \frac{1}{3}\delta_{ik} \text{Tr}I \quad (2)$$

with the inertia tensor defined by:

$$I_{ik} = \int_V \rho x_i x_k dx \quad (3)$$

where  $\rho$  is the density and  $V$  the volume of the star (cf. Chandrasekhar 1961). We should like to note that the expression of the potential adopted in Eq. (1) is coherent with Chandrasekhar's (1961) definition and differs in sign from that assumed by Applegate (1992).

For simplicity sake, we assume that the primary star can be treated as a point mass and consider only the distortion of the secondary component. Its equilibrium figure is determined by a balance among the acting forces, in particular the centrifugal and the tidal forces as well as the Lorentz forces due to the magnetic field. We shall regard the figure distortions arising from centrifugal and Lorentz forces as independent from those arising from tidal forces. Such an assumption is justified when limiting the description of the stellar equipotentials up to the fourth order zonal harmonics, and it is adequate to treat our problem because the gravitational quadrupole term is related to the second order harmonics (cf. Kopal 1959, 1978). Following our discussion in Paper I, we shall neglect the effects of the meridional currents which, in a differentially rotating star, are driven by the Lorentz and the non-potential centrifugal forces.

We shall focus on the effects of the internal perturbing forces in the secondary, i.e., the centrifugal and Lorentz forces because the tidal forces give a constant contribution to the gravitational quadrupole moment to the order of accuracy of our treatment. Therefore, the figure of equilibrium of the secondary can be assumed to be symmetric about its rotation axis and its equatorial plane, with the rotation axis orthogonal to the orbital plane. We shall specify our analysis in an orthogonal Cartesian reference frame with the origin at the center of mass of the secondary star, the  $x$  axis along the line joining the centers of the two components, the  $y$  axis lying in the orbital plane and the  $z$  axis along the star's rotation axis.

In the framework of the above assumptions, the quadrupole moment tensor is diagonal and it is specified by only one component, e.g.,  $Q_{xx} = Q$  (cf. Landau & Lifshitz 1962, Applegate 1992). For a uniformly rotating star, the value of  $Q$  is (cf. Paper I):

$$Q = \frac{2}{9} k_2 \frac{\Omega^2 R^5}{G} \quad (4)$$

where  $k_2$  is the apsidal motion constant as given by, e.g., Kopal (1959), Claret & Gimenez (1992) and Claret (1995),  $\Omega$  is the angular velocity,  $R$  the star's radius and  $G$  the gravitational constant.

The suggestion that a cyclic change of  $Q$  may produce a modulation of the orbital period was put forward by Matese & Whitmire (1983). In our formalism the relationship between cause and effect may be written as:

$$\frac{\Delta P}{P} = -9 \frac{\delta Q}{Ma^2} \quad (5)$$

where  $a$  is the semi-major axis of the orbit (cf. Applegate 1992). It is interesting to note that the orbital motion will immediately react to any change of  $Q$  because a variation of the gravitational potential will affect the relative orbital acceleration of the component stars. Thus the orbital period change occurs at constant orbital angular momentum without any spin-orbit coupling, which would require significantly longer time scales.

In the case of the RS CVn binaries, the observed amplitudes of  $\Delta P/P$  imply a variation of the quadrupole moment of the secondary  $\delta Q$  of the order of  $10^{51} - 10^{52} \text{ g cm}^2$ , whereas for the cataclysmic variables it is of the order of  $\delta Q \sim 10^{49} \text{ g cm}^2$ .

Matese & Whitmire (1983) showed that a relative change of the radius of the secondary  $\Delta R/R$  of the order of  $10^{-3} - 10^{-2}$  may induce the variation of the quadrupole moment needed for RS CVn and Algols, whereas a variation as small as  $\Delta R/R \sim 10^{-4}$  would suffice for CVs. Their suggestion was further developed by Applegate & Patterson (1987) and Warner (1988), who proposed that an isotropic radius change may, in principle, be produced by a variation of the magnetic pressure at the base of the convection zone. However, a serious difficulty was pointed out by Marsh & Pringle (1990), who showed that the invoked isotropic expansion of the secondary requires more energy to expand the star against its own gravitational field than it is radiated by the star itself during a cycle of the period modulation. Applegate (1992) overcame the energetic problem by showing

that a non-isotropic distortion of the stellar shape may well produce the required quadrupole moment change without doing work against the gravitational field (in a first order approximation). He proposed that a quasi-periodic exchange of angular momentum between different layers in the stellar interior may induce a modulation of the stellar oblateness and consequently of its quadrupole moment. A variation of the angular velocity of the outer convection zone of the order of  $\Delta\Omega/\Omega \sim 10^{-3} - 10^{-2}$  is sufficient to explain the amplitude of the orbital period variations observed in RS CVn's and CVs. In Paper I, Applegate's model was reconsidered and it was shown that its energetic requirements can be further reduced by a factor of two, considering the effect of the Lorentz force on the stellar mechanical equilibrium. The new approach was applied to model the orbital period change of the prototype active binary RS CVn, with a substantial improvement of the agreement between theory and observation.

However, our treatment in Paper I was limited to the consideration of a purely azimuthal magnetic field and did not address the problem of its stability. In the next sections, we shall address this question by considering also a more general magnetic field geometry.

### 3. The virial theorem and the quadrupole moment of a star

In the light of the considerations of Sect. 2, we shall restrict our discussion to the case of an axisymmetric star, assuming an inertial reference frame with the origin at its barycentre, the  $z$  axis along its rotation axis and the  $x$  and  $y$  axes lying in the equatorial plane. The tensorial virial theorem in our inertial reference frame is expressed by the relationship (Chandrasekhar 1961):

$$\frac{1}{2} \frac{d^2 I_{ik}}{dt^2} = 2\mathcal{T}_{ik} - 2\mathcal{M}_{ik} + \mathcal{G}_{ik} + \delta_{ik} \{(\gamma - 1)\mathcal{U} + \mathcal{G}\} \quad (6)$$

where the inertia tensor  $I_{ik}$  has already been introduced in Sect. 2, and

$$\mathcal{T}_{ik} = \frac{1}{2} \int_V \rho u_i u_k d\mathbf{x}, \quad (7)$$

$$\mathcal{M}_{ik} = \frac{1}{8\pi} \int_V B_i B_k d\mathbf{x}, \quad (8)$$

$$\mathcal{G}_{ik} = -\frac{1}{2} G \int_V \int_V \rho(\mathbf{x}) \rho(\mathbf{x}') \frac{(x_i - x'_i)(x_k - x'_k)}{|\mathbf{x} - \mathbf{x}'|^3} d\mathbf{x} d\mathbf{x}' \quad (9)$$

are the kinetic energy tensor, the magnetic energy tensor and the gravitational energy tensor, respectively, where  $t$  is the time,  $\mathbf{u}$  the velocity of the plasma,  $\mathbf{B}$  the magnetic field,  $V$  the volume of the star,  $\gamma$  the ratio of the specific heats,  $\mathcal{U}$  the internal energy of the star and  $\mathcal{G}$  its gravitational potential energy, given by the contraction of the gravitational energy tensor:  $\mathcal{G} = \mathcal{G}_{ii}$ .

For the sake of simplicity, we shall assume that the stellar equipotential surfaces and the velocity and magnetic fields are symmetric about the rotation axis and the equatorial plane.

Moreover, we shall assume that the velocity field is entirely arising from stellar rotation with  $\mathbf{u} = (u_x, u_y, 0)$  and  $\mathcal{T}_{zz} = 0$ . The axisymmetry condition also implies:  $\mathcal{T}_{xx} = \mathcal{T}_{yy}$ ,  $\mathcal{M}_{xx} = \mathcal{M}_{yy}$  and  $\mathcal{G}_{xx} = \mathcal{G}_{yy}$ . If we denote by  $\mathcal{M}$  the total magnetic energy of the star and by  $\eta$  ( $0 \leq \eta \leq 1$ ) the fraction associated with the component of the magnetic field parallel to the  $z$  axis, then:  $\mathcal{M}_{zz} = \eta\mathcal{M}$  and  $\mathcal{M}_{xx} + \mathcal{M}_{yy} = (1 - \eta)\mathcal{M}$ .

In the above hypotheses it is possible to derive an expression for the second time derivative of the quadrupole moment of the star starting from Eq. (2), by a simple application of the virial theorem and a little algebra:

$$\frac{d^2Q}{dt^2} = \frac{2}{3}\{\mathcal{T} - (1 - 3\eta)\mathcal{M} - \tilde{\mathcal{G}}\} \quad (10)$$

where  $\mathcal{T}$  is the total kinetic energy of rotation,  $\mathcal{M}$  the total magnetic energy and  $\tilde{\mathcal{G}} \equiv \mathcal{G}_{zz} - \mathcal{G}_{xx}$ . Eq. (10) generalizes Eq. (A2) of the Appendix of Paper I.

If the quadrupole moment has a sinusoidal dependence on time with amplitude  $\delta Q$  and frequency  $\omega$ , we have:

$$\omega^2 \delta Q \sin \omega t = \frac{2}{3}\{\mathcal{T} - (1 - 3\eta)\mathcal{M} - \tilde{\mathcal{G}}\} \quad (11)$$

For RS CVn systems,  $\omega \sim (1-5) \times 10^{-9} \text{ s}^{-1}$ , and  $\delta Q \sim 10^{51} - 10^{52} \text{ g cm}^2$ , which yields  $\omega^2 \delta Q \sim 10^{33} - 10^{34} \text{ erg}$ . For comparison, the typical value of the rotational kinetic energy on the r.h.s. of Eq. (11) is of the order of  $\sim 10^{45} \text{ erg}$ , which implies that a very good approximation is to neglect the l.h.s. of Eq. (10), assuming that all the changes occur through a sequence of static equilibria characterized by the balance:

$$\mathcal{T} - (1-3\eta)\mathcal{M} = \tilde{\mathcal{G}} \quad (12)$$

The value of the gravitational energy term  $\tilde{\mathcal{G}}$  can be related to the quadrupole term of the gravitational potential  $Q$  for a slightly distorted configuration. We shall assume that the departure from spherical symmetry is small and the density  $\rho$  and the potential  $\Phi$  can be expressed as:

$$\rho = \rho_0(r) + \rho_2(r)P_2(\cos \theta) \quad (13)$$

$$\Phi = \Phi_0(r) + \Phi_2(r)P_2(\cos \theta) \quad (14)$$

where  $r$  is the distance from the barycentre of the star,  $\theta$  the colatitude measured from the rotation axis and  $P_2(\cos \theta)$  the second order Legendre polynomial, with  $|\rho_2(r)| \ll |\rho_0(r)|$ ,  $|\Phi_2(r)| \ll |\Phi_0(r)|$ . The value of  $\Phi_2(r)$  is assumed to vanish at  $r = 0$ , whereas at the outer boundary of the configuration  $r = R$ , it is related to the gravitational quadrupole moment  $Q$  by the condition of the continuity of the potential (cf., e.g., Ulrich & Hawkins 1981):  $\Phi_2(R) = -3GQ/R^3$ .

In the above hypotheses, following Chandrasekhar & Roberts (1963), it can be shown that:

$$\tilde{\mathcal{G}} = -\frac{2}{5} \int_0^R \frac{M(r)}{r^3} \frac{d}{dr} \{r^3 \Phi_2(r)\} dr \quad (15)$$

where  $M(r) = \int_0^r 4\pi \rho_0(s) s^2 ds$  is the mass of the unperturbed spherical configuration inside the radius  $r$  and  $R$  its outer ra-

dius. The derivative in integral (15) can be written as (cf. Chandrasekhar & Roberts 1963):

$$\frac{d}{dr} \{r^3 \Phi_2(r)\} = 4\pi G r^4 \int_r^R \frac{\rho_2(s)}{s} ds \quad (16)$$

For a configuration whose isopotential surfaces are oblate spheroids,  $\rho_2(r) < 0$  for  $0 \leq r \leq R$ , and the sign of the derivative is always negative. If we define  $f(r) \equiv -r^3 \Phi_2(r)$ , we can express  $r$  as a function of  $f$ , since the function  $f(r)$  can be inverted for  $\frac{df}{dr} > 0$  in  $0 \leq r \leq R$ . Therefore, we can change the variable of integration in Eq. (15), which becomes:

$$\tilde{\mathcal{G}} = \int_0^{3GQ} \frac{M(f)}{[r(f)]^3} df. \quad (17)$$

Since for a centrally condensed star with the density decreasing monotonously outward:

$$\frac{4\pi}{3} \langle \rho \rangle < \frac{M(r)}{r^3} < \frac{4\pi}{3} \rho_c \quad (18)$$

where  $\langle \rho \rangle = 3M/4\pi R^3$  is the mean stellar density and  $\rho_c$  the central density, we can constrain the value of  $\tilde{\mathcal{G}}$  as:

$$\tilde{\mathcal{G}} = \frac{8\pi}{5} G \xi \langle \rho \rangle Q \quad (19)$$

where  $1 \leq \xi \leq (\rho_c / \langle \rho \rangle)$ . An expression for the quadrupole moment  $Q$  can be derived by comparing Eqs. (12) and (19):

$$Q = \frac{5}{6\xi} \frac{R^3 [(\mathcal{T} - (1-3\eta)\mathcal{M})]}{GM} \quad (20)$$

It is interesting to compare this expression with that for a uniformly rotating star in the absence of magnetic field, given by Eq. (4). Since  $\mathcal{T} = \frac{1}{2} \beta^2 M R^2 \Omega^2$ , where  $\beta$  is the fractional gyration radius, for a uniformly rotating star without magnetic field we have:  $\xi = 15\beta^2/8k_2$ , independently of its kinetic energy. Typical values for a star with a sizeable outer convective envelope are:  $\beta = 0.38$  and  $k_2 = 0.07$ , which give  $\xi = 3.9$ .

The adimensional value of  $\Phi_2(R)$ , appearing in the usual expression of the exterior gravitational potential is (Ulrich & Hawkins 1981, Paternò et al. 1996):  $J_2(R) = 3Q/MR^2$ . For the Sun, in the above hypotheses and with  $k_{2\odot} = 0.0194$  and  $\beta_{\odot} = 0.206$ , we find:  $J_2 = 2.49 \times 10^{-7}$ , in agreement with the value derived from recent measurements of solar oblateness and oscillations (cf. Paternò et al. 1996).

#### 4. Energetics of the quadrupole moment variations

A study of the energetic balance of the quadrupole moment variation can be performed starting from Eqs. (12) and (19). Let us consider a star which changes its quadrupole moment with a contemporaneous variation of the energy term  $\tilde{\mathcal{G}}$ . Differentiating Eq. (19) it follows:

$$\delta \tilde{\mathcal{G}} = \frac{8\pi}{5} G \langle \rho \rangle (\xi \delta Q + Q \delta \xi) \quad (21)$$

It is important to note that the factor  $\xi$  may also change during the course of the  $\delta \tilde{\mathcal{G}}$  variation, in a way that depends on the

details of the redistribution of the kinetic and magnetic energies within the star. For the sake of simplicity, we consider  $\xi$  to be independent of  $\mathcal{T}$  for nearly uniform rotation and we shall restrict ourselves to the case in which  $\delta\xi/\xi \ll \delta Q/Q$ , so that Eq. (21) can be simplified as it follows:

$$\delta\tilde{\mathcal{G}} = \frac{8\pi}{5} G \langle \rho \rangle \xi \delta Q \quad (22)$$

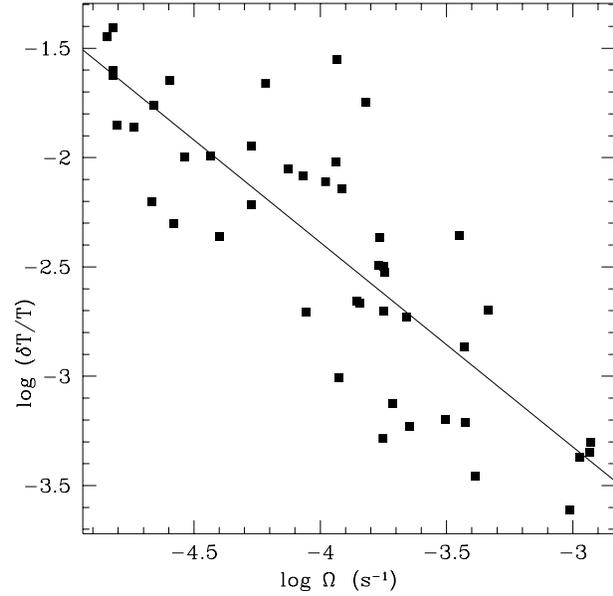
Eq. (22) shows that, in the simple assumptions we have adopted, the quadrupole moment change is proportional to the variation of the energetic term. The variation of the quadrupole moment  $\delta Q$  can be derived from the observed  $\Delta P/P$  through Eq. (5) and, therefore, our formalism can be used to estimate the energetic changes associated with the quadrupole moment change.

## 5. Application to observed systems

We have extracted from the literature as much information as possible on the orbital period modulation of 46 close binaries belonging to the Algol, RS CVn, W UMa and CV groups. These systems were selected according to the following criteria: a) the orbital period changes show alternate sign, and b) at least one component is of spectral type F or later, i.e., with a sizeable convective envelope that can support magnetic activity. Systems with independent evidence that a third body may explain the observed  $O-C$  residuals (e.g., XY Leo, SW Lyn, SW Lac) have been excluded from our data base (Chambliss 1992, Hendry & Mochnacki 1998, Ogloza et al. 1998). It is not possible to exclude that some of the systems in our sample still show an orbital period modulation due to a light-time effect arising from the presence of a third body. However, for about one third of our systems the available observations cover more than one cycle of the  $O-C$  modulation and there is some indication of a non-periodic variation, contrary to the third-body hypothesis.

The amplitude and the period of the  $O-C$  modulation are usually rather uncertain, even up to a factor of about 2–2.5 when only one cycle is observed, as for most of the systems in our sample. When multiple periodicities are present, we always considered the periodicity which is best determined. This procedure obviously favours short-term modulations. In view of such uncertainties and our assumptions in Sect. 4, a detailed comparison between model and observation for individual systems was not pursued and we prefer to apply a statistical approach to derive some general properties of the active systems in our selected sample.

In Table 1 we report the relevant data for the selected 46 systems, which are listed in order of increasing orbital angular velocity. In the first three columns we give the variable star name, the class of variability and the spectra of the components, respectively. In the fourth, fifth and sixth columns we give the radius  $R$ , the mass  $M$  and the luminosity  $L$  in solar units of the active component (that we considered to be responsible for the orbital period change), respectively. In the last five columns, the orbital semi-major axis  $a$ , the amplitude of the orbital period modulation  $\Delta P/P$ , the period of the modulation  $P_{mod}$ , the orbital angular velocity  $\Omega$  and the literature references on



**Fig. 1.** The relative amplitude of the kinetic energy variation  $\delta\mathcal{T}/\mathcal{T}$  vs.  $\Omega$ , calculated according to Eq. (22) for the systems in our sample. The linear regression between  $\delta\mathcal{T}/\mathcal{T}$  vs.  $\Omega$  is also shown (see Eq. (24)).

the period variability and the physical parameters of the system are listed, respectively. The amplitudes of the orbital period modulations are computed by the formula:

$$\frac{\Delta P}{P} = \frac{4\pi A_{O-C}}{P_{mod}} \quad (23)$$

where  $A_{O-C}$  is the semiamplitude of a sinusoidal fit to the observed  $O-C$  modulation (cf. Applegate 1992).

The data in Table 1 allow us to compute the variation of the gravitational quadrupole moment  $\delta Q$  and the variation of the energetic term according to Eq. (22) for all the systems in our sample. We assumed the magnetic field to be azimuthal ( $\eta = 0$ ) and that  $\delta\mathcal{T} = -\delta\mathcal{M}$ . The former assumption is well verified in the case of an  $\alpha$ - $\Omega$  solar-like dynamo, as discussed in Paper I. The latter assumption comes from the hypothesis that the sum of the kinetic and magnetic energy is conserved during the operation of a stellar dynamo. It implies:  $\delta\tilde{\mathcal{G}} = 2\delta\mathcal{T}$ , and we can estimate the amount of kinetic energy converted into magnetic energy during the course of the quadrupole moment modulation by assuming  $\xi = 4$ . In Fig. 1, we plot the relative variation of the kinetic energy  $\delta\mathcal{T}/\mathcal{T}$  versus the stellar angular velocity  $\Omega$  for the active components in our selected sample of binaries, because rotation is a key ingredient of the dynamo action.

The regression relationship between  $\delta\mathcal{T}/\mathcal{T}$  and  $\Omega$ , is:

$$\log \frac{\delta\mathcal{T}}{\mathcal{T}} = -0.93(\pm 0.10) \log \Omega - 6.13 \quad (24)$$

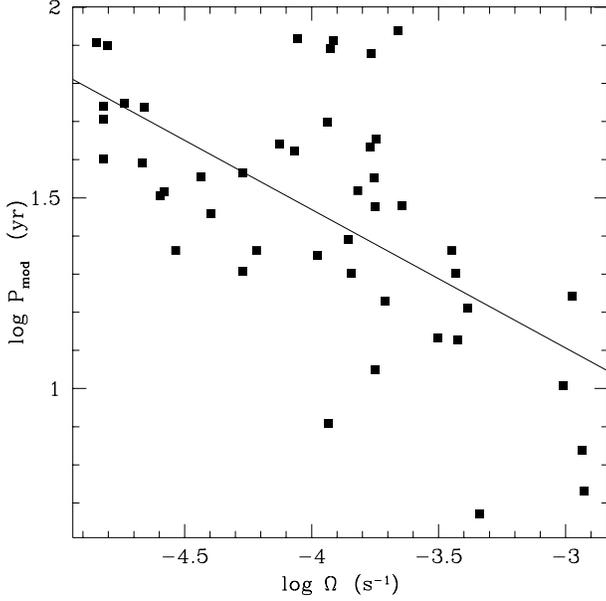
with a correlation coefficient of  $|r| = 0.83$ . As uncertainty of the correlation slope we give in Eq. (24), its standard deviation was assumed.

Dynamo theory suggests a correlation between the length of the activity cycle and the angular velocity for active stars

**Table 1.** Orbital Period Modulation for a Sample of Eclipsing Binaries

<i>Name</i>	<i>Class</i>	<i>Spectrum</i>	<i>R</i> ( $R_{\odot}$ )	<i>M</i> ( $M_{\odot}$ )	<i>L</i> ( $L_{\odot}$ )	<i>a</i> ( $R_{\odot}$ )	$\Delta P/P$ ( $\times 10^6$ )	<i>P</i> <sub>mod</sub> (yr)	$\Omega$ ( $\times 10^4 \text{ s}^{-1}$ )	<i>Ref.</i>
RT Lac	RS CVn	G5+G9IV	4.40	1.56	18.06	15.8	20.0	80.7	0.14	1, 2
RS CVn	RS CVn	F5V+K2IV	4.61	1.44	10.96	18.7	24.0	40.0	0.15	2, 3
SS Cam	RS CVn	F5IV+K0I	6.39	1.80	19.04	18.7	60.0	55.0	0.15	4
W Del	Algol	A0+K2	4.61	0.60	12.02	18.7	37.0	50.9	0.15	5, 6
WW Dra	RS CVn	G2IV+K0I	3.89	1.34	5.65	15.8	6.0	79.3	0.16	7, 2
SZ Psc	RS CVn	F8V+K1IV	5.10	1.62	13.99	14.4	31.0	56.0	0.18	8
U Sge	Algol	B8V+G4IV	5.00	1.65	16.97	17.2	12.0	39.0	0.21	9, 10
WW Cyg	Algol	B7V+G8IV	5.10	1.90	12.31	18.7	28.0	54.5	0.22	11, 6
Algol	Algol	B8+K4	3.50	0.80	6.99	13.9	32.0	32.0	0.25	4
RW Tau	Algol	B8V+K0IV	3.00	0.55	2.11	12.2	6.6	32.8	0.26	12, 13
U Cep	Algol	B7+G4	5.22	3.25	14.38	11.5	50.0	23.0	0.29	14, 15
AR Lac	RS CVn	G2IV+K0I	3.10	1.30	5.49	9.1	24.0	36.0	0.37	16, 17, 2
TV Cas	Algol	B9+K3	3.29	1.53	7.25	10.9	9.6	28.8	0.40	18, 19
XZ And	Algol	A1+G9	2.60	1.30	5.54	8.6	26.0	36.8	0.54	20
Z Dra	Algol	A5+K4	1.92	0.50	1.48	7.3	11.0	20.3	0.54	21, 22, 6
RZ Cas	Algol	A2V+	1.90	0.75	2.28	6.9	36.0	23.0	0.61	23, 6
X Tri	Algol	A3+K0	1.93	2.47	3.91	6.2	9.0	43.8	0.75	18, 24
RT Per	Algol	F2+K3	1.27	0.42	0.76	4.9	13.0	41.9	0.86	25, 26, 6
WY Cnc	RS CVn	G5V+[M2]	1.00	0.93	3.16	2.3	2.0	82.9	0.88	7, 2
VV UMa	Algol	A2V+	1.35	0.96	1.04	3.6	20.0	22.3	1.05	27
CG Cyg	RS CVn	K0V+K3V	0.87	0.52	0.21	3.2	7.9	50.0	1.15	28, 2
RT And	RS CVn	F8V+K0V	0.84	0.99	2.95	4.2	6.0	8.1	1.16	7, 2
BX And	W UMa	F2V	1.78	1.52	6.17	4.0	6.6	78.0	1.19	29
SV Cam	RS CVn	G3V+K4V	1.11	0.93	0.89	4.2	7.2	82.0	1.22	4
V471 Tau	RS CVn	wd+K2V	0.86	0.80	0.69	3.0	1.8	24.6	1.39	30, 4
OO Aql	W UMa	G5V	1.39	1.04	1.82	3.3	13.0	20.0	1.43	31
XY UMa	RS CVn	G5V+K5V	0.73	0.70	0.52	3.0	8.7	33.0	1.52	32
AP Leo	W UMa	F8V	1.36	1.53	1.54	3.0	20.0	43.0	1.70	33, 24
AK Her	W UMa	F2V+F6V	0.88	0.51	0.62	3.2	8.6	75.7	1.72	34, 35
V566 Oph	W UMa	F4V	1.51	1.56	4.17	2.9	6.4	35.6	1.77	36, 37
AH Vir	W UMa	K0V	1.48	1.47	1.50	2.9	24.0	11.2	1.78	38, 39
V839 Oph	W UMa	F8V	1.08	1.20	1.17	2.9	9.7	30.0	1.78	40, 41, 24
SS Ari	W UMa	F8	1.34	1.21	1.95	2.7	30.0	45.0	1.79	42, 43
U Peg	W UMa	G2V	1.40	1.80	3.37	2.9	6.6	17.0	1.94	44
AB And	W UMa	G5	1.01	0.93	0.36	1.7	22.0	86.5	2.19	45, 46, 47
YY Eri	W UMa	G5	1.17	1.30	1.05	2.4	5.6	30.2	2.26	48, 49
RW Tri	CV	wd+M0V	0.60	0.63	0.09	1.6	2.0	13.6	3.13	50, 51, 52, 53
T Aur	CV		0.53	0.63	0.02	0.7	45.0	23.0	3.56	54, 55, 56
UX UMa	CV	wd+M6V	0.50	0.48	0.04	1.4	4.0	20.0	3.70	57
DQ Her	CV	wd+M3V	0.49	0.40	0.02	1.4	2.0	13.4	3.76	58, 59
U Gem	CV	wd+M5V	0.51	0.58	0.01	1.6	0.9	16.3	4.11	60, 61
IP Peg	CV	wd+M4V	0.49	0.64	0.01	1.4	6.0	4.7	4.60	62, 63
Z Cha	CV	wd+M6V	0.17	0.12	0.0001	0.7	0.3	10.2	9.76	64, 65
EX Hya	CV		0.16	0.13	0.0001	0.7	0.6	17.5	10.60	66, 67, 68
V2051 Oph	CV		0.14	0.11	0.0001	0.5	0.7	6.9	11.60	69, 70
V4140 Sgr	CV		0.12	0.07	0.0001	0.5	0.6	5.4	11.80	71, 72

References: 1: Keskin et al. 1994; 2: Strassmeier et al. 1993; 3: Rodonò et al. 1995; 4: Applegate 1992; 5: Plavec 1959; 6: Giuricin et al. 1983; 7: Albayrak et al. 1999; 8: Kalimeris et al. 1995; 9: Simon 1997a; 10: Sarna & De Greve 1994; 11: Hall & Wawrukiewicz 1972; 12: Simon 1997b; 13: Plavec & Dobias 1983; 14: Hall 1975; 15: Rafert & Markworth 1991; 16: Jetsu et al. 1997; 17: Lanza et al. 1998b; 18: Friboes-Conde & Herczeg 1973; 19: Kalesseh & Hill 1992; 20: Demircan et al. 1995; 21: Kreiner 1971; 22: Rafaert 1982; 23: Hall et al. 1976; 24: Brancewicz & Dworak 1980; 25: Panchatsaram 1981; 26: Edalati & Zeinali 1996; 27: Simon 1996; 28: Hall 1991; 29: Bell et al. 1990; 30: İbanoğlu et al. 1994; 31: Demircan & Gürol 1996; 32: Erdem & Güdür 1998; 33: Zhang et al. 1989; 34: Tinca et al. 1987; 35: Rovithis-Livaniou et al. 1992; 36: Kalimeris et al. 1994a; 37: Niarchos et al. 1993; 38: Demircan et al. 1991; 39: Niarchos et al. 1983; 40: Akalin & Derman 1997; 41: Wolf et al. 1996; 42: Demircan & Selam 1993; 43: Rainger et al. 1992; 44: Zhai et al. 1984; 45: Demircan et al. 1994; 46: Kalimeris et al. 1994b; 47: Hrivnak 1988; 48: Maceroni & Van't Veer 1994; 49: Budding et al. 1997; 50: Robinson et al. 1991; 51: Africano et al. 1978; 52: Richman et al. 1994; 53: Smak 1995; 54: Bevermann & Pakull 1984; 55: Warner 1995; 56: Bianchini 1980; 57: Rubenstein et al. 1991; 58: Patterson et al. 1978; 59: Horne et al. 1993; 60: Smak 1993; 61: Friend et al. 1990; 62: Wolf et al. 1993; 63: Marsch et al. 1988; 64: Robinson et al. 1995; 65: Wade & Horne 1988; 66: Hellier & Sproats 1992; 67: Bond & Freeth 1988; 68: Hellier et al. 1987; 69: Echevarria & Alvarez 1993; 70: Watts et al. 1986; 71: Baptista et al. 1992; 72: Baptista et al. 1989



**Fig. 2.** The period of the orbital period modulation  $P_{mod}$  vs.  $\Omega$  for the systems in our sample. The linear regression between  $P_{mod}$  and  $\Omega$  is also shown (see Eq. (28)).

(cf., e.g., Brandenburg et al. 1994, Baliunas et al. 1996). More precisely, the average length of the cycle is given by:

$$P_{cyc} = \frac{2\pi R^2 D^{-n}}{\eta_t} \quad (25)$$

where  $R$  is the stellar radius,  $D$  the dynamo number,  $\eta_t$  the turbulent magnetic diffusivity and  $n$  a scaling exponent ( $n \sim 0.15$  for a dynamo working in a thin shell and  $n \sim 0.5-0.6$  for a distributed dynamo). The dynamo number  $D$  can be written as:

$$D = \alpha_D \frac{\Omega' R^4}{\eta_t} \quad (26)$$

where  $\alpha_D$  is a parameter describing the regeneration of a poloidal field from a toroidal field by cyclonic convection (the so-called  $\alpha$  effect of the dynamo theory, cf., e.g., Moffatt 1978, Parker 1979), and  $\Omega' = \partial\Omega/\partial r$  is the radial differential rotation. The internal rotation regime of active stars is not known. Following the approach by Brandenburg et al. (1994), we assumed some parametrization for  $\alpha_D$ ,  $\eta_t$  and  $\Omega'$ . For fast rotation that characterises the very active stars we have considered, we may assume  $\alpha_D \sim const.$ ,  $\eta_t \sim \Omega^{-1}$  and  $\Omega' \sim \Omega^m/R$ , where  $m = 0.15$  was adopted, according to the observed differential rotation law determined by Hall (1991a) and Henry et al. (1995).

In our sample, a strong correlation is found between  $R$  and  $\Omega$  ( $\log R = -0.767 \log \Omega + const.$ , with a correlation coefficient  $|r| = 0.956$ ). This implies that we can express  $R$  as a function of  $\Omega$  in Eqs. (25) and (26). Adopting such an approach, Eq. (25) can be rewritten as:

$$P_{cyc} \propto \Omega^q \quad (27)$$

with  $q = -0.4$  for a distributed dynamo and  $q = 1.9$  for a shell dynamo. If we assume that  $P_{cyc} = P_{mod}$ , then we can compare Eq. (27) with the observations.

In Fig. 2 the length of the modulation cycle  $P_{mod}$  is plotted versus  $\Omega$  for the systems in our sample. The following regression relationship results:

$$\log P_{mod} = -0.36(\pm 0.10) \log \Omega + 0.018 \quad (28)$$

with a correlation coefficient  $|r| = 0.62$ . Such a result seems to support the hypothesis of a distributed dynamo in the active components of close binaries. However, this result must be taken with caution, not only for the quite small correlation coefficient of regression (28), but above all for the large uncertainty that may arise from the parametrizations adopted in Eqs. (25) and (26), being these based on a simplified turbulence theory and indirect observations (Moss et al. 1995, Moss & Tuominen 1997).

It is interesting to compare the present results with those by Baliunas et al. (1996), who studied the activity cycles in a sample of 34 solar-like, presumably single stars observed within the Mt. Wilson Ca II H&K monitoring program. These stars are somewhat less active than those in our sample. However, using their data and the present formalism, a slope  $q = -0.26 \pm 0.13$  is found for the regression (28), with a regression coefficient  $r = 0.68$ . Therefore, the slopes of the regressions obtained from two quite independent star samples appear to be comparable within the uncertainty due to the quite large scatter of the data. Moreover, the probability that there is no correlation between  $P_{cyc}$  and  $\Omega$  in the present and Baliunas et al.'s (1996) samples is only of a few times  $10^{-6}$ .

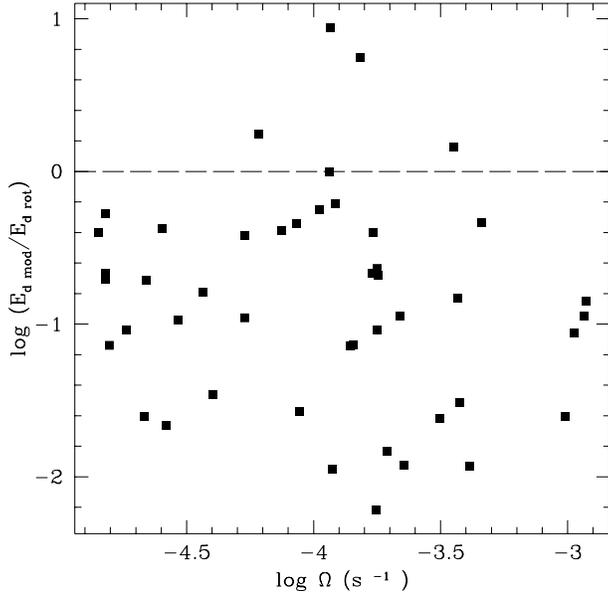
## 6. Discussion

In the framework of the hypothesis that magnetic activity is responsible for orbital period modulation, as presented in this paper, the results obtained in the previous section provide us with interesting clues on the energetics of hydromagnetic dynamos in very active stars.

The quite high correlation coefficient of the regression  $\delta\mathcal{T}/\mathcal{T}$  versus  $\Omega$  indicates that the time variable part of the kinetic energy is strongly correlated with the angular velocity, as one would expect in the dynamo scenario. The variation of the kinetic energy is associated with a time-variable differential rotation  $\delta\Omega/\Omega \simeq (1/2)\delta\mathcal{T}/\mathcal{T}$  and Eq. (24) implies that  $\delta\Omega/\Omega$  decreases with increasing  $\Omega$ . Such a result is similar to the dependence of the latitudinal differential rotation on the angular velocity, as derived from a sample of spotted stars by Hall (1991a) and Henry et al. (1995). Specifically, Hall (1991a) found that:

$$\left(\frac{\Delta\Omega}{\Omega}\right)_\phi = -0.85(\pm 0.06) \log \Omega - 5.82(\pm 0.06) \quad (29)$$

where  $\Omega$  is expressed in  $s^{-1}$ . The similarity of the slopes of regressions (24) and (29) suggests that the amplitude of the time-variable part of the differential rotation may be, on average, a



**Fig. 3.** The ratio of the energy dissipated during a cycle of the quadrupole moment modulation to that dissipated to support the latitudinal differential rotation  $E_{dmod}/E_{drot}$  vs.  $\Omega$  for the systems in our sample. The dashed line indicates  $E_{dmod}/E_{drot} = 1$ .

constant fraction of the latitudinal differential rotation. Unfortunately, observational data on differential rotation are available for only a few systems in Table 1. Therefore, a detailed comparison between theory and observations is not always possible as for the case of RS CVn discussed in Paper I. However, it is worthwhile to compare the average relationships given by Eqs. (24) and (29) by considering that Eq. (24) was obtained adopting  $\xi = 4$  and  $\delta\mathcal{T} = -\delta\mathcal{M}$ . In the light of these assumptions, we find that the time-variable part of the stellar differential rotation can account for  $\sim 25\%$  of the amplitude of the latitudinal differential rotation. A different value of  $\xi$  does not affect the slope of the  $\log \delta\mathcal{T}/\mathcal{T}$  vs.  $\log \Omega$  regression, but only the relative amplitude of the derived variations of the differential rotation (cf. Eq. (22)).

It is interesting to note that Eq. (29) is based on the observed variations of the light modulation period induced by starspots (cf. Hall 1991a). The latitude of the starspots can not be reliably estimated from light curves alone, thus Eq. (29) provides us only with a lower limit for the amplitude of the average differential rotation in latitude. Moreover, the variations of the spot angular velocity are usually observed over time scales from several months to years, significantly shorter than the time scales that generally characterise the orbital period modulation. Therefore, it is reasonable to assume that Eq. (29) describes the behaviour of the average latitudinal differential rotation, without any significant effect by the time-variable component of the differential rotation associated with quadrupole moment changes.

In the original Applegate (1992) model, the quadrupole moment change is associated with a variation of the kinetic energy of the star. Such a variation is assumed to be equal to the energy effectively lost by the star to support a cycle of the modula-

tion, and it is to be supplied by the stellar luminosity. However, such a view is probably too extreme, because the kinetic energy exchanges and the conversion of kinetic into magnetic energy are cyclic processes, which are, to some extent, reversible. Therefore, Applegate's approach can provide us with an upper limit for the energy required to produce a given change of the quadrupole moment. As already shown in Paper I, the energy actually needed may be, indeed, significantly smaller and the difficulties found in applying Applegate's mechanism to some systems may probably be overcome.

In the present work, we adopt a different approach and compare the time-variable part of the differential rotation, associated with the quadrupole moment change, with the differential rotation given by Eq. (29), which represents a time averaged stellar differential rotation in latitude. Since the convective envelope of an active star is in a turbulent state, the energy of any velocity shear is dissipated and must be ultimately supplied by the stellar luminosity. Our purpose is to compare the order of magnitude of the energy dissipated to support the time-averaged differential rotation of Eq. (29) with that needed to support the time-variable differential rotation, associated with the quadrupole moment variation.

The energy dissipated per unit time and volume in a turbulent fluid is given by the dissipation function (cf., e.g., Landau & Lifshitz 1959, Chandrasekhar 1961):

$$\mathcal{K} = 2\rho_0\nu_t r^2 \sin^2 \theta \left( \frac{\partial\Omega}{\partial r} \right)^2 \quad (30)$$

where  $\nu_t \sim \frac{1}{3}l_t v_t$  is the turbulent kinematic viscosity with  $l_t$  and  $v_t$  typical values of the length scale and velocity of the turbulent motions, respectively. An order-of-magnitude integration of Eq. (30) over the stellar volume gives the energy dissipated along a cycle of the time-variable differential rotation:

$$E_{diss} = \frac{\nu_t}{\omega_{cyc} R^2} \left( \frac{\delta\mathcal{T}}{\mathcal{T}} \right) \delta\mathcal{T} \quad (31)$$

where  $\omega_{cyc} = 2\pi/P_{mod}$  and the kinetic energy variation can be estimated with the parameters adopted above. The ratio of the two dissipated energies does not depend on the highly uncertain value of  $\nu_t$  and it is plotted in Fig. 3 versus the angular velocity  $\Omega$ . It is interesting to note that, on average, only  $\sim 10\%$  of the energy needed to maintain the latitudinal differential rotation is required to support the quadrupole moment change. Only for five systems out of 46 the energy dissipated in the change is larger than the energy required to maintain the differential rotation. However, if we consider that the observed  $\Delta\Omega/\Omega$  may deviate up to  $\sim 1$  order of magnitude from the average values given by Eq. (29), the difficulty does not appear to be a serious one (cf. Fig. 7 in Hall 1991a). It is important to notice that no detectable variation of the stellar luminosity is expected by the dissipation and the energy conversion processes, because the thermal inertia of a convective envelope is very large and its characteristic thermal time scale corresponds to the Kelvin-Helmoltz time scale, i.e.,  $\sim 10^6 - 10^8$  yr in the objects we are considering. Therefore, even fluctuations of the energy input at

the base of the convection zone of several tens of percent are effectively smoothed out and produce negligible change of the stellar luminosity on the time scale of a few decades that characterise orbital period modulations. The observed variations of the stellar luminosity on such a time scale can be attributed to the blocking effects of the starspots and it can be used as a proxy for the total area covered by the spots themselves (Spruit 1982, Spruit & Weiss 1986, Messina & Rodonò 1999).

A fundamental issue to be clarified by future observations concerns the relationship between the orbital period modulation and the stellar activity cycle, as derived from the starspot area changes. There seems to be two different scenarios: a) for RS CVn it is found that the period of the starspot cycle is half that of the orbital period modulation (Rodonò et al. 1995), a behaviour which may be tentatively interpreted by the torsional oscillator model proposed in Paper I, and there are preliminary indication of a similar behavior for AR Lac (Lanza et al. 1998b) and possibly also for RT Lac (Keskin et al. 1994, İbanoğlu et al. 1998) and AB And (Demircan et al. 1994); on the contrary, b) for V471 Tau, and possibly CG Cyg, the period of the light modulation equals that of the orbital period modulation (İbanoğlu et al. 1994, Hall 1991b), suggesting a more conventional  $\alpha$ - $\Omega$  dynamo model. If both kinds of behaviours are present in our sample, this may explain the quite low correlation coefficient of the regression between  $\log P_{mod}$  and  $\log \Omega$ , the bimodality of the distribution being hidden by the uncertainty in the determination of  $P_{mod}$ .

The geometry of the mean magnetic field determines the factor  $\eta$  in Eq. (12) which, for consistency with our previous analysis (cf. Paper I), has been assumed to be  $\eta = 0$ , corresponding to a purely azimuthal field. However, the stability of such a field against magnetic buoyancy in a superadiabatic convective envelope, poses a serious problem. In the linear regime, an azimuthal field may be stable if  $\partial(B/\rho)/\partial s > 0$ , where  $s$  is the distance from the rotation axis, but non-linear perturbations are likely to induce instabilities (Acheson 1978). A magnetic field  $B > 10^4$  G is capable of significantly modifying the local temperature gradient and, possibly, to stabilize itself in the deepest layers of a stellar convective envelope (Spiegel & Weiss 1980). The model of Paper I predicts mean fields with intensities of the order of  $10^5$  G, which may well change the local gradient, but the question concerning their stability against doubly diffusive and non-linear effects remains open.

A possible solution to the problem of the structure and geometry of the field in the stellar convective envelopes is offered by the observations which indicate the presence of huge starspots in very active stars, with filling factors up to 30%–50% of the photosphere (O’Neal et al. 1996). Starspots may be regarded as proxies for strong magnetic fields organized into nearly vertical flux tubes extending from the base of the convective envelope up to the photosphere. When the flux tubes emerge on the photosphere, they can significantly modify the local energy balance affecting the surface brightness. The intensity of the field within the convection zone is of the order of  $B \sim 10^4 - 10^5$  G and the vertical flux tubes are stable to magnetic buoyancy effects (van Ballegoijen 1988). Therefore, if the mean field is

organized in the form of many starspot flux tubes, the stability problem may be solved. From the point of view of our integral approach, only the total magnetic energy and the geometric factor  $\eta$  are changed. If the mean latitude of starspot forming regions changes during the activity cycle, the mean field gradient within the convective envelope may vary and produce a change of the gravitational quadrupole moment of the star. A detailed study of such an effect requires the use of the approach described in Paper I, but the present integral approach may also be used to derive some general information. In particular, it may be possible to change the quadrupole moment without any variation of the kinetic and magnetic energies, simply by changing the value of  $\eta$ , which depends on the average latitude of the radial mean field (cf. Eq. (20)). Evidence for a long-term migration of the starspots on the active component of HR 1099 has recently been presented by Vogt et al. (1999). Such a study, in connection with more detailed hydromagnetic dynamo models, may indeed provide the information required to develop such a qualitative idea into a detailed model.

## 7. Conclusion

The analysis of the orbital period modulation in 46 close binaries with late-type components gives support to Hall’s suggestion that orbital period modulation is related to magnetic activity and, in light of such a hypothesis, allows us to elaborate some interesting conclusions on the energetics of hydromagnetic dynamos in very active stars.

The time-variable part of the kinetic energy involved in the quadrupole moment variation appears to be a roughly constant fraction of the kinetic energy of the stellar differential rotation, independent of the star angular velocity. This supports the idea that a self-similar mechanism is at work in all stars. The relatively sizeable values of the variations of the differential rotation indicate that the hydromagnetic dynamos strongly affect the differential rotation, contrary to the solar case (Ulrich & Bertello 1996). The hydromagnetic non-linear mean-field models by Brandenburg et al. (1991), Moss et al. (1995), including the back reaction of the Lorentz force of the mean magnetic field on the differential rotation, predict such effects, at least qualitatively.

Moreover, the observation of orbital period modulations among CVs below the period gap, with characteristics comparable to the other types of active systems in our sample, suggests that the phenomenon is produced by a distributed dynamo working in the convection zone, because CV systems below the period gap have secondaries which are regarded to be fully convective. In more massive stars, a thin-shell dynamo may also be at work in the overshoot layer, but it does not seem to play a relevant role in the orbital period modulation phenomenon, even if it may be relevant for other manifestations of stellar activity (cf. also Paper I).

Our previous work suggested the important role of the magnetic field structure in the convective envelope of the active components. Future observations may help to clarify this fundamental point and the related problem of the field stability

and time variation (cf. Donati 1999). A long-term monitoring of both the activity level and the orbital period of an extended sample of eclipsing binaries is needed for properly addressing these questions. The active close binaries of the RS CVn group appear to be the appropriate astrophysical laboratories for this type of investigation, because their activity level and rotation can be studied without the additional complications of heavy mass-transfer and circumstellar envelope phenomena, that instead characterise the close binaries of other magnetically active groups. Moreover, through an extensive and long-term study of such systems, other unsolved problems may be addressed, in particular, the relationship between the lengths of activity cycles, as measured by the modulations of the spot area, and the orbital period, respectively. This relationship is of fundamental importance to constrain hydromagnetic dynamo models.

*Acknowledgements.* The authors wish to thank R. Rosner, L. Paternò, J.-F. Donati, M. Schüssler, O. Demircan and C. İbanoğlu for useful discussions. The extensive use of the SIMBAD and ADS databases operated by the CDS center, Strasbourg, France, is also gratefully acknowledged. Active star research at Catania Astrophysical Observatory and the Institute of Astronomy of Catania University is funded by MURST (*Ministero della Università e della Ricerca Scientifica e Tecnologica*), CNAA (*Consorzio Nazionale per l'Astronomia e l'Astrofisica*) and the *Regione Siciliana*, whose financial support is gratefully acknowledged.

## References

- Acheson D. J., 1978, *Phil. Trans. Roy. Soc. London*, A289, 459  
 Africano J. L., Nather R. E., Patterson J., et al., 1978, *PASP* 90, 568  
 Akalin A., Derman E., 1997, *A&AS* 125, 407  
 Albayrak B., Ozeren F. F., Ekmekci F., Demircan O., 1999, *Rev. Mex. Astron. Astrophys.* 35, 3  
 Applegate J. H., 1992, *ApJ*, 385, 621  
 Applegate J. H., Patterson J., 1987, *ApJ*, 322, L99  
 Baliunas S. L., Nesme-Ribes E., Sokoloff D., Soon W. H., 1996, *ApJ* 460, 848  
 Baptista R., Jablonski F. J., Steiner J. E., 1989, *MNRAS* 241, 631  
 Baptista R., Jablonski F. J., Steiner J. E., 1992, *AJ* 104, 1557  
 Bell S. A., Rainger P. P., Hill G., Hilditch R. W., 1990, *MNRAS* 244, 328  
 Bevermann K., Pakull M. W., 1984, *A&A* 136, 250  
 Bianchini A., 1980, *MNRAS* 192, 127  
 Bond I. A., Freeth R. V., 1988, *MNRAS* 232, 753  
 Borkovits T., Hegedüs T., 1996, *A&AS* 120, 63  
 Brancewicz H. K., Dworak T. Z., 1980, *AcA* 30, 501  
 Brandenburg A., Moss D., Rüdiger G., Tuominen I., 1991, *GAFD* 61, 179  
 Brandenburg A., Charbonneau P., Kitchatinov L. L., Rüdiger G., 1994, *ASP Conf. Ser.* 64, J.-P. Caillault (Ed.), p. 354  
 Budding E., Kim C.-H., Demircan O., et al., 1997, *Ap&SS* 246, 229  
 Chambliss C. R., 1992, *PASP* 104, 663  
 Chandrasekhar S., 1961, *Hydrodynamic and hydromagnetic stability*, Oxford Univ. Press, Oxford  
 Chandrasekhar S., Roberts P. H., 1963, *ApJ* 138, 801  
 Claret A., 1995, *A&AS*, 109, 441  
 Claret A., Gimenez A., 1992, *A&AS* 96, 255  
 DeCampli W. M., Baliunas S. L., 1979, *ApJ* 230, 815  
 Demircan O., Akalin A., Selam S., et al., 1995, *A&AS* 114, 167  
 Demircan O., Derman E., Akalin A., 1991, *AJ* 101, 201  
 Demircan O., Derman E., Akalin A., Selam S., Müyesseroğlu Z., 1994, *MNRAS* 267, 19  
 Demircan O., Gürol B., 1996, *A&AS* 115, 333  
 Demircan O., Selam S., 1993, *A&A* 267, 107  
 Donati J.-F., 1999, *MNRAS* 302, 457  
 Echevarria J., Alvarez M., 1993, *A&A* 275, 187  
 Edalati M. T., Zeinali F., 1996, *Ap&SS* 243, 275  
 Erdem A., Güzür N., 1998, *A&AS* 127, 257  
 Fekel F. C., 1981, *ApJ* 246, 879  
 Frieboes-Conde H., Herczeg T., 1973, *A&AS* 12, 1  
 Friend M. T., Martin J. S., Connon-Smith R., Jones D. H. P., 1990, *MNRAS* 246, 637  
 Giuricin G., Mardirossian F., Mezzetti M., 1983, *ApJS* 52, 35  
 Hall D. S., 1975, *AcA* 25, 1  
 Hall D. S., Keel W. C., Neuhaus G. H., 1976, *AcA* 26, 239  
 Hall D. S., 1989, *Space Sci. Rev.*, 50, 219  
 Hall D. S., 1990, in Ibanoglu C., ed., *Active close binaries*, Kluwer Academic Publ., Dordrecht, p. 95  
 Hall D. S., 1991a, in Tuominen I., Moss D., Rüdiger G., eds., *The Sun and cool stars: activity, magnetism, dynamos*, IAU Coll. 130, Springer-Verlag, Berlin, p. 353  
 Hall D. S., 1991b, *ApJ* 380, L85  
 Hall D. S., Wawrukiewicz A. S., 1972, *PASP* 84, 541  
 Hellier C., Sproats L. N., 1992, *IBVS* 3724, 1  
 Hellier C., Mason K. O., Rosen S. R., Cordova F. A., 1987, *MNRAS* 228, 463  
 Hendry P. D., Mochnecki S. W., 1998, *ApJ* 504, 978  
 Henry G. W., Eaton J. A., Hamer J., Hall D. S., 1995, *ApJS* 97, 513  
 Horne K., Welsh W. F., Wade R. A., 1993, *ApJ* 410, 357  
 Hrivnak B. J., 1988, *ApJ* 335, 319  
 İbanoğlu C., Keskin V., Akan M., Evren S., Tunca Z., 1994, *A&A*, 281, 811  
 İbanoğlu C., Pekünlü E. R., Keskin V., et al., 1998, *A&SS* 257, 11  
 Jetsu L., Pagano I., Moss D., Rodonò M., Lanza A. F., Tuominen I., 1997, *A&A*, 326, 698  
 Kalimeris A., Rovithis-Livaniou H., Rovithis P., 1994a, *A&A* 282, 775  
 Kalimeris A., Rovithis-Livaniou H., Rovithis P., et al. 1994b, *A&A* 291, 765  
 Kalimeris A., Mitrou C. K., Doyle J. G., Antonopoulou E., Rovithis-Livaniou H., 1995, *A&A* 293, 317  
 Keskin V., Ibanoglu C., Akan M., Evren S., Tunca Z., 1994, *A&A* 287, 817  
 Khalessheh B., Hill G., 1992, *A&A* 257, 199  
 Kopal Z., 1959, *Close binary systems*, J. Wiley & Sons, Inc., New York  
 Kopal Z., 1978, *Dynamics of close binary systems*, D. Reidel Publ. Co., Dordrecht  
 Kreiner J. M., 1971, *AcA* 21, 365  
 Landau L. D., Lifshitz E. M., 1959, *Fluid Mechanics*, Pergamon Press, Oxford  
 Landau L. D., Lifshitz E. M., 1962, *The Classical Theory of Fields*, Pergamon Press, Oxford  
 Lanza A. F., Catalano S., Cutispoto G., Pagano I., Rodonò M., 1998b, *A&A*, 332, 541  
 Lanza A. F., Rodonò M., Rosner R., 1998a, *MNRAS* 296, 893  
 Maceroni C., Van't Veer F., 1994, *A&A* 289, 871  
 Mayer P., 1990, *BAICz* 41, 231  
 Marsh T. R., 1988, *MNRAS* 231, 1117  
 Marsh T. R., Pringle J. E., 1990, *ApJ*, 365, 677  
 Matese J. J., Whitmire D. P., 1983, *A&A*, 117, L7  
 Messina S., Rodonò M., 1999, *A&A* submitted  
 Moffatt H. K., 1978, *Magnetic field generation in electrically conducting fluids*, Cambridge University Press, Cambridge

- Moss D., Barker D. M., Brandenburg A., Tuominen I., 1995, *A&A*, 294, 155
- Moss D., Tuominen I., 1997, *A&A* 321, 151
- Niarchos P. G., 1983, *A&AS* 53, 13
- Niarchos P. G., Rovithis-Livaniou H., Rovithis P., 1993, *Ap&SS* 203, 197
- Ogloza W., Zola S., Tremko J., Kreiner J. M., 1998, *A&A* 340, 81
- O'Neal D., Saar S. H., Neff J. E., 1996, *ApJ* 463, 766
- Panchatsaram T., 1981, *Ap&SS* 77, 179
- Parker E. N., 1979, *Cosmical magnetic fields*, Clarendon Press, Oxford
- Paternò L., Sofia S., Di Mauro M. P., 1996, *A&A* 314, 940
- Patterson J., Robinson E. L., Nather R. E., 1978, *ApJ*, 224, 570.
- Plavec M., 1959, *BAICz* 10, 185
- Plavec M. J., Dobias J. J., 1983, *ApJ* 272, 206
- Rafaert J. B., 1982, *PASP* 94, 485
- Rafert J. B., Markworth N. L., 1991, *ApJ* 377, 278
- Rainger P. P., Bell S. A., Hilditch R. W., 1992, *MNRAS* 254, 568
- Richman H. R., Applegate J. H., Patterson J., 1994, *PASP* 106, 1075
- Robinson E. L., Shetrone M. D., Africano J. L., 1991, *AJ* 102, 1176
- Robinson E. L., Wood J. H., Bless R. C., et al., 1995, *ApJ* 443, 295
- Rodonò M., Lanza A. F., Catalano S., 1995, *A&A* 301, 75
- Rosner R., Weiss N. O., 1992, in Harvey K. L. (Ed.), *The solar cycle*, ASP Conf. Ser. vol. 27, San Francisco, p. 511
- Rovithis-Livaniou H., Rovithis P., Bitzaraki O., 1992, *Ap&SS* 189, 237
- Rubenstein E. P., Patterson J., Africano J. L., 1991, *PASP* 103, 1258
- Sarna M. J., De Greve J.-P., 1994, *A&A* 281, 433
- Šimon V., 1996, *A&A* 311, 915
- Šimon V., 1997a, *A&A* 327, 1087
- Šimon V., 1997b, *A&A* 319, 886
- Šimon V., 1999, *A&AS* 134, 1
- Smak J., 1993, *AcA* 43, 121
- Smak J., 1995, *AcA* 45, 259
- Söderhjelm S., 1980, *A&A*, 89, 100
- Spiegel E. A., Weiss N. O., 1980, *Nat* 287, 616
- Spruit H. C., 1982, *A&A* 108, 348
- Spruit H. C., Weiss A., 1986, *A&A* 166, 167
- Strassmeier K. G., Hall D. S., Fekel F. C., Scheck M., 1993, *A&AS* 100, 173
- Tunca Z., Keskin V., Akan M. C., Evren S., İbanoğlu C., 1987, *Ap&SS* 136, 63
- Ulrich R. K., Bertello L., 1996, *ApJ* 465, L65
- Ulrich R. K., Hawkins G. W., 1981, *ApJ* 246, 985
- van Ballegooijen A. A., 1982, *A&A* 106, 43
- Vogt S. S., Hatzes A. P., Misch A., Kürster M., 1999, *ApJS* 121, 547
- Wade R. A., Horne K., 1988, *ApJ* 324, 411
- Warner B. W., 1988, *Nat* 336, 129
- Warner B., 1995, *Cataclysmic Variable Stars*, Cambridge Univ. Press, Cambridge
- Watts D. J., Bailey J., Hill P. W., et al., 1986, *A&A* 154, 197
- Weiss N. O., 1994, in *Lectures on Solar and Planetary Dynamos*, M. R. E. Proctor and A. D. Gilbert (Eds.), Cambridge Univ. Press, Cambridge, p. 59
- Wolf M., Molik P., 1996, *IBVS* 4304, 1
- Wolf S., Mantel K. H., Horne K., et al., 1993, *A&A* 273, 160
- Zahn J.-P., 1989, *A&A* 220, 112
- Zhai D., Zhang R., Leung K.-C., 1984, *A&AS* 57, 487
- Zhang J.-T., Zhang R.-X., Zhai D.-S., Li Q.-S., Jin T.-L., 1989, *IBVS* 3302, 1