Astron. Astrophys. 349, 1017–1026 (1999)

ASTRONOMY
AND
ASTROPHYSICS

On the physics of cold MHD winds from oblique rotators

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Received 29 March 1999 / Accepted 18 June 1999

Abstract. I show that the self-consistent solution of the problem of MHD plasma flow in the magnetosphere of an oblique rotator with an initially split-monopole magnetic field is reduced to the solution of a similar problem for the axisymmetric rotator. All properties of the MHD cold plasma flows from the axisymmetric rotators with the initial split-monopole magnetic field are valid for the oblique rotators as well. Rotational losses of the oblique rotator do not depend on the inclination angle and there is no temporal evolution of this angle. Self - consistent analytical and numerical solutions for the axisymmetric plasma flows obtained earlier show that the rotators can be divided on both fast rotators ($\sigma_0/U_0^2 > 1$) and slow rotators ($\sigma_0/U_0^2 < 1$), where σ_0 is the ratio of the Poynting flux over the matter energy flux in the flow at the equator on the surface of the star, $U_0 = \gamma_0 v_0/c$, v_0 and γ_0 being the initial velocity and the Lorentz-factor of the plasma. The self-consistent approximate analytical solution for the plasma flow from the oblique rotator is obtained under the condition $\sigma_0/U_0^2 \ll 1$. Implications of these results for radio pulsars are discussed. In particular, I argue that all radio pulsars are apparently the slow rotators ejecting the Poynting dominated relativistic wind.

Key words: Magnetohydrodynamics (MHD) – stars: pulsars: general – ISM: jets and outflows

1. Introduction

An analysis of the relativistic plasma flow is necessary to understand the processes taking place in radio pulsar magnetospheres, compact galactic objects and in AGNs (Arons 1996; Mirabel & Rodriguez 1996; Pelletier et al. 1996). In the present paper, as in a previous one (Bogovalov 1997), we concentrate fully on the problem of the relativistic plasma flow in the conditions typical to radio pulsars.

In spite of systematic research in the field of the physics of radio pulsars in the last years, the structure of the magnetosphere and the mechanisms for the acceleration of plasma in these objects remain to a large extent vague (Lyubarsky 1995). One of the most important unsolved problems of the physics of radio pulsars is the problem of the energetics of the ejected wind of relativistic plasma. For example, the kinetic energy of the plasma accelerated in the inner magnetosphere of the Crab pulsar is not sufficient to explain the energetics of the relativistic wind exiting the synchrotron nebula surrounding the pulsar (Arons 1996).

The basis of the theory of plasma production in radio pulsar magnetospheres was initiated by Sturrock (1971). This theory was developed in more detail for different conditions on the stellar surface by Ruderman & Sutherland (1975) and by Arons (1981). Primary electrons are accelerated in the so-called "electrostatic gaps" and produce dense relativistic plasma of secondary particles with Lorentz-factor $\gamma_0 \sim 10^2$–10^3. This plasma screens the accelerating electric field and limits the potential drop by the value $\gamma_{gap}mc^2$ with $\gamma_{gap} \sim 10^7$. The total flux of the kinetic energy of all particles appears to be far interior to the total rotational losses of the fast rotating radio pulsars due to this limitation. Almost all energy is carried out by the electromagnetic field. A wind with similar characteristics is formed in the outer gap model (Cheng et al. 1986). The wind of relativistic plasma from radio pulsars can be characterized by the ratio of the Poynting flux over the flux of the kinetic energy of the plasma σ. The plasma is Poynting flux dominated, when $\sigma > 1$, and is kinetic energy dominated, if $\sigma < 1$. Electrostatic gaps give $\sigma > 1000$ for the Crab pulsar. At the same time, interpretation of the observations of the Crab Nebula compels us to conclude that this ratio is very small at large distances from the pulsar. This conclusion is based on the assumption that the subsonic flow of the plasma terminated by the shock wave at the interaction of the wind with the interstellar medium can be considered as the flow of ideal plasma with the only dissipative process of synchrotron cooling. Under this assumption, the observed expansion of the outer edge of the Crab Nebula, the synchrotron and TeV gamma-ray emission of the nebula produced via Inverse Compton Scattering of the relativistic particles on 2.7-K MBR emission can be explained under the unique choice of $\sigma = 3 \cdot 10^{-3}$ (Atoyan & Aharonian 1996).

Recently Begelman (1998) reviewed the key assumption of the theory by Kennel & Coroniti (1984). He argued that even a wind with $\sigma \sim 1$ in the pre-shock region can give the observable properties of the Crab Nebula if one takes into account dissipative processes in the nebula. It follows from the theory of relativistic MHD shocks that a wind with $\sigma \sim 1$ in the pre-shock region creates the flow in the post-shock region with $\sigma > 1$. But this flow must be strongly unstable. The instability provides an effective transformation of the magnetic field into the kinetic energy of the plasma accompanied by acceleration of particles in the nebula. In both (Kennel & Coroniti or Begelman) scenarios it follows that some unspecified mechanism for the transformation of the Poynting flux into the flux of the kinetic energy exists as the plasma travels from the star to infinity. This mechanism is apparently the basic mechanism for plasma acceleration since it ensures the transformation of at least 50% (in the Begelman scenario) and up to 99.7% (in the Kennel & Coroniti model) of the rotational energy of the neutron star into the kinetic energy of the relativistic wind.

One of the possible mechanisms of the acceleration is the magnetic acceleration of the plasma by the rotating magnetosphere. Unfortunately this mechanism appeared non effective for the axisymmetric rotators with a typical pulsar's parameters. No effective acceleration was found neither in the nearest zone (Bogovalov 1997), nor in the far zone of the rotator (Bogovalov & Tsinganos 1999). At the same time, until now no solution describing the MHD plasma flow from the oblique rotator exists. Usually it is believed that the inclination of the magnetic moment of the star to the axis of rotation could be important for the plasma acceleration. In this paper we study this possibility in the model of the oblique rotator with the initial split-monopole magnetic field on the surface of the star.

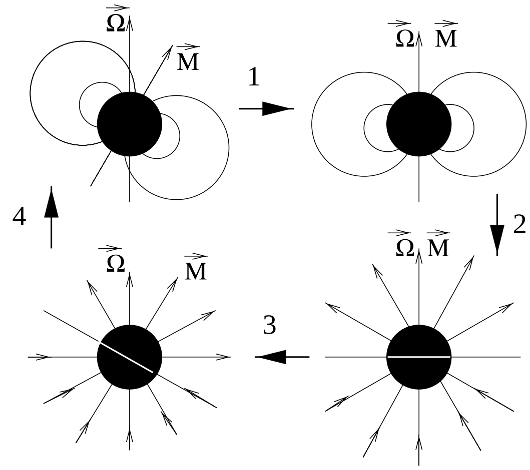

Fig. 1. The sequence of models introduced to simplify the solution of the problem of the plasma outflows from radio pulsars. Step 1 is the transition from the oblique rotator with a dipole magnetic field to the axisymmetric rotator. Step 2 is the transition from the axisymmetric rotator with the dipole magnetic field to the axisymmetric rotator with the split-monopole magnetic field. In this paper, the transition from the axisymmetric split-monopole to the oblique split-monopole model is done (step 3) together with step 4, connecting this model with real pulsars.

2. Basic assumptions

The axis of rotation for real radio pulsars is not directed along the magnetic moment. The solution of the problem of the plasma flow in the magnetosphere of this object is extremely complicated. To simplify the problem, several models of radio pulsars were proposed. The sequence of these models is presented in Fig. 1. Firstly Goldreich & Julian (1969) proposed an axisymmetrically rotating star with an initially dipole magnetic field (step 1 in Fig. 1). The rotational losses of the axisymmetrically rotating star ejecting relativistic plasma are comparable to the rotational losses of the oblique dipole in vacuum. The energy of rotation is carried out near the surface of the axisymmetric rotator by the Poynting flux. This is why this model can be used for study of the process of the Poynting flux transformation into the kinetic energy of the plasma.

However, even with such simplification, the structure of the axisymmetric flow of the plasma from the star with the initial dipole magnetic field appears too complicated. There were a lot of attempts to solve this problem in massless approximation (Michel 1973b; Beskin et al., 1983; Lyubarskii 1990; Contopoulos et al. 1999). To simplify the problem, Michel (1969) was the first to use the rotator with the prescribed split-monopole poloidal magnetic field for the investigation of the relativistic plasma flow in the magnetosphere of the axisymmetrical rotator (step 2). It follows from Fig. 1 that there are no closed field lines in the split-monopole model. All lines go to infinity. This allows one to simplify the analysis of the flow considerably.

Michel (1969) considered the flow of the plasma in the prescribed split-monopole magnetic field. The affect of the moving plasma on the poloidal magnetic field was not taken into account. In the self-consistent solution, the plasma and the electromagnetic field affect each other. The problem of the self-consistent plasma flow from the rotator with the initial split-monopole magnetic field was investigated in nonrelativistic and relativistic limits in the papers by Sakurai (1985), Bogovalov (1992) Bogovalov (1997), Bogovalov & Tsinganos (1999). The phrase "initial split-monopole magnetic field" means that the normal component of the magnetic field on the surface of the star is constant but changes sign on the magnetic equator. In this paper we firstly consider the model of the oblique rotator with the split-monopole magnetic field on the surface of the star (step 3). This model allows one to investigate the plasma flow in conditions more typical to real radio pulsars than it occurs in the axisymmetric model and to connect the model with the split-monopole magnetic field with the real pulsars (step 4).

Since the density of the relativistic plasma produced in the pulsar magnetosphere is high enough to screen the electric field parallel to the magnetic field, the magnetohydrodynamical approximation is used in this paper to describe the flow of the plasma. The plasma is considered cold as a first approximation (Lyubarsky 1995).

3. Reduction of the oblique rotator problem to the axisymmetrical problem

System of time dependent equations defining the temporal evolution of the flow of the relativistic plasma (Akhiezer et al. 1975)

$$mn(\frac{\partial \gamma \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\gamma\mathbf{v}) = q \cdot \mathbf{E} + \frac{1}{c}\,\mathbf{j} \times \mathbf{H}, \qquad (1)$$

$$\frac{\partial \mathbf{H}}{c\partial t} = curl\,\mathbf{E}, \tag{2}$$

$$curl\,\mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{\partial \mathbf{E}}{c\partial t}, \tag{3}$$

$$div\,\mathbf{H} = 0, \tag{4}$$

$$div\,\mathbf{E} = 4\pi q, \tag{5}$$

$$\frac{\partial n}{\partial t} + div(\mathbf{v}n) = 0. \tag{6}$$

together with frozen-in condition $\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H} = 0$ gives, in spherical coordinate system, the equations of motion

$$mn(\frac{\partial \gamma v_r}{\partial t} + (\mathbf{v}\nabla)\gamma v_r - \frac{\gamma(v_\theta^2 + v_\varphi^2)}{r}) = q\cdot E_r + \frac{1}{4\pi}\Big\{(\frac{1}{r\sin\theta}\frac{\partial H_r}{\partial\varphi} - \frac{1}{r}\frac{\partial(rH_\varphi)}{\partial r})H_\varphi - -(\frac{\partial(rH_\theta)}{r\partial r} - \frac{\partial H_r}{r\partial\theta})H_\theta + \frac{1}{c}(H_\theta\frac{\partial E_\varphi}{\partial t} - H_\varphi\frac{\partial E_\theta}{\partial t})\Big\}, \tag{7}$$

$$mn(\frac{\partial \gamma v_\theta}{\partial t} + (\mathbf{v}\nabla)\gamma v_\theta + \frac{\gamma(v_r v_\theta - v_\varphi^2\cot\theta)}{r}) = q\cdot E_\theta + \frac{1}{4\pi}\Big\{(\frac{\partial(rH_\theta)}{r\partial r} - \frac{\partial H_r}{r\partial\theta})H_r - -\frac{H_\varphi}{r\sin\theta}(\frac{\partial\sin\theta H_\varphi}{\partial\theta} - \frac{\partial H_\theta}{\partial\varphi}) + +\frac{1}{c}(H_\varphi\frac{\partial E_r}{\partial t} - H_r\frac{\partial E_\varphi}{\partial t})\Big\}, \tag{8}$$

$$mn(\frac{\partial \gamma v_\varphi}{\partial t} + (\mathbf{v}\nabla)\gamma v_\varphi + \frac{\gamma(v_r v_\varphi + v_\theta v_\varphi\cot\theta)}{r}) = q\cdot E_\varphi + \frac{1}{4\pi}\Big\{\frac{1}{r\sin\theta}(\frac{\partial\sin\theta H_\varphi}{\partial\theta} - \frac{\partial H_\theta}{\partial\varphi})H_\theta - -(\frac{1}{r\sin\theta}\frac{\partial H_r}{\partial\varphi} - \frac{\partial rH_\varphi}{r\partial r})H_r + +\frac{1}{c}(H_r\frac{\partial E_\theta}{\partial t} - H_\theta\frac{\partial E_r}{\partial t})\Big\}, \tag{9}$$

and the induction equations:

$$\frac{\partial H_r}{c\partial t} = -\frac{1}{r\sin\theta}(\frac{\partial(\sin\theta E_\varphi)}{r\partial\theta} - \frac{\partial E_\theta}{\partial\varphi}), \tag{10}$$

$$\frac{\partial H_\theta}{c\partial t} = -(\frac{1}{r\sin\theta}\frac{\partial E_r}{r\partial\varphi} - \frac{\partial(rE_\varphi)}{r\partial r}), \tag{11}$$

$$\frac{\partial H_\varphi}{c\partial t} = -(\frac{\partial(rE_\theta)}{r\partial r} - \frac{\partial E_r}{r\partial\theta}). \tag{12}$$

The laws of conservation of the magnetic field and the matter flux, together with the Coulomb law take the following form

$$\frac{1}{r^2}\frac{\partial(r^2H_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial\sin\theta H_\theta}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial H_\varphi}{\partial\varphi} = 0. \tag{13}$$

$$\frac{\partial n}{\partial t} + \frac{1}{r^2}\frac{\partial(r^2 n v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial\sin\theta n v_\theta}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial n v_\varphi}{\partial\varphi} = 0. \tag{14}$$

$$\frac{1}{r^2}\frac{\partial(r^2E_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial\sin\theta E_\theta}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial E_\varphi}{\partial\varphi} = 4\pi q. \tag{15}$$

The frozen-in conditions are

$$E_r + \frac{1}{c}(v_\theta H_\varphi - v_\varphi H_\theta) = 0, \tag{16}$$

$$E_\theta + \frac{1}{c}(v_\varphi H_r - v_r H_\varphi) = 0, \tag{17}$$

$$E_\varphi + \frac{1}{c}(v_r H_\theta - v_\theta H_r) = 0. \tag{18}$$

In this system q is the induced space electric charge density, θ is the polar angle and φ is the azimuthal angle.

Boundary conditions on the surface of the star with radius R_* for the system of equations above are:

1. The Lorentz-factor γ_0 is specified constant;
2. The normal component of the density of the matter flux is specified constant and uniform;
3. The normal component of the poloidal magnetic field H_0 does not depend on coordinates or time for every point of the surface of the star and it changes sign on the magnetic equator.
4. The tangential component of the electric field is continuous on the star's surface;

It is assumed that the velocity of the plasma exceeds fast magnetosonic velocity (the flow is super sonic) at the infinity. This system of equations together with the boundary conditions describes the stationary axisymmetric flow as well.

Let's assume that the self-consistent solution for the problem of the plasma flow from the axisymmetric rotator with an initial monopole-like poloidal magnetic field is specified. The phrase "initial monopole-like magnetic field" means that the normal component of the poloidal magnetic field on the surface of the star is constant and does not change sign on the magnetic equator. Actually this field has no magnetic equator at all. Although such magnetic fields can not be created in reality, this rather artificial mathematical model is convenient to construct solutions for more realistic plasma flows. The monopole-like magnetic field differs from the split-monopole magnetic field only by direction of the field lines in one of the hemispheres and was introduced by Michel (1969). In the model with the monopole-like magnetic field, all field lines are coming out from the surface of the star as it is shown in Fig. 2. There is no current sheet in this flow in contrast to the model with the split-monopole magnetic field which contains the current sheet in the equatorial plane. The current sheet is a tangential discontinuity (Landau & Lifshitz 1963) in ideal MHD approximation. The magnetic field changes direction during the passage through the current sheet. It is obvious that the solution for the split-monopole can be obtained from the solution for the monopole-like magnetic

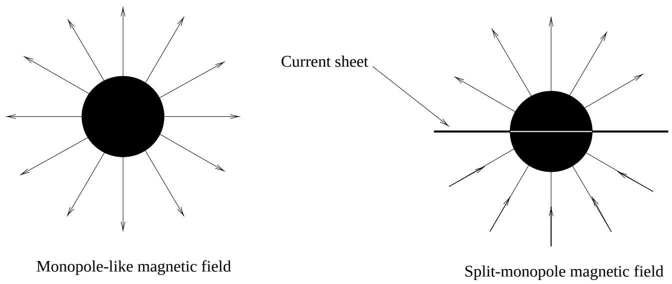

Fig. 2. The solution in the model with the split-monopole magnetic field can be obtained from the solution for the monopole-like magnetic field by a change of sign of the magnetic field in the lower hemisphere. The same is valid for the reverse operation. There is a current sheet at the equatorial plane in the split-monopole model which is a tangential discontinuity in an ideal MHD approximation

field by formally reversing the sign of the magnetic fields in the lower hemisphere. Similar operation can be done in the case of the oblique rotator as well.

The flow and the monopole-like poloidal magnetic field are perturbed by rotation in the self-consistent axisymmetric flow (Bogovalov & Tsinganos 1999). We assume that this perturbed self-consistent solution is known. Let's introduce the velocity $\mathbf{V}(\mathbf{r})$, the poloidal $\mathbf{B_p}(\mathbf{r})$ and toroidal $B_\varphi(\mathbf{r})$ magnetic fields and the density of the plasma $N(\mathbf{r})$ in the self-consistent axisymmetric flow with the initial monopole-like magnetic field. The poloidal electric field $\mathbf{E}$ and the poloidal magnetic field are connected in the axisymmetric flow $\mathbf{E} = (r\sin\theta\Omega/c)\mathbf{B_p}\times\mathbf{e}_\varphi$ (Weber & Davis 1967), where $\mathbf{e}_\varphi$ is the unit vector directed in the azimuthal direction. The toroidal electric field is equal to 0. These variables depend only on coordinates. Dependence on time is absent for the stationary axisymmetric flow. This solution automatically satisfies all the equations of the system (7-18) and corresponding boundary conditions.

Now we show how the self-consistent solution for the plasma flow in the magnetosphere of the oblique rotator with the initial split - monopole magnetic field can be obtained from the known solution for the axisymmetric problem. Let's consider the following transformation of the axisymmetric solution. The same velocity and the density values as for the axisymmetric rotator are used:

$$v(\mathbf{r},t) = V(\mathbf{r}),\ n(\mathbf{r},t) = N(\mathbf{r}). \tag{19}$$

The poloidal magnetic field $\mathbf{H_p}(\mathbf{r},t)$ for the oblique rotator is obtained from the poloidal magnetic field of the axisymmetric rotator as follows

$$\mathbf{H_p}(\mathbf{r},t) = \eta(\mathbf{r},t)\mathbf{B_p}(\mathbf{r}). \tag{20}$$

The same procedure gives us the toroidal magnetic field

$$H_\varphi(\mathbf{r},t) = \eta(\mathbf{r},t)B_\varphi(\mathbf{r}), \tag{21}$$

where $\eta(\mathbf{r},t)$ is an unknown function to be specified. The poloidal electric field E is defined as in the axisymmetric case

$$E_\theta = -\frac{r\sin\theta\Omega}{c}H_r, \tag{22}$$

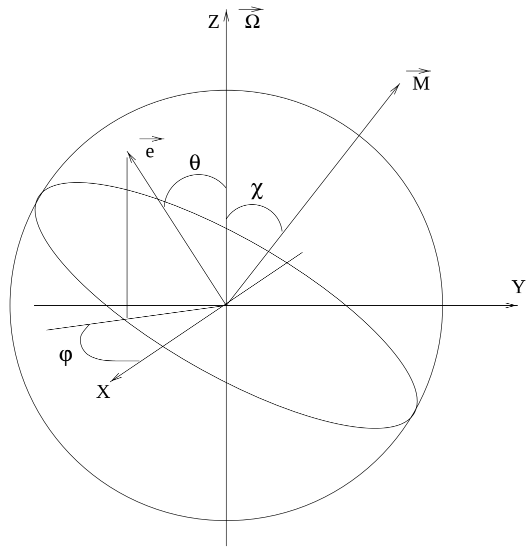

Fig. 3. The initial position of the star and the geometry used in Eq. (29).

$$E_r = \frac{r\sin\theta\Omega}{c}H_\theta, \tag{23}$$

and the toroidal electric field $E_\varphi = 0$. It is easy to show that this is indeed the solution of the problem for the oblique rotator and to specify the function η.

The frozen-in conditions (16-18) are fulfilled automatically for the solution (19-23). At the stationary rotation, the dependence of η on φ and t can be presented by one variable $\xi = \varphi - \Omega t$ in the spherical system of coordinates. Thus, η depends on three variables r, θ and ξ. Then, the induction equation (10) is reduced to the equation

$$\frac{\partial}{\partial\xi}\left(\frac{E_\theta}{r\sin\theta} + \frac{\Omega H_r}{c}\right) = 0 \tag{24}$$

which is fulfilled due to condition (22). The same is valid for the induction equation (11) which is reduced to the equation

$$\frac{\partial}{\partial\xi}\left(\frac{E_r}{r\sin\theta} - \frac{\Omega H_\theta}{c}\right) = 0 \tag{25}$$

which is satisfied due to condition (23). The third induction equation (12) after the substitution of the conditions (22,23) is reduced to the flux conservation condition (13) which takes the form

$$(\mathbf{B}\cdot\nabla\eta) = 0. \tag{26}$$

since the total magnetic field $\mathbf{B}$ of the axisymmetric flow satisfies the equation $\nabla\cdot\mathbf{B} = 0$. This implies that function η is constant on the field line of the magnetic filed of the axisymmetric flow.

Fig. 3 shows the initial position of the star with the magnetic moment $\mathbf{M}$ inclined to the axis of rotation at the angle χ at $t = 0$. It follows from Eq. (20) and the boundary conditions that function η is equal to 1 for the points on the surface of the star where the field lines leave the surface of the star and is equal to -1 for the points where the field lines enter the surface. It is convenient to introduce the following function

$$D(x) = \begin{cases} 1, & \text{if} x \geq 1 \\ -1, & \text{if} x < 1. \end{cases} \quad (27)$$

The sign of the function η on the surface of the star varies with the sign of the product $(\mathbf{e} \cdot \mathbf{e}_M)$, where $\mathbf{e}$ is the unit vector directed to the point on the surface of the star, $\mathbf{e}_M$ is the unit vector directed along the magnetic moment. This product can be presented as

$$(\mathbf{e} \cdot \mathbf{e}_M) = \sin\chi \sin\theta \sin\varphi + \cos\chi \cos\theta. \quad (28)$$

Then, on the surface of the star, the function η is

$$\eta(\theta, \varphi - \Omega t) = D(\sin\chi \sin\theta \sin(\varphi - \Omega t) + \cos\chi \cos\theta). \quad (29)$$

Actually this is the boundary condition for Eq. (26). The equation can be presented in the form

$$B_r \frac{\partial \eta}{\partial r} + B_\theta \frac{\partial \eta}{r \partial \theta} + B_\varphi \frac{\partial \eta}{r \sin\theta \partial \varphi} = 0. \quad (30)$$

The equations for the characteristics of this equation are

$$\frac{dr}{B_r} = \frac{r \sin\theta d\varphi}{B_\varphi} \quad (31)$$

and

$$\frac{dr}{B_r} = \frac{r d\theta}{B_\theta}. \quad (32)$$

Therefore the general solution is

$$\eta(r, \theta, \varphi, t) = f(\theta - \int^r \frac{B_\theta dr}{r B_r}, \varphi - \int^r \frac{B_\varphi dr}{r \sin\theta B_r}), \quad (33)$$

where the integrals over r are taken along the field line of the poloidal magnetic field of the axisymmetric solution, f is an arbitrary function. The solution satisfying the boundary condition (29) has the form:

$$\eta(r, \theta, \varphi, t) = D\Big(\sin(\chi) \sin(\theta - \int_{R_*}^r \frac{B_\theta dr}{r B_r}) \times \\ \times \sin(\varphi - \int_{R_*}^r \frac{B_\varphi dr}{r \sin\theta B_r} - \Omega t) + \\ + \cos(\theta - \int_{R_*}^r \frac{B_\theta dr}{r B_r}) \cos\chi\Big). \quad (34)$$

It follows from this solution that $\eta^2 = 1$ and η changes sign when the magnetic field changes direction. It is easy to show now that equations of motion (7-9) are also satisfied for the solution (19-23) at the function η defined by (34). Notice that on the left hand side of these equations there is no function η. In the right hand side of the equations of motion, function η comes in the combination $A_i \eta \frac{\partial \eta B_k}{\partial x_l}$, where A_i and B_k are arbitrary components of fields of the axisymmetric solution, and x_l is a spatial or time coordinate in 4-space. This relationship can be presented as

$$A_i \eta \frac{\partial \eta B_k}{\partial x_l} = \eta^2 A_i \frac{\partial B_k}{\partial x_l} + A_i B_k \frac{1}{2} \frac{\partial \eta^2}{\partial x_l} = A_i \frac{\partial B_k}{\partial x_l}. \quad (35)$$

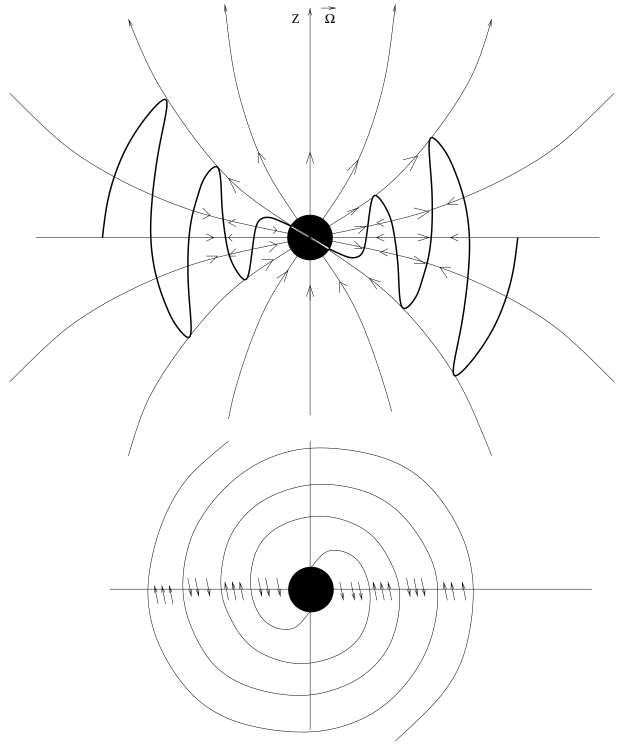

Fig. 4. Top panel shows the structure of field lines and the current sheet (thick wave-like line) in the poloidal plane. The lower panel shows the same in the equatorial plane. Arrows show the direction of the magnetic field lines. The direction of the field lines changes on the current sheet.

Therefore, the function η disappears in the equations of motion. Here we ignore the difference in the dynamics of the current sheet and the surrounding plasma assuming that the current sheet is the mathematical discontinuity as usual in ideal MHD. This assumption can be violated for the oblique rotators at large distances from the star. But, at large distances, the dynamics of the current sheet can be considered particularly in WKB approximation (Coroniti 1990; Michel 1994).

Thus, we obtain the self-consistent solution for the oblique rotator from the known self-consistent solution for the axisymmetric rotator. The sketch demonstrating the structure of the cold wind from the oblique rotator is presented in Fig. 4. The structure of the plasma flow is symmetric in relation to the equator. The form of the poloidal field lines is the same as for the axisymmetric rotator. In general there is a collimation of the plasma flow to the axis of rotation, although the effect of the collimation depends on the parameters of the problem (Bogovalov & Tsinganos 1999). In the axisymmetric flow the current sheet dividing the magnetic fluxes of opposite directions is located on the equator. In the wind from the oblique rotator, the current sheet takes the form of a wave. In the poloidal plane the poloidal magnetic field lines change direction on the current sheet. At first glance it seems that this behavior contradicts magnetic flux conservation. The bottom panel of Fig. 4 shows the structure of the field lines in the equatorial plane. It is seen that there is no contradiction with the magnetic flux conservation since the total magnetic field depends on the azimuthal angle φ. The velocity and the density of the plasma do not depend on χ and φ and are the same as for the axisymmetric rotator. Only the magnetic and the electric fields are modulated with the period of rotation. The sign of these fields is modulated in the sector limited by the field lines which have roots on the magnetic equator. The squares of these fields are not modulated. The wave is simply the wave of the tangential discontinuity (Landau & Lifshitz 1963) which is convected with the velocity of the plasma.

The rotational losses of the oblique rotator can be calculated as the total Poynting flux through a sphere surrounding the star. The Poynting flux is defined by the formula $c\frac{\mathbf{E}\times\mathbf{H}}{4\pi}$. This formula is biquadratic on the components of the electromagnetic field. Since for the oblique rotator these components differ from the component for the axisymmetric rotator only by function η, the Poynting flux appears independent on function η and, therefore, the rotational losses are independent on the angle χ.

There is also no alignment of the magnetic moment along the axis of rotation as it happens in vacuum approximation (Ostriker & Gunn 1969, Pachini 1968, Michel & Goldwire 1970, Davis & Goldstein 1970) or disalignment as is seen in the solution by Beskin et al. (1983). The evolution of the inclination angle χ is defined by torques on the surface of the star (Michel & Goldwire 1970)

$$L_y = \int [T_{r\theta}\cos\varphi - T_{r\varphi}\cos\theta\sin\varphi] d^2S, \quad (36)$$

$$L_x = -\int [T_{r\theta}\cos\varphi - T_{r\varphi}\cos\theta\sin\varphi] d^2S, \quad (37)$$

and

$$L_z = \int T_{r\varphi}\sin\theta \; d^2S,, \quad (38)$$

where $T_{r\theta}$ and $T_{r\varphi}$ are the components of the energy-momentum tensor $T_{r\theta}$. Integration in (37) is performed over the surface of the star. The energy-momentum tensor is also biquadratic on the components of the electromagnetic field (Landau & Lifshitz 1975). Therefore it does not depend on the function η and the angles χ and φ. The torques for the oblique rotator are the same as for the axisymmetric rotator. Integration in Eq. (37) over the surface of the star gives $L_x = L_y = 0$. There are no torques which can change the inclination angle of the oblique rotator with the initial split-monopole magnetic field. This is not a totally unexpected conclusion. The torques on the star rotating in a vacuum differ from the torque on the star ejecting ideal plasma. The torque on the star rotating in a vacuum always align the rotational and magnetic axes (Soper 1972). The temporal evolution of the inclination angle χ of the star ejecting plasma depends on the distribution of the magnetic field on the surface of the star. It was shown by the perturbation theory in first order approximation on Ω that magnetic and rotation axes tend to align when the magnetic flux is more concentrated at the magnetic poles and evolve to $\chi = 90°$ when there is less flux density at the poles than at the equator (Mestel & Selley 1970). Thus, the result obtained here for the arbitrary angular velocity of the rotator with the split-monopole magnetic field is due to the combination of two conditions: by the ejection of the plasma simultaneously with the uniform magnetic flux distribution on the surface of the star.

4. The flow of the plasma at $\frac{\sigma_0}{U_0^2} \ll 1$

The solution of the problem of the cold plasma flow from the rotator with the initial split-monopole magnetic field is defined by the following parameters and variables: H_0, mn_0, v_0, Ω, c, r, θ, φ, R_*, χ and t. Lower index “0” denotes the values on the surface of the star. Since the dependence on χ, φ and t is known, we can consider only the dependence of the solution for the axisymmetric flow on the other 8 parameters. There are 3 parameters in this set with independent dimensions. Therefore, according to the theory of dimensions (Barenblatt 1979), the solution in dimensionless variables depends only on 5 parameters. Let us write this dependence as

$$\mathbf{A} = \mathbf{f}(R_{star}/a, \alpha 1, \alpha 2, r/a, \theta). \quad (39)$$

Here $\mathbf{A}$ is the vector of the state of the plasma including density, velocity and magnetic field, $\mathbf{f}$ is the unknown vector-function and a is some parameter with the dimension of length. It is reasonable to consider a limiting case when the dimensionless radius of the star goes to zero. It allows one to decrease the number of variables in the solution by one. Now the solution crucially depends only on two parameters and two variables in the dimensionless presentation.

Let us measure the velocity of the plasma in the units of the initial velocity v_0 and all geometrical variables measured with the initial radius of the fast mode surface (FMS) r_f. The FMS is the surface where the poloidal velocity of the plasma equals the local velocity of the fast mode MHD perturbations. In the comoving coordinate system where the electric field is equal to zero, this condition takes the form

$$v_p = \frac{H'^2}{4\pi m n' c^2 + H'^2}, \quad (40)$$

where n' and H' are the density of the plasma and the magnetic filed in the comoving system. Taking into account that $H^2 - E^2$ is invariant (Landau & Lifshitz 1975), r_f can be calculated with $\Omega = 0$ as follows

$$r_f^2 = \frac{H_0^2 R_*^2}{4\pi m c n_0 v_0 U_0}. \quad (41)$$

In these units the radius-vector is $\mathbf{X} = \mathbf{r}/r_f$. The electric and magnetic fields can be measured in the units of the initial poloidal magnetic field on the FMS. The density of the plasma can also be measured in the units of the initial density on the FMS.

The parameters $\alpha 1$ and $\alpha 2$ can be chosen rather arbitrarily. In particular, it is convenient to choose $\alpha 1 = \alpha = (r_f\Omega/v_0\gamma_0)$ and $\alpha 2 = \epsilon = (r_f\Omega/c)$. Let's consider the dependence of the solution on these parameters.

A nonrelativistic limit corresponds to $\epsilon \to 0$ and $v_0 \ll c$. The solution depends only on one parameter in this case. The axisymmetrical nonrelativistic plasma flow was investigated numerically by Bogovalov & Tsinganos (1999) for the parameter α achieving the value up to 4.5. It was found that there is a clear division of the rotators between fast and slow ones with a dependence on the value of α. The slow rotators correspond to $\alpha < 1$ and the fast rotators correspond to $\alpha > 1$. There is a strong physical difference between them. The plasma in outflows from the slow rotators is not accelerated considerably and is not collimated in the subsonic region. In the case of the fast rotators the plasma is effectively accelerated and collimated to the axis of rotation already in the subsonic region of the flow.

The flow of the relativistic plasma depends crucially on two parameters. But in this case it is also possible to divide the rotators between slow and fast ones depending on the value of the parameter α. The solution can be presented as a vector function $\mathbf{A}$ with components A_l which are the density of plasma, the components of velocity, the magnetic field, etc. Let's consider the expansion of this function by powers of α in the point $\alpha = 0$ at fixed $\epsilon \neq 0$. Physically, this means that we consider the relativistic limit of the problem at $\gamma_0 \to \infty$. The expansion can be presented as

$$A_l = f_l^{(0)}(\epsilon, \mathbf{X})(1 + \alpha \mathbf{f}^{(1)}(\epsilon, \mathbf{X}) + (\alpha)^2 \mathbf{f}^{(2)}(\epsilon, \mathbf{X}) + ...). \quad (42)$$

The first term $f_l^{(0)}(\epsilon, \mathbf{X})$ in this expansion has a very simple presentation when using the arbitrary parameter $\epsilon \neq 0$. It was shown that, at the limit under consideration, the function $f_l^{(0)}(\epsilon, \mathbf{X})$ is as follows using ordinary (dimension) variables (Bogovalov, 1997)

$$U_r = U_0,\ U_\varphi = 0,\ U_\theta = 0,\ n(r) = n_0(\frac{r_0}{r})^2. \quad (43)$$

$$H_\varphi = -(\frac{r\Omega \sin\theta}{c})H_p,\ E = (\frac{r\Omega \sin\theta}{c})H_p, \quad (44)$$

and

$$H_r = \begin{cases} H_0(\frac{r_0}{r})^2,\ \text{if}\theta \le \pi/2 \\ -H_0(\frac{r_0}{r})^2,\ \text{if}\theta > \pi/2 \end{cases},\ \ H_\theta = 0. \quad (45)$$

This is exactly the solution obtained by Michel (1973a) for the massless plasma. It is easy to show that the full system of the equations of motion is satisfied for this solution except in the case of the frozen-in condition (18). The residual in this condition on the FMS is

$$\delta v_\varphi = \epsilon \frac{(\alpha/\epsilon)^2}{(1 + v_0/c)}. \quad (46)$$

The residual shows that the corrections to the solution are indeed proportional to powers of α, and can be neglected in the region limited by the FMS, provided that $\alpha \ll 1$. This residual also shows that expansion (42) contains the terms $(\alpha/\epsilon)^k$, where k is integer number. Therefore the first term in expansion (42) gives an incorrect result for the limit $\alpha \to 0$, $\epsilon \to 0$ provided that $\alpha/\epsilon = const$ which corresponds to the slow rotation of the star at arbitrary velocity v_0. Mathematically, this means that the point ($\alpha = 0$, $\epsilon = 0$) is a particular point of the expansion. The terms $(\epsilon/\alpha)^k$ can be eliminated from expansion (42). Let's to define the function

$$g_l(\alpha/\epsilon, \mathbf{X}) = \lim_{\substack{\alpha \to 0 \\ \alpha/\epsilon = const}} \frac{A_l(\alpha, \epsilon, \mathbf{X})}{f_l^{(0)}(\epsilon, \mathbf{X})}. \quad (47)$$

Then the expansion can be presented as

$$A_l = f_l^{(0)}(\epsilon, \mathbf{X})(g_l(\alpha/\epsilon, \mathbf{X}) + \sum_{k=1} \alpha^k g_l^{(k)}(\epsilon, \mathbf{X})). \quad (48)$$

The expansion $\sum_{k=1} \alpha^k g_l^{(k)}(\epsilon, \mathbf{X})$ already does not contain terms $(\alpha/\epsilon)^k$ and function $F_l = f_l^{(0)}(\epsilon, \mathbf{X})(g_l(\alpha/\epsilon, \mathbf{X})$ gives a correct first term for $\alpha \to 0$ with the arbitrary parameter ϵ including point $\epsilon = 0$. This function was calculated analytically by Bogovalov (1992). The corrected solution of the problem for the limit $\alpha \to 0$ with arbitrary ϵ coincides with the solution (43-45) with the exception of the toroidal magnetic field. It is replaced by

$$H_\varphi = -(\frac{r\Omega \sin\theta}{v_0})H_r. \quad (49)$$

According to this solution the plasma moves radially in the split-monopole poloidal magnetic field with constant poloidal velocity and without any toroidal motion. At large distances the collimation of the plasma to the axis of rotation and the corresponding acceleration of the plasma are also very weak (Bogovalov & Tsinganos 1999). It is reasonable to call all rotators with $\alpha < 1$ slow rotators, and all rotators with $\alpha > 1$ fast rotators, independent of the value of ϵ.

Let us consider the physical sense of the parameters α and ϵ. It is clear that ϵ is the ratio of the initial radius of the fast mode surface over the light cylinder radius. On the other hand

$$\epsilon^2 = \sigma_0 = \frac{H_0^2}{4\pi n_0 mc^2 \gamma_0}(\frac{R_* \Omega}{v_0})^2. \quad (50)$$

For the slow rotators σ_0 is the ratio of the Poynting flux over the matter energy flux (kinetic energy flux for the relativistic plasma) at the equator and coincides with the well known magnetization parameter (Arons 1996) for the slow rotators. It is easy to show that

$$\alpha^2 = \frac{\sigma_0}{U_0^2}. \quad (51)$$

Therefore the slow rotators correspond to the condition $\frac{\sigma_0}{U_0^2} \ll 1$.

The diagram in Fig. 5 shows different regimes of the flow from rotators with the initial split-monopole magnetic field in coordinates σ_0 and U_0. The line $\sigma_0 = 1$ divides the rotators with the Poynting flux dominated flow near the surface of the star (above this line) from the rotators with the matter energy dominated flow (below this line). The slow rotators are located below the line $\sigma_0 = U_0^2$ while the fast rotators are located above this line. In the wide range of the parameters, the regimes of the axisymmetric flows have been already investigated. Now it is evident that the obtained solutions are valid for the oblique

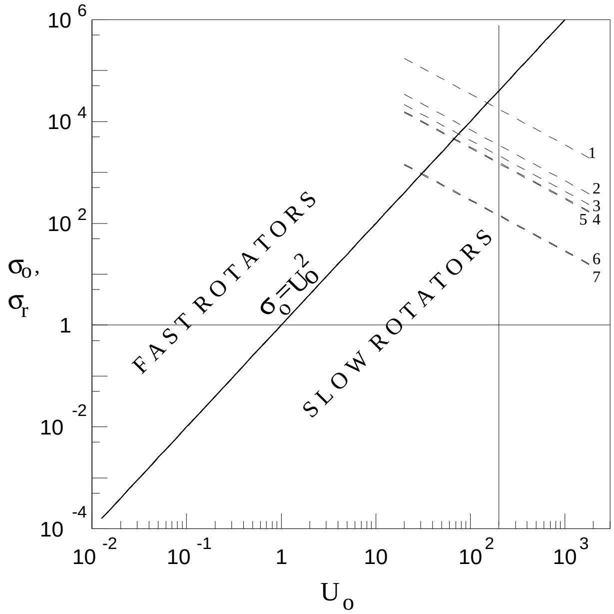

Fig. 5. Regimes of the cold plasma flows. The locations of 6 pulsars emitting observable gamma-rays above 100 MeV: Crab -1, PSR 1508-59 - 2, Vela - 3, PSR 1706-44 - 4, PSR 1951+32 - 5, Geminga -6 and PSR 1055-52 - 7 are shown by dashed lines for all possible values of U_0. All the pulsars are located in the region of slow rotators ejecting the Poynting flux dominated wind provided that $U_0 > 200$.

rotators also. For the slow rotators with the strong matter energy dominated flow, ($\sigma_0 \to 0$) an approximate analytical solution with accuracy on the order α^2 was obtained by Bogovalov (1992). With the increase of σ_0 at constant U_0, rotators with the nonrelativistic flow firstly cross the line $\sigma_0 = U_0^2$ and become fast rotators. Numerical solutions for the nonrelativistic plasma flows ($\sigma_0 \to 0$) from fast rotators were obtained up to $\alpha = 4.5$ (Bogovalov & Tsinganos 1999). It was found that due to the strong collimation of the plasma to the axis of rotation, the flow becomes subsonic in some region along the axis of rotation and therefore should be absolutely unstable well before the rotator reaches the line $\sigma_0 = 1$ (see also Lyubarskii (1992) and Begelman (1998)).

The rotators with the initial relativistic plasma flow first cross the line $\sigma_0 = 1$ with the increase of σ_0 at constant U_0 and became the slow rotators ejecting the Poynting flux dominated wind. With the increase of σ they also cross the line $\sigma = U_0^2$ and become the fast rotators. The relativistic stationary plasma flow from the fast rotators was investigated recently by Beskin et al. (1998). Surprisingly, the authors found that the perturbation of the poloidal magnetic field go to zero as $\sigma_0/U_0^2 \to 0$, and acceleration of the plasma remains very ineffective (like in the model by Michel 1969 with the prescribed monopole-like poloidal magnetic field). The Lorentz-factor of the plasma reaches the value $\gamma_\infty \sim (\sigma_0\gamma_0)^{1/3}$.

The plasma flow from the slow oblique rotator becomes especially simple. In particular, the function η for the slow rotators ($\alpha \ll 1$) is

$$\eta(r,\theta,\varphi,t) = D(\sin\chi\sin\theta\sin(\varphi - \Omega(t - r/v_0)) + \cos\theta\cos\chi). \quad (52)$$

Rotational losses of the slow rotators with the initial split-monopole magnetic field are defined by the total Poynting flux through the surface of the star

$$\dot{E}_{rot} = \frac{2}{3}\frac{H_0^2 R_*^4 \Omega^2}{v_0}, \quad (53)$$

and they do not depend on the angle χ. It is useful also to express the rotational losses through the total magnetic flux ψ of the open field lines of one direction (for example leaving the star).

$$\dot{E}_{rot} = \frac{1}{6\pi^2}\frac{\psi^2\Omega^2}{v_0}, \quad (54)$$

where $\psi = 2\pi H_0 R_*^2$ for the split-monopole magnetic field.

5. Implications for radio pulsars

How the model considered above is connected with the real radio pulsars? It is reasonable to assume that the rotator with the split-monopole magnetic field is equivalent to a real radio pulsar if it provides the plasma flow similar to the plasma flow from the real pulsar at distances larger than the light cylinder. Therefore the equivalent split-monopole rotator should have the same matter flux, the same poloidal magnetic field flux from one of the poles, the same γ_0, v_0, Ω, and the same inclination angle χ as the real pulsar.

The magnetic flux from one of the poles of the split-monopole rotator is $\psi = 2\pi R_*^2 H_0$. The magnetic flux of the open field lines from the radio pulsar with a dipole magnetic field is defined by the radius of the polar cap $R_p = R_*\sqrt{\frac{R_*\Omega}{c}}$ (Goldreich & Julian 1969). Therefore, the flux of the open field lines from one of the polar caps of the radio pulsar is estimated as

$$\psi = \pi H_0 R_*^2 (\frac{R_*\Omega}{c}). \quad (55)$$

The ratio of the poloidal magnetic field over the matter flux density H_p/nv_p is constant on the field line. Therefore this ratio for the split-monopole rotator and for the radio pulsar should be the same. These conditions allow one to define all parameters. The split-monopole rotator, equivalent to the pulsar with the magnetic field on the polar cap H_0, angular velocity Ω and ejecting plasma with initial density n_0 and γ_0, has the magnetization parameter

$$\sigma_r = \frac{H_0^2}{8\pi n_0 mc^2\gamma_0}(\frac{R_*\Omega}{c})^3. \quad (56)$$

Below we consider the relativistic plasma with $v_0 = c$. The rotational losses of this object are

$$\dot{E}_{rot} = \frac{1}{6}\frac{H_0^2 R_*^6\Omega^4}{c^3} \quad (57)$$

These rotational losses equal the losses of the dipole rotating in vacuum at the angle of inclination $\chi = 30^0$.

The initial density of the plasma n_0 is defined by the processes of multiplication of plasma in the electromagnetic cascades initiated by primary particles with the Lorentz-factor $\gamma_{gap} \sim 2 \cdot 10^7$ in the magnetosphere of the pulsar. In the inner gap theories, (Ruderman & Sutherland 1975; Arons 1981) the primary beam has Goldreich-Julian density

$$n_{GJ} \sim \frac{\Omega H_0}{2\pi ec}, \qquad (58)$$

We neglect here the dependence of n_{GJ} on the inclination angle. The primary particles produce λ secondary electrons and positrons. Therefore the initial density of the plasma is $n_0 = \lambda n_{GJ}$. Calculations of cascades performed by Daugherty & Harding (1982) and by Gurevich & Istomin (1985) show that $\lambda \sim 10^4$. The outer gap model (Cheng & Ruderman 1986; Romani 1996) gives similar values of n_0. After substitution of (58) in (56), σ_r takes a form

$$\sigma_r = \frac{eH_0}{4\lambda mc\Omega\gamma_0}(\frac{R_*\Omega}{c})^3. \qquad (59)$$

The estimates of γ_0 are less definite. The flow of the plasma formed in the electromagnetic cascade consists of two components. One of them is the beam of the primary particles which can lose a remarkable amount of energy. Another component is the plasma of the secondary particles. It is reasonable to take the average Lorentz-factor of the secondary particles as γ_0 because it is this plasma which screens the electric field and ensures the frozen-in condition. The average Lorentz-factor of the secondary particles strongly depends on the model of the gap. This parameter lies in the range $20 < \gamma_0 < \gamma_{gap}/\lambda$. To be definite, we assume here that

$$\gamma_0 = 0.1\frac{\gamma_{gap}}{\lambda} \sim 200. \qquad (60)$$

This value agrees closely with the results of the cascade simulations performed by Daugherty & Harding (1982). For other γ_0's the magnetization parameter is scaled as $\sigma_r = \sigma_{200}(200/\gamma_0)$. The location of all radio pulsars observed by EGRET in gamma-rays above 100 MeV (Crab, Vela, Geminga, PSR 1706-44, PSR 1509-58, PSR 1951+32 and PSR 1055-55; Thompson 1998) in the parameter space σ_r, U_0 for all possible U_0 is shown in Fig. 5 by dashed lines. For the estimates of the magnetic field H_0, Eq. (57) was used. It was assumed also that the momentum of inertia of all pulsars is $10^{45}\mathrm{g}\cdot\mathrm{cm}^2$. It is seen that all the pulsars appear as slow rotators provided that $U_0 \geq 200$. This implies that they do not accelerate plasma and do not collimate it considerably to the axis of rotation beyond the light cylinder if the flow of the plasma is dissipativeless (Bogovalov 1998; Bogovalov & Tsinganos 1999).

6. Conclusion

This paper clearly demonstrates that the oblique rotators do not differ strongly from the axisymmetric rotators. In the particular case considered in this paper, the dynamics of the plasma ejected by the oblique rotator is exactly the same as the plasma dynamics of the wind from the axisymmetric rotator. It allows us to apply all results obtained for the cold axisymmetric plasma flows to the oblique rotators. But these results show that for parameters typical of radio pulsars the acceleration of the relativistic plasma is not effective. Apparently radio pulsars are basically slow rotators. The obliqueness of the rotators does not change this conclusion.

Our results indicate that only violation of the ideal MHD can provide effective acceleration of the winds from the radio pulsars since neither the magneto-centrifugal mechanism nor the acceleration by the gradient of the toroidal magnetic field at the collimation of the plasma to the axis of rotation are effective for the pulsars. The role of the dissipative processes in the relativistic winds has been under discussion for many years (Coroniti 1990, Michel 1994, Mestel & Shibata 1994). Recently Melatos & Melrose (1996) tried to argue that the violation of the frozen-in condition takes place in the wind from an oblique rotator and it can provide the acceleration of the wind. Our self-consistent solution of the problem for the particular case of the oblique rotator with the initial split-monopole magnetic field shows no evidence of the violation of the ideal MHD in the wind.

Mestel & Shibata (1994) argued that the frozen-in condition can be violated right after the light cylinder due to the formation of singularities in the stationary axisymmetric flow. This assumption is not confirmed by Bogovalov's (1997) and by Contopoulos et al. (1999) latest results.

It follows from all data obtained recently in the physics of relativistic plasma outflows that only dissipative processes can ensure plasma acceleration under pulsar conditions. One of these mechanisms was proposed by Coroniti (1990) and Michel (1994). They argued that, due to geometrical reasons, the dissipative processes can ensure the transformation of the Poynting flux into the kinetic energy of the plasma, provided that the inclination angle is high enough ($\sim 80°$). The Poynting flux is transformed into the thermal energy of the plasma which is than accelerated by the thermal pressure gradient.

Another possibility is the Poynting flux transformation in the volume of the flow. This can happen if the Poynting flux dominated plasma flow is unstable in relation to MHD perturbations. The anomalously low conductivity due to the MHD turbulence would provide the violation of the ideality in the flow with the corresponding transformation of the Poynting flux into the energy of the plasma. The advantage of this approach is that it can provide effective acceleration even with the axisymmetric rotation. But the question about the stability of the Poynting flux dominated plasma flow is still open and should be investigated in detail separately.

Acknowledgements. The author acknowledges discussions with colleagues from the Astrophysics group of the MPI für Kernphysik and Dr. Yu.E.Lyubarsky. The author also thanks Dr. F.A.Aharonian for reading the manuscript and useful comments, as well as the MPI für Kernphysik for their warm hospitality and support given during the work on the paper. This work is partially supported by the Ministry of Education of Russia in framework of the program "Universities of Russia - basic research", project N 897. The author is also grateful to unknown referee for useful remarks on the paper.

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