

Efficiency of soft X-ray radiation reprocessing in supersoft X-ray sources

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Abstract. We evaluate the reprocessing efficiency for soft X-rays in irradiated accretion discs of supersoft X-ray sources. Our investigation is based on the assumption that the structure of the atmosphere of the flat inner accretion disk and the elevated disk rim is close to that of a stellar atmosphere. We have constructed an analytical two-color X-ray irradiated model atmosphere of the accretion disc and consider its main features. We compute X-ray irradiated stellar model atmospheres using the modified Kurucz code ATLAS5. We obtained an analytical expression for the X-ray albedo A_x . We show that spectral line acceleration is larger than gravity in surface layers of X-ray irradiated model atmospheres.

We found that the reprocessing efficiency for soft X-rays is rather small due to the fact that the soft X-rays are absorbed high in the atmosphere and cannot penetrate to the visual light formation depth. This results contradicts the observation of supersoft sources as very bright blue objects. Since the disc rim is the only possible source of this visual light we suspect that the blobby structure of the elevated rim and the spray area, both connected with the impinging accretion stream, might be the reason for this discrepancy.

Key words: accretion, accretion disks – radiative transfer – stars: novae, cataclysmic variables – X-rays: stars

1. Introduction

The luminous X-ray sources with extremely soft spectra are known as supersoft sources (SSS) (see Hasinger 1994 and Kahabka & van den Heuvel 1997 for reviews). First objects of this class, CAL83 and CAL87, were discovered by the *Einstein* X-ray observatory in the Large Magellanic Cloud (Long et al. 1981), but most sources were found and investigated with ROSAT (Hasinger 1994). Now 30 to 40 sources are known (Greiner 1996), most sources in the Magellanic Clouds and the Andromeda Nebulae. SSS are characterized by the very high luminosity ($\approx 10^{37} - 10^{38}$ erg/s) and the very soft spectra (most of the X-ray emission below 0.5 keV). In the now established model of van den Heuvel et al. (1992) the white dwarf primary

star accretes matter at a high rate ($\sim 10^{-7} M_{\odot}/yr$) and steady state nuclear burning on the surface causes the supersoft radiation. The modelling of the *BeppoSAX* observations for CAL87 (Parmar et al. 1997) agrees with nuclear burning at the white dwarf surface. Also the modelling of the X-ray spectrum by Hartmann & Heise (1997) shows that it corresponds to that of a hot white dwarf atmospheres.

Investigations of supersoft X-ray sources in the optical band of the spectrum revealed the binary nature of these systems. Periods found range from hours to days (Cowley et al. 1990, Pakull et al. 1988, Greiner 1996.) Light curve modelling of the eclipsing source CAL87 (Schandl et al. 1997) based on the investigation of van den Heuvel et al. (1992) has led to the conclusion that the accretion disc in this system has an elevated rim at the outer edge and that most visual light must originate from X-rays reprocessed at this rim. The geometrically thickening of the outer edge near the impact point is caused by the matter overflowing from the companion star, impinging on the disc. Such a rim structure seems to exist in all SSS (Meyer-Hofmeister et al. 1997, hereafter MH97). An elevated outer disc rim was also found for low mass X-ray binary (LMXB) systems (White et al. 1995). Reprocessing of X-rays to optical light in LMXB was discussed by van Paradijs (1981). But the elevated rim structure is more pronounced in SSS due to the higher mass accretion rate in these systems.

The importance of X-ray heating in accretion discs was pointed out earlier (Shakura & Sunyaev 1973, Meyer & Meyer-Hofmeister 1982). Up to now the light curve modelling of the sources CAL87, RX J0513.9 and RX J0019.8 was performed only using a blackbody approximation for the irradiated disc and the rim (disc structure consistently determined with the irradiation, reprocessing efficiency η assumed equal to 0.5 (MH97)). For a detailed investigation of the X-ray heating one has to take into account a number of physical processes, including radiation transfer for ionisation and thermal structure of the accretion disc atmospheres. The general problem demands non-LTE models with line-blanketing. Even stellar atmospheres without X-ray irradiation are not yet modelled in sufficient detail with non-LTE and line-blanketing (Hubeny & Lanz, 1995). Better knowledge of the reprocessing would allow a more adequate light curve fitting and would provide constraints for the investigation of the nature of the disc rim.

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The aim of the present work is the investigation of the effect of soft X-ray heating on the atmospheric structure of the accretion disc and the disc rim and the evaluation of the reprocessing efficiency. The soft X-ray irradiation can not significantly change the emergent optical flux of a stellar or accretion disc atmosphere. The reason is the following. The opacity in the soft X-ray part of the spectrum is larger than in the optical part and the soft X-ray flux can not penetrate to optical flux formation depth. Therefore the reprocessing efficiency is low.

In Sect. 2 we discuss different approaches to study the irradiated atmospheres. In Sect. 3 we develop a simple analytical two-color model for the temperature structure of the irradiated accretion disk, investigate main features of this model and compare with stellar atmospheres. In Sect. 4 we describe the method used for the X-ray irradiated LTE model stellar atmosphere calculations with frequency and depth dependent opacities and show our results. In Sect. 5 we discuss the reprocessing efficiency. In Sect. 6 conclusions are given.

2. Methods for the modelling of irradiated accretion disc atmospheres

At present two different approaches are used for modelling X-ray heated atmospheres. The first is known as “nebular” approximation. Ionisation, recombination, Auger effect, charge-exchange and Compton processes are treated well, but the radiation transfer problem is solved in an escape probability approximation (Raymond 1993, Ko & Kallman 1994). The resulting uncertainty is about a factor of two (Dumont & Collin-Souffrin, 1985) as compared with non-LTE calculations for classical stellar atmospheres. The second way is connected with model stellar atmosphere calculations. The radiation transfer is considered explicitly, but often only the LTE-approximation is used (Barret & Smith 1993, Sakhbullin & Shimansky 1996a, 1996b, 1997). If departure from LTE is included the cooling by spectral lines is not taken into account (Milgrom & Salpeter 1975, London et al. 1981). Sincell & Krolik (1997) have considered an X-ray heated skin layer between hot corona and cool accretion disc using stellar atmosphere methods too in LTE and without spectral lines.

The first kind of model gives the temperature structure of the Compton heated corona and transition zone together with extreme ultraviolet line and continuum fluxes, but does not consider the temperature structure of the photosphere, where the optical spectrum originates. Therefore it is necessary to use the model atmosphere calculations for the investigation of the visual flux emerging from SSS.

We use the LTE approximation in our investigations. As already pointed out the opacities are important. Can a departure from LTE change the relation between opacities in soft X-ray and optical parts of the spectrum? This is possible, if matter is fully ionized due to outside irradiation. In this case the thermal balance is determined by Compton processes (Compton heated corona). But in a partially ionized medium the soft X-ray opacity is larger than the optical one, even in non-LTE, since ionization

thresholds of the important ions are located in the ultraviolet and X-ray spectral regions.

It is known that in non-LTE the ionization balance is determined by statistical equilibrium equations. Four processes are important: collisional ionization and recombination, and photoionization and photorecombination. In the outer parts of the atmosphere, where the density is low ($n_e \leq 10^{12} \text{ cm}^{-3}$) we can neglect the collisional processes, the approach is called “nebular approximation”. In this case the ionization balance depends only on the ratio of photoionization and photorecombination rates. This relation, expressed in terms of X-ray flux and gas pressure is the “ionization parameter”. So, if an ionization parameter is larger than unity and a density is small enough a Compton heated corona exists (see Krolik et al., 1981).

The LTE assumption is valid, when the collisional rates are higher than the radiative rates (electrons have a Maxwellian velocity distribution). The collisional rates are higher than the radiative rates if $n_e \geq 10^{13} - 10^{14} \text{ cm}^{-3}$. Therefore strong non-LTE effects can not occur in deep layers. In the stellar atmosphere the density increases with depth very quickly and the corona can not have a significant optical depth. The emergent optical flux is formed in deeper partially ionized layers. The opacity of the soft X-ray spectral region is larger there than in the optical, and our main conclusion about reprocessing efficiency should be valid.

We investigate in the first approximation the non-LTE effect for the Compton corona and we demonstrate that this corona can not change our result. If we would use the nebular approximation we would obtain more accurate values for the temperature distribution in the corona and transition zone between corona and cool atmosphere and the emergent far ultraviolet flux. But our aim is different. We want to obtain the emergent optical flux, which can not be found from the “nebular approximation”. So, we prefer to use the model atmosphere method in LTE and we expect our conclusions for the reprocessing efficiency for soft X-rays should be qualitatively right.

3. Simple models of X-ray heated accretion disc atmosphere

As a first attempt to study the effects of irradiation we take a simple two-color analytical model for the vertical structure of the X-ray heated accretion disc. “Two-color” means that we consider the X-ray band and the optical/ultraviolet band of the spectrum separately and assume that opacities do not depend on the frequency in each band (“two-color” models are in the stellar atmospheres literature also known as “multi-grey” or “two-step grey” models). Such an approach was suggested by Sakhbullin & Shimansky (1996a) for X-ray irradiated stellar atmospheres, but only with pure absorption, without scattering in both bands. The temperature structure of the X-ray heated accretion disc as function of z (vertical height above midplane of a disc) is determined by the radiation transfer equations in these two bands and energy conservation, and energy generation laws. We use the two-stream approximation (see Popham & Narayan 1995)

which leads to two moments of the radiation transfer equation (Mihalas, 1978) for each color:

$$\frac{dH}{dm} = k_a(J - B) \quad (1)$$

$$\frac{dJ}{dm} = 3(k_a + \sigma)H \quad (2)$$

$$\frac{dH_x}{dm} = k_x J \quad (3)$$

$$\frac{dJ_x}{dm} = \frac{1}{\mu_0^2}(k_x + \sigma)H_x \quad (4)$$

where m [g/cm²] is the column mass

$$\frac{dm}{dz} = -\rho, \quad (5)$$

H and J are Eddington flux and mean intensity in the optical part, H_x and J_x are the same values for X-rays, B is the integral Planck function, k_a and k_x are the absorption opacity in optical and X-ray bands and σ is the electron scattering. We do not consider Compton processes here, but this effect will be discussed below.

The optical radiation is considered in two rays with $\mu = \cos \theta = \pm 1/\sqrt{3}$ and the X-ray radiation in the rays with $\pm \mu_0$, where μ_0 is the incident angle of the X-ray flux.

We consider here the accretion disc illuminated by a parallel beam of X-rays with specific intensity I_x^0 under the incident angle μ_0 without an X-ray source in the disc.

Outer regions of accretion discs in SSS (irradiation is important for outer region of accretion discs) have not significant intrinsic X-ray radiation, therefore we neglect the thermal emissivity of the accretion disc gas in the X-ray spectral band in this simple analytical model, but we take it into account in the numerical models (see next section).

We solve Eqs. (1–4) under the simplifying assumption that the opacity does not depend on the column mass. We take the energy conservation law in the form

$$H(m) + H_x(m) = H_0(m). \quad (6)$$

The energy generation rate due to viscous heating in the disc we take from Shakura & Sunyaev (1973)

$$\frac{dH_0(m)}{dm} = -\frac{2H_0(0)}{\Sigma} \quad (7)$$

where Σ is the surface density and $H_0(0) \equiv H_0$ is the integral Eddington flux radiated from disc surface at a given radius.

The solution of Eqs. (1–4) and (6–7) with corresponding boundary conditions at the surface and at the midplane (see Appendix) yields:

$$B = 3\tau H_0 \left(1 - \frac{m}{\Sigma}\right) + 2\frac{H_0}{\Sigma k_a} + \sqrt{3}H_0 + \sqrt{3}H_x^0(1 - A_x) + \frac{k_x}{k_a}J_x + 3\mu_0^2 \frac{k_a + \sigma}{k_x + \sigma} (J_x(0) - J_x) \quad (8)$$

where $H_x^0 = \mu_0 I_x^0/2$ is the incident X-ray flux and A_x is the X-ray albedo (fraction of X-ray flux reflected back due to scattering without thermalisation (see Appendix)). We introduce an optical depth $\tau = (k_a + \sigma)m$. A stellar atmosphere is usually considered as a semi-infinite atmosphere. So we can obtain the temperature structure of an X-ray irradiated stellar atmosphere from the X-ray irradiated accretion disc taking $\Sigma \rightarrow \infty$.

$$B = 3\tau H_0 + \sqrt{3}H_0 + \sqrt{3}H_x^0(1 - A_x) + \frac{k_x}{k_a}J_x + 3\mu_0^2 \frac{k_a + \sigma}{k_x + \sigma} (J_x(0) - J_x) \quad (9)$$

The first three terms in (8) determine the temperature structure of the accretion disc without irradiation (compare with Laor & Netzer (1989), Hubeny (1990)). Note that $B = \sigma T^4/\pi$ and $H_0 = \sigma T_{\text{eff}}^4/4\pi$.

The last three terms in (8) and (9) arise from X-ray heating. The first means that the boundary condition on the surface is changed, the second is the direct heating due to X-ray absorption and the third is the reradiation of the absorbed X-ray flux in the optical band (see Sakhibullin & Shimansky, 1996a). Direct heating plays the most important role and leads to a chromospheric layer at $\tau_x \sim 1$.

Eq. (8) shows that X-ray heating can change the temperature stratification of the disc only if $H_x^0(1 - A_x) \sim \tau_0 H_0(0)$, where τ_0 is an optical half-thickness of the disc, or if the disc has an X-ray optical half-thickness $\tau_x^0 \sim 1$ (see also Ko & Kallman 1991).

The outer regions of the discs in SSS are important for the radiation. There we have $\Sigma \approx 10^3$ g/cm² and $k_a \approx 10^2$ cm²/g, which means $\tau_0 \sim 10^5$. $R_x = H_x^0/H_0$ is only about a few tenths in the flat part, but at the disc rim the X-ray flux is much stronger than the flux caused intrinsically by viscosity (R_x equal several tens). For soft X-rays ($kT \approx 100$ eV) $\tau_x \approx 1$ is reached for $m \leq 10^{-2}$ g/cm² (see Sect. 4). Therefore the soft X-ray irradiation cannot change the temperature (and the scaleheight z_0) essentially at the midplane of accretion discs with rims in SSS. A more realistic consideration with real opacities taken into account shows that soft X-ray irradiation cannot change the temperature structure at the midplane of accretion discs in SSS (see Sect. 5).

Let us consider the differences of the temperature stratification between stellar atmosphere and disc atmosphere. In the disc the temperature grows more slowly and, in addition, a corona (viscously heated) can exist in the upper layers of the atmosphere when $k_a \approx 1/\Sigma$. In the case of SSS where Σ is high due to the high mass flow rate in the disc, these differences are small at the photosphere of the disc ($\tau \sim 1$ at $m < 1$ g/cm²) (see also Suleimanov, 1992). Therefore we can expect that accretion disc atmospheres of SSS in optical light locally radiate like stellar atmospheres with the same effective temperature, surface gravity and incident X-ray flux. In the next section we consider the properties of such stellar atmospheres in more details.

4. Model of an X-ray heated stellar atmosphere

In this section we compute numerical X-ray heated model stellar atmospheres with frequency and depth dependent opacity and investigate features depending on the X-ray hardness.

4.1. Method of calculation

For our calculation of X-ray heated stellar atmospheres we consider the optical/ultraviolet (called stellar from now on) band and the X-ray band of the spectrum separately and construct model stellar atmospheres with an additional source of heating due to X-ray irradiation. For the dividing energy $h\nu_b$ between stellar and X-ray bands 0.1 keV was taken corresponding to approximately 120 Å.

An X-ray heated model stellar atmosphere is characterised by the following parameters: effective temperature T_{eff} , surface gravity g , chemical composition A (solar $A = 1$), ratio R_x of outer X-ray flux H_x^0 to stellar flux $H_0 = \sigma T_{\text{eff}}^4 / 4\pi$, incidence angle of the X-rays $\mu_0 = \cos \theta_0$ and the X-ray spectrum.

For quantitative results differential equations which describe hydrostatic equilibrium, radiative equilibrium, radiation transfer, statistical equilibrium, charge conservation and particle number conservation have to be solved.

Hydrostatic equilibrium is defined as follows

$$\frac{dP_g}{dm} = g - \frac{4\pi}{c} \int_0^{\nu_b} (k_\nu + \sigma_\nu) H_\nu d\nu - \frac{4\pi}{c} \int_{\nu_b}^{\infty} (k_\nu^x + \sigma_\nu^x) H_\nu^x d\nu_x, \quad (10)$$

and radiative equilibrium as

$$\int_0^{\nu_b} k_\nu B_\nu d\nu = \int_0^{\nu_b} k_\nu J_\nu d\nu + \int_{\nu_b}^{\infty} k_\nu^x J_\nu^x d\nu_x, \quad (11)$$

At the LTE assumption atomic level populations and the ionization balance are determined from Boltzman and Saha equations. The Eddington flux H_ν and the mean intensity J_ν are found from solving the radiation transfer equation (RTE):

$$\mu \frac{dI_\nu}{dm} = (k_\nu + \sigma_\nu)(I_\nu - S_\nu), \quad (12)$$

where $S_\nu = \epsilon B_\nu + (1 - \epsilon) J_\nu$ is a source function, $\epsilon = k_\nu / (k_\nu + \sigma_\nu)$, and I_ν is the specific intensity. Flux and mean intensity are determined from I_ν as the zero and first moments (Mihalas, 1978).

We have used the modified Kurucz code ATLAS5 (Kurucz, 1970). In this code additional opacities were taken into consideration (Suleimanov, 1991). Opacities in the X-ray band of the spectrum were taken from Verner & Yakovlev (1995). About 500 of the strongest spectral lines of the most abundant chemical elements (Sakhibullin & Shimansky, 1997) were included. We used about 3500 frequency points and 100 depth points for the calculation of $I_\nu(m)$ and to solve the radiation and hydrostatic equilibrium equations. The radiation transfer equation was solved by two different methods for the stellar and the X-ray band of the spectrum. In the stellar band the RTE was solved

by the Hermitian method (Auer, 1976) for three angles μ . For the X-ray radiation field the RTE was solved using the Feautrier method for the two rays only with angles $\pm \mu_0$ (Mihalas, 1978). Such an approach gives good results if the isotropic component of X-ray radiation is small compared to the incident X-ray beam specific intensity. This is the case in our calculation because there is no intrinsic X-ray flux of the stellar atmosphere and the scattering is less than the absorption for soft X-rays. The diffusion approximation $J_\nu = B_\nu$ was used as lower boundary condition for the RTE. The upper boundary condition is different for the stellar and the X-ray band. The lack of the incoming specific intensity gives the upper boundary condition for the stellar band, $I_\nu(\mu < 0) = 0$, whereas the incident specific intensity for the X-ray band is defined from the outer X-ray flux

$$I_\nu^x(-\mu_0) = \frac{2H_x^0}{\mu_0}. \quad (13)$$

The temperature structure was found by two temperature correction procedures, the integral Λ -iteration for the optically thin part of the atmosphere and the Avrett-Krook flux correction for the optically thick one (Kurucz, 1970), both related to the two different forms of the radiative equilibrium equation, equivalent to (11)

$$\frac{dH}{dm} = \frac{d}{dm} \int_0^{\nu_b} H_\nu d\nu + \frac{d}{dm} \int_{\nu_b}^{\infty} H_\nu^x d\nu = 0, \quad (14)$$

and

$$H = \int_0^{\nu_b} H_\nu d\nu + \int_{\nu_b}^{\infty} H_\nu^x d\nu = \frac{\sigma T_{\text{eff}}^4}{4\pi}. \quad (15)$$

Therefore the temperature distribution is estimated from the conditions

$$\frac{d}{dm} \int_0^{\nu_b} H_\nu d\nu = - \int_{\nu_b}^{\infty} k_\nu^x J_\nu^x d\nu_x, \quad (16)$$

and

$$\int_0^{\nu_b} H_\nu d\nu = \frac{\sigma T_{\text{eff}}^4}{4\pi} - \int_{\nu_b}^{\infty} H_\nu^x d\nu_x. \quad (17)$$

So, the flux integrated over the stellar band at each depth point in an X-ray heated atmosphere must be larger than in a non-irradiated atmosphere by the value of the flux integrated over the X-ray band at the same point. The X-ray flux is a negative, because it has an opposite direction compared with stellar flux. So, the flux integrated over the both bands at the each depth point is the same as in the non-irradiated model stellar atmosphere.

The scheme of our calculations is the following. We use a grey atmosphere (Eq. (9) without X-ray heating) as an initial temperature structure of a stellar model atmosphere and then solve the hydrostatic equilibrium equation (10) (without radiative acceleration) and compute the opacities at all the frequencies and depth points. Further we solve RTE (12) and find J_ν and H_ν at all depths and frequencies. In the next step we may find that radiative equilibrium (16) and (17) is not satisfied and calculate the flux error

$$\Delta H(m) = \left(\int_0^{\nu_b} H_\nu(m) d\nu + \int_{\nu_b}^{\infty} H_\nu^x(m) d\nu_x - H_0 \right) / H_0, \quad (18)$$

and the error in the flux derivative determined in a similar way. Then using these errors we find the new temperature distribution from the procedures of temperature correction. This scheme is repeated iteratively (further we include the radiative terms in the hydrostatic equilibrium equation) until the required accuracy is achieved. We perform this iterative scheme three times for each model constructed. First we find a model without spectral lines and without X-ray heating. In the second step we find a model with lines, but without X-ray heating using the first model as an initial one. Finally we find the model with spectral lines and X-ray heating taken into account. The flux errors in the models without X-ray irradiation are less than 1%, and depend on the X-ray spectrum and R_x for models with X-ray irradiation (see next subsection). In the outer layers of some models a radiation pressure exceeds the total pressure. In this case we take the gas pressure equal 0.1 of the total pressure to $P_g = 0.1P_{tot}$.

4.2. Results

Using this method X-ray heated model stellar atmospheres were calculated. For our examples we choose the parameters $T_{eff} = 17\,000\text{ K}$ and $\log g = 3.5$ as values relevant for the atmosphere of the accretion disc rim of the SSS CAL 87. In our calculations we used two different spectral distributions of the X-rays: (1) an optically thin plasma spectrum $F_\nu \sim \exp(-h\nu/kT_x)$ with $kT_x = 0.2\text{ keV}$ and (2) the same with $kT_x = 5\text{ keV}$. (1) corresponds to the X-ray spectra of supersoft X-ray sources and (2) to classical binary X-ray sources with a neutron star primary.

We computed models for the ratio R_x of the incident X-ray flux H_x^0 to the disc produced “stellar” flux H_0 varying from 0 to 1. This corresponds to flat disc irradiation (for larger R_x , corresponding to the rim structure, this iterative procedure does not converge). We took $\mu_0 = 0.4$ for the incident angle of X-ray flux simulating the situation at the disc rim surface of CAL 87. The effect of non-solar chemical abundance as in the Magellanic Clouds was also considered.

We present the main results of the calculations in Figs. 1 to 4. Figs. 1a and 1b show the temperature distributions for the model atmospheres, with and without X-ray heating. The main features are the same as for the simple analytical model, namely: a hot chromosphere-like layer and a higher temperature value near the temperature minimum. The comparison of our results with the temperature structure of the stellar atmosphere without irradiation from Kurucz (1993) (Fig. 1a), shows only a small difference between the two temperature distributions and proves that our model describes the structure sufficiently well. The difference is due to the larger number of lines taken by Kurucz. An accuracy of the calculated models depends on the value of R_x and the hardness of the X-ray spectrum. The maximum flux error is $\leq 0.3 R_x$ for the soft spectrum and $\leq 0.1 R_x$ for the hard one. These errors are negative in the hot chromospheric layer and positive in the vicinity of the temperature minimum. This means that our models underestimate the temperature of the hot surface skin and overestimate the value of the temperature at temperature minimum.

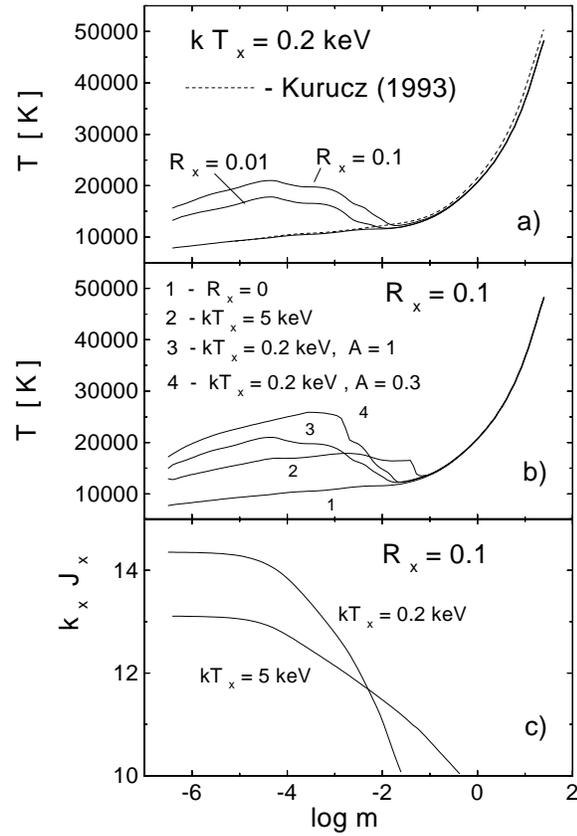


Fig. 1a–c. Distributions of temperature and X-ray heating function against column mass m ($[m] = \text{g/cm}^2$) for X-ray heated and non-irradiated model stellar atmospheres. (Here and later $T_{eff} = 17000\text{ K}$ and $\log g = 3.5$). **a** Temperature structures of our non-irradiated and irradiated ($R_x = 0.01$ and $R_x = 0.1$) atmospheres compared with Kurucz (1993). **b** Effect of X-ray hardness ($kT_x = 0.2$ and 5 keV) and chemical composition ($A=1$ and 0.3) on the temperature structure. **c** Effect of hardness on the X-ray heating function.

A higher X-ray flux (Fig. 1a with $R_x=0.01$ and 0.1) leads to a temperature increase in the hot layer, but only a small increase of the depth of the “hot skin”, a small change of the minimum temperature. The effect of the hardness of X-rays is shown in Fig. 1b. In Fig. 1c the dependence of the distribution of the X-ray heating $\int_{\nu_b}^{\infty} k_\nu^x J_\nu^x d\nu$ along depth on the X-ray spectrum is given. The harder X-rays ($kT_x = 5\text{ keV}$) cause a deeper hot layer, but its temperature is lower. This is due to the deeper penetration of hard X-rays into the atmosphere. In the Fig. 2a the dependence of penetration depth on the X-ray photon energy is shown. The penetration depth is defined via the column mass where the optical depth equals 1 (Sincell & Krolik, 1997). The column mass where visual light is formed is 0.5 g/cm^2 for our models. Photons with energy higher than 3 keV can penetrate to this depth and deeper. From Fig. 1b also the dependance on the chemical composition can be seen. A low amount of heavy elements, $A=0.3$, results in an significant increase of the hot layer temperature, but an only small growth of the layer depth. This is due to the fact that heating by X-rays occurs mostly via He absorption, but cooling is connected with the formation of

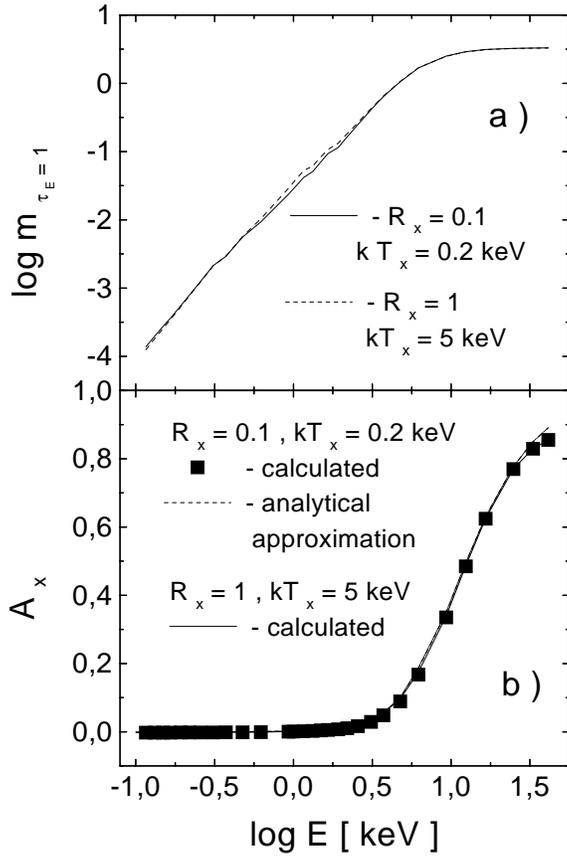


Fig. 2. **a** X-ray flux penetration depth as function of the photon energy. Penetration depth defined as column mass at optical depth equal 1 for given photon energy. **b** X-ray albedo for different hardness and strong irradiation as function of the photon energy.

spectral lines of ions of heavy elements (see Fig. 3b). The *He* abundance is constant, and if heavy element abundances are low, a higher temperature is necessary for the same cooling rate. Therefore we can conclude that a temperature structure of an optical flux formation depth depends weakly on heavy element abundances. In the following we will consider only models with solar abundance.

There is another reason for less effective X-ray heating by hard radiation, the net X-ray albedo A_x . In Fig. 2b we compare A_x computed by solving RTE in the X-ray band with values determined by the analytical approximation (A21) with absorption k_x and scattering σ taken at the optical depth 1. From Fig. 2b it is obvious that A_x is very close to zero for energies less than 3 keV, grows near linear with $\log E$ (E in keV) for a wide energy range up to 30 keV and is close to 0.9 for higher energies. The linear part can be expressed as

$$A_x = 0.85(\log E - 0.5). \quad (19)$$

Such a dependence of the X-ray albedo on the frequency is due to the corresponding dependence of the opacity on the frequency. This means that a significant part of the X-ray photons with $h\nu > 3$ keV are back scattered without thermalization. From Fig. 2 it is obvious that penetration depth and X-ray albedo does

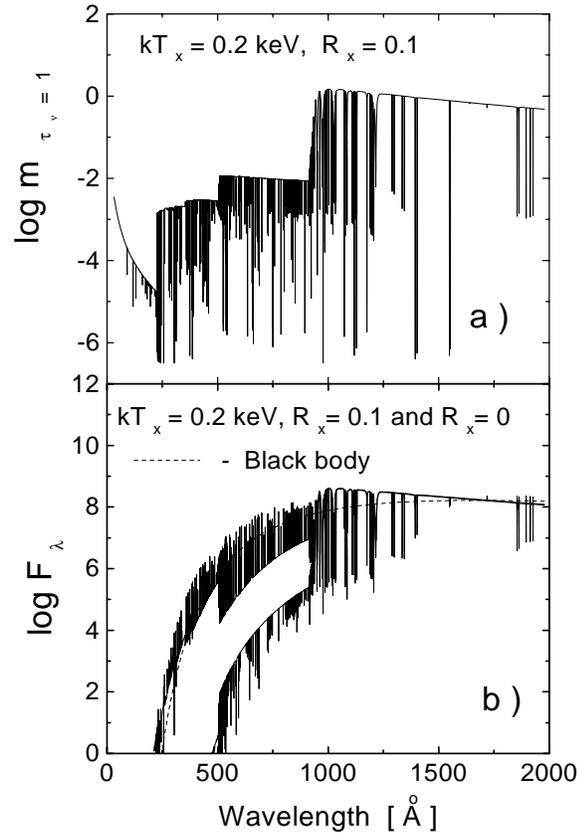


Fig. 3. **a** Wavelength dependence of flux formation depth with and without irradiation Flux formation depth is defined as column mass at optical depth equal 1. **b** Spectra of model stellar atmospheres with and without soft X-ray irradiation

not depend on R_x and the X-ray spectrum and that the analytical expression for A_x gives a good approximation. The spectra of irradiated and non-irradiated model stellar atmospheres are shown on Fig. 3b. The largest part of the radiation from the non-irradiated atmosphere is in the Balmer continuum. The Lyman discontinuity in the spectrum is large. The emergent flux in the Lyman continuum is weak and all lines are absorption lines. In case of the irradiated atmospheres most of the additional flux comes from the Lyman continuum of *H* and *HeI* and from heavy ion emission lines. The flux in the Balmer continuum is changed to a minor degree. These features of the spectra are connected with the opacity in the atmosphere. The opacity in the Balmer continuum is small and most of the intrinsic stellar flux originates from deep layers ($m \approx 1g/cm^2$) in this band (Fig. 3a). The Lyman continuum is more opaque and the radiation emerges from surface layers ($m = 10^{-3.0} - 10^{-2.0}g/cm^2$). Note that soft X-rays penetrate to the same depth and heat these layers (compare Fig. 2a and Fig. 1a). Therefore the soft X-ray flux cannot reach Balmer and optical light formation depth. In the next section we will discuss the reprocessing efficiency in more detail. But we point out one special feature here.

Radiative acceleration due to lines becomes more important than gravity in the surface layers of X-ray irradiated model atmospheres (Fig. 4), as noted by London et al. (1981). This line

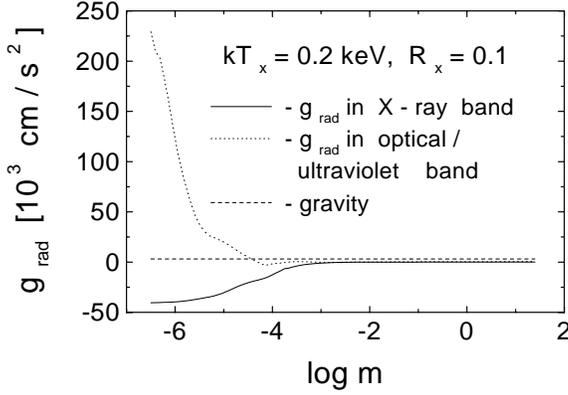


Fig. 4. Acceleration due to line radiation pressure and X-ray radiation pressure vs column mass for soft X-ray heated model atmospheres

radiation pressure may cause a wind from the surface of the irradiated atmosphere. This is different from a thermal wind due to Compton heating of the corona (Basko & Sunyaev, 1973). It is rather a wind similar to line-driven winds from OB stars. This needs further consideration.

5. The derivation of the reprocessing efficiency

In this section we evaluate which fraction of the soft X-ray flux is reradiated as visual light. We consider a qualitative picture based on the numerical results discussed in the previous section, which allows a first judgement of the reprocessing efficiency.

5.1. Conclusions from the two-color approximation and explicit models

Using the results of the simple two-color model we find for the reprocessing efficiency that more than half of the irradiation flux is thermalized (if $A_x=0$, compare Fig. 2b). But comparing with the numerical results in the last section we recognize that the simple two-color method does not provide results detailed enough. From the first three terms of Eq. (9) and changing to temperatures we have

$$T^4 = T_{\text{eff}}^4 \left(\frac{3}{4} \tau + \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} R_x (1 - A_x) \right). \quad (20)$$

The last term shows that about half of the incident flux produces a general rise of the temperature. This is due to the upper boundary condition, $H(0) = H_0 - H_x^0$. An emergent flux is formed at optical depths from 0.1 to 2, the effect of the surface layers on the flux is very weak. Therefore, this upper condition implies that all X-ray flux reaches the main stellar flux formation depth. But the calculated models in the previous section show that the shape of the X-ray spectrum determines the penetration depth because this depth depends on the X-ray photon energy (see Fig. 2a). Only X-ray photons with energy above 3 keV can reach the main stellar flux formation depth (compare Figs. 2a and 3a).

Therefore the reprocessing efficiency derived from the two-color model is in conflict with the explicit numerical models.

We consider the more realistic qualitative picture for a clearer illustration of the numerical model physics. We take into consideration that opacity in soft X-ray and far ultraviolet spectral bands are larger than opacity in hard X-ray and soft ultraviolet and optical bands.

This qualitative picture is following. The soft part of the X-ray flux ($h\nu < 2 \text{ keV}$) heats the outer layers of the atmosphere and causes a chromospheric layer. The soft X-ray flux is absorbed almost totally and reradiated in the far ultraviolet (“opaque”) part of the stellar spectrum and changes only weakly the intrinsic soft ultraviolet and optical stellar flux. The hard part of the X-ray flux ($h\nu > 2 \text{ keV}$), not reflected back due to scattering, heats the deeper atmospheric layers where the intrinsic flux is formed. This part of the X-ray flux is thermalized and changes the stellar fluxes at all frequencies.

Therefore the reprocessing efficiency to visual light can be estimated as follows:

$$\eta \approx (0.5-1)f, \quad (21)$$

where f is the relative part of absorbed hard X-ray flux:

$$f = \frac{\int_{2 \text{ keV}}^{\infty} H_{\nu}^x (1 - A_x(\nu_x)) d\nu_x}{\int_{2 \text{ keV}}^{\infty} H_{\nu}^x d\nu_x}. \quad (22)$$

It means that the reprocessing efficiency in stellar atmospheres irradiated by soft X-ray flux, typical for the flat disc in supersoft X-ray sources is very small.

5.2. Reprocessing for strong irradiation

To study the different reprocessing efficiencies we have computed irradiated stellar model atmospheres also for stronger irradiation. Such atmosphere models are needed for an investigation of the accretion disc rim in supersoft X-ray sources with powerful irradiation, but cannot be computed using the temperature correction method. We have used the method by Sakhbullin & Shimansky (1996b). The method is based on the two-color temperature distribution with three additional terms due to irradiation (Eq. 9). Each term is added with a separate scaling factor. These factors are found from conservation of the integral flux at each depth (Eq. 15). The stellar model atmosphere for our standard parameters ($T_{\text{eff}} = 17000 \text{ K}$, $\log g = 3.5$) was calculated with soft X-rays ($kT_x = 0.2 \text{ keV}$) but for strong irradiation ($R_x = 20$, $\mu = 0.4$). In Fig. 5a we show the resulting temperature distribution in the atmosphere with the temperature distribution of the Compton heated corona taken into account (see below). For the atmospheric structure of the elevated disc rim in the SSS CAL 87 these parameters are adequate. The intrinsic flux is negligible compared to the irradiation. In this computation the flux error is about 30% in the layers where the temperature gradient is large and is less than 10% otherwise. The calculated spectrum of this model shows (see Fig. 6a) that the additional flux mainly emerges at wavelengths below 912 \AA and that the additional visual flux is relatively small. The optical flux of the irradiated model is higher by factor of 1.5 compared to the non-irradiated model. This means the reprocessing efficiency is about 0.07 for the irradiation 20 times the

intrinsic stellar flux. This agrees with our qualitative results for a low efficiency of soft X-ray reprocessing.

We have also investigated the effect of Compton heating of the corona on the reprocessing efficiency. Such a corona may exist in the upper surface layers of strongly irradiated atmospheres where the ionization parameter $\xi = 4\pi H_x/cp_g$ is larger than 1–10 (Krolik et al. 1981).

The temperature of a plasma with only Compton heating and cooling is equal to the Compton temperature

$$kT_c \approx \frac{\langle h\nu \rangle}{4}. \quad (23)$$

For our standard parameters and $kT_x = 0.2$ keV and $R_x = 20$ this formula gives a coronal temperature of about 500000 K. Since plasma cooling by spectral lines of multiple ionized atoms is important at this temperature, this line cooling might cause a lower temperature. Therefore this estimate is probably an upper limit for the corona temperature.

For the following estimates we take $T_c = 500000$ K for the coronal temperature. We assume the corona exists above the depth at which the ionization parameter is equal 10 or $\log n_e = 13.5$. These values have been adopted by the following reasons. Krolik & Kallman (1984) have shown that the transition to a Compton heated corona occurs for soft X-ray spectra at a larger value of the ionization parameter. Further, collisional processes dominate the ionization equilibrium at $\log n_e > 13.5$.

The model with the corona is constructed by the above mentioned method with total pressure $P_{\text{tot}} = gm_c$, where m_c is a surface density of the corona, used as upper boundary condition for the hydrostatic equilibrium equation. The value of m_c was found iteratively.

The corona extends from the surface down to the depth of $m_c \sim 0.1$ g/cm² (see Fig. 5a). The Thompson optical depth of the corona is about 0.03 ($\tau_e \approx 0.03$) and the true absorption optical depth is about 10^{-5} . Therefore the corona provides only a small amount of additional optical flux and the reprocessing efficiency is not affected. A large extension of the corona or a thermal coronal wind are not considered.

The explicit model for the same standards parameters T_{eff} , $\log g$ and μ , but with $R_x = 1$ and $kT_x = 5$ keV has also been computed by the temperature correction method for the reprocessing efficiency for hard X-rays. The temperature distribution and the spectrum are shown in Figs. 5b and 6b. The optical flux for this model is higher by a factor of 1.5 compared to the non-irradiated atmosphere. Therefore the reprocessing efficiency is 0.5 in this case, close to the value calculated from Eq. (21).

5.3. The effect of surface structure on the reprocessing efficiency

From the modelling of light curves of supersoft X-ray sources it was found that the high amount of visual light apparently originates from a geometrically high accretion disc rim (MH97). The elevated rim is formed by the matter flowing over from the companion star and impinging on the outer disc. This spray area of only marginal optical thickness $\tau \approx 1$ has a structure different

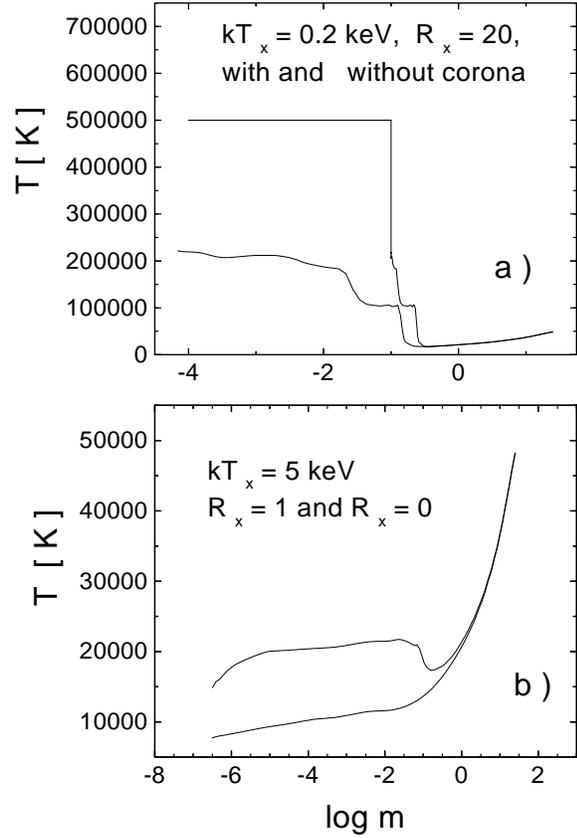


Fig. 5. **a** Temperature structure of model atmospheres, strongly irradiated by soft X-rays, with and without Compton heated corona. **b** Temperature structure for irradiation by hard X-rays, for comparison also without irradiation.

from that of an atmosphere, it might appear as an area consisting of blobs. Bulges at an accretion disc rim were already found earlier from observations of low mass X-ray binaries (Mason & Cordova 1982). These dips point to a clumpy structure of the disc bulge. This feature is even more pronounced in supersoft sources due to the higher mass overflow rate. The repeated scattering of X-rays in such a blobby area might lead to a higher reprocessing efficiency. The radiation transfer under such conditions will be investigated in further work.

6. Conclusions

In this work we have attempted to derive the efficiency of reprocessing of soft X-rays to visual light in accretion discs of supersoft X-ray sources using irradiated stellar atmosphere modelling. The following results have been obtained.

As a first attempt a simple analytical two-color model for an X-ray irradiated accretion disc atmosphere with electron scattering was considered. It has been shown that accretion disc atmospheres at the outer disc in supersoft X-ray binaries can be approximated by stellar model atmospheres with the same effective temperature and surface gravity.

Examples for irradiated stellar atmospheres with a ratio R_x of X-ray flux to intrinsic flux ($R_x \sim 0.01$ –1) have been com-

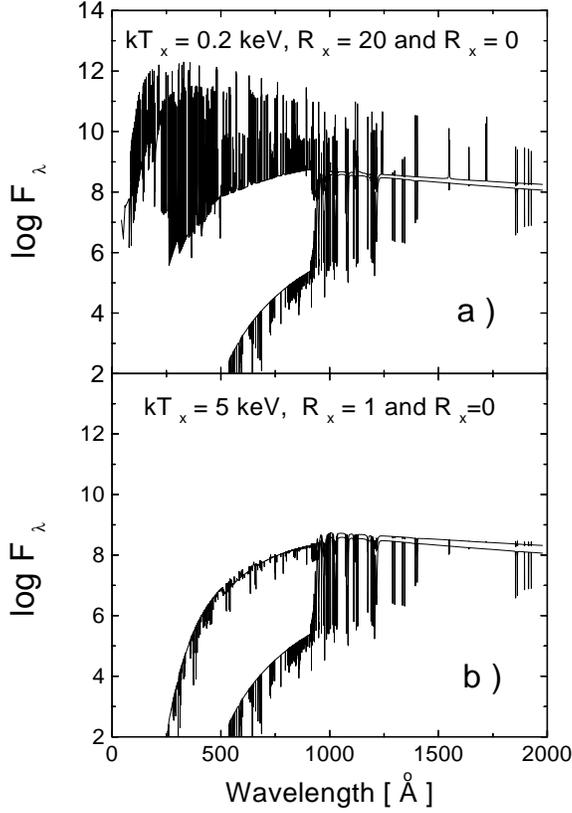


Fig. 6a and b. Spectra of model stellar atmospheres, irradiated and non-irradiated. **a** irradiation by soft X-rays, **b** by hard X-rays.

puted using the modified Kurucz code ATLAS5. Such values R_x are adequate for the flat part of the disc. The resulting efficiency is small, close to zero. The dependence of temperature stratification in the atmosphere and emergent spectra on the hardness of the X-ray flux and on the chemical composition have been investigated. It has been shown that softer X-ray irradiation leads to higher temperatures in the upper layers, less thick chromosphere-like layers, and lower temperature increase in the deeper layers compared to less soft X-rays. A lower abundance of heavy elements also leads to a temperature increase in the hot surface layer. It has been shown that the analytical relation for the X-ray albedo with frequency dependent opacity k_ν^x and scattering σ taken at $\tau_\nu^x = 1$ gives values close to the results of the numerical computations. The spectral line radiation pressure in X-ray irradiated atmospheres exceeds gravity and may initiate a line-driven wind.

For a simulation of the elevated disc rim one has to study much higher values of R_x since the disc flux caused by viscosity is negligible compared to the X-ray flux. We computed atmosphere models for strong irradiation, $R_x = 20$. The efficiency of soft X-ray reprocessing into visual light was found to be about $\eta \sim 0.07$ due to the fact that the soft X-rays cannot penetrate to the optical flux formation depth. The effect of Compton heating in the corona was studied. For the reprocessing efficiency for irradiation by hard X-rays ($kT_x = 5$ keV) a value of $\eta \approx 0.5$ was found. A simple relation for the reprocessing efficiency has

been suggested depending on the X-ray spectrum and the X-ray albedo. All evaluations do not include convection and are only adequate for hot stellar atmospheres ($T_{\text{eff}} \geq 8000\text{--}9000$ K).

The main conclusion of our investigation is that the efficiency of the reprocessing of soft X-rays is very low in irradiated stellar atmospheres. The efficiency could be higher if the structure of the disc rim is different from an irradiated atmosphere as discussed in Sect. 5.4.

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Appendix A: temperature distribution of the X-ray heated disc in the two-color gray model

We have two moments of the radiation transfer equation at each spectral band

$$\frac{dH}{dm} = k_a(J - B) \quad (\text{A1})$$

$$\frac{dJ}{dm} = 3(k_a + \sigma)H \quad (\text{A2})$$

$$\frac{dH_x}{dm} = k_x J \quad (\text{A3})$$

$$\frac{dJ_x}{dm} = \frac{1}{\mu_0^2}(k_x + \sigma)H_x, \quad (\text{A4})$$

the equation of energy generation rate

$$\frac{dH_0}{dm} = -\frac{2H_0}{\Sigma}, \quad (\text{A5})$$

and the energy balance equation

$$H(m) + H_x(m) = H_0\left(1 - \frac{m}{2\Sigma}\right). \quad (\text{A6})$$

Boundary conditions for these equations follow, at midplane of disc, as

$$H\left(\frac{\Sigma}{2}\right) = 0, H_x\left(\frac{\Sigma}{2}\right) = 0, \quad (\text{A7})$$

at the surface for the optical band,

$$J(0) = \sqrt{3}H(0), \quad (\text{A8})$$

and at both surfaces for the X-ray band,

$$J_x(0) = I_x^0 + \frac{H_x(0)}{\mu_0}, J_x(\Sigma) = I_x^0 - \frac{H_x(\Sigma)}{\mu_0}. \quad (\text{A9})$$

It is necessary to mention that $H_x(0) = -H_x(\Sigma)$. The energy balance equation can be rewritten in equivalent form

$$k_a B = k_a J + \frac{2H_0}{\Sigma} + k_x J_x. \quad (\text{A10})$$

Integrating the X-ray part equations yields

$$J_x = A(\exp(-bm) + \exp(-b(\Sigma - m))), \quad (\text{A11})$$

$$H_x = -\frac{k_x}{b} A(\exp(-bm) - \exp(-b(\Sigma - m))), \quad (\text{A12})$$

$$A = \frac{\mu_0 b I_x^0}{\mu_0 b + k_x + \exp(-b\Sigma)(\mu_0 b - k_x)} \quad (\text{A13})$$

where $b = \sqrt{k_x(k_x + \sigma)}/\mu_0$. It is possible also obtain the X-ray flux at the surface

$$H_x(0) = -H_x^0(1 - A_x) \quad (\text{A14})$$

where $H_x^0 = \mu_0 I_x^0/2$, and the X-ray albedo

$$A_x = \frac{\sigma + k_x(1 - C^2)}{(\sqrt{k_x C} + \sqrt{k_x + \sigma})^2} \quad (\text{A15})$$

where $C = (1 - \exp(-b\Sigma))/(1 + \exp(-b\Sigma))$.

Integrating the equations for the optical part yields

$$J = 3(k_a + \sigma)mH_0(1 - \frac{m}{\Sigma}) + \sqrt{3}(H_0 - H_x(0)) + 3\mu_0^2 \frac{k_a + \sigma}{k_x + \sigma} (J_x(0) - J_x), \quad (\text{A16})$$

and from the energy equation we have finally:

$$B = 3\tau H_0(1 - \frac{m}{\Sigma}) + 2\frac{H_0}{\Sigma k_a} + \sqrt{3}H_0 + \sqrt{3}H_x^0(1 - A_x) + \frac{k_x}{k_a} J_x + 3\mu_0^2 \frac{k_a + \sigma}{k_x + \sigma} (J_x(0) - J_x). \quad (\text{A17})$$

For a stellar atmosphere (Σ equal infinity) we have:

$$J_x = A \exp(-bm), \quad (\text{A18})$$

$$H_x = -\frac{k_x}{b} A \exp(-bm), \quad (\text{A19})$$

$$A = \frac{\mu_0 b}{\mu_0 b + k_x} I_x^0 \quad (\text{A20})$$

$$A_x = \frac{\sigma}{(\sqrt{k_x} + \sqrt{k_x + \sigma})^2} \quad (\text{A21})$$

and the temperature stratification

$$B = 3\tau H_0 + \sqrt{3}H_0(0) + \sqrt{3}H_x^0(1 - A_x) + \frac{k_x}{k_a} J_x + 3\mu_0^2 \frac{k_a + \sigma}{k_x + \sigma} (J_x(0) - J_x). \quad (\text{A22})$$

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