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Masses and densities of the four inner major Saturn's satellites

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Abstract. Firstly in this paper, values for the masses and densities of the four inner major Saturn's satellites are determined from the recent theories of motion of these satellites developped by Dourneau (1993). The masses respectively obtained for Mimas, Enceladus, Tethys and Dione, expressed in units of Saturn's mass, are:
$m_1 = (0.651 \pm 0.021)10^{-7}$,
$m_2 = (2.02 \pm 0.54)10^{-7}$,
$m_3 = (1.094 \pm 0.031)10^{-6}$,
$m_4 = (1.923 \pm 0.057)10^{-6}$.
Satellite densities are then derived from satellite masses given just above and from radii determined via Voyagers pictures. Their values, expressed in g/cm^3, are:
$d_1 = 1.123 \pm 0.047$,
$d_2 = 1.77 \pm 0.48$,
$d_3 = 1.032 \pm 0.059$,
$d_4 = 1.494 \pm 0.085$.
Secondly in this paper, we discuss masses and densities of these atellites, in terms of their method of determination. It appears that the method using satellite theories of motion provides quite reliable values, like those proposed above in this paper, for all satellites, Enceladus excepted. For this satellite, determinations from theories of motion are not as accurate as those obtained from the planetological work of Dermott & Thomas (1994).

Key words: planets and satellites: individual: Saturn – celestial mechanics, stellar dynamics

1. Introduction

The recent fit of the theories of motion of the eight major Saturn's satellites to a large time span of a century of high quality observations by Dourneau (1993), has provided an opportunity to determine the corresponding masses of these satellites, especially the four inner ones whose masses are strongly linked with orbital elements and other theorical parameters. This paper clearly presents the method used to determine masses from theories of motion, using peculiar properties of both resonant couple of satellites Mimas-Tethys and Enceladus-Dione. Then, satellite densities are derived from their masses determined above in this paper. A comparison of the new values of masses and densities with previous ones is proposed next, by considering the different methods of determination of satellite masses, in order to point out the more reliable values available now.

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2. Determination of satellite masses and densities

2.1. Satellite masses

The method used in this paper to determine satellite masses is related to the resonance of both satellites Mimas-Tethys and Enceladus-Dione. In the theories of motion of these resonant satellites, some expressions involve satellite masses which are consequently linked with some other libration parameters. These parameters, which directly depend on satellite motion, were recently adjusted to a large century of high quality observations by Dourneau (1993) who did not propose any value of satellite masses. So, in this paper, masses from Dourneau are calculated in order to compare them with those from other previous works. Now, let us successively consider both of these couple of resonant satellites, Mimas-Tethys and Enceladus-Dione.

2.1.1. Mimas and Tethys

The basical theories of these satellites were first developped by H. Struve (1898). The longitude libration ΔL_i of Mimas ($i = 1$) and Tethys ($i = 3$) can be expressed as follows:

$$\Delta L_i = A_i sin\mu_0 \Delta\tau + B_i sin3\mu_0 \Delta\tau + C_i sin5\mu_0 \Delta\tau \quad (1)$$

where $\Delta\tau = \tau - \tau_0$ in which τ is the argument of date, expressed in julian years and τ_0 is the time origin. A_i, B_i and C_i are the amplitudes of the libration and μ_0 its frequency. The libration amplitudes for Mimas and Tethys are related by:

$$\frac{\Delta L_3}{\Delta L_1} = -\frac{x_{13}}{2} \quad (2)$$

in which:

$$x_{13} = \frac{1}{\alpha}\frac{m_1}{m_3} \quad (3)$$

and where:

$$\alpha = \frac{a_1}{a_3} \quad (4)$$

Table 1. Orbital elements and libration parameters of the four inner Saturn's satellites from Dourneau (1993)

Mimas	Tethys
$a_1 = (26''.818 \pm 0''.012)$[a]	$a_3 = (42''.6393 \pm 0''.0045)$[a]
$\gamma_1 = 1^o.563 \pm 0^o.030$	$\gamma_3 = 1^o.0976 \pm 0^o.0063$
$n_1 = (381.994497 \pm 0.000014)^o/d$	
$N'_1 = (-365.072 \pm 0.029)^o/yr$	
$A_1 = -43^o.57 \pm 0^o.13$	
$\tau_0 = (1866.39 \pm 0.21\,)yr$	
$\mu_0 = (5.095 \pm 0.014)^o/yr$	
$x_{13} = 0.09469 \pm 0.00040$	
$q = 0.04739 \pm 0.00031$	
Enceladus	**Dione**
$a_2 = (34''.4301 \pm 0''.0056)$[a]	$a_4 = (54''.5876 \pm 0''.0045)^{(a)}$
$e_2 = 0.00485 \pm 0.00012$	
$n_2 = (262.731900 \pm 0.0000012)^o/d$	$n_4 = (131.53493193 \pm 0.00000049)^o/d$
$p_2 = (15'.4 \pm 1'.0)$	$p_4 = (-1'.29 \pm 0'.42)$
$q_2 = (12'.53 \pm 0'.73)$	$q_4 = (-1'.04 \pm 0'.34)$
$\nu = (32.39 \pm 0.13)^o/yr$	
$\nu' = (92.62 \pm 0.16)^o/yr$	
$\mu = 74^o.4 \pm 5^o.6$	
$\mu' = 134^o.3 \pm 2^o.8$	
$x_{24} = -0.0833 \pm 0.0221$	

[a] At Saturn's mean distance from the Earth $9.538937 A.U.$

a_i and m_i, indexed as in expression (1) above, respectively represent semi-major axis and masses for Mimas and Tethys. Tethys' mass m_3 is computed from the following relation derived from G. Struve (1930):

$$(\frac{2K}{\Pi})^2(\frac{\mu_0}{n_1})^2 = 12m_3\gamma_1\gamma_3\alpha^2 B_3^{(3/2)}(1+x_{13})(1+\frac{N'_1}{2n_1}) \quad (5)$$

γ_1 and γ_3 are the inclinations on Saturn's ring of the respective orbital planes of Mimas and Tethys, n_1 is the mean motion and N'_1 is the secular motion of the node of Mimas. The complete elliptic integral of the first kind K is computed from both following relations:

$$\frac{2K}{\Pi} = 1 + 4\sum_{s=1}^{\infty}\frac{q^s}{1+q^{2s}} \quad (6)$$

where q is the Jacobi's parameter of the pendulum oscillation so that:

$$\frac{\sqrt{q}}{1+q} = -\frac{n_1(1+x_{13})}{2(2n_1-N_1)} \quad (7)$$

Finally, from the theoretical parameters of Dourneau (1993) given in Table 1, we obtain $K = 1.8845$ and $B_3^{3/2} = 2.5809$. Then are derived the following masses of Tethys and Mimas, expressed in units of Saturn's mass:

$$m_3 = (1.094 \pm 0.021)10^{-6}$$
$$m_1 = (0.651 \pm 0.014)10^{-7}$$

2.1.2. Enceladus and Dione

The masses of the other resonant couple of satellites Enceladus and Dione also are determined from their basical theory. Librations in longitude ΔL_i for Enceladus ($i = 2$) and Dione ($i = 4$) are:

$$\Delta L_i = p_i sin(\mu + \nu t) + q_i sin(\mu' + \nu' t) \quad (8)$$

in which p_i and q_i represent the amplitudes of both libration arguments, μ and μ' their respective phases, ν and ν' their frequencies. Values for μ' and ν' are computed from:

$$\mu' + \nu' t = 2L_4 - L_2 - P_4 \quad (9)$$

which involves L_2 and L_4, the respective mean longitudes of Enceladus and Dione, and P_4 the longitude of Dione's pericenter. According to G. Struve's (1930) theory, the mass of Dione m_4 is a solution of the following second order polynomial:

$$(\frac{\alpha A}{e_2})^2 m_4^2 + 3\alpha A e_2(1–8x_{24}(\frac{n_4}{n_2})^2)m_4 - (\frac{\mu}{n_2})^2 = 0 \quad (10)$$

where e_2 is the eccentricity of Enceladus and with

$$\alpha = \frac{a_2}{a_4} \quad (11)$$

and

$$x_{24} = \frac{\Delta L_4}{\Delta L_2} = -\frac{m_2}{2\alpha m_4} \quad (12)$$

ai and n_i, indexed as above, respectively represent semi-major axis and mean motions for Enceladus and Dione. The coefficient A is computed from the relation:

$$A = 2B_2^{1/2} + \frac{\alpha}{2}\frac{d}{d\alpha}(B_2^{1/2}) \quad (13)$$

Finally, the values of orbital parameters given in Table 1 lead to $A = 1.194$ and to the following mass m_4 of Dione, expressed in units of Saturn's mass, as a solution of Eq. (10):

Table 2. Masses of the four inner major saturnian satellites determined in this paper and compared to previous ones. All values are expressed in units of Saturn's mass x 10^{-6}

Authors	Mimas	Enceladus	Tethys	Dione
G. Struve (1930)	0.0669 ± 0.0040	0.151 ± 0.030	1.141 ± 0.054	1.847 ± 0.092
Kozaï (1957)	0.0659 ± 0.0015	0.148 ± 0.061	1.095 ± 0.022	2.039 ± 0.053
Kozaï (1976)		0.134 ± 0.061		1.85 ± 0.06
Tyler et al. (1982)	0.0800 ± 0.0095		1.33 ± 0.16	
Dourneau (1987)	0.0648 ± 0.0021	0.206 ± 0.055	1.088 ± 0.031	1.954 ± 0.058
Campbell & Anderson (1989)			1.19 ± 0.26	
Harper & Taylor (1993)	0.0646 ± 0.0011	0.213 ± 0.046	1.075 ± 0.018	1.916 ± 0.036
Dermott & Thomas (1994)[a]		0.114 ± 0.003		
Dermott & Thomas (1994)[b]		0.128 ± 0.006		
Vienne & Duriez (1995) TASS 1.6	0.0634 ± 0.0025	0.069 ± 0.088	1.060 ± 0.042	1.963 ± 0121
Vienne & Duriez (1995) TASS 1.5	0.0640 ± 0.0025	0.107 ± 0.088	1.068 ± 0.042	1.950 ± 0.121
This paper	0.0651 ± 0.0021	0.202 ± 0.054	1.094 ± 0.031	1.923 ± 0.057

[a] Average value
[b] Upper limit value

$m_4 = (1.923 \pm 0.057)10^{-6}$.

Then, the mass m_2 of Enceladus is derived from Eq. (12) so that:

$m_2 = (2.02 \pm 0.60)10^{-7}$.

Masses for each of the four inner satellites determined in this paper are presented in Table 2 with their root-mean-square (RMS) errors computed from formal errors of satellite orbital elements and libration parameters as given in Table 1.

2.2. Satellite densities

Satellite densities are derived from satellite masses determined above. We have used satellite radii obtained from analysis of pictures taken by Voyagers 1 and 2 spacecrafts. The more recent values were proposed by Dermott & Thomas for Mimas (1988) and for Enceladus (1994):

$$r_1 = (198.9 \pm 0.6)km$$
$$r_2 = (249.4 \pm 0.2)km$$

and for both other satellites, by Davies & Katayama (1983):

$$r_3 = (524 \pm 5)km$$
$$r_4 = (559 \pm 5)km.$$

We also have adopted Saturn's mass value of $(5.6845 \pm 0.0014)10^{29}g$ from Campbell & Anderson (1989). Satellite densities, so determined in this paper, are given in Table 3.

3. Discussion of results

Table 2 presents the new values of satellite masses determined in this paper, with most of the previous values. These values were generally determined from satellite theories of motion.

However, for Mimas and Tethys, Tyler et al. (1982) and Campbell & Anderson (1989) proposed values derived from Pioneer 11 and Voyagers 1 and 2 spacecrafts, gravitational data analysis. Also, Enceladus mass was derived from planetological assumptions by Dermott & Thomas (1994) who propose an average value and an upper limit value, both affected with very low RMS errors. Oppositely, Mimas and Tethys values obtained from spacecraft gravitational perturbations are significantly different from other ones and present much higher RMS errors. This can be explained because spacecrafts did not approach inner saturnian satellites closely enough for being significantly gravitationnally perturbed. In addition, there were some technical problems concerning the stability of the on-board crystal oscillator of Voyager 2, mentionned by Campbell & Anderson. So, for both satellites Mimas and Tethys, the method of mass determination using dynamical theories is still providing the most accurate values.

In addition, our new values for satellite masses presented in Table 2 are in good agreement with most previous ones for all satellites, Enceladus excepted. Our RMS errors can sometimes appear slightly high, compared to some other ones derived from similar works. This can be related to the method of adjustment to observations of the theoretical constants of satellites, simultaneously including, with satellite masses, more or less other parameters. Precisely, Dourneau's adjustment includes a very large number of these parameters.

For Enceladus, three values are significantly higher than all other ones: Dourneau's (1987), Harper & Taylor's (1993) and that proposed in this paper. This can be explained as these values are derived from rather similar dynamical theories and sets of observations. So, an adjustment of satellite orbital elements including new CCD astrometric observations (Harper & Taylor 1994) and (Harper et al. 1997) should be useful in the future in order to analyse the so derived mass of Enceladus. Inversely for this satellite, Vienne & Duriez (1995), in a dynamical work including additional theoretical matter, propose very low masses, especially that derived from their TASS 1.6 dynamical theory. But, from their other TASS 1.5 theory, they obtain a mass in quite good agreement with the average value of the planetological work by Dermott & Thomas (1994). We also note that

Table 3. Densities of the four inner Saturn' satellites, determined in this paper and compared with previous ones. Densities calculated from author's masses are mentionned "author". All values are expressed in g/cm^3.

Authors	Mimas	Enceladus	Tethys	Dione
"Struve" (1930)	1.154 ± 0.079	1.32 ± 0.27	1.076 ± 0.082	1.44 ± 0.11
"Kozaï" (1957)	1.137 ± 0.037	1.30 ± 0.54	1.033 ± 0.054	1.58 ± 0.11
"Kozaï" (1976)		1.17 ± 0.49		1.44 ± 0.11
Dourneau (1987)	1.15 ± 0.04	1.77 ± 0.47	1.03 ± 0.03	1.52 ± 0.05
Campbell & Anderson (1989)	1.14 ± 0.03	1.12 ± 0.55	1.03 ± 0.04	1.44 ± 0.006
"Harper&Taylor" (1993)	1.114 ± 0.029	1.86 ± 0.41	1.014 ± 0.046	1.489 ± 0.046
Dermott & Thomas (1994)a		1.00 ± 0.03		
Dermott & Thomas (1994)b		1.12 ± 0.05		
"Vienne & Duriez" (1995) TASS 1.6	1.093 ± 0.053	0.60 ± 0.77	1.000 ± 0.040	1.525 ± 0.094
"Vienne & Duriez" (1995) TASS 1.5	1.190 ± 0.054	0.93 ± 0.77	1.007 ± 0.040	1.515 ± 0.094
This paper	1.123 ± 0.047	1.77 ± 0.48	1.032 ± 0.059	1.494 ± 0.085

Kozaï's (1976) revised theory of Enceladus has led to a mass in rather good agreement with the upper value of this planetological work.

Table 2 shows that the planetological method developped by Dermott & Thomas (1994) proposes a couple of Enceladus masses, slightly different but affected with the lower RMS errors. So, this method appears, for this satellite, the most reliable one. Therefore, as Kozaï's (1976) Enceladus dynamical theory and TASS 1.5 Vienne & Duriez's theory lead to masses of this satellite in rather good agreement with those derived from the planetological method, the additional terms which are included in both of these theories should be quite realistic.

Now, let us discuss satellite densities. Table 3 presents values determined in this paper, as well as most previous ones. Some of these previous values were calculated in this paper from previous authors' masses given in Table 2, and from the radii adopted above in this paper to determine our new values. They are mentionned "authors" in Table 3. As satellite densities are quite directly related to satellite masses, the discussion about densities will present a great similarity with that concerning masses. So, as for masses, densities proposed in this paper are in a rather good agreement with previous ones for all satellites, Enceladus excepted. For this satellite, the planetological work of Dermott & Thomas (1994) also provides the most accurate values. Similarly, the densities calculated from Kozaï (1976) and Vienne & Duriez TASS 1.5 analytical theories appear the more coherent with the accurate values derived from the planetological work.

When we analyse mass and density RMS errors respectively in Table 2 and Table 3, we can see that the method of determination using satellite theories of motion leads to values affected with significantly higher errors for Enceladus than for all other three satellites. In addition for this satellite, we already have seen that errors are significantly higher when derived from dynamical works than from the planetological study of Dermott & Thomas (1994).

Consequently, the method of determination of satellite mass and density using dynamical theories can be considered quite satisfactory for the three satellites Mimas, Tethys and Dione. But for Enceladus, the planetological method appears significantly more accurate. So the values for Enceladus mass and density proposed by Dermott & and Thomas (1994) seems to be the most reliable ones available now. Moreover, as these accurate values are in good agreement with those derived from the analytical theories developped by Kozaï (1976) and by Vienne & Duriez (TASS 1.5), these theories should be now the more realistic ones.

4. Conclusion

Firsly in this paper, the masses of the four inner Saturn's satellites have been determined from Dourneau's (1993) analytical theories of the motion of these satellites. These masses have then been used to derive correspondant satellite densities, via satellite radii determined from Voyager 1 and Voyager 2 spacecrafts. Secondly, this paper discusses the different values of these satellite masses and densities available now. It appears that the method of determination of these physical parameters from satellite dynamical theories is quite accurate for all satellites, Enceladus excepted.

Consequently, for the three satellites Mimas, Tethys and Dione, masses and densities determined in this paper from recent dynamical theories of these satellites are quite reliable. But for Enceladus, the planetological method proposed by Dermott & Thomas (1994) and leading to values affected with lower errors, appears the most reliable one.

In conclusion, the discussion developped in this paper has led to the proposal of the more reliable values available now for the masses and for the densities of the four inner saturnian satellites. However, in the future, these values will have to be confirmed, probably by next spatial missions, as the Casini-Huyghens one, launched in 1997 and which is planned to approach the saturnian system in 2004.

References

Campbell J.K., Anderson J.D., 1989, AJ 97, 5
Davies M.E., Katayama F.Y., 1983, J. Geophys. Res. 88, A11, 8729
Dermott S.F., Thomas P.C., 1988, Icarus 73, 25
Dermott S.F., Thomas P.C., 1994, Icarus 109, 241

Dourneau G., 1987, Thèse de Doctorat d'Etat, Université Bordeaux 1
Dourneau G., 1993, A&A 267, 292
Harper D., Taylor D.B., 1993, A&A 268, 326
Harper D., Taylor D.B., 1994, A&A 284, 619
Harper D., Murray C.D., Beurle K., et al., 1997, A&AS 121, 65
Kozaï Y., 1957, Ann. Tokyo Astron. Obs. V, 73
Kozaï Y., 1976, PASJ 28, 675
Struve H., 1898, Publ. Obs. Central Nicolas, Saint Petersbourg Serie II, vol XI
Struve G., 1930, Veröff. der Sternwarte zu Berlin-Babelsberg Band VI, Heft 4
Tyler G.L., Eshleman V.R., Anderson J.D., et al., 1982, Sci 215, 553
Vienne A., Duriez L., 1995, A&A 297, 588