Astron. Astrophys. 350, 685–693 (1999)

ASTRONOMY AND ASTROPHYSICS

Kuiper Belt evolution due to dynamical friction

A. Del Popolo[1,2], **E. Spedicato**[2], **and M. Gambera**[1,3]

[1] Istituto di Astronomia dell'Università di Catania, Viale A. Doria 6, 95125 Catania, Italy
[2] Dipartimento di Matematica, Università Statale di Bergamo, Piazza Rosate 2, 24129 Bergamo, Italy
[3] Osservatorio Astrofisico di Catania and CNR-GNA, Viale A. Doria 6, 95125 Catania, Italy

Received 23 September 1998 / Accepted 24 June 1999

Abstract. In this paper we study the role of dynamical friction on the evolution of a population of large objects ($m > 10^{22}$ g) at heliocentric distances > 70 AU in the Kuiper Belt. We show that the already flat distribution of these objects must flatten further due to non-spherically symmetric distribution of matter in the Kuiper Belt. Moreover the dynamical drag, produced by dynamical friction, causes objects of masses $\geq 10^{24}$g to lose angular momentum and to fall through more central regions in a timescale $\approx 10^9$yr. This mechanism is able to transport inwards objects of the size of Pluto, supposing it was created beyond 50AU, according to a Stern & Colwell's (1997b) suggestion.

Key words: minor planets, asteroids – comets: general – planets and satellites: general – solar system: general

1. Introduction

The current model for comets in the solar system supposes that a vast cloud of cometary objects orbits the Sun. This cloud consists of three components. The inner one, referred to as the Kuiper Belt (hereafter KB) (Edgeworth 1949; Kuiper 1951), is a disc like structure of $\geq 10^{10}$ comets extending from $40-10^3$AU from the Sun (Weissman 1995; Luu et al. 1997). The KB has been proposed as the source of the Jupiter-family short-period (hereafter SP) comets. The second component, referred to as the Oort inner cloud, or the Hills cloud (Hills 1981), is supposed to be a disc, thicker than KB, containing $10^{12}-10^{13}$ objects lying $\sim 10^3 - 2\times 10^4$AU from the Sun. It is proposed as a source of long-period (hereafter LP) and Halley-type SP comets (Levison 1996). The last component, the Oort cloud (Oort 1950), is a spherical cloud of $10^{11} - 10^{12}$ cometary objects with nearly isotropic velocity distribution extending from 2×10^4 to 2×10^5AU. Even if the Oort cloud was considered in the past as the fundamental reservoir of LP comets which have been brought into the inner solar system by perturbations due to the galactic tidal field, molecular clouds and passing stars, nowadays it has been shown that it can contribute only for a small part to the LP population of comets (Duncan et al. 1988; Wiegert & Tremaine 1997).

Observational confirmation of the KB was first achieved with the discovery of object 1992QB1 by Jewitt & Luu (1993). To date over 40 KB objects (hereafter KBO) with diameters between 100 and 400 km have been discovered and the detection statistics obtained suggest that a complete ecliptic survey would reveal 7×10^4 such bodies orbiting between 30 and 50AU. Such a belt of distant icy planetesimals could be a more efficient source of SP comets than the Oort cloud (Fernandez 1980; Duncan et al. 1988). Dynamical simulations have shown that a cometary source with low initial inclination distribution was more consistent with the observed orbits of SP comets than the randomly distributed inclinations typical of the comets in the Oort cloud (Quinn et al. 1990; Levison & Duncan 1993). According to other simulations the greatest part of objects in the KB should be stable for the age of the solar system. However if the population of the KB is $\sim 10^{10}$ objects, weak gravitational perturbations provide a large enough influx to explain the current population of SP comets (Levison & Duncan 1993; Holman & Wisdom 1993; Duncan et al. 1995). In particular Levison & Duncan (1993) and Holman & Wisdom (1993), studied the long term stability of test particles in low-eccentricity and low-inclination orbits beyond Neptune, subject only to the gravitational perturbations of the giant planets. They found orbital instability on timescales $< 10^7$yr interior to 33–34AU, regions of stability and instability in the range 34–43AU and stable orbits beyond 43AU.

A study by Malhotra (1995a) showed that the KB is characterized by a highly non uniform distribution: most of the small bodies in the region between Neptune and 50AU would have been swept into narrow regions of orbital resonance with Neptune (the 3:2 and 2:1 orbital resonances, respectively located at distances from the Sun of 39.4AU and 47.8AU). The orbital inclinations *i* of many of these objects would remain low ($i < 10^{\rm o}$) but the eccentricities *e* would have values from 0.1 to 0.3. At the same time many of the trans-neptunian objects discovered lie in low-inclination orbits, as predicted by the dynamical models of Holman & Wisdom (1993) and Levison & Duncan (1993). A more detailed analysis of this distribution reveals that most objects inside 42AU reside in higher-e, i orbits locked in mean motion resonance with Neptune, but most objects beyond this

Send offprint requests to: A. Del Popolo (antonino@astrct.ct.astro.it);
E. Spedicato (emilio@ibguniv.unibg.it);
M. Gambera (mga@sunct.ct.astro.it)

distance reside in non-resonant orbits with significantly lower eccentricities and inclinations.

After the previously quoted discovery of 100–200 km sized objects (Jewitt & Luu 1993; Jewitt & Luu 1995; Weissmann & Levison 1997), proving that the KB is populated, Cochran et al. (1995) have reported Hubble Space Telescope results giving the first direct evidence for comets in the KB. Cochran's observations imply that there is a large population ($> 10^8$) of Halley-sized objects (radii ~ 10 km) within ~ 40AU of the Sun, made up of low inclination objects ($i < 10^{\rm o}$). At the time of the publication, Cochran's et al. (1995) results were criticized on two grounds:

1) the detections were statistical in nature, and the authors were not able to fit orbits to their objects;
2) the number of detections did not agree with extrapolation of the size distribution of large KBOs determined from early ground-based observations (but Weissman & Levison 1997 showed that Cochran's et al. 1995 results were in agreement with the number of KBOs needed to populate the Jupiter-family comets); moreover Brown et al. 1997 contended that detections reported in Cochran et al. (1995) were not possible, based on an analysis of the noise properties of the data. In a recent paper, Cochran et al. 1998, confirmed the early results by means of a new analysis.

The spatial dimensions and mass distribution in the KB are poorly known. Yamamoto et al. (1994) have applied a planetesimal model to the trans-neptunian region, finding that the maximum number density of the planetesimal population should be about at 100–200AU and the planetesimal disc itself can extend up to distances $\approx 10^3$AU. This is in agreement with detection by IRAS of discs around main sequence stars, Vega (Aumann et al. 1984), β Pictoris (Smith & Terrile 1984), extending to several hundred AU. From the available radio and infrared data, Beckwith & Sargent (1993) conclude that disc masses may range between $10^{-3} M_\odot$ to $1 M_\odot$ and extend from a few hundred AU to more than 10^3AU. In short, both theoretical arguments and observations strengthen the view that our solar system is surrounded by a flattened structure of planetesimals, extending perhaps to several hundred AU.

Although originally it had been thought that the population might be collisionless, recent work (Stern 1995) has shown that the collisional effects cannot be neglected over 4.5 Gyr. As shown by Stern (1995, 1996a,b) and Stern & Colwell (1997a,b), the collisional evolution is an important evolutionary process in the disc as a whole, and moreover, it is likely to be the dominant evolutionary process beyond 42AU. In the case of larger planetesimals the evolution is connected to the energy loss due to dynamical friction, which transfers kinetic energy from the larger planetesimals to the smaller ones. This mechanism, in the early solar system, provides an energy source for the small planetesimals that is comparable to that provided by the viscous stirring process (Stewart & Wetherill 1988; Weidenschilling et al. 1997).

The objective of this paper is to examine the role of dynamical friction, in the primordial KB, in the orbital evolution of the largest planetesimals that lie at a heliocentric distance > 70AU (at this distance the effects of the planets decline rapidly to zero and only a small fraction of objects is influenced by planetary perturbations - Wiegert & Tremaine 1999; Stern & Colwell 1997b). While the importance of dynamical friction in planetesimal dynamics was demonstrated in several papers, (Stewart & Kaula 1980; Horedt 1985; Stewart & Wetherill 1988) and in particular in the case of the planetary accumulation process, the role of this effect on the orbital evolution of the largest planetesimals and the consequent change of mass distribution in KB was never studied.

The paper is organized as follows. In Sect. 2, we review the role of encounters and collisions in KB. In Sect. 3, we introduce the equations to calculate dynamical friction effects. In Sect. 4 we describe how we use these equations to determine the evolution of the largest bodies population in KB. In Sect. 5 we discuss the results of the calculation and we also show (supposing the scenario proposed by Stern & Colwell 1997b of Pluto formation beyond 50AU to be correct) how dynamical friction is able to transport an object of the size of Pluto from 50AU to the actual position. In Sect. 6 we give our conclusions.

2. Encounters and collisions in KB

As shown in several N-body simulations (Stern 1995), the structure of KB in the region 30–50AU is fundamentally due to two processes:

1) dynamical erosion due to resonant interactions with Neptune (Holman & Wisdom 1993; Levison & Duncan 1993; Duncan et al. 1995; Malhotra 1995a). In this region the KB has a complex structure. Objects with perihelion distances $\simeq 35$AU are unstable. For orbits with $e \geq 0.1$ and with semi-major axis $a < 42$AU the only stable orbits are those in Neptune resonances. Between 40AU and 42AU at low inclinations and between 36AU and 39AU with $i \simeq 15^o$ the orbits are unstable.
2) collisional erosion (Stern 1995, 1996a,b; Stern & Colwell 1997a,b). Starting from a primordial disc having a mass of $40 M_\odot$ collisions are able to reduce its mass to 0.1–$0.3 M_\odot$ in 10^9yr if $\langle e \rangle \geq 0.1$.

Then the 30–50AU zone is both collisionally and dynamically evolved, since dynamics acted to destabilize most orbits with $a < 42$AU and were able to induce eccentricities that caused collisions out to almost 50AU. The dynamically and collisionally evolved zone might extend as far as $\simeq 63$AU, if Malhotra's (1995a) mean motion resonance sweeping mechanism is important. Beyond this region one expects there to be a collisionally evolved zone where accretion has occurred but eccentricity perturbations by the giant planets have been too low to initiate erosion. Beyond that region a primordial zone is expected in which the accretion rates have hardly modified the initial population of objects. Supposing that the last assumption is correct and that the radial distribution of heliocentric surface mass density in the disc, $\Sigma(R)$, can be described by $\Sigma(R) \propto R^{-2}$ (Tremaine 1990; Stern 1996a,b) and supposing that in the zone 30–50AU of the primordial disc 40–$50 M_{\oplus}$ of matter was present, we expect $\simeq 35 M_{\oplus}$ in the zone 70–100AU of the present KB. As the effects of planets are negligible for planetesimals beyond 70AU, the orbital motion there can be considered not far from Keplerian and circular (Brunini & Fernandez 1996). This last assumption is more strictly satisfied by the largest objects, which should, through energy equipartition, evolve to the lowest eccentricity in the swarm (Stewart & Wetherill 1988; Stern & Colwell 1997b)

Although the main motion of KB objects (KBOs) is Keplerian rotation around the Sun, the motion is perturbed by encounters with other objects and by collisions. Encounters influence the structure of the system in several ways:

a) Relaxation;
b) Equipartition;
c) Escape;
d) Inelastic encounters.

Each of the quoted effects has greater or smaller importance in a system evolution according to the system characteristics. In the case of a system like KB, inelastic encounters have a fundamental role because KBOs have on average a much smaller escape speed than the rms velocity dispersion, σ. If r_* is the radius of a KBO, $v_* = \sqrt{2Gm/r_*}$ is the escape speed from the KBO surface, $\Theta = \frac{v_*^2}{4\sigma^2}$ is the Safronov number, n the number density and σ the velocity dispersion, then the collision time $t_{\rm coll}$ for a population of planetesimals with a Gaussian distribution in dispersion velocity is given by:

$$t_{\rm coll} = \frac{1}{16\sqrt{\pi} n\sigma r_*^2 (1+\Theta)} \quad (1)$$

(Binney & Tremaine 1987; Palmer et al. 1993).

Within 1AU from the Sun, a population of a few hundred km-sized planetesimals, with several Earth masses in total, would have $t_{\rm coll} \simeq 10^4$yr, while at distances > 50AU $t_{\rm coll}$ becomes comparable with the age of the solar system. Indeed by means of N-body simulations, Stern (1995) showed how collisional evolution plays an important role through KB and that it has a dominant role at $r > 42$AU. The effect of collisions is that of inducing energy dissipation in the system but at the same time collisions are important in the growth of QB1 objects, Pluto-scale and larger objects starting from 1 to 10 km building blocks in a time that in some plausible circumstances is as little as $\simeq 100$–200Myr.

Mutual gravitational scattering induces random velocity in two different ways: one is viscous stirring which converts solar gravitational energy into random kinetic energy of planetesimals. Energy is transferred from circular, co-planar orbits with zero random velocities to eccentric, mutually inclined orbits with nonzero random velocities. The other is dynamical friction which transfers random kinetic energy from the larger planetesimals to the smaller ones (Stewart & Wetherill 1988; Ida 1990; Ida & Makino 1992; Palmer et al. 1993). Unlike viscous stirring, the exchange of energy does not depend on the differential rotation of the mean flow for its existence. Dynamical friction would drive the system to a state of equipartition of kinetic energy but viscous stirring opposes this tendency. In the dispersive regime, the time scales of stirring and dynamical friction are almost equal to the two-body relaxation time

$$T_{\rm 2B} \simeq \frac{1}{\pi r_{\rm G}^2 n\sigma \ln\Lambda} \quad (2)$$

where $r_{\rm G}^2$ is the gravitational radius, n the number density,

$$\Lambda = \frac{b_{\rm max}\overline{|v|^2}}{G(m1+m2)} \quad (3)$$

$b_{\rm max}$ being the largest impact parameter and $\overline{|v|^2}$ the mean square velocity of the objects, which for the typical values of the parameters in the KB is of the order of $\approx 10^{11}$ and $\log\Lambda \approx 25$.

In conclusion the random velocity of the smaller planetesimals is increased by viscous stirring while the larger planetesimals suffer dynamical friction due to smaller planetesimals.

3. Dynamical friction in KB

The equation of motion of a KBO can be written as:

$$\ddot{\mathbf{r}} = \mathbf{F}_{\odot} + \mathbf{F}_{\rm planets} + \mathbf{F}_{\rm tide} + \mathbf{F}_{\rm GCM} + \mathbf{F}_{\rm stars} + \mathbf{R} + \mathbf{F}_{\rm other} \quad (4)$$

(Wiegert & Tremaine 1997). The term $\mathbf{F}_{\odot}$ represents the force per unit mass from the Sun, $\mathbf{F}_{\rm planets}$ that from planets, $\mathbf{F}_{\rm tide}$ that from the Galactic tide, $\mathbf{F}_{\rm GCM}$ that from giant molecular clouds, $\mathbf{F}_{\rm stars}$ that from passing stars, $\mathbf{F}_{\rm other}$ that from other sources (e.g. non-gravitational forces), while **R** is the dissipative force (the sum of accretion and dynamical friction terms - see Melita & Woolfson 1996). If we consider KBOs at heliocentric distances > 70AU then $\mathbf{F}_{\rm planets}$ may be neglected. We also neglect the effects of non-gravitational forces and the perturbations from Galactic tide, GCMs or stellar perturbations, which are important only for objects at heliocentric distances $>> 100$AU (Brunini & Fernandez 1996). We assume that the planetesimals travel around the Sun in circular orbits and we study the orbital evolution of KBOs after they reach a mass $> 10^{22}$g. Moreover we suppose that the role of collisions for our KBOs at distances > 70AU can be neglected. We know that the role of collisions would be progressively less important with increasing distance from the Sun because the collision rate, $nv\sigma'$ (σ' is the collision cross section), decreases due to the decrease in the local space number density, n of KBOs and the local average crossing velocity, v, of the target body. As stated previously, using Eq. (1) at distances larger than 50AU the collision time, $t_{\rm coll}$, is of the order of the age of the solar system. Besides, the energy damping is not dominated by collisional damping but by dynamical friction damping; also, artificially increasing the collisional dampings hardly change the dynamics of the largest bodies (Kokubo & Ida 1998).

To take account of dynamical friction we need a suitable formula for a disk-like structure such as KB. Following Chandrasekhar & von Neumann's (1942) method, the frictional force which is experienced by a body of mass m_1, moving through a homogeneous and isotropic distribution of lighter particles of mass m_2, having a velocity distribution $n(v_2)$ is given by:

$$\mathbf{F} = -4\pi m_1 m_2 (m_1+m_2) G^2 \int_0^{v_1} n(v_2) dv_2 \frac{\mathbf{v_1}}{v_1^3} \log\Lambda \quad (5)$$

(Chandrasekhar 1943); where $\log\Lambda$ is the Coulomb logarithm, m_1 and m_2 are, respectively, the masses of the test particle and that of the field one, and v_1 and v_2 the respective velocity, $n(v_2)dv_2$ is the number of field particles with velocities between v_2, v_2+dv_2.

If the velocity distribution is Maxwellian Eq. (5) becomes:

$$\mathbf{F} = -4\pi n m_1(m_1+m_2)\rho G^2\frac{\mathbf{v_1}}{v_1^3}\cdot \log\Lambda[erf(X)-2X\exp(-X^2)/\sqrt{(\pi)}] \quad (6)$$

(Chandrasekhar 1943, Binney & Tremaine 1987),where ρ is the density of field particles and $X=v_1/\sqrt{(2\sigma)}$, σ being the velocity dispersion. Eq. (6) cannot be used for systems not spherically symmetric except for the case of objects moving in the equatorial plane of an axisymmetric distribution of matter. These objects, in fact, have no way of perceiving that the potential in which they move is not spherically symmetric.

We know that KB is a disc and consequently for objects moving away from the disc plane we need a more general formula than Eq. (6). Moreover dynamical friction in discs differs from that in spherical isotropic three dimensional systems. First, in a disc close encounters give a contribution to the friction that is comparable to that of distant encounters (Donner & Sundelius 1993; Palmer et al. 1993). Collective effects in a disc are much stronger than in a three-dimensional system. The velocity dispersion of particles in a disc potential is anisotropic. N-body simulations and observations show that the radial component of the dispersion, $\sigma_{\rm R}$, and the vertical one, $\sigma_{\rm z}$, are characterized by a ratio $\sigma_{\rm R}/\sigma_{\rm z}\simeq 0.5$ for planetesimals in a Keplerian disc (Ida et al. 1993). The velocity dispersion evolves through gravitational scattering between particles. Gravitational scattering between particles transfers the energy of the systematic rotation to the random motion (Stewart & Wetherill 1988). In other words the velocity distribution of a Keplerian particle disc is ellipsoidal with ratio 2:1 between the radial and orthogonal (*z*) directions (Stewart & Wetherill 1988). According with what previously told, we assume that the matter-distribution is disc-shaped, having a velocity distribution:

$$n(\mathbf{v},\mathbf{x}) = n(\mathbf{x})\left(\frac{1}{2\pi}\right)^{3/2}\exp\left[-\left(\frac{v_\parallel^2}{2\sigma_\parallel^2}+\frac{v_\perp^2}{2\sigma_\perp^2}\right)\right]\frac{1}{\sigma_\parallel^2\sigma_\perp} \quad (7)$$

(Hornung & al. 1985, Stewart & Wetherill 1988) where $v_\parallel$ and $\sigma_\parallel$ are the velocity and the velocity dispersion in the direction parallel to the plane while $v_\perp$ and $\sigma_\perp$ are the same in the perpendicular direction. We suppose that $\sigma_\parallel$ and $\sigma_\perp$ are constants and their ratio is simply taken to be 2:1. Then according to Chandrasekhar (1968) and Binney (1977) we may write the force components as:

$$F_\parallel = k_\parallel v_{1\parallel} = B_\parallel v_{1\parallel}\cdot \left[2\sqrt{2\pi}\overline{n}G^2\lg\Lambda m_1m_2\,(m_1+m_2)\,\sqrt{1-e^2}\frac{1}{\sigma_\parallel^2\sigma_\perp}\right] \quad (8)$$

$$F_\perp = k_\perp v_{1\perp} = B_\perp v_{1_\perp}\cdot \left[2\sqrt{2\pi}\overline{n}G^2\lg\Lambda m_1m_2\,(m_1+m_2)\,\sqrt{1-e^2}\frac{1}{\sigma_\parallel^2\sigma_\perp}\right] \quad (9)$$

where

$$B_\parallel = \int_0^\infty dq\exp\left[-\frac{v_{1\parallel}^2}{2\sigma_\parallel^2}\frac{1}{1+q}-\frac{v_{1\perp}^2}{2\sigma_\parallel^2}\frac{1}{1-e^2+q}\right]\cdot \frac{1}{\left[(1+q)^2(1-e^2+q)^{1/2}\right]} \quad (10)$$

$$B_\perp = \int_0^\infty dq\exp\left[-\frac{v_{1\parallel}^2}{2\sigma_\parallel^2}\frac{1}{1+q}-\frac{v_{1\perp}^2}{2\sigma_\parallel^2}\frac{1}{1-e^2+q}\right]\cdot \frac{1}{\left[(1+q)^2(1-e^2+q)^{3/2}\right]} \quad (11)$$

and

$$e = \sqrt{(1-\frac{\sigma_\perp^2}{\sigma_\parallel^2})} \quad (12)$$

while $\overline{n}$ is the average spatial density. When $B_\perp > B_\parallel$ the drag caused by dynamical friction will tend to increase the anisotropy of the velocity distribution of the test particles. The frictional drag on the test particles may be written:

$$\mathbf{F} = -k_\parallel v_{1\parallel}\mathbf{e}_\parallel - k_\perp v_{1\perp}\mathbf{e}_\perp \quad (13)$$

where $\mathbf{e}_\parallel$ and $\mathbf{e}_\perp$ are two unity vectors parallel and perpendicular to the disc plane.

This result differs from the classical Chandrasekhar (1943) formula. Chandrasekhar's result tells that dynamical friction force, $\mathbf{F}$, is always directed as $-\mathbf{v}$. This means that if a massive body moves, for example, in a disc in a plane different from the symmetry plane, dynamical friction causes it to spiral through the center of the mass distribution always remaining in its own plane. It shall reach the disc plane only when it reaches the centre of the distribution. Eq. (13) shows that a massive object suffers drags $-k_\parallel v_{1\parallel}$ and $-k_\perp v_{1\perp}$ in the directions within and perpendicular to the equatorial plane of the distribution. This means that the object shall find itself confined to the plane of the disc before it reaches the centre of the distribution (this means that also inclinations are damped). In other words the dynamical drag experienced by an object of mass m_1 moving through a non-spherical distribution of less massive objects of mass m_2 is not directed in the direction of the relative movement of the massive particle and the centre of mass of the less massive objects (as in the case of spherically symmetric distribution of matter). As a consequence the already flat distribution of more massive objects will be further flattened during the evolution of the system (Binney 1977). The objects lying in the plane, as previously told, have no way to perceive that they are moving in a not spherically symmetric potential. Hence we expect that the dynamical drag is directed in the direction opposite to the motion of the particle:

$$\mathbf{F} \simeq -k_\parallel v_\parallel \mathbf{e}_\parallel \quad (14)$$

4. Parameters used in the simulation

To calculate the effects of dynamical friction, introduced in the previous section, on a disc-like structure as KB, we cannot use the classical Chandrasekhar (1943) theory but we need equations specified for distributions like KB. These equations were written in the previous section (Eqs. 8 − 11). To calculate the effect of dynamical friction on the orbital evolution of the largest bodies we suppose that $\sigma_\perp$ and $\sigma_\parallel$ are constants and that $\sigma_\parallel$=2$\sigma_\perp$. We need also the mass density distribution in the disc. We assume a heliocentric (R) distribution in surface mass density $\Sigma \propto R^{-2}$ and a total primordial mass, $M = 50M_\oplus$ in the region 30–50AU (Stern & Colwell 1997b). To evaluate the dynamical friction force we need the spatial distribution of the field objects, $\overline{n}$. To reproduce the quoted surface density we need a number density decreasing as a function of distance from the Sun, R, to the third power (Levison & Duncan 1990):

$$n = n_o R^{-3} \tag{15}$$

We remember also that the KB is a disc and then we use the mass distribution given by a Myiamoto-Nagai disc (Binney & Tremaine 1987; Wiegert & Tremaine 1997):

$$n(R,z) = \left(\frac{b^2 M}{4\pi}\right)\cdot \frac{aR^2 + \left(a + 3\sqrt{z^2+b^2}\right)\left(a+\sqrt{z^2+b^2}\right)^2}{\left[R^2 + \left(a+\sqrt{z^2+b^2}\right)^2\right]^{5/2}\left(z^2+b^2\right)^{3/2}} \tag{16}$$

which in the disc plane reproduces the quoted surface density and Eq. (15) $[n(R,0) \propto R^{-3}]$. Here M is the disc mass, a and b are parameters describing the disc characteristic radius and thickness. Because there is presently no information on the way in which ensemble-averaged inclinations ($\langle i\rangle$) and eccentricities ($\langle e\rangle$) vary in the KB, we adopt a disc with a width $\langle i\rangle = \frac{1}{2}\langle e\rangle$ (Stern 1996b). The equations of motion were integrated in heliocentric coordinates using the Bulirsch-Stoer method.

5. Results

The model described was integrated for several values of masses, starting from $m = 10^{22}$g, supposing the KBOs move on circular orbits. We studied the motion of KBOs both on inclined orbits, in order to study the evolution of the inclination with time, and on orbits on the plane of the disc, in order to study the drift of the KBOs from their initial position. The masses of the KBOs, M, were considered constant during the whole integration in order to reduce the number of differential equations to solve. Moreover we use $M << Nm$ and $m << M$, N and m being the total number and the mass of the swarm of field particles in which the KBO moves. The assumption that field particles have all equal masses, m, does not affect the results, since dynamical friction does not depend on the individual masses of these particles but on their overall density. The results of our calculations are shown in Figs. 1–4.

In Fig. 1 we plot the values of $\langle z\rangle/z_o$ versus time for planetesimals having masses 10^{22}g and 10^{23}g. The z coordinate

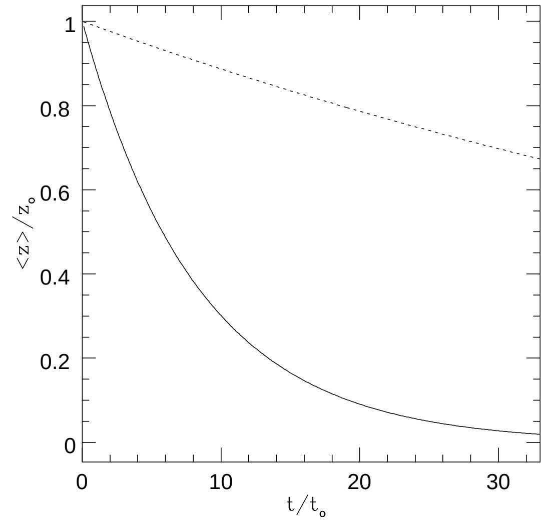

Fig. 1. Height from the plane versus time for a planetesimal of 10^{22}g, dotted line, and one of 10^{23}g, full line. Time is measured in units of $t_\mathrm{o} = 10^8$yr while height is measured in $z_o = 7$AU.

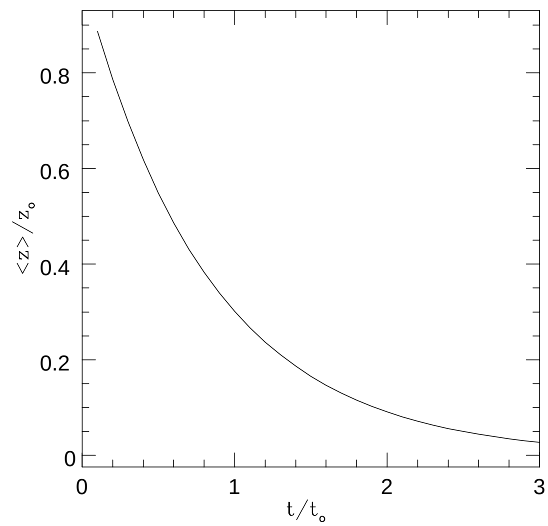

Fig. 2. Height from the plane versus time for a planetesimal of 10^{25}g. Time is measured in units of $t_\mathrm{o} = 10^8$yr while height is measured in $z_o = 7$AU.

is orthogonal to the plane of the disc and $z_o = 7$AU while $t_\mathrm{o} = 10^8$yr. The brackets $\langle\rangle$ are mean values obtained averaging over a suitable number of orbital oscillations. As shown in the figure the decay of the inclination of the planetesimal of 10^{22}g (dotted line) has a timescale larger than the age of the solar system and on the order of 3×10^{10}yr. The inclination of the more massive planetesimal (full line) decays in a time $\simeq 3 \times 10^9$yr. This is due to the fact that dynamical friction effects increase with the mass M of the KBO. In fact (see Fig. 2) when the mass of the KBO is 10^{25}g the decay time reduces to 3×10^8yr. Dynamical friction makes the orbits of KBOs undergo some collimation along the z direction, characterized by a low value of the dispersion velocity. This was expected because it is known that the larger the velocity dispersion along the direction

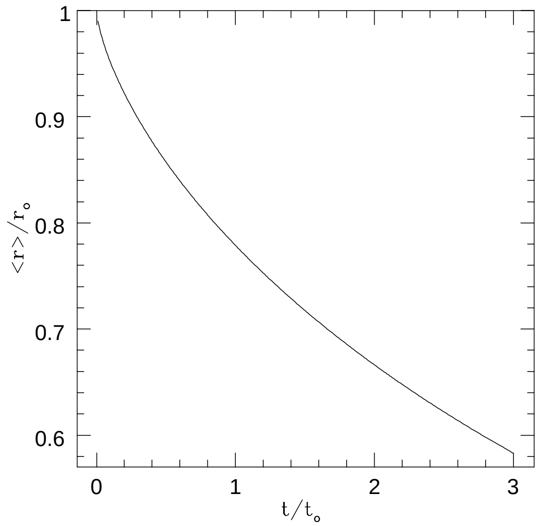

Fig. 3. Heliocentric distances in the disk plane for a planetesimal of $10^{23}g$. Time is measured in units of $t_{\rm o} = 10^{10}$yr while distances are measured in $r_{\rm o} = 70$AU.

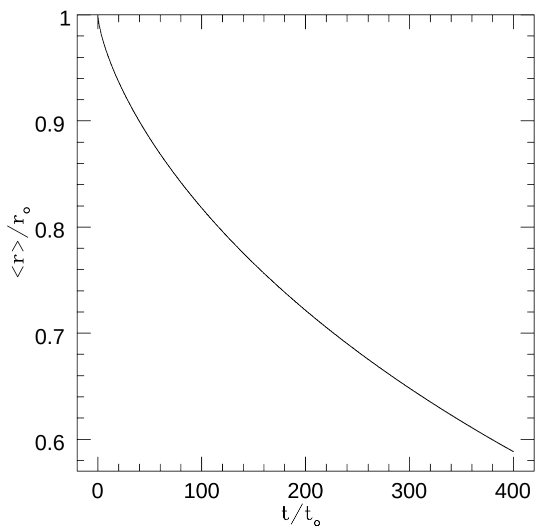

Fig. 4. Heliocentric distances in the disk plane for a planetesimal of 10^{25}g. Time is measured in units of $t_{\rm o} = 10^{6}$yr while distances are measured in $r_{\rm o} = 70$AU.

of motion, the lesser the effect of dynamical friction (Pesce et al. 1992). Binney (1977) has found an efficient collimation of orbits along the main axis of the velocity dispersion tensor in the case of an anisotropic axisymmetric system, in which the principal velocity dispersions have constant values.

The result obtained is in agreement with previous studies of the damping of the inclinations of very massive objects by Ida (1990), Ida & Makino (1992) and Melita & Woolfson (1996). In fact in the semi-analytical theory by Ida (1990) the timescale for inclinations damping due to dynamical friction is almost equal to the two-body relaxation time

$$T_{\rm damp} \simeq T_{\rm 2B} \simeq \frac{1}{nG^2M^2\, ln\Lambda/\sigma^3} \simeq \frac{1}{\frac{\Sigma}{m}\frac{G^2M^2}{\sigma^4}\Omega \ln\Lambda} \qquad (17)$$

where Σ is the surface density of small objects and Ω is the Keplerian frequency. The timescales given by Ida's theory for our planetesimals of 10^{22}g, 10^{23}g, 10^{25}g are respectively 4×10^{10}yr, 4×10^{9}yr, 4×10^{8}yr, in agreement with our results.

The effect of dynamical friction on the semi-major axis is plotted in Fig. 3 and Fig. 4. In Fig. 3 we plot $\langle r\rangle/r_{\rm o}$ versus time for a planetesimal of mass 10^{23}g. Here r is the in-plane radial heliocentric distance of the planetesimal while $r_{\rm o} = 70$AU and $t_{\rm o} = 10^{10}$yr. As shown, the time required to a planetesimal of the quoted mass to reach 40AU is $\simeq 3\times10^{10}$yr, which is an order of magnitude larger than the damping timescale. This is due, in agreement with what was previously told, to the fact that in the plane the dispersion velocity is larger than that in the z direction. Increasing the mass to 10^{25}g the time needed for a planetesimal to reach 40AU decreases to 4×10^{8}yr (here $t_{\rm o} = 10^{6}$yr) (see Fig. 4). The threshold planetesimal mass, $M_{\rm threshold}$, that starts orbital migration is $\sim 10^{24}$ g. This mass scales with the disk density as:

$$M_{\rm threshold} \sim \frac{10^{24}}{(\rho/10^{-16}{\rm g/cm^3})}\, g \qquad (18)$$

We recall that we do not take into account the effects of the planets because we considered planetesimals initially at distances > 70AU, but when the planetesimal moves towards the region of influence of planets the role of these must be taken into account.

We have some difficulty to compare this result with previous studies because the problem of the decay of the semi-major axis has not been particularly studied. So far, many people have assumed a priori that radial migration due to dynamical friction is much slower than damping of velocity dispersion due to dynamical friction. Therefore most studies of dynamical friction were concerned only with damping of velocity dispersion (damping of the eccentricity, e, and inclination, i), adopting local coordinates. Analytical work by Stewart & Wetherill (1988) and by Ida (1990) adopted local coordinates. N-body simulation by Ida & Makino (1992) adopted non-local coordinates, but did not investigate radial migration. Only the density wave approach by Goldreich & Tremaine (1979, 1980) and Ward (1986) considered radial migration. However, the relation of this approach to the particle orbit approach is not clear. Furthermore, a few numerical simulation has been devoted to investigate radial migration.

In any case we shall compare our result with that by Ward (1986) supposing that this density approach describes correctly the radial migration. Following Ward (1986) the characteristic orbital decay time of a disc perturber is given by:

$$T_{\rm dec} = \frac{1}{\Omega|C|\mu}\frac{M_\odot}{\Sigma a^2}\left(\frac{c}{a\Omega}\right)^2 \qquad (19)$$

where $\mu = M/M_\odot$, and the nondimensional factor C is $\simeq -18$, depending on the disc's surface density gradient, k, and the adiabatic index, s, for a disc with $Q = \infty$, c is the gas sound speed (in a planetesimal system this must be replaced by the velocity dispersion). The timescale for the perturber to drift out of a region of radius r is given by (Ward 1986):

$$T_{drift} \simeq \frac{c}{r\Omega} T_{dec} \tag{20}$$

Using $\Omega = \sqrt{GM_{\odot}/r^3}$, calculating Σ supposing that the disc mass is uniformly spread in the region 40–70AU and $c = 20000\,\mathrm{cm/s}$ we find $T_{\mathrm{drift}} \simeq 7 \times 10^8\mathrm{yr}$ for a planetesimal having $M = 10^{25}\mathrm{g}$, a little larger than the value previously found. We have to remember that Ward's (1986) model supposes that the solar nebula is two-dimensional and that in a finite thickness disc other damping mechanisms may come into play perhaps invalidating Ward's result.

This last calculation shows that in a timescale less than the age of the solar system, objects of the mass of Pluto $\simeq 0.002M_{\oplus}$ may move from the region $> 50\mathrm{AU}$ towards the position actually occupied by the planet. This opens a third possibility to the standard scenarios for Pluto formation (in situ formation, formation at 20–30AU and transport outwards) namely that Pluto was created beyond 50AU, and then transported inwards. One of the problems of Pluto formation in the first two scenarios is that the growth of Neptune caused accretion to be inhibited and then it is necessary that the time for accreting an object of Pluto mass was shorter than the timescale of Neptune formation. This problem is not present in the third scenario because at heliocentric distances larger than 50AU Neptune never induced significant eccentricities on most orbits in the region. Hence the dynamical conditions necessary for growth may have persisted for the whole age of the solar system and consequently Pluto could have formed later than in the other two quoted scenarios. The mechanism responsible for the transport of Pluto from the quoted region to that nowadays occupied might have been dynamical friction.

Three possible objections to this last model are:

1) Moving to greater and greater distances both the disc density and the velocity dispersions decrease and consequently the accretion times increase. Can a Pluto-scale body form at distances of 70 AU?
2) A possible explanation for Pluto's orbital parameters is connected to outwards migration of Neptune (Malhotra 1993). If this works for Neptune, could it also work for Pluto-like objects in the KB? How much might Pluto have moved out? Could that compensate for the effect of the frictional motion?
3) How can one explain the odd orbital parameters of Pluto's orbit ($e = 0.25$, $i = 17$ degrees to the ecliptic)?

Surely, as made clear by the first objection, formation of large bodies is more and more difficult moving away from the inner parts of Solar system. Although growth times at 70 AU are about 4–5 times longer than at 40 AU, Pluto-mass bodies can indeed be grown at this distance, from 1 to 10 km building blocks, in $\sim 1\mathrm{Gyr}$, if the mean disc eccentricity, $< e > \simeq 0.001$, and if the KB mass interior to 50 AU was, as previously stated in Sect. 4, 30–50 $M_{\oplus}$ and continued outwards with $\Sigma \propto R^{-2}$ (see Stern & Colwell 1997b).

The answer to the second objection is the following.

Orbital migration of a planet can be accomplished through two mechanisms. In the first mechanism a planet and the circumstellar disc interact tidally which results in angular momentum transfer between the disc and the planet (e.g. Goldreich & Tremaine 1980; Ward 1997). The planet's motion in the disc excites density waves both interior and exterior to the planet. If the planet is large enough (at least several Earth masses), it is able to open and sustain a gap. It establishes a barrier to any radial disc flow due to viscous diffusion and it becomes locked to the disc and must ultimately share its fate (this is known as $type$ II drift). In this case both inwards and outwards planet migration are allowed. In fact in a viscous disc, gas inside the radius of maximum viscous stress, r_{mvs}, drifts inwards as it loses angular momentum while gas outside r_{mvs} expands outwards as it receives angular momentum (Lynden-Bell & Pringle 1974). Neptune's outwards migration is due to the fact that the gas in the Neptune forming region has a tendency to migrate outwards (Ruden & Lin 1986).

If the planet is not able to sustain a gap, the net torque from the disc is still not zero and it migrates inwards in a shorter timescale ($type$ I drift).

Pluto being a low mass planet can migrate only by means of $type$ I drift, this means that it can only migrate inwards.

In the second mechanism a planet can undergo orbital migration as a consequence of gravitational scattering between itself and residuals planetesimals. If a planetesimal in a near-circular orbit similar to that of the planet is ejected into a Solar system escape orbit, the planet suffers a loss of orbital angular momentum and a corresponding change of orbital radius. Conversely, planetesimals scattered inwards would cause an increase of orbital radius and angular momentum of the planet. A single massive planet scattering a population of planetesimals in near-circular orbits in the vicinity of its own orbit would suffer no net change of orbital radius as it scatters approximately equal numbers of planetesimals inwards and outwards. However in some peculiar situations, such as that encountered in the region of Jovian planets, things go differently from this picture (Fernandez & Ip 1984). In particular, as Jupiter preferentially removes the inwards scattered Neptune planetesimals, the planetesimal population encountering Neptune at later times is increasingly biased towards objects with specific angular momentum larger than Neptune's. Encounters with this planetesimal population produce a net gain of angular momentum, hence an increase in its orbital radius. Evidently this situation is not the one present in the outer Solar system region, occupied by Pluto in our model. In other words, there is no reason to suppose that Pluto moved outwards like Neptune.

For what concerns the third objection, a possible answer is that Pluto gained high eccentricity and inclination in a similar way to that described by Malhotra (1993, 1995b). There has, of course, been much speculation as to the origin of the extraordinary orbit of Pluto (Lyttleton 1936; Farinella et al. 1979; Olsson-Steel 1988; Malhotra 1993). All but one (Malhotra (1993, 1995b)) of these speculations require one or more low-probability "catastrophic" events. In Malhotra's (1993, 1995b) model, Neptune's orbit may have expanded considerably, and its exterior orbital resonances would have swept through a large region of trans-Neptunian space. During these resonances sweeping, Pluto could have been captured into the 3:2 orbital period resonance with Neptune and its eccentricity and inclination would have been pumped up during the subsequent evolution.

The phenomenon of capture into resonance as result of some dissipative forces is common in nature. Weidenschilling & Davies (1985) studied resonance trapping of planetesimals by a protoplanet in association with gas drag. Many of the characteristics of this effect have been studied. Patterson (1987) and Beauge et al. (1994) have investigated its cosmogonic implications. The stability of the orbits and capture probabilities have been studied by Beauge & Ferraz-Mello (1993), Gomes (1995). Melita & Woolfson (1996) showed that a three-body system (Sun and two planets) under the influence of both accretion and dynamical friction forces, evolve into planetary resonance when the inner body is more massive. In general capture into a stable orbit-orbit resonance is possible when the orbits of two bodies approach each other as a result of the action of some dissipative process. The transition from a non-resonant to a resonant orbit depends sensitively upon initial conditions and the rate of orbital evolution due to the dissipative effects. Borderies & Goldreich (1984) showed that for single resonance and in the limit of slow "adiabatic" orbit evolution, the probability of capture for the 3:2 Neptune resonance is 100% for initial eccentricity less than ~ 0.03 and reduces to less than 10% for initial eccentricities exceeding 0.15 (see Malhotra 1995b).

In our model Pluto reaches its actual position in a time ($> 10^8$ yr) larger than Neptune's orbital migration timescale (10^6–10^7 yr; see Ida et al. 1999). Then Neptune was in its actual position when Pluto reached its own. Before the resonance encounter, Pluto has an initial low eccentricity ~ 0.03, because of eccentricity and inclination damping due to dynamical friction, and its migration is very slow ($> 10^8$ yr). The capture probability is then very high. The increase in eccentricity and inclination is naturally explained by Malhotra's theory.

6. Conclusions

In this paper we studied how dynamical friction due to small planetesimals influence the evolution of KBOs having masses larger than 10^{22}g. We find that mean eccentricity of large mass particles is reduced by dynamical friction by small mass particles in timescales shorter than the age of the solar system for objects of mass equal or larger than 10^{23}g. We also studied the effect of dynamical friction on the evolution of the semi-major axis of the largest planetesimals. We find that even if dynamical friction is less effective in transferring planetesimals towards the inner part of the solar system, with respect to the damping of inclinations that it is able to produce, the timescale for radial migration is shorter than the age of the solar system for large enough masses ($\geq 10^{24}$g).

Finally, our calculation show the dynamical friction may be the mechanism responsible for the transport of objects like Pluto from regions with $r > 50$AU towards the position nowdays occupied and this opens a third possibility for Pluto formation that eliminates the problem of the Neptune formation.

Acknowledgements. We would like to thank the referee C. Dominik for helpful comments which led to improve our paper. Besides we are grateful to S. Ida for helpful and stimulating discussions during the period in which this work was performed; to A. Stern for interesting informations and to E. Recami for some useful comments.

References

Aumann H.H., Gillette F.C., Beichman C.A., et al., 1984, ApJ 278, L23
Beauge C., Ferraz-Mello S., 1993, Icarus 103, 301
Beauge C., Aarseth S.J., Ferraz-Mello S., 1994, MNRAS 270, 21
Beckwith G., Sargent A.I., 1993, In: Levy E.H., Lunine J.I., (eds.) Protostars and Planets III. The University of Arizona Press, Tucson, p. 521
Brown M.E., Kulkarni S.R., Ligget T.J, 1997, ApJ 490, L119
Brunini A., Fernandez J.A., 1996, A&A 308, 988
Binney J., 1977, MNRAS 181, 735
Binney J., Tremaine S., 1987, Galactic Dynamics. In: Princeton Series in Astrophysics. Princeton University Press
Borderies N., Goldreich P., 1984, Cel. Mech. 32, 127
Chandrasekhar S., von Neumann J., 1942, ApJ 95, 489
Chandrasekhar S., 1943, ApJ 97, 255
Chandrasekhar S., 1968, Ellipsoidal figures of equilibrium. Yale University Press
Cochran A.L., Levison H.F., Stern S.A., Duncan M.J., 1995, AJ 455, 342
Cochran A.L., Levison H.F, Tamblyn P., Stern S.A., Duncan M.J., 1998, ApJ 503, L89 (see also SISSA preprint astro-ph/9806210)
Donner K.J., Sundelius B., 1993, MNRAS 265, 88
Duncan M.J., Quinn T., Tremaine S., 1988, ApJ 328, L69
Duncan M.J., Levion H.F., Budd S.M., 1995, AJ 110, 373
Edgeworth K.E., 1949, MNRAS 109, 600
Farinella P.A., Milani A., Nobili M., Valsecchi G.B., 1979, The Moon and the Planets 20, 415–421
Fernandez J.A., 1980, MNRAS 192, 481
Fernandez J.A., Ip W.H., 1984, Icarus 58, 109
Goldreich P., Tremaine S., 1979, ApJ 233, 857
Goldreich P., Tremaine S., 1980, ApJ 241, 425
Gomes R.S., 1995, Icarus 115, 47
Hills J.G., 1981, AJ 86, 1730
Holman M., Wisdom J., 1993, AJ 105, 1987
Horedt G.P., 1985, Icarus 64, 448
Hornung P., Pellat R., Borge P., 1985, Icarus 64, 295
Ida S., 1990, Icarus 88, 129
Ida S., Makino J., 1992, Icarus 98, 28
Ida S., Kokubo E., Makino J., 1993, MNRAS 263, 875
Ida S., Bryden G., Lin D.N.C., Tanaka H., 1999, submitted to ApJ, (preprint available from http://www.geo.titech.ac.jp/nakazawalab/ida/DG.html)
Jewitt D., Luu J.X., 1993, Nat 362, 730
Jewitt D., Luu J.X., 1995, AJ 109, 1867
Kokubo E., Ida S., 1998, Icarus 131, 171
Kuiper G.P., 1951, In: Hynek J.A. (ed.) Astrophysics: A Topical Symposium. McGraw Hill, New York, p. 357
Levison H.F., 1996, In: Rettig T.W., Hahn J.M. (eds.) ASP Conf. Proc. 107, Completing the Inventory of the Solar System. ASP, San Francisco, p. 173

Levison H.F., Duncan M.J., 1990, AJ 100, 1669
Levison H.F., Duncan M.J., 1993, ApJ 406, L35
Linden-Bell D., Pringle J.E., 1974, MNRAS 168, 603
Luu J., Marsden B.G., Jewitt D., et al., 1997, Nat 387, 573
Lyttleton R.A., 1936, MNRAS 97, 108
Malhotra R., 1993, Nat 365, 819
Malhotra R., 1995a, AJ 110, 420
Malhotra R., 1995b, SISSA preprint astro-ph/9504036
Melita M.D., Woolfson M.M., 1996, MNRAS 280, 854
Olsson-Steel D.I, 1988, A&A 195, 327
Oort J.H., 1950, Bull. Astr. Inst. Netherlands 11, 91
Palmer P., Lin D.N.C., Aarseth S.J., 1993, ApJ 403, 336
Patteron C.W., 1987, Icarus 70, 319
Pesce E., Capuzzo-Dolcetta R., Vietri M., 1992, MNRAS 254, 466
Quinn T.R., Tremaine S., Duncan M.J., 1990, ApJ 355, 667
Ruden S.P, Lin D.N.C., 1986, ApJ 308, 883
Smith B.A., Terrile R.J., 1984, Sci 226, 1421
Stern S.A., 1995, AJ 110, 856
Stern S.A., 1996a, A&A 310, 999
Stern S.A., 1996b, AJ 112, 1203
Stern S.A., Colwell J.E., 1997a, ApJ 490, 879
Stern S.A., Colwell J.E., 1997b, AJ 114, 841
Stewart G.R., Kaula W.M., 1980, Icarus 44, 154
Stewart G.R., Wetherill G.W., 1988, Icarus 74, 542
Tremaine S., 1990, In: Lynden-Bell D., Gilmore G. (eds.) Baryonic Dark Matter. Kluwer, Dordrecht, p. 37
Ward W.R., 1986, Icarus 67, 164
Ward W.R., 1997, ApJ 482, L211
Weidenschilling S.J., Davies D.R., 1985, Icarus 62, 16
Weidenschilling S.J., Spaute D., Davis D.R., Marzari F., Ohtsuky K., 1997, Icarus 128, 429
Weissmann P.R., 1995, ARA&A 33, 327
Weissmann P.R., Levison H.F., 1997, In: Stern S.A., Tholen D.J (eds.) Pluto and Charon. University of Arizona Press, Tucson, in press, 559
Wiegert P., Tremaine S., 1999, Icarus 137, 84
Yamamoto T., Mizutani H., Kadota A., 1994, PASJ, L5