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Cosmic ray acceleration and nonlinear relativistic wavefronts

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Abstract. The usual Fermi processes are not efficient enough to account for the high energy cosmic ray acceleration in AGNs, Jet hot spots, GRBs, and in other compact object environments. It is proposed to accelerate particles by large perturbations in the relativistic plasmas of compact object environments where the Alfvén velocity is close to the velocity of light. Relativistic wavefronts are far more efficient in the acceleration of particles; however, the theory of acceleration requires some new developments. In this paper, the essential points of the acceleration theory, as well as the main points of the theory of nonlinear relativistic wavefronts, are sketched. One important aspect of the physics of those nonlinear fronts is the large scale inertial effect produced by the cosmic rays, which makes the waves significantly dispersive.

Key words: acceleration of particles – Magnetohydrodynamics (MHD) – relativity – galaxies: active – gamma rays: bursts – stars: pulsars: general

1. Introduction

The origin of high energy cosmic rays is one of the main enigmas of modern astrophysics. Supernovae remnants cannot account for the cosmic ray spectrum beyond an energy of $10^5 GeV$, although it continues with almost the same power law isotropic distribution upto $10^{11} GeV$ with an index close to 3. The extragalactic origin is thus very likely, and the Auger project should be able to discriminate an extragalactic origin because of the GZK-effect. According to the GZK effect (Aharonian & Cronin 1994), a proton, with energy superior to the threshold for its interaction with the photons of the cosmological microwave background, undergoes a photo-pion production. This threshold is at $3 \times 10^{10} GeV$ and the energy of protons coming from an extragalactic source located at more than $100 Mpc$ decreases to the threshold energy, so that the Auger experiment should detect a bump in the distribution around the GZK-threshold (Aharonian & Cronin 1994). All the events at energy values greater than the GZK-threshold would come from the galactic environment. Few such events have been detected yet, but their origin was not identified. The GZK-effect is accompanied by a neutrino emission that is also expected to be detected by the neutrino telescopes in the course of development (AMANDA and ANTARES). In fact the GZK-effect with the UV-bump of an AGN has a lower threshold ($10^7 GeV$) and would produce a neutrino flux which, if detected with enough angular resolution, would allow to identify AGNs as cosmic rays accelerators.

Indeed AGNs are the most promising accelerators of high energy cosmic rays because of the concentration of an intense magnetic field in a large volume, not only the nucleus itself but also possibly the jets, the hot spots and the extended lobes. Gamma ray bursts and topological defects are the other candidates (Gibbs 1997, Bonazzola & Peter 1997). Large shocks between galaxy clusters have also been considered recently (Norman et al. 1995). Apart from the topological defects, these candidates have been selected for the high value of their product BR, B being the intensity of the magnetic field in a region of size R; which means a large electromotive force. Indeed the capability of these accelerators is limited by their size since a charge particle that acquires a Larmor radius comparable to the size of the accelerator can no longer be accelerated and it escapes from the source, which leads to the energy limit:

$$\epsilon_{max} = ZeBR \simeq 10^{12} Z \frac{B}{1G} \frac{R}{1pc} GeV \ . \qquad (1)$$

AGNs are good candidates for the production of protons upto $10^{12} GeV$ either in the black hole environment, because we can expect a magnetic field of $10^4 G$ within 10 gravitational radii, i.e. $10^{-4} pc$ for a black hole of 10^8 solar masses, or in their hot spots, because the magnetic field is about $10^{-4} G$ in $10 kpc$.

But considering ordinary Fermi acceleration processes, a more severe limitation rises because their time scale is not short enough to reach the previous limit before the particles leave the accelerator with the background flow (Henri et al. 1999). Therefore the main goal, adressed in this paper, is to find a more efficient version of the Fermi acceleration capable of reaching an energy closer to the size limit. In this paper, it is proposed to assume that regions in the environment of compact objects, or in jets and hot spots, where the mass and pressure is dominated by the cosmic ray population, exist. Furthermore, these regions are supposed to be magnetically confined, so that the relativistic plasma has an Alfvén velocity close to the velocity of light

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(Pelletier & Marcowith 1998). The relativistic Alfvénic disturbances are far more efficient in the acceleration of particles than the non-relativistic ones. But they require a different theoretical treatment because the expansion in V_A/C cannot be done as in the case of the ordinary Fermi processes. The assumption of the existence of such regions is reasonable in AGNs, GRBs and Pulsar nebulae.

The shape of the cosmic ray distribution in each acceleration region turns out to be less important in comparison to the high energy cut off, for it will be shown that the cosmic ray power law distribution could result from the accumulation of the contributions of all the acceleration regions. In other words, the power law spectrum could result from the distribution of the products BR, in the each extragalactic object itself and/or in the Universe.

This work presents two aspects: one is the sketch of the kinetic theory of particle acceleration in relativistic disturbances, the other is the theory of the nonlinear relativistic wavefronts. The paper is organized as follows. In Sect. 2, the scheme of Fermi acceleration is presented and the deep difference between the non-relativistic regime (first and second order) and the relativistic regime is emphasized. In Sect. 3, the astrophysical consequences, such as the efficiency of the acceleration process, the spectrum and the neutrino emission, are adressed. Sect. 4 is devoted to a brief derivation of the nonlinear relativistic wavefronts and the relativistic solitons as a particular case, with a presentation of their main properties following previous work by Pelletier & Marcowith (1998, hereafter PM). The generation of the parallel electric field is presented in Sect. 5 and it is shown how the fronts can be modifyied by kinetic effects to become collisionless shocks, and how they are seen by making the plasma then shine as they pass through. The acceleration of low energy particles (injection problem) is solved through the intensification of the compression effects close to equipartition, as predicted in (PM). Since no Fermi type acceleration works if the pitch angle scattering of the suprathermal particles off wavefronts is not efficient, the important issue of the scattering is examined in Sect. 6, which opens some nontrivial problems of nonlinear dynamics, postponed for future works. The main points are summarized in the concluding Sect. 7 together with a discussion of the astrophysical impact of these results.

2. Relativistic Fermi acceleration

By "relativistic Fermi acceleration" I mean the acceleration of suprathermal particles by relativistic MHD disturbances that propagate at a velocity close to the velocity of light. Those disturbances are very likely in the form of delocalized Alfvén waves or localized nonlinear Alfvén wavefronts, because the magnetosonic perturbations are more damped in astrophysical plasmas that have a pressure comparable to the magnetic pressure. The general Alfvén velocity, which includes the displacement current effect, is defined by (see PM):

$$V_* = \frac{C}{\sqrt{1+\frac{e+P}{2P_m}}} \ , \tag{2}$$

where e is the internal energy-mass density, P the plasma pressure and P_m the magnetic pressure. In non-relativistic plasmas, $e \simeq \rho C^2$ and is usually much larger than P_m and one has $V_* \simeq V_A = B_0/\sqrt{\mu_0 \rho}$. In an ultrarelativistic plasma, $e \simeq 3P$ and

$$V_* = \frac{C}{\sqrt{1+\frac{2P}{P_m}}} \ , \tag{3}$$

so that the propagation velocity is relativistic for a confined plasma that has $P < P_m$, since V_* is larger than the relativistic sound velocity $C/\sqrt{3}$. A degeneracy occurs at pressure equipartition since the Alfvén and fast magnetosonic waves propagate at the same speed as the sound waves (slow magnetosonic waves) along the magnetic field.

2.1. The acceleration scheme

The interaction with forward waves can be presented in the following way:

$$(p_1,\mu_1) \overset{L_{\beta_*}}{\longmapsto} (p_1',\mu_1') \overset{S}{\longmapsto} (p_2',\mu_2') \overset{L_{\beta_*}^{-1}}{\longmapsto} (p_2,\mu_2) \tag{4}$$

The Lorentz transform L_{β_*} is such that

$$p_1' = \gamma_* \left(1-\beta_*\mu_1\right) p_1 \tag{5}$$

$$\mu_1' = \frac{\mu_1-\beta_*}{1-\beta_*\mu_1} \tag{6}$$

The scattering S does not change the energy, the pitch angle is changed randomly: $p_2' = p_1'$ and $\mu_1' \mapsto \mu_2'$, with a conditional probability density $K^+_{\Delta t'}\left(\mu_2'|\mu_1'\right)$ during a time $\Delta t'$ measured in the wave-frame. This kernel is normalized such that

$$\int_{-1}^{+1} K^+_{\Delta t'}\left(\mu_2'|\mu_1'\right) d\mu_2' = 1 \ , \ \forall \mu_1' \ . \tag{7}$$

Moreover

$$\lim_{\Delta t' \to 0} K^+_{\Delta t'}\left(\mu_2'|\mu_1'\right) = \delta\left(\mu_2'-\mu_1'\right) \ . \tag{8}$$

Then the reversed Lorentz transform gives the momentum after the interaction in the plasma frame:

$$p_2 = \gamma_* \left(1+\beta_*\mu_2'\right) p_1' \tag{9}$$

$$\mu_2 = \frac{\mu_2'+\beta_*}{1+\beta_*\mu_2'} \tag{10}$$

Because μ_1 and μ_2' can take any value between -1 and $+1$, some particles can undergo a large energy gain by a factor of order γ_*^2:

$$p_2 = \gamma_*^2 \left(1-\beta_*\mu_1\right)\left(1+\beta_*\mu_2'\right) p_1 \tag{11}$$

Then, the energy jump in a progressive wave is

$$\Delta p^+ = \beta_* \frac{\mu_2-\mu_1}{1-\beta_*\mu_2} p_1 \tag{12}$$

The jump is large for most particles because the Lorentz transform concentrates the pitch-angle α_2 in a narrow cone of half-angle $\alpha_* = arcsin\left(1/\gamma_*\right)$. This is similar to the inverse Compton effect in the Thomson regime.

If only forward waves (linear or nonlinear) are considered, then they would tend to isotropize the tail of the suprathermal distribution with respect to the wave-frame. Fermi acceleration works only if both forward and backward waves come into play. If a mixture of forward and backward waves (in proportion a^+ and a^- respectively, with $a^+ + a^- = 1$) is considered, then the distribution function is changed during Δt according to the following law:

$$\begin{aligned} &2\pi p^2 f(p,\mu,t) \qquad (13)\\ &= \int_{-1}^{+1} d\mu_1 \int_0^\infty dp_1 \left[K^+_{\Delta t}(\mu|\mu_1)\, a^+ \delta\left(p - p_1 - \Delta p^+\right)\right.\\ &\left. + K^-_{\Delta t}(\mu|\mu_1)\, a^- \delta\left(p - p_1 - \Delta p^-\right)\right] 2\pi p_1^2 f(p_1,\mu_1,t-\Delta t) \end{aligned}$$

The probability density $K'^-_{\Delta t'}$ in the backward frame is not significantly different from the forward probability density. However the probability density $K^-_{\Delta t}$ differs from $K^+_{\Delta t}$ because of the different composition with the Lorentz transformations.

$$K^\pm_{\Delta t}(\mu|\mu_1) = \frac{1}{\gamma_*^2 (1 \mp \beta_* \mu)^2} K'^\pm_{\Delta t'}(\mu'|\mu'_1) \ . \qquad (14)$$

This expression (13) holds if the interaction of a particle with a forward wave is independent from its interaction with a backward wave. This is true because the interaction occurs under a resonance condition that differs for forward and backward waves (see Sect. 6 for details).

Several types of interaction can occur.

– i) Mirror reflection:

$$\begin{aligned} &K'^\pm_{\Delta t'}(\mu'|\mu'_1)\\ &= \delta(\mu' - \mu'_1)\left(1 - \frac{c\Delta t'}{\bar{l}'}\right) + \frac{c\Delta t'}{\bar{l}'}\delta(\mu' + \mu'_1) \ , \qquad (15) \end{aligned}$$

where $\bar{l}'$ is the mean free path of the particle colliding with magnetic perturbations. This was the assumption used by Fermi in his historical paper 1949. However he assumed that the perturbations (clouds) were propagating in every direction, whereas Alfvén perturbations propagate along the field lines.

– ii) Fast random scattering:

$$K'^\pm_{\Delta t'}(\mu'|\mu'_1) = \delta(\mu' - \mu'_1)\left(1 - \frac{c\Delta t'}{\xi}\right) + \frac{c\Delta t'}{\xi}\frac{1}{2} \qquad (16)$$

That kind of scattering is relevent to describe the interaction with large localized wavefronts of width ξ (see Sect. 6).

– iii) Angular diffusion:

$$K'^\pm_{\Delta t'}(\mu'|\mu'_1) = \frac{1}{\sqrt{2\pi}\sigma_{\mu'_1}} exp\left[-\frac{(\mu' - \mu'_1)^2}{2\sigma^2_{\mu'_1}}\right] \qquad (17)$$

When a diffusion process occurs, it is entirely governed by the pitch angle frequency in the wave-frame:

$$\nu'_s \equiv \frac{\langle \Delta\alpha'^2 \rangle}{\Delta t'} \ . \qquad (18)$$

and $\sigma^2_{\mu'_1} = \nu'_s \left(1 - \mu_1'^2\right)\Delta t'$. This diffusive approximation is used in the usual 1st and 2nd Fermi processes since the seventies (Jokipi 1966, Melrose 1968). This is based on the so-called quasi-linear theory leading to a Fokker-Planck equation. This cannot be used in the relativistic regime.

– iv) Anomalous diffusion:

$$K'^\pm_{\Delta t'}(\mu'|\mu'_1) = \alpha \frac{|\mu' - \mu'_1|^{\alpha-1}}{(\nu'_a \Delta t')^\beta} g\left(\frac{|\mu' - \mu'_1|^\alpha}{(\nu'_a \Delta t')^\beta}\right) \ ; \qquad (19)$$

where the function g is normalized to unity:

$$\int g(y)dy = 1 \ . \qquad (20)$$

If g has the momentum $\int |y|^{2/\alpha} g(y)dy = s < \infty$, then the anomalous diffusion is such that:

$$\left\langle (\Delta\mu')^2 \right\rangle = s\,(\nu'_a \Delta t')^{2m} \ , \qquad (21)$$

with $m = \beta/\alpha$ and the process is said to be subdiffusive if $m < 1/2$, diffusive for $m = 1/2$ and superdiffusive for $m > 1/2$. To describe pitch angle scattering of particles in magnetic disturbances, the usual diffusion description does not work at all pitch angles, because the quasi-linear pitch angle frequency vanishes at 90^0, indicating that a more nonlinear theory must be developed. The problem was pointed out by R. Schlickeiser (1994) who proposed a linear remedy by introducing a damping broadening of the resonance. Another linear possibility to cure this problem is to take into account the inertial dispersion effect. In fact, passing through 90^0 is essential for Fermi acceleration to work. This issue will be seen in Sect. 6.

2.2. The non-relativistic second and first order processes

The concept of first and second order Fermi processes holds only in the case of non-relativistic MHD disturbances, when the Alfvén velocity is much smaller than the velocity of light: $V_A \ll C$. Indeed these processes are derived by expanding the "collision" operator to the second order in powers of β_*; especially the delta-function in (13) is expanded to the second order with respect to the momentum jumps, leading to a Fokker-Planck operator. When the cosmic rays are not carried by a decelerating flow, like in shock, the acceleration process is of the second order and a pitch angle diffusion in wave-frame leads to energy diffusion in plasma frame.

$$\Delta p_0 \ = \ \gamma_* \left(\Delta p'_0 + \beta_* \Delta p'_\parallel\right) \qquad (22)$$

$$\Delta p_\parallel \ = \ \gamma_* \left(\Delta p'_\parallel + \beta_* \Delta p'_0\right) \qquad (23)$$

with $\Delta p'_0 \simeq 0$ and $\Delta p'_\parallel \simeq p'_0 \Delta\mu'$, and the following diffusion coefficients are then derived, knowing that $\Delta t = \gamma_* \Delta t'$:

$$\begin{aligned} \Gamma_\parallel \ &\equiv \ \frac{\left\langle \Delta p_\parallel^2 \right\rangle}{2\Delta t}\\ &= \ \frac{1}{2}\gamma_* p_\perp^2 \left(1 + \beta_*^2 \mu^2 - 2\beta_* \left(a^+ - a^-\right)\mu\right)\nu'_s \ ; \qquad (24) \end{aligned}$$

$$\Gamma_0 \ \equiv \ \frac{\left\langle \Delta p_0^2 \right\rangle}{2\Delta t} = \beta_*^2 \Gamma_\parallel \ ; \qquad (25)$$

the result (24) is obtained by averaging over pitch angle jumps with the Gaussian probability distribution (17). The last result clearly shows the superiority of relativistic waves having $\beta_* \simeq 1$, compared to non-relativistic ones that have $\beta_* \ll 1$, for the purpose of particle acceleration. However, the expansion in power of β_* cannot be done; in particular there is no longer a scale separation between a fast isotropization and a longer energy diffusion, the energy diffusion being as fast as the pitch angle diffusion.

In a decelerating flow, there is a well known first order contribution to the acceleration. It can be described by the inertial force in the waveframe $F'_j = -p'_i \nabla_i u_j$, whose angular average power is as follows, expressed in the nonrelativistic front-frame:

$$P_a = -\frac{1}{3} p v div \mathbf{u} \,. \tag{26}$$

It is often said that the first order process is more efficient than the second order process because at each shock crossing the relative gain is such that $\Delta p/p \sim (u_1 - u_2)/c$ whereas the second order process leads to V_A^2/c^2; this is not plainly true. Schlickeiser et al. (1993) realized that the second order process is never negligible behind a shock. Jones (1994) argued that first and second Fermi processes are not very different in terms of efficiency, the interest of the first order acceleration at adiabatic shocks is to give universal power law distribution downstream. This can be seen as follows (see Henri et al. 1999 for details). The rate of change of the energy of a particle crossing a non-relativistic shock many times is given by:

$$\frac{\langle \Delta p \rangle}{\Delta t} = \frac{r-1}{3t_r} p \,, \tag{27}$$

where r is the shock compression ratio and t_r the residence time of a relativistic particle behind the shock. This depends on the diffusion coefficient D with $t_r = 2D/u_2^2$, u_2 being the downstream flow speed (actually in a more realistic theory one distinguishes between the usual compression ratio r and the compression ratio r_s experienced by the scattering center flow (Bell 1978), which is the correct one in the calculation the particle distribution, leading to harder spectra. See Vainio & Schlickeiser (1999) for the most recent development of this issue). For $D \simeq D_\parallel \simeq \frac{1}{3}\frac{v^2}{\nu_s}$, the first order acceleration time is thus

$$t_1 = t_r \sim \frac{c^2}{u_2^2} \nu_s^{-1} \,, \tag{28}$$

whereas the second order acceleration time is:

$$t_2 \sim \frac{c^2}{V_A^2} \nu_s^{-1} \,. \tag{29}$$

These two time scales are not significantly different because u_2 is often on the order of V_A. The result is simple enough. At each scattering there is a small energy diffusion; the energy jump is larger when the particle crosses the shock front, but these crossings are rare, since their frequency $\nu_c \sim (u_2/c)\nu_s$. When it can work at quasi perpendicular shocks, the first order acceleration becomes more efficient because t_1 is changed into $t_{1\perp} = t_1 (\nu_s/\omega_s)^2$ with transverse diffusion.

The results for t_1 and t_2 indicate of course that first and second order Fermi acceleration tend towards the same maximum efficiency when V_A and u_2 are close to the velocity of light, which once more emphasizes the interest in working with relativistic waves.

2.3. The efficiency of the relativistic Fermi acceleration

The efficiency of relativistic acceleration has two origins: first, as seen in Sect. 2.1, the momentum variation during scattering is much larger than in nonrelativistic waves; second, as seen in Sect. 2.2, the acceleration rate reaches its highest possible value which is given by the pitch angle frequency. Therefore relativistic waves lead to an acceleration time close to the pitch angle scattering time and this time becomes closer (but still longer) to the gyro-period T_g of the considered particle for large wave amplitude. Acceleration at relativistic shocks with non-relativistic Alfvén velocity was analysed by Kirk & Schneider (1988). They calculated the particle distribution by assuming a pitch angle diffusion of the particles in the scattering center frame, the acceleration having been obtained with a single shock crossing. Indeed a detailed discussion of the possibility of accelerating particles by repeated crossings of a relativistic shock was just published by Gallant & Achterberg (1999). They have shown that the acceleration is efficient at the first Fermi cycle with an energy gain by a factor γ_s^2, γ_s being the Lorentz factor of the shock, and weak for the next cycles. The reason is that the scattered particles that come back upstream are distributed in a narrow cone of angle $1/\gamma_s$ in momentum space and they should be scattered out of this cone in order to undergo another acceleration. But the shock front reaches them before they have time to be significantly scattered. Thus relativistic shocks accelerate particles mostly in a single step with an energy gain by a factor γ_s^2.

Large amplitude relativistic waves are good candidates for the achievement of the expected efficiency of the acceleration process to explain high energy cosmic rays with an acceleration time scale $t_{acc} = AT_g(\epsilon)$, with $A \sim 10$. If the very high energy cosmic rays are not accounted for by these physical assumptions, then exotic phenomena such as topological defects could be invoked.

3. Astrophysical consequences

The detailed theory, not completed in this paper, must develop both the case of delocalized waves and the case of localized fronts. Some preliminary results will be presented for both cases. However the general statements presented in Sect. 2 allow for the anticipation of some astrophysical consequences already, that will motivate further theoretical developments.

3.1. Relativistic expansion and head on front collisions

The efficiency gain with delocalized relativistic waves is interesting per se; however spectacular acceleration could be achieved with head on collisions of relativistic fronts. This is somehow a rather primitive concept of particle accelerator which consists in colliding heavy particles (here the relativistic solitons) with each other, thus producing new energetic particles. Indeed the head on collision of two solitons of bulk Lorentz factor γ_* produces the following energy amplification of protons

$$p_2 \sim \gamma_*^4 p_1 \tag{30}$$

within a time of a few gyro-periods.

Is it reasonable to expect such head on collisions in Nature? Yes, if we believe in the existence of relativisitic expansion flows. Any relativistic expansion of a relativistic plasma emating from the environment of a compact object (single or twin in coalescence) will produce both forward fronts because of perturbations at the source and backward fronts because of the interaction with the ambient medium.

A forward front observed in a relativistic flow of bulk reduced velocity β_J has a reduced velocity:

$$\beta_+ = \frac{\beta_J + \beta_*}{1 + \beta_J \beta_*} , \tag{31}$$

whereas for a backward:

$$\beta_- = \frac{\beta_J - \beta_*}{1 - \beta_J \beta_*} . \tag{32}$$

When the expansion is superAlfvénic, they are both seen to advance, but with different speed, and of course, they collide. Such events could be expected in relativistic extragalactic and galactic jets, and very likely in Gamma Ray Bursts and in pulsar nebulae where wisps are observed (Scargle 1969). Recent observations of gamma ray burts (Kulkarni et al. 1999) strongly suggest that the relativistic flow is beamed. VLBI observations of extragalactic jets (Guirado et al. 1995) revealed motions of relativistic knots that could possibly lead to front collisions; events of that kind are currently analysed in several sources.

Relativistic motions of such fronts could occur in hot spots as well, even if there are no more relativisitic bulk motions in them because they result from the terminal shocks of the jets. Indeed disturbances with relativistic motions will develop if they are dominated by the cosmic ray pressure, which is what could make hot spots interesting sites of very high energy cosmic ray generation. Extended lobes could also have the same properties if they are still magnetically confined.

To get these cosmic ray generation events, ideal solitons are not necessary; relativistic fronts are sufficient, even if they are destroyed after the collision. The only advantage of the soliton is that it can survive after a collision and the ground conditions, in particular the magnetic topology, are restored in the plasma after it passed away.

3.2. Spectrum

Assuming that the acceleration process is efficient enough to locally reach a maximum energy that is a significant fraction of the accelerator size limit such that $\epsilon_m = aZeBR$ (with $a \sim 0.1$, say) and assuming that the integrated energy distribution results from the distribution of the product BR, and, if the local energy distribution function $f(\epsilon, \epsilon_m(z))$ is not a power law, but any self-similar function of the following form:

$$f(\epsilon, \epsilon_m(z)) = \frac{n_*}{\epsilon_m(z)} h\left(\frac{\epsilon}{\epsilon_m(z)}\right) , \tag{33}$$

thus the probability density of finding a particle of energy ϵ in the global flow of cross section $S(z)$ is given by

$$\rho(\epsilon) = \int dz n_* S \frac{1}{\epsilon_m(z)} h\left(\frac{\epsilon}{\epsilon_m(z)}\right) . \tag{34}$$

Setting $v = \epsilon/\epsilon_m$, for a relativistic flow that keeps most of its mass $n_* S = constant$, one gets an energy distribution of cosmic rays:

$$\rho(\epsilon) = n_* S \int |\frac{d\epsilon_m}{dz}(v)|^{-1} h(v) \frac{dv}{v} . \tag{35}$$

Assuming a field distribution such that $B \propto R^{-m}$ ($1 < m < 2$ is expected in a jet) and $dR/dz \propto R^{-n}$ for the flow (for a conical jet $n = 0$, a confined jet $n > 0$, a widening flow $n < 0$) and supposing the same escape probability along the flow, the energy distribution of the cosmic ray leaving the source is

$$p(\epsilon) \propto \epsilon^{-\frac{m+n}{m-1}} . \tag{36}$$

For instance, a confined jet, such that $m = 2, n = 0.5$, gives $\rho(\epsilon) \propto \epsilon^{-2.5}$; a conical jet with $m = 1.5$ gives ϵ^{-3}. In the case of a dominant poloidal field ($m \simeq 2$) and thus weak collimation ($n \ll 1$), the spectrum index is close to 2 since $\propto \epsilon^{-(2+\delta)}$, where $\delta = m - 2 + n \ll 1$; and the product BR varies over many decades (close to R^{-1}).

3.3. Neutrino emission by AGNs

Compared to proton-proton collisions, the photo-production of pions is expected to be more efficient in the production of neutrinos in AGNs through the $\Delta - resonance$ (the so-called GZK effect):

$$p + \gamma \to \Delta^+ \begin{array}{l} \stackrel{(2/3)}{\to} p + \pi^0 \to p + \gamma + \gamma \\ \\ \stackrel{(1/3)}{\to} n + \pi^+ \to ... \to p + e^+ + e^- \\ \qquad + \nu_e + \bar{\nu}_e + \nu_\mu + \bar{\nu}_\mu \end{array} \tag{37}$$

For a head-on collision, the threshold energy of the proton is

$$\epsilon_{th} = \frac{m_\Delta^2 - m_p^2}{4\epsilon_\gamma} ; \tag{38}$$

and the cross section $\sigma_{p\gamma} \simeq 5.4 \times 10^{-28}\, cm^2$. The process can occur in the AGNs with UV-photons where the threshold energy is on the order of $10^{16}\, eV$, since it was shown in the previous subsection that the protons can reach much higher energies.

A neutrino emission is thus possible (Protheroe & Stanev 1992) and

$$\begin{aligned} L_\nu &= \int\int \epsilon_\nu n_{ph} \sigma_{p\gamma} c f_p d^3p d^3V \\ &\simeq \frac{\bar{\epsilon}_\nu}{\bar{\epsilon}_{ph}} L_{UV} \sigma_{p\gamma} \frac{R}{3} n_p(> \gamma_{th}) . \end{aligned} \tag{39}$$

The neutrino luminosity can be compared to the UV-luminosity of an accretion disk having a numerical density of particles in the inner region n_*. The neutrino luminosity depends on the density of protons of momentum larger than p:

$$n(> p) = \chi n_* (\frac{p}{p_0})^{1-\eta} , \tag{40}$$

where χ is the fraction of proton number above the threshold p_0. The following estimate was obtained (Henri et al., 1999):

$$\frac{L_\nu}{L_{UV}} \sim 10^{14-8\eta} . \tag{41}$$

For $\eta = 2$ there are enough protons above the high threshold ($\gamma_{th} \sim 10^7$) to have $L_\nu \sim 10^{-2} L_{UV}$, whereas for $\eta = 3$ the ratio is only 10^{-10}! If the estimate of the neutrino flux is based on the assumption of a local power law cosmic ray spectrum in the AGNs, the success of the future neutrino astronomy of AGNs would be unpredictable... But if the global power law spectrum results from a superposition of local "pile-up" distributions, as proposed in the previous subsection, the hope of detecting the neutrino emission of AGNs is greater, since an estimate comparable to the case $\eta = 2$ could be expected.

The GZK-effect occurs also during the propagation of cosmic rays in the intergalactic medium where they collide with the cosmological black body. The threshold is of the order $3 \times 10^{19}\, eV$ and cosmic rays of larger energy cannot come from sources beyond 100 Mpc (see Aharonian & Cronin 1994).

4. The nonlinear relativistic wavefront

The interest of relativistic MHD waves has been emphasized in the previous sections. Linear waves are of course well known, but the localized nonlinear relativistic waves, in particular relativistic solitons, are not well known. Such relativistic fronts have been studied recently (PM). It was argued that their dissipation by resonant interactions would lead to an efficient acceleration process, but the kinetics of suprathermal particles in these fronts were not adressed. In this section, the main properties are recalled and their derivation is indicated without entering into the so-called "reductive perturbation expansion" technique; some results are even indicated as non-perturbative results to stress their robustness.

Another important aspect of the kinetic corrections of these solitons is their modification as relativistic collisionless parallel shocks. The concept of collisionless parallel shock is not easily grasped, but must be addressed in astrophysics, especially in the physics of relativistic jets and gamma ray bursts. The following approach allows for its precise ground.

4.1. Inertial effect in a cosmic ray plasma

Localized nonlinear wavefronts often result from a balance between nonlinear steepening (wave braking) and dispersion, which builds solitons or solitary waves. In standard MHD, the waves are not dispersive and a dispersion effect is obtained on a small scale by taking into account the inertial correction (often called "Hall effect"). The relative size of the correction for a mode of wavelength λ is on the order of r_0/λ, where $r_0 \equiv V_A/\omega_{ci}$. Cosmic rays that have much larger Larmor radii than r_0 couple to the MHD of the thermal medium through resonant interactions with MHD waves. However, in a plasma containing proton cosmic rays, that is for large scale dynamics, more cosmic rays participate in the MHD and the radius r_0 is replaced by an energy dependent radius r_* that can be much larger than r_0 (PM). It could be thought that r_* is simply r_0 multiplied by the averaged Lorentz factor of the cosmic ray population, $\bar{\gamma} \equiv \langle \epsilon \rangle / mc^2$. This is the result that is found with a multi-fluid description. But the evaluation from kinetic theory (with Vlasov equation) gives a different estimate: $r_* = \left(\langle \gamma^2 \rangle / \bar{\gamma} \right) r_0$.

So for a given magnetic field, a relativistic plasma has a much larger inertial effect, because of its content of particles with high relativistic mass. The inertial effect is described with the generalized Ohm's law which, in relativistic MHD, reads:

$$F_{\mu\nu} u^\nu = \eta J_\mu + \frac{1}{nq_p} F_{\mu\nu} J^\nu . \tag{42}$$

The general dispersion relation of Alfvén waves, including relativistic populations and inertial effects has been derived by Pelletier and Marcowith (PM):

$$\omega^2 \left(1 + \sum_a \frac{e_a + P_a}{2P_m} \left(1 \pm \chi_a(\omega) \right) \right) - k_\parallel^2 C^2 = 0 ; \tag{43}$$

where e_a is the energy-mass density of the population labeled by "a" (thermal or relativistic electrons, thermal or relativistic protons) and P_a the pressure; the sign $\pm$ relates to right or left polarisation. For a non-relativistic population, the inertial correction reads:

$$\chi_a(\omega) = sgn(q_a) \frac{\omega}{\omega_{ca}} , \tag{44}$$

where ω_{ca} is the cyclotron pulsation; of course the dominant contribution comes from the proton population (the heaviest). For relativistic populations, the fluid theory leads to $\chi_a(\omega) = sgn(q_a) \bar{\gamma} \omega / \omega_{ca}$ whereas the kinetic theory with Vlasov equation leads to the correct result:

$$\chi_a(\omega) = sgn(q_a) \frac{1 + 5\beta_*^2}{4\beta_*^2} \frac{\langle \gamma^2 \rangle}{\bar{\gamma}} \frac{\omega}{\omega_{ca}} . \tag{45}$$

Notice that the inertial effect vanishes at this order for a pure electron-positron plasma and the next order corrections need to be taken into account (as done in PM). Relativistic plasma dominated by the electron component (i.e. more massive than the protons) can also be envisaged in some situations. Note also that the cosmic rays can dominate the inertial effect even in a non-relativistic plasma like the interstellar medium or stellar winds, because even if the cosmic rays contribute only negligibly to the mass ($n_* \bar{\gamma} \ll n_{th}$, n_* being the numerical density of relativistic protons, and n_{th} the numerical density of thermal protons), they can bring a major contribution to the inertial effect if $n_* \langle \gamma^2 \rangle / \bar{\gamma} \gg n_{th}$. At scales between r_0 and r_*, cosmic rays are not statistically coupled to MHD, they participate at scales beyond r_* and a modification of the propagation through the dispersive effect stems from the passage to this new dynamical regime. But in this paper, only a plasma dominated by the relativistic protons is considered.

The fact that the cosmic rays produce an inertial scale $r_* \sim \left(\left\langle\gamma^2\right\rangle/\bar{\gamma}\right) r_0$ far greater than the usual one (r_0) is of great importance, because this means that localized large perturbations can be maintained at observable scales. Also, shocks giving entropy to the cosmic ray population cannot have a front width smaller than this scale r_*; they can also display oscillations due to the inertial effect. Thus, a typical time scale (lower bound) for this MHD is $\tau_* \equiv r_*/V_* = T_g(\left\langle\gamma^2\right\rangle/\bar{\gamma})$.

One exemple of localized nonlinear perturbations is the well-known family of MHD solitons (Mjölhus 1976, Roberts 1985, Kaup & Newell 1978). In those solitons, the nonlinear steepening of the wavefront is balanced by the inertial dispersion effect. Strictly speaking, it is difficult to assert that there are such MHD solitons in Nature, because it is an ideal concept based on fully integrable conservative PDEs. So when excitation and dissipation are taken into account, there are no more solitons but solitary waves, that do not have the same stability properties as ideal solitons. It is still interesting to start with a soliton solution of a problem and then introduce corrections in order to describe a more realistic localized nonlinear wave, even possibly some kind of shock. In the purpose of cosmic rays acceleration, the emphasis is more on nonlinear relativistic fronts than on ideal solitons, especially when they collide. Ideal solitons are not affected by collisions, but of course real solitary waves are affected by collisions, mostly because of the dissipative effects. They are however rarely destroyed when the dissipation remains smooth. Stability against transverse perturbations is also an important issue that deserves further investigations.

4.2. Relativistic MHD

Relativistic fronts are studied with the relativistic MHD. By "relativistic MHD", I understand a fluid description of fully electromagnetic disturbances such that the electromagnetic interaction with matter is mostly magnetic in the rest-frame of each disturbance (delocalized or localized, forward or backward wave). I consider only fronts that have a thickness smaller than their transverse size; thus I will analyse 1D relativistic dynamics only. The 1D relativistic MHD-system is written for the 4-specific momentum (u^0, u^1, u^2, u^3) (the velocity times the Lorentz factor of the flow, also called "unitary 4-velocity"), coupled with the transverse electromagnetic wave described by its reduced magnetic component $\mathbf{b} : (b_1, b_2)$; the transverse flow is described by $\mathbf{u}_\perp : (u^1, u^2)$ and the longitudinal flow due to compression is $u \equiv u^3$, and $u^0 = (1 + u^2 + \mathbf{u}_\perp^2)^{1/2}$.

In the front-frame, $u = u_* + \tilde{u}$ with $u_* = -\gamma_*\beta_*$ for a forward front, the compression is supposed either quasi static (off equipartition) or a wave (close to equipartition) calculated at second order in $\tilde{u}$, the system reads

– Parallel motion:

$$\frac{8}{3}u^0 P\partial_t u + \left(1 - 2u^2\right)\frac{\partial P}{\partial u}\partial_x u + P_m \partial_x |b|^2 = 0 \, . \tag{46}$$

– The transverse motion:

$$2u^0 P\partial_t \mathbf{u}_\perp + 2\partial_x\left(uP\mathbf{u}_\perp\right) = P_m \partial_x \mathbf{b} \, . \tag{47}$$

– Generalized transverse Ohm's law:

$$\mathbf{u}_\perp = u\mathbf{b} - \alpha \mathbf{e}_x \times \partial_x \mathbf{b} - \nu_m \partial_x \mathbf{b} \, , \tag{48}$$

where the coefficient α measures the strength of the inertial effect, and ν_m represents the magnetic diffusivity. The system is closed by inserting a barotropic law $P(\tilde{u})$. In this frame, because the magnetic pressure increases the velocity when $P < P_m$, and decreases it when $P > P_m$ (see later on), solitary waves exist only for $|b|^2 < 1$. It is therefore suitable to calculate the solitary wave in a perturbative theory, even if they can still exist for an amplitude $b_0 < 1$ but close to unity.

A localized wavefront undergoes an exponential decay, and some results can be derived from the asymptotic conditions in the linear approximation. The asymptotic solution is of the form, that involves four parameters a priori:

$$\mathbf{b} = b_0 e^{-a|y|}\left[\mathbf{e}_1 cos\left(\kappa y - \Omega t\right) \pm \mathbf{e}_2 sin\left(\kappa y - \Omega t\right)\right] \, . \tag{49}$$

with $y \equiv x - vt$. From Eqs. (47) and (48), two relations can be found between these parameters:

$$\gamma_* v = -2\alpha_0 \kappa \, , \tag{50}$$

that relates κ with the nonlinear modification of the soliton velocity; and the nonlinear oscillation is related to the width (a) and the nonlinear velocity (κ) such that

$$\gamma_* \Omega = \alpha_0\left(a^2 + \kappa^2\right) \, , \tag{51}$$

where $\alpha_0 \equiv \alpha/\gamma_*\beta_*$.

However the relations of the width and the velocity with the amplitude is derived from the nonlinear theory. Off equipartition, the reductive perturbation expansion method allows for the derivation of the relativistic DNLS-equation from the previous system. The properties of the DNLS-equation and its solitons are summarized in the next subsection.

4.3. Relativistic DNLS equations and properties

The simplest nonlinear equation is obtained by a perturbative method applied to the system (46)–(48) when off equipartition. The nonlinear compression $\tilde{u}$ is of the order of $|b|^2$ in the front-frame. This is the relativistic version of the so-called DNLS-equation (Derivative NonLinear Schrödinger equation): (Mjölhus 1976, Mio et al. 1976):

$$\gamma_* \partial_t b - \frac{1}{2\delta}\partial_x |b|^2 b + i\alpha_0 \partial_x^2 b = 0 \, . \tag{52}$$

The coefficient $\alpha_0 = \frac{4}{3}\beta_*^2\gamma_*^2$ which is equal to $2/3$ at equipartition. But the most important coefficient is δ:

$$\delta \equiv \sigma\frac{\sqrt{2}}{3}\frac{P - P_m}{P_m} \, ; \tag{53}$$

it measures the deviation to equipartition, and $\sigma = +1$ for forward propagation, -1 for backward propagation. This equation has been intensively studied (Kaup & Newell 1978, Mjölhus 1978, Spangler & Sheerin 1982, Kennel et al. 1988), for it is one of the most famous example of soliton equation. The solitons have a width ξ inversely proportional to the square of their amplitude, an exponential decay of their envelope such that

$$a = \zeta b_0^2 \ . \tag{54}$$

They do not propagate at the same speed; there is a nonlinear shift of their velocity proportional to the square of their amplitude, such that the larger solitons run faster than the weaker:

$$\kappa = \zeta' b_0^2 \ . \tag{55}$$

There exists a simple relation between the order one numbers ζ and ζ', ζ' being bounded between two values (see appendix). They can cross each other and have head on collisions without destroying themselves as long as dissipation effects are neglected. These soliton properties are interesting for some astrophysical phenomena. For instance, the GRBs light curve and afterglow (Meszaros & Rees 1997) is explained if the relativistic expansion is organized such that the larger disturbances are faster and are the first to interact with the ambient medium and this organization of the flow must be realized without significant energy lost during the expansion; the concept of a soliton helps to account for such behaviour. In pulsar nebulae, at $0.1\,pc$, transient phenomena probably triggered by an MHD instability have been observed and those wisps could be interpreted as relativistic solitons transfering energy from the pulsar wind to relativistic particles in the nebulae (Klein et al. 1996). Moreover Gallant & Arons (1994) argued that the electron-positron wind could be loaded by some amount of protons which would be efficiently accelerated also by the magnetic disturbances and the terminal shock.

4.4. Relativistic Hada's system

As can easily be seen, the DNLS-equation is not valid close to equipartition (δ small). It turns out that the compression effect becomes stronger and the scaling such that $\tilde{u}$ is of the order of b instead of $|b|^2$. Hada derived the correct system (Hada, 1993) with the appropriate reductive expansion method for the case of a non-relativistic plasma. This was done for relativistic plasma in PM and can be derived from the system (46)–(48). The relativistic system reads:

$$\frac{4}{3}\gamma_* \partial_t \tilde{u} + \partial_x \left(\frac{2}{3}\tilde{u}^2 + \delta\tilde{u} + \frac{1}{2}|b|^2 \right) = 0 \tag{56}$$

$$\gamma_* \partial_t b + \partial_x (\tilde{u} b) + i\alpha_0 \partial_x^2 b = 0 \tag{57}$$

The interest of the system is to describe correctly the intensification of the compression effect of the Alfvén waves when approaching equipartition, which strengthens the absorption of energy by relativistic particles as claimed in PM and numerically studied in Baciotti et al. 1997. The kinetic understanding of that "anomalous" absorption will be explained in the next section and is related with the generation of a significant parallel component of the electric field.

5. Generation of a parallel electric field

The nonlinear behaviour of Alfvén waves produce a compression that intensifies tremendously when approaching equipartition as shown in PM. These compression effects generate parallel components of the electric field that are more intense than those induced by the usual weak turbulence theory situation. In the case of delocalized waves, this generation allows the Landau absorption to work and to accelerate low energy particles efficiently, thus solving the injection problem. In the case of solitons, it modifies them into soliton-shocks.

5.1. Electrostatic potential in the wavefront

The main nonlinear effect of the parallel Alfvénic perturbation is the localized magnetic pressure that produces a local compression of the plasma. Since the inertia of the electrons is negligible, their pressure variation along the field line must be balanced by a parallel electric force:

$$\partial_\parallel P_e = n_e q_e E_\parallel \ . \tag{58}$$

This effect can be described by the generalized Ohm's law in parallel direction. Since the electrons are in a quasi-static equilibrium, their pressure variation is such that $\delta P_e = T_e \delta n$. Now from the continuity equation, $\delta n/n_0 = -\tilde{u}/(u_* + \tilde{u})$. Since for a forward front $u_* < 0$, the solitary front exits only for $\tilde{u} < |u_*|$. For moderate perturbations, $\tilde{u}$ gives the electrostatic potential directly because

$$\frac{q_e V}{T_e} = -\frac{\tilde{u}}{|u_*|} \tag{59}$$

which leads to the formation of a potential well for the electrons and a potential barrier for the protons in the front-frame.

The protons, having a motion close to the front motion within an energy band fixed by the potential barrier, are reflected by the solitary wave. They are more numerous to be reflected ahead. This leads to the formation of a collisionless parallel soliton-shock.

Electrons experience a static potential well in the front-frame. They cross the well as long as they do not suffer energy loss by radiation. The radiation loss can lead to trapping a part of the electron population, those having a motion close to the front motion. Thus the soliton makes the plasma shine locally.

The previous features are typical of a localized BGK-mode (Bernstein et al. 1957). BGK-modes are exact nonlinear solutions of the 1D Vlasov-Poisson system, where the potential profile and the particle distributions are calculated self-consistently. It is particularly stimulating to envisage that such localized structures of phase-space can be generated in Nature by Alfvénic solitons. Similar structures were observed and analysed very recently in space (Goldman, 1999).

Such structures can be viewed as a kind of coherent and localized realization of the nonlinear Landau effect.

5.2. Nonlinear Landau damping of waves

The parallel electric field generated by magnetic compression is sensitive to that sort of Landau effect called "transit time magnetic damping", acting through the resonance $\omega - k_\parallel v_\parallel = 0$. In a wave-frame, from (59) one has:

$$q_e E_\parallel = \frac{T_e}{\gamma_* \beta_*} \partial_x \tilde{u} \; . \qquad (60)$$

For an ensemble of waves, this leads to a parallel momentum diffusion coefficient (in the plasma frame) of the form:

$$\Gamma_\parallel = \frac{T_e^2}{\gamma_*^2 \beta_*^2} < (\partial_x \tilde{u})^2 > \tau_c \; , \qquad (61)$$

where τ_c is the effective correlation time of the parallel electric field experienced by the particle in their motions. Off equipartition, in the wave frame, $\tilde{u}$ is a response to the magnetic pressure $|b|^2$ and oscillates at Alfvén wave beating frequency. This is a possible mechanism of generation of fast magnetosonic waves and Ragot & Schlickeiser (1998) have proved their efficiency in the injection of electrons in the cosmic ray population. Close to equipartition, there is no such well defined oscillation. The parallel electric field has a broadened frequency spectrum such that the resonance has a relaxation time by phase mixing. Since these intense compressions occur at short scales (at scales larger than r_* but not very much), they are driven by magnetic perturbations that are dispersed by the inertial effect; therefore the characteristic time scale of the inertial dispersion effect surely measures the correlation time which should be of order τ_*. This issue would deserve a specific numerical simulation, postponed for future works.

Detailed calculation shows that the diffusion coefficient depends on the pitch angle of the particle, not on its energy! Moreover the interaction does not require a high energy threshold like the Landau-synchrotron resonance necessary in the Fermi processes. This means that this process could probably solve the long standing issue of the injection problem of cosmic rays.

6. Cosmic rays scattering off magnetic disturbances

High energy particles undergo pitch angle scattering in the wave-frame (forward or backward) and this leads to important energy variation in the plasma frame. A preliminary investigation of this scattering is presented in this section. Since the plasma electromagnetic waves give energy to the particles through this Fermi process, in turn, a kind of collisionless damping occurs, which is nothing but a generalization of the Landau-synchrotron absorption. This requires some resonance between the gyro-motion of the particle and the wave oscillations, as will be recalled further on.

The motion of a particle that interacts with quasi parallel propagating magnetic modes, characterized by a vector potential $\mathbf{A}(z) \equiv B_0 \xi \mathbf{a}(z)$, ξ being a typical variation length along the average magnetic field B_0, is governed by a simple nonlinear system that reads:

$$\dot{\alpha} \;=\; \frac{qB_0\xi}{m\gamma} \mathbf{e}_\perp(t) . \frac{\partial}{\partial z} \mathbf{a}(z(t)) \qquad (62)$$

$$\dot{z} \;=\; v cos\, \alpha \qquad (63)$$

This is an Hamiltonian system of one degree of freedom, but depending on time through the gyro-motion described by the transverse unit vector $\mathbf{e}_\perp(t)$ that rotates at almost the gyro-pulsation ω_s. The Hamilton function can be written in terms of the two conjugate variables (α, x):

$$H(\alpha, x) = sin\alpha - \bar{\omega} \mathbf{e}_\perp(t).\mathbf{a}(x) \; , \qquad (64)$$

where $x \equiv z/\xi$, the time unit is the travel time over the characteristic length ξ, namely $\tau_c \equiv \xi/v$, $\bar{\omega} \equiv \omega_g \tau_c = 2\pi\tau_c/T_g$. The dynamics are different for an ensemble of plane waves and for a localized soliton.

6.1. Pitch angle scattering by delocalized waves

For an ensemble of Fourier modes in the wave-frame (forward propagation say):

$$\mathbf{a}(x) = \sum_n a_n (\mathbf{e}_1 cos(k_n x) + \varepsilon_c \mathbf{e}_2 sin(k_n x)) \; , \qquad (65)$$

where $\varepsilon_c = +1$ for right handed polarized modes and -1 for left handed polarized modes. In fact these Fourier components are not necessarly eigen modes of the plasma; they are eigen modes for quasi parallel propagation only; but for oblique propagation the Alfvén waves have a linear polarization. However as long as $k_\perp r_g \ll 1$, r_g being the Larmor gyro-radius, the effect of the transverse wave length is unimportant in the particle dynamics. So, even in case of linearly polarized Alfvén waves, the previous expansion (65) is assumed. The Hamilton function is thus

$$H(\alpha, x) = sin\alpha + \bar{\omega} \sum_n a_n cos(k_n x - \varepsilon \bar{\omega} t + \phi_0) \; , \qquad (66)$$

where $\varepsilon = \varepsilon_c sgn(q)$. A resonance occurs for different values α_n of α such that $k_n \dot{x} = \varepsilon \bar{\omega}$, or $k_n \mu_n = \varepsilon \bar{\omega}$ with $\mu_n = cos\alpha_n$. A negative charge moving forward resonates with a right mode ($\epsilon = 1$) whereas it resonates with a left mode if it moves backward and vice versa for a positive charge. The opposite conclusions held for backward wave propagation. These are the synchrotron resonances.

When the resonances are isolated by separatrix, the hamiltonian can be approximated by a pendulum resonance Hamiltonian in the vicinity of each resonance: setting the canonical transform

- $\theta = k_n x - \varepsilon \bar{\omega} t + \phi_0$,
- $J = (\alpha - \alpha_n)/k_n$,
- $H' = H - \varepsilon \bar{\omega} \alpha / k_n + Cst$,

the approximate Hamiltonian is

$$H'(J, \theta) = -k_n^2 sin\alpha_n \left(\frac{J^2}{2} - \Omega_n^2 cos\theta \right) \; , \qquad (67)$$

where the nonlinear pulsation Ω_n is such that

$$\Omega_n^2 = \frac{\bar{\omega} a_n}{k_n^2 sin\alpha_n} \; . \qquad (68)$$

The pendulum approximation differs from the exact Hamiltonian by oscillating terms.

The half-width of the n^{th} nonlinear resonance in (J, θ) phase-space is $\Delta J = 2\Omega_n$ and resonances overlap when this half-width is larger than the half-spacing between resonances $\Delta\alpha_n/k_n$; which leads to the Chirikov criterium (Zaslavsky & Chirikov 1972) for stochasticity:

$$\bar{\omega} a_n sin\alpha_n > (\Delta\mu_n)^2/4 . \quad (69)$$

As is well known, chaos occurs even at a lower threshold. The dynamics described by H' differ from the exact one by oscillating contributions, among those is the contribution of the backward waves. The particle cannot resonate simultaneously with a forward and a backward wave.

The smallest value of μ_n control the jump around the pitch angle of 90^0. The particle can jump from the resonance with the right mode ($\varepsilon = 1$) to the resonance with the left mode ($\varepsilon = -1$) if $k_n^2 a_n > \bar{\omega}/4$. This is the nonlinear solution to the momentum turn over problem.

When the mode amplitudes are sufficiently above the stochasticity threshold, the chaotic jumps of the pitch angle behave like a diffusion process. Only the momentum turn over might be slowed down by sticky regimes around 90^0, leading to subdiffusion.

The angular operator describing the evolution of the pitch angle distribution is obtained by setting $\Delta p^{\pm} = 0$ in the evolution equation (13). The following result is obtained:

$$\frac{\partial}{\partial t} f \mid_{ang} = \sum_{\pm} a^{\pm} (1 \mp \beta_* \mu)^2 \frac{\partial}{\partial \mu} \frac{\nu_s^{\pm} \left(1 - \mu^2\right)}{(1 \mp \beta_* \mu)^2} \frac{\partial}{\partial \mu} f . \quad (70)$$

In (70) it can easily be seen that at an angle larger than α_* the angular diffusion is slowed down by a factor γ_*^2 in comparison to the diffusion inside the cone of half-angle α_*. The μ-diffusion coefficient vanishes for $\mu = \pm 1$ as expected and also at $\mu = \pm\beta_*$ where $\nu_s^{\pm}$ vanishes. In the vicinity of these points, the diffusion process is anomalous.

The acceleration process is suitably described by the response to an initial monoenergetic distribution of the form $\delta(p - p_0)/4\pi p_0^2$. The solution at time t is given by Eq. (13):

$$f_0(p, \mu, t) = \frac{1}{4\pi p^2 p_0 \beta_*} \sum_{\pm} a^{\pm} (1 \mp \beta_* \mu) P^{\pm}(p, \mu, t) , \quad (71)$$

where

$$P^{\pm}(p, \mu, t) = \frac{1}{\sqrt{2\pi}\sigma(t)} exp\left[-\frac{(\Delta\mu^{\pm})^2}{2\sigma(t)^2}\right] , \quad (72)$$

with

$$\Delta\mu^{\pm} \equiv \frac{p - p_0}{\beta_* p_0} (1 \mp \beta_* \mu) . \quad (73)$$

The standard deviation is such that $\sigma(t)^2 = (1 - \mu^2)\nu_s^{\pm} t$ for μ sufficiently different from ± 1; whereas for μ close to ± 1, $\sigma(t)^2 \sim (\nu_s^{\pm} t)^2$ (because of the superdiffusion of μ). This response explicitely shows that in the cone, $\alpha < \alpha_*$, the particle gains an energy of the order $\gamma_*^2 p_0$ in a scattering time, whereas outside the cone, the gain factor is only β_*^2. Indeed the isocountours (defined by constants C_{ic}) of the distribution are such that, for $\alpha > \alpha_*$,

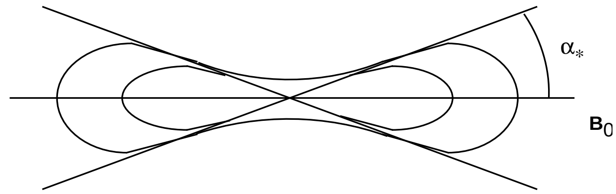


Fig. 1. After a scattering time, the distribution function is elongated along the magnetic axis. Inside the cone particles gain energy by a factor γ_*^2.

$$p_{ic}(t) \simeq p_0 \left(1 + C_{ic} \beta_*^2 \sigma(t)\right) , \quad (74)$$

whereas for $\alpha < \alpha_*$,

$$p_{ic}(t) \simeq p_0 \left(1 + C_{ic} 2\gamma_*^2 \sigma(t)\right) , \quad (75)$$

(see Fig. 1).

Let us end this subsection with some remarks about anomalous diffusion. Obviously, if the pitch angle scattering is an anomalous diffusion process, then the energy variations also undergo an anomalous diffusion with the same secular behaviour. In that case the stationary energy spectrum would be significantly modified as compared to the ordinary Fermi processes. Spatial diffusion derived from anomalous pitch angle scattering is also anomalous. Recently Kirk et al. (1996) analyzed the consequences of assumed braided magnetic field lines behind a shock. In particular, the index of the cosmic ray power law could be changed because of the change of the effective compression ratio due to anomalous diffusion (Ragot & Kirk 1997). Anomalous pitch angle scattering would also produce these deviations from the normal laws in the shock acceleration theory. On the subset of anomalous diffusion points, the distribution function is not differentiable.

6.2. *Scattering by a localized nonlinear wavefront*

In the frame of a soliton, the cosmic rays undergo pitch angle variations that are governed by a similar system.

$$\dot{\alpha} = \bar{\omega}\phi(x) cos\left[\theta(x) - \varepsilon\bar{\omega}t\right] \quad (76)$$

$$\dot{x} = cos\,\alpha \quad (77)$$

The profile decays exponentially on the scale ξ and the phase has a nonlinear variation: $\theta'(x) = \kappa\xi + c_0\xi\phi^2(x)$, c_0 being a coefficient of order unity. The dynamics are then controlled by three parameters $\bar{\omega}$, b_0 and κ ($\propto b_0^2$); it is useful to bear in mind that $\bar{\omega} = \xi/r_g$ and is assumed much larger than Ω. The particle interacts with the soliton during few time units and numerical computations exhibit few typical behaviours. First, even for large amplitude perturbations (say $b_0 \sim 1$), particles having a large Larmor radius compared to the soliton width are characterized by a small parameter $\bar{\omega}$. Therefore they clearly suffer a small deterministic variation of their pitch angle. Second, in the opposite extreme, particles that have a small Larmor radius ($\bar{\omega}$ large) undergo tremendous pitch angle variations, unless the amplitude b_0 is very small. They behave for a while like in a broad bank spectrum of plane waves and the pitch angle undergoes a diffusion process because of the resonances. The third interesting case is that of particles having a Larmor radius comparable to the width of a soliton ($b_0 \sim 1$). Numerical solutions indicate the following results. The motions are regular when sufficiently off resonance and tremendous pitch angle jumps occur when close to resonance with some sensitivity to initial conditions. Also, reflection occurs in some interval of initial pitch angle in the vicinity of $\pi/2$. When the soliton propagates at a velocity smaller than the linear Alfvén velocity (i.e. $\kappa > 0$), as in the case of Figs. 2a–c, forward particles ($\mu(0) > 0$) resonate with the soliton and undergo strong scattering. When the soliton propagates at a velocity larger than the Alfvén velocity (i.e. $\kappa < 0$), backward particles ($\mu(0) < 0$) resonate and undergo strong scattering, as in the case of Figs. 3a–c.

In summary, a forward soliton transforms the distribution function of cosmic rays having a Larmor radius r_g larger than r_* upto a few ξ, cosmic rays having a Larmor radius much larger than ξ are not scattered ($\bar{\omega} \ll 1$); that part of the distribution function is fastly isotropized (rough isotropization) in the soliton frame and thus this suprathermal tail has the bulk motion of the solitons $\simeq \beta_*$. The scattering is suitably described by the fast random scattering of Sect. 2.1. In the plasma frame, this tail is concentrated in a narrow cone of half angle $\alpha_* \simeq 1/\gamma_*$ and the energy of the particle in this cone has been amplified by a factor γ_*^2. No further acceleration is possible with other forward solitons. Further acceleration is possible with an incoming backward soliton that produces a new amplification of the energy by a factor γ_*^2, changing thus the initial energy of the cosmic ray by a factor γ_*^4. If several head on collisions could occur, the energy gain would be by a factor γ_*^4, each time within a few Larmor periods.

7. Discussion

Two kinds of acceleration were presented, one associated with the Landau absorption of the parallel electric field generated by compression, which is large close to equipartition; the other is an extension of Fermi acceleration based on resonnant scattering of particles in the frame of relativistic waves, and is associated with Landau-synchrotron absorption. The former injects low energy particles, the latter accelerates high energy cosmic rays.

The main physical points are the following.

i) Presumably, confined relativistic plasma, whose electromagnetic disturbances propagate at relativistic speed, exists in the environment of relativistic objects. Some properties of such relativistic MHD fronts have been reviewed.

ii) When a plasma is dominated by the cosmic ray pressure, MHD description is significantly modified by a large scale inertial effect. This effect is responsible for a dispersive correction to the propagation velocity of the waves even at long wavelengths.

(a)

(b)

(c)

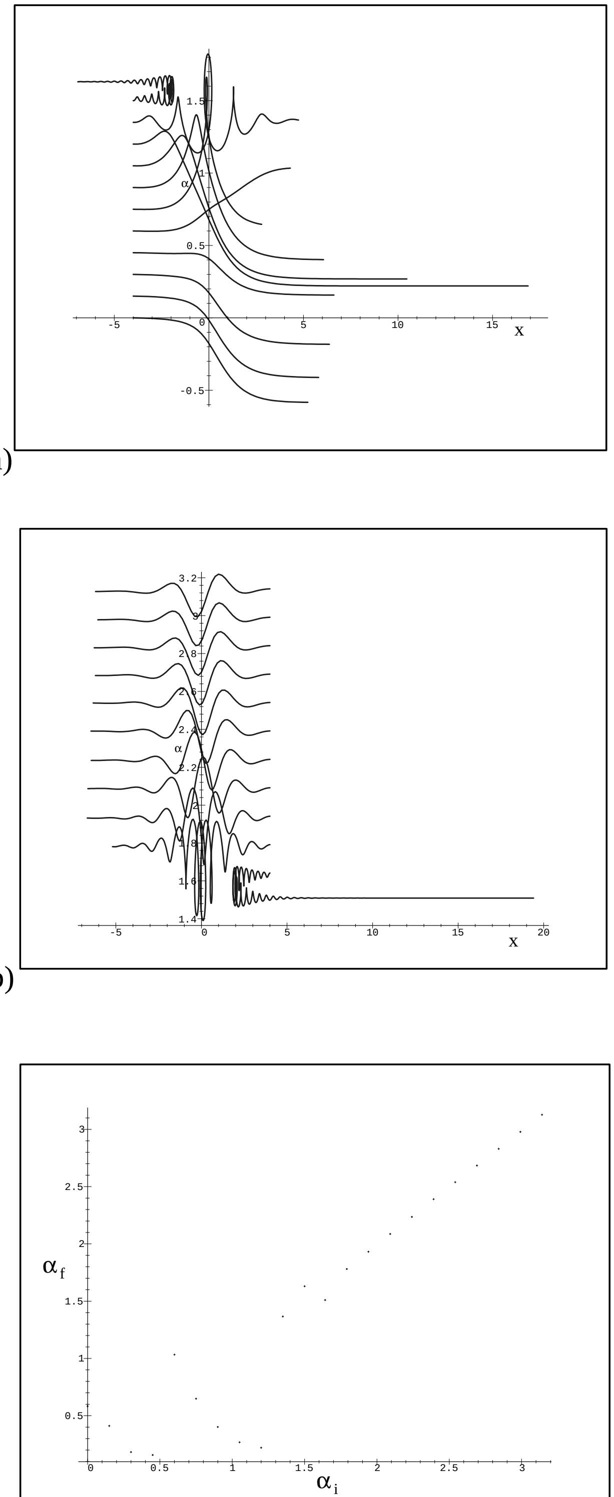
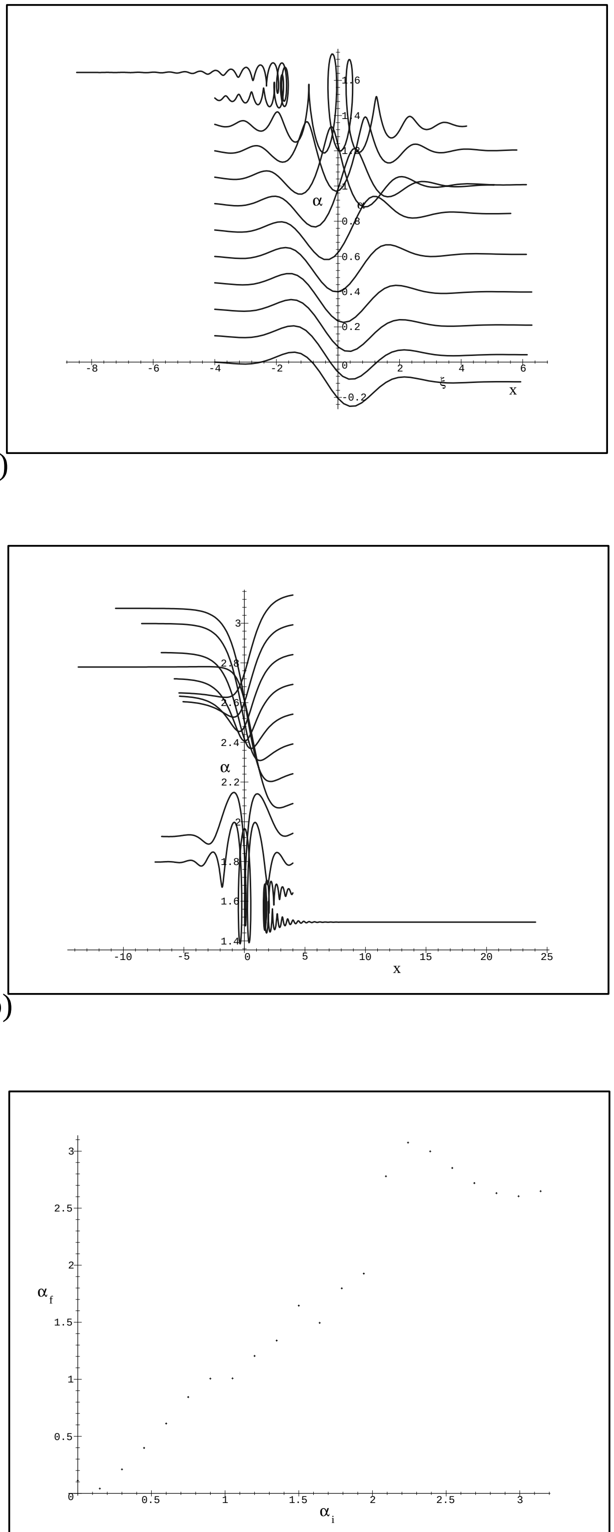


Fig. 2a–c. In these figures the amplitude $b_0 = 0.3$, $\kappa = 1$ (subAlfvénic soliton), $\bar{\omega} = 1$. In **a**, the phase portrait (x, α) shows that forward particles undergo strong irregular scattering; some undergo reflection when the initial pitch angle is close to $\pi/2$. In **2b**, backward particles do not resonate and the scaterring is very weak, only reflection occurs in the vicinity of $\pi/2$. **c** represents the mapping giving the final pitch angle versus the initial pitch angle.



(a)

(b)

(c)

Fig. 3a–c. In these figures the amplitude $b_0 = 0.3$, $\kappa = -1$ (super-Alfvénic soliton), $\bar{\omega} = 1$. In **a**, the phase portrait (x, α) shows that forward particles do not resonate and the scaterring is very weak, only reflection occurs in the vicinity of $\pi/2$. In **b**, backward particles resonate and undergo strong irregular scattering; some undergo reflection when the initial pitch angle is close to $\pi/2$. **c** represents the mapping giving the final pitch angle versus the initial pitch angle.

iii) The fluid description of the nonlinear front needs to be modified by unavoidable kinetic effects due to the development of a parallel electric field. The kinetics transform the solitons into BGK modes driven by the magnetic compression, so they can be considered as parallel collisionless shocks. This is a proper understanding of this physical concept.
iv) The essential microphysics ingredient of the acceleration theory is the pitch angle scattering through chaotic resonant interactions in the waveframe (forward or backward). In delocalized waves, the momentum turn over is controlled by the stochastic jump from one resonance to another, especially to pass the 90^0 barrier; this is an important issue in cosmic ray acceleration. In strong localized wave packets, the pitch angle jumps randomly (except at small angles) and momentum turn over occurs also randomly, provided that the Larmor radius is smaller than the front width.
v) In relativistic waves, the energy evolution is no longer governed by a Fokker-Planck equation but by an integro-differential equation; the expansion in first and second order contribution does not make sense. This is the most efficient regime of the generalized Fermi acceleration process because particles energy can be amplified by a factor γ_*^2 during a scattering time and the scattering time, for strong waves, can be as short as several times the gyro-period (typically ten). The spectrum is not a universal powerlaw; however, a global power law can easily be obtained.

The main astrophysical points are the following.
vi) A strong relativistic shock accelerate particles more efficiently by amplifying their energy by a factor of γ_s^2, with a shock Lorentz factor γ_s that can be much larger than the Alfvén Lorentz factor γ_*. However, as shown by Gallant & Achterberg (1999), this cannot be repeated. This is not the case with relativistic waves and solitary waves; even if a solitary wave has some shock like behaviour, this is not a strong shock that produces a deep modification of the medium. Thus, each particle can experience interactions with several fronts of that kind.
vii) A head on collision of relativistic fronts seems to be an unavoidable event in relativistic expansion, since perturbations at the origin of the flow are expected and propagate forward and also backward fronts which are generated by the interaction of the flow with the ambient medium. At each head on collision, the energy of the particle is amplified by a factor γ_*^4. Such events could be observed with the VLBI technique.
viii) The efficiency of this new version of Fermi acceleration can account for the generation of the very high energy cosmic rays in AGNs and FR2 radiogalaxy hot spots upto $10^{21}\ eV$. However this process is probably more relevant for Gamma Ray Bursts and detailed estimates will be given in a forthcoming paper. Moreover, the observations of wisps in pulsar nebulae could be accounted for by the formation of relativistic solitons triggered by a kink instability; the cosmic ray generation by these events (Gallant & Arons 1994) deserves a specific investigation.

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Appendix

When the pressure is not close to equipartition, $\delta = o(1)$, $\tilde{u}$ must be considered as of the second order and $\tilde{u} = -|b|^2/2\delta$. The DNLS-equation 52) is obtained with the following ordering, $b_0 = o(\varepsilon^{1/2})$, $\partial_x = o(\varepsilon)$, $\partial_t = o(\varepsilon^2)$. This equation has been intensively studied (Kaup & Newell 1978, Mjölhus 1978, Spangler & Sheerin 1982, Kennel et al.1988), for it is one of the most famous example of a soliton equation. It illustrates the capability of the dispersion effect to stabilize the nonlinear steepening.

Let us summarize the main properties. A circularly polarized plane wave of the form $b = b_0 exp(ik_0 x - i\omega_0 t)$ undergoes self-modulation instability only if $\delta_P k_0 < 0$ (where $\delta = \sigma\delta_P$, such that $\delta_P < 0$ for a pressure below equipartition) in ordinary plasmas (Hada, 1993), whereas it is the opposite sign in RE-plasma. The instability disappears for vanishing δ_P when the complete system is taken into account instead of the DNLS equation only.

The soliton solutions of DNLS equation are of the form:

$$u(x,t) = u(y) \tag{A.1}$$

$$b(x,t) = \phi(y) e^{i(\theta(y) - \Omega t)} , \tag{A.2}$$

where $y \equiv x - vt$. Circularly polarized DNLS-solitons have been calculated by Mjölhus (1976) and Spangler & Sheerin (1982). All these solitary solutions can be gathered in the folowing results.

The soliton phase has a local variation related to its amplitude:

$$\theta'(y) = \kappa + c_0 \phi^2(y) , \tag{A.3}$$

with $c_0 = -3/(8\alpha_0\delta)$. The velocity is such that $\gamma_* v = -2\alpha_0\kappa$. The localized amplitude is the solution of the differential equation:

$$\frac{1}{2} \left(\partial_y \phi\right)^2 + U\left(\phi\right) = E_0 . \tag{A.4}$$

where the pseudo-potential U is given by

$$U\left(\phi\right) = -\frac{1}{2} a^2 \phi^2 - \frac{\kappa}{8\alpha_0\delta}\phi^4 + \frac{1}{128\alpha_0^2\delta^2}\phi^6 . \tag{A.5}$$

All the classes of solitary solutions require $E_0 = 0$. The amplitude b_0 are such that $U(b_0) = 0$ and thus:

$$b_0^2 = -8\alpha_0\delta\left(\kappa \pm \sqrt{a^2 + \kappa^2}\right) . \tag{A.6}$$

Four classes of solutions can be discriminated, they depend on two parameters Ω and κ or v and a. There are ionic solutions corresponding to $\alpha_0 > 0$ and electron dominated solutions for $\alpha_0 < 0$, and there are also supersonic solutions corresponding to $\delta_P < 0$, as well as subsonic solutions for $\delta_P > 0$. The Mjölhus (1976) solutions are recovered by taking $\alpha_0 = 1$ and $\delta_P = -1/2$. Spangler & Sheerin (1982) showed that subsonic solutions similar to the supersonic ones also exist. The relation between the size and the nonlinear velocity is such that:

$$\zeta^2 = \frac{1}{(8\alpha_0\delta)^2} - \frac{\zeta'}{4\alpha_0\delta} . \tag{A.7}$$

All the amplitude profiles can be written in this form obtained by Mjölhus:

$$\phi^2(y) = 8|\alpha_0\delta| \frac{a^2}{\left(a^2 + \kappa^2\right)^{1/2} ch\left(2ay\right) + |\kappa|} . \tag{A.8}$$

Now considering the system, one can see that the solitary waves have an amplitude of the order of δ_P; they disappear at equipartition. The system is certainly not completely integrable. Sonic perturbations intensify when δ_P is small, as shown by numerical simulations (Baciotti et al. 1997).

The relativistic Hada system and the DNLS equation themselves are conservation equations for the integrals $\int \tilde{u} dx$ and $\int b dx$. The systems are invariant under the gauge transformation $b \rightarrow e^{i\theta_0} b$ which implies that $\int \left(|b|^2 + u^2\right) dx$ is a constant of the motion. They keep the magnetic helicity constant: $\frac{1}{2}\int \left(a^* b + a b^*\right) dx$. However only the asymptotic approximation for $\delta_P = o(1)$ (DNLS) has an infinite sequence of constants of motion. Those invariants are adiabatic invariants of the system when the plasma is significantly off equipartition. The DNLS equation passes the so-called "Painlevé test", is subject to a Bäcklund transformation (Weiss et al. 1983) that allows for the construction of an integration scheme by the inverse scattering transform (Kaup & Newell 1978).

References

Aharonian F.A., Cronin J.W., 1994, Phys. Rev. D 50, 1892
Baciotti F., Chiuderi P., Pouquet A., 1997, ApJ 478, 594
Bell A.R., 1978, MNRAS 178, 147
Bernstein I.B., Green J.M., Kruskal M.D., 1957, Phys. Rev. 108, 546
Bonazzola S., Peter P., 1997, Astropart. Phys. 7, 161
Fermi E., 1949, Phys. Rev. 75, 1169
Gallant Y.A., Achterberg A., 1999, MNRAS 305, L6
Gallant Y.A., Arons J., 1994, ApJ 435, 230
Gibbs K.G., 1997, Proceedings of the 32nd rencontres de Moriond: Very high energy phenomena in the Universe. p. 203
Goldman M., 1999, In: Passot, Sulem (eds.) Nonlinear MHD waves and turbulence. Proceedings Nice workshop 1998, Springer-Verlag
Guirado J.C., Marcaide J.M., Alberdi A., et al., 1995, AJ 110, 2586
Henri G., Pelletier G., Petrucci P.O., Renaud N., 1999, Astropart. Phys., in press
Hada T., 1993, GRL 20, 2415
Jokipii J.R., 1966, ApJ 146, 480; 1987, ApJ 313, 842
Jones F., 1994, ApJS 90, 561
Kaup D.J., Newell A.C., 1978, J. Math. Phys. 11, 798
Kennel C.F., Buti B., Hada T., Pellat R., 1988, Phys. Fluids 31, 1949
Kirk J.G., Schneider P., 1988, ApJ 328, 269
Kirk J.G., Duffy P., Gallant Y.A., 1996, A&A 314, 1010
Klein R.I., Jermigan J.G., Arons J., et al., 1996, ApJ 469, L119
Kulkarni S.R., Djorgovski S.G., Odewahn S.C., et al., 1999, Nat 398, 389
Mannheim K., Biermann P., 1992, A&A 253, L21
Melrose D.B., 1968, Plasma Astrophysics. vol. 2, Gordon and Breach
Melrose D.B., 1986, Instabilities in space and laboratory plasmas. Cambridge Univ. Press
Meszaros P., Rees M.J., 1997, ApJ 482, L29
Mio K., Ogino T., Minami K., Takeda S., 1976, J. Phys. Soc. Jpn. 41, 265
Mjölhus E., 1976, J. Plasma Phys. 16, 321

Mjölhus E., 1978, J. Plasma Phys. 19, 437
Norman C.A., Melrose D.B., Achterberg A., 1995, ApJ 454, 60
Pelletier G., Marcowith A., 1998, ApJ 502, 598
Protheroe R.J., Stanev T., 1992, In: Stenger V.J., et al. (eds.) High Energy Neutrino Astronomy. World Scientific, Singapore, p. 40
Ragot B.R., Kirk J.G., 1997, A&A 327, 432
Ragot B.R., Schlickheiser R., 1998, APh 9, 79
Roberts B., 1985, Phys. Fluids 28, 3280
Scargle J.D., 1969, ApJ 156, 401
Schlickeiser R., Campeanu A., Lerche L., 1993, A&A 276, 614
Schlickeiser R., 1994, ApJS 90, 929
Spangler S.R., Sheerin J.P., 1982, J. Plasma Phys. 27, 193
Vainio R., Schlickeiser R., 1999, A&A 343, 303
Weiss J., Tabor M., Carnevale G., 1983, J. Math. Phys. 24, 522
Zaslavsky G.M., Chirikov B.V., 1972, Sov. Phys. Uspekhi 14, 549; Chirikov B.V., 1979, Phys. Reports 52, 265