Astron. Astrophys. 350, L47–L50 (1999)

ASTRONOMY
AND
ASTROPHYSICS

Letter to the Editor

Diquark condensates and the magnetic field of pulsars⋆

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Received 20 May 1999 / Accepted 19 September 1999

Abstract. We study the consequences of superconducting quark cores in neutron stars for the magnetic fields of pulsars. We find that within recent nonperturbative approaches to the effective quark interaction the diquark condensate forms a superconductor of second kind whereas previously quark matter was considered as a first kind superconductor. In both cases the magnetic field which is generated in the surrounding hadronic shell of superfluid neutrons and superconducting protons can penetrate into the quark matter core since it is concentrated in proton vortex clusters where the field strength exceeds the critical value. Therefore the magnetic field will not be expelled from the superconducting quark core with the consequence that there is no decay of the magnetic fields of pulsars. Thus we conclude that the occurence of a superconducting quark matter core in pulsars does not contradict the observational data which indicate that magnetic fields of pulsars have life times larger than 10^7 years.

Key words: dense matter – Magnetohydrodynamics (MHD) – stars: interiors – stars: magnetic fields – stars: neutron – stars: pulsars: general

Recently, the possible formation of diquark condensates in QCD at finite density has been reinvestigated in a series of papers following Refs. (Alford et al. 1998; Rapp et al. 1998). It has been shown that in chiral quark models with a nonperturbative 4-point interaction motivated from instantons (Carter & Diakonov 1999) or nonperturbative gluon propagators (Blaschke & Roberts 1998, Bloch et al. 1999) the anomalous quark pair amplitudes in the color antitriplet channel can be very large: of the order ≈ 100 MeV. Therefore, in two-flavor QCD, one expects this diquark condensate to dominate the physics at densities beyond the deconfinement/chiral restoration transition and below the critical temperature (≈ 50 MeV) for the occurence of this "color superconductivity" (2SC) phase. In a three-flavor theory it has been found (Alford et al. 1999a, Schäfer & Wilczek 1999) that there can exist a color-flavor locked (CFL) phase for not too large strange quark masses (Alford et al. 1999b) where color superconductivity is complete in the sense that diquark condensation produces a gap for quarks of all three colors and flavors, which is of the same order of magnitude as that in the two-flavor case.

The high-density phases of QCD at low temperatures are most relevant for the explanation of phenomena in rotating compact stars - pulsars. Conversely, the physical properties of these objects (as far as they are measured) could constrain our hypotheses about the state of matter at the extremes of densities. In contrast to the situation for the cooling behaviour of compact stars (Blaschke et al. 1999) where the CFL phase is dramatically different from the 2SC phase, we don't expect qualitative changes of the magnetic field structure between these two phases. Consequently, we will restrict ourselves here to the discussion of the simpler two-flavor theory first.

According to Bailin and Love (1984) the magnetic field of pulsars should be expelled from the superconducting interior of the star due to the Meissner effect and decay subsequently within $\approx 10^4$ years. If their arguments would hold in general, the observation of lifetimes for the magnetic field as large as 10^7 years (Makashima 1992; Baym et al. 1969) would exclude the occurence of an extended superconducting quark matter phase in pulsars. For their estimate, they used a perturbative gluon propagator which yielded a very small pairing gap and they made the assumption of a homogeneous magnetic field. Since both assumptions seem not to be valid in general, we perform a reinvestigation of the question whether presently available knowledge about the lifetime of magnetic fields of pulsars might contradict the occurence of a color superconducting phase of QCD at high densities.

The free energy density in the superconducting quark matter phase with ud diquark pairing ($J^P = 0^+$ and color antitriplet index p) is given by (Bailin & Love 1984)

$$f = f_n + \alpha d_p^* d_p + \frac{1}{2}\beta(d_p^* d_p)^2 + \gamma(\nabla d_p^* + iq\boldsymbol{A} d_p^*)(\nabla d_p - iq\boldsymbol{A} d_p) + \frac{B^2}{8\pi}\,, \quad (1)$$

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⋆ Research supported in part by the Volkswagen Stiftung under grant no. I/71 226

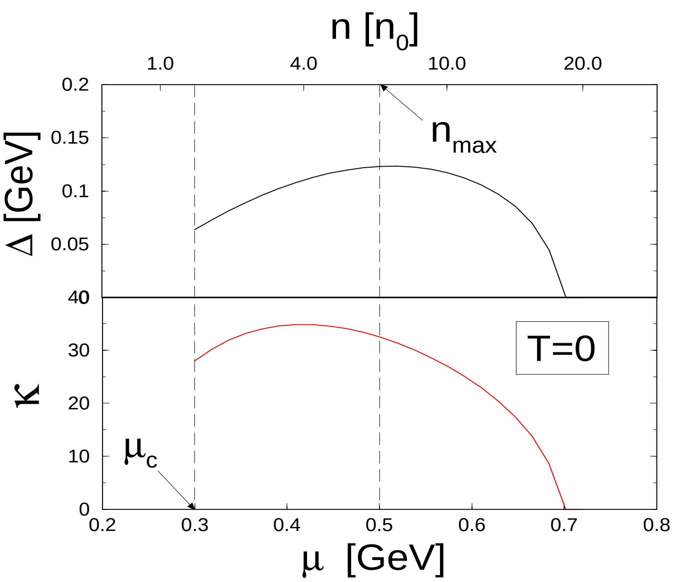

Fig. 1. The dependence of diquark energy gap Δ and Ginzburg-Landau parameter κ on chemical potential μ and density n for a NJL-type quark interaction (Berges & Rajagopal 1999); $n_0 = 0.16\,\mathrm{fm}^{-3}$ is the nuclear saturation density. The left dashed line denotes the critical chemical potential of the onset of quark superconductivity (the corresponding baryon number densities are given on the upper scale), the right dashed line - the maximal values of chemical potential and density for stable stellar configurations.

where $\boldsymbol{B} = \mathrm{rot}\boldsymbol{A}$ is the magnetic induction, q the charge of the ud pair and the coefficients of the free energy are given by the following expressions

$$\begin{aligned} \alpha &= \frac{dn}{dE} t \,, \\ \beta &= \frac{dn}{dE} \frac{7\zeta(3)}{8(\pi k_{\mathrm{B}} T_{\mathrm{c}})^2} \,, \\ \gamma &= \frac{dn}{dE} \frac{7\zeta(3)}{48(\pi k_{\mathrm{B}} T_{\mathrm{c}})^2} \frac{p_{\mathrm{F}}^2}{\mu^2} = \frac{1}{6} \frac{p_{\mathrm{F}}^2}{\mu^2} \beta \,, \end{aligned} \tag{2}$$

where $t = (T - T_{\mathrm{c}})/T_{\mathrm{c}}$. Here T_{c} is the critical temperature, p_{F} the quark Fermi momentum, $\mu = \sqrt{p_{\mathrm{F}}^2 + m^2}$ is the chemical potential (in zeroth order with respect to the coupling constant), $dn/dE = \mu p_{\mathrm{F}}/\pi^2$.

The Ginzburg-Landau equations for relativistic superconducting quarks are obtained in the usual way

$$\begin{aligned} 0 &= \alpha d_p + \beta(d_p^* d_p) d_p + \gamma(-i\nabla - q\boldsymbol{A})^2 d_p \,, \\ \boldsymbol{j} &= iq\gamma(d_p \nabla d_p^* - d_p^* \nabla d_p) - 2q^2 \gamma d_p d_p^* \boldsymbol{A} \,. \end{aligned} \tag{3}$$

In deriving the expression for the current $\boldsymbol{j}$ we have also used the Maxwell equation

$$\mathrm{rot}\boldsymbol{B} = 4\pi \boldsymbol{j} \,. \tag{4}$$

The first of the Ginzburg-Landau equations (3) has a solution which corresponds to the Meissner effect ($\nabla d_p = 0, \ \boldsymbol{A} = 0$ inside of the superconductor):

$$\Delta^2 = |d_p|^2 = -\frac{\alpha}{\beta} = -\frac{8t(\pi k_{\mathrm{B}} T_{\mathrm{c}})^2}{7\zeta(3)} \,. \tag{5}$$

For the case of weak fields ($H < H_{c2}$) one obtains the London equation:

$$\boldsymbol{B} + \lambda_{\mathrm{q}}^2 \mathrm{rot}\,\mathrm{rot}\boldsymbol{B} = 0 \,, \tag{6}$$

where λ_{q} is the penetration depth of the magnetic field into the superconducting quark condensate. The region of the change of the order parameter d_p can also be determined from (3) via

$$\xi_{\mathrm{q}}^2 = -\frac{\gamma}{\alpha} = -\frac{7\zeta(3)}{48t(\pi k_{\mathrm{B}} T_{\mathrm{c}})^2} \left(\frac{p_{\mathrm{F}}}{\mu}\right)^2 \,. \tag{7}$$

In the diquark condensate phase with a nonperturbative interaction the energy gap is $\Delta \approx 100$ MeV at $\mu \approx 400$ MeV[1], see Fig. 1. We obtain for the coherence length $\xi_{\mathrm{q}} = 0.8 \times 10^{-13}$ cm. For the penetration depth of the magnetic field we have

$$\lambda_{\mathrm{q}} = \frac{1}{\sqrt{8\pi\gamma q d_0}} = \sqrt{\frac{-3\pi\mu}{4q^2 t p_{\mathrm{F}}^3}} \approx 3.6 \times 10^{-12}\mathrm{cm} \,. \tag{8}$$

The thermodynamical critical field H_{cm} that fully destroys the superconducting state in the case of a superconductor of the first kind is given (Bailin & Love 1984)

$$H_{\mathrm{cm}}^2 = \frac{32\pi\mu p_{\mathrm{F}} (k_{\mathrm{B}} T_{\mathrm{c}} t)^2}{7\zeta(3)} \,. \tag{9}$$

For the parameter values given above the critical field is $H_{\mathrm{cm}} \approx 8.7 \times 10^{17}$ G, i.e. by two orders of magnitude larger than in Ref. (Bailin & Love 1984).

The Ginzburg-Landau parameter κ which determines the behaviour of the superconductor in an external magnetic field is given by (Bailin & Love 1984)

$$\kappa = \frac{\lambda_{\mathrm{q}}}{\xi_{\mathrm{q}}} = \sqrt{\frac{\beta}{8\pi\gamma^2 q^2}} = 132 \frac{\Delta}{\mu} \left(\frac{\mu}{p_{\mathrm{F}}}\right)^{5/2} \,. \tag{10}$$

For values of $\mu \sim p_{\mathrm{F}} \sim 400$ MeV/c and $\Delta = 100$ MeV we obtain $\kappa = 34$, see also Fig. 1. Therefore the superconducting quark condensate appears as a superconductor of the second kind into which the external magnetic field can penetrate by forming quantized vortex lines in the interval $H_{c1} < H < H_{c2}$. The upper critical field H_{c2} is determined by

$$H_{c2}^{\mathrm{q}} = -\frac{\alpha}{q\gamma} = \frac{6\Delta^2}{q} \left(\frac{\mu}{p_{\mathrm{F}}}\right)^2 \approx 3 \times 10^{19}\mathrm{G} \,. \tag{11}$$

The magnetic flux of the quark vortex lines Φ_{q} amounts to

$$\Phi_{\mathrm{q}} = \frac{2\pi\hbar c}{q} = \frac{2\pi\hbar c}{e/3} = 6\frac{\pi\hbar c}{e} = 6\Phi_0 \,, \tag{12}$$

where $\Phi_0 = 2 \times 10^{-7}$ G cm^2 is the quantum of the proton magnetic flux. The lower critical field for the occurence of quark vortex lines is then

$$H_{c1}^{\mathrm{q}} = \frac{\Phi_{\mathrm{q}}}{6\pi\lambda_{\mathrm{q}}^2} \ln \frac{\lambda_{\mathrm{q}}}{\xi_{\mathrm{q}}} = 1.8 \times 10^{16}\mathrm{G} \,. \tag{13}$$

[1] within a dynamical confining quark model the diquark gaps can be even larger than this estimate (Blaschke & Roberts 1998).

Here we have taken into account the spherical shape of the quark core in a neutron star (Sedrakian et al. 1984).

Now we can describe the magnetic structure of a superconducting quark condensate in a pulsar and its time evolution. When during the cooling of the protoneutron star with a quark matter core the critical temperature for the transition to the superconducting state is reached in the presence of a magnetic field, then this field remains in the quark phase in the form of quantized vortex lines.

At some point in the further rapid cooling of the star due to neutrino emission the neutrons in the hadronic phase ("npe"-phase) of the star become superfluid. Since the basic interaction between isolated protons resembles that of the neutrons, the protons in the hadronic phase become superfluid too. Since the density of protons in "npe"-phase is only few per cent of the neutron density, the protons will pair in $^1S_0$ pairing state (Chao et al. 1972; Amundsen & Ostgaard 1985; Baldo et al. 1992). The neutrons take part in the rotation, forming a lattice of quantized vortex lines. Because of the strong interaction of the neutrons with the protons a part of the superconducting protons will be entrained by the neutrons (Sedrakian & Shahabasian 1980; Alpar et al. 1984) and create in the region of the neutron vortex a magnetic field of strength $H(r)$ given by (Sedrakian & Shahabasian 1980; Sedrakian et al. 1983)

$$H(r) = \hat{\nu}_{\rm n} \frac{k\Phi_0}{2\pi\lambda_{\rm p}^2} \ln \frac{b}{r} \;, \tag{14}$$

where $b = \sqrt{\pi\hbar/\sqrt{3}m_{\rm n}\Omega}$ is the lattice spacing of the neutron vortex lattice, $k = (m_{\rm p}^* - m_{\rm p})/m_{\rm p}$ is the entrainment coefficient with the effective mass $m_{\rm p}^*$ and the bare mass $m_{\rm p}$ of the protons; $\hat{\nu}_{\rm n}$ is the unit vector in the direction of the vortex axis, r is the distance from the center of the vortex and Ω is the angular velocity of the rotation of the star. This field, whose magnitude is determined by the rotation of the star, acts as an external field for the non-entrained protons and creates a cluster of proton vortices with the fluxes Φ_0 in the region around the axis of the neutron vortex where $H(r) > H_{c1}^{\rm p}$. The radius of this region, $\delta_{\rm n}$, equals (Sedrakian et al. 1984)

$$\delta_{\rm n} = b(\xi_{\rm p}/\lambda_{\rm p})^{\frac{1}{3|k|}} \;. \tag{15}$$

For the pulsar Vela PSR 0833-45 with $\Omega = 70$ rad s^{-1} and $b = 10^{-3}$ cm, we have $\delta_{\rm n} = 10^{-5}$ cm.

While the mean magnetic induction in the star due to proton vortex clusters is of the order 10^{12} G, the mean magnetic induction within the cluster reaches values of 4×10^{14} G (Sedrakian & Shahabasian 1991; Sedrakian & Sedrakian 1995).

The magnetic field strength $H(r)$ which occurs in the "npe"-phase is the strength of the external field relative to the superconducting quark condensate. It reaches the maximum value $H(0)$ close to the center of the neutron vortex, i.e.

$$H(0) = \frac{k\Phi_0}{2\pi\lambda_{\rm p}^2} \ln \frac{b}{\xi_{\rm n}} \approx 4.7 \times 10^{16}{\rm G} \;, \tag{16}$$

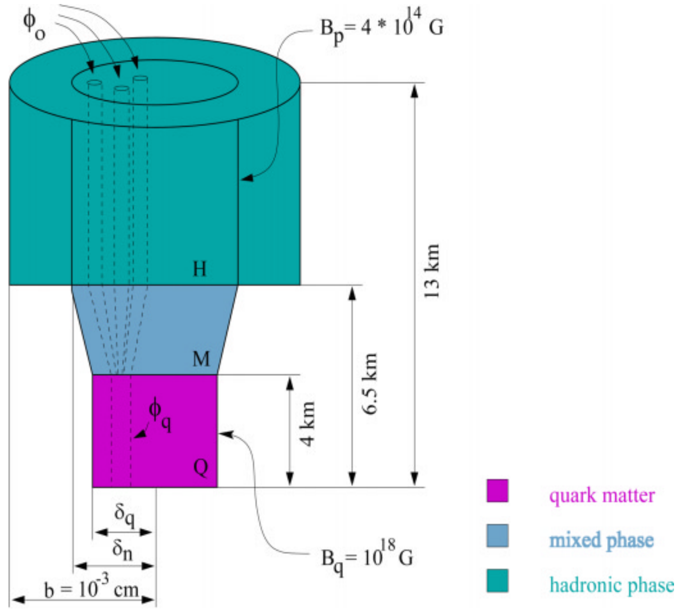

Fig. 2. Magnetic field structure in the interior of a hybrid star with $M = 1.4 M_\odot$; b is the radius of the neutron vortex $\delta_{\rm n} = 10^{-5}$ cm is the radius of the proton vortex cluster, and $\delta_{\rm q} = 4.3 \times 10^{-7}$ cm is that of the quark vortex cluster. For details, see text.

for $\lambda_{\rm p} = 30$ fm, $k = 0.7$ and a coherence length $\xi_{\rm n} = 30$ fm of the neutron.

This external field generates quark vortex lines when the condition $H(r) \geq H_{c1}^{\rm q}$ is fulfilled. The radius of this region is

$$\delta_{\rm q} = b(\xi_{\rm q}/\lambda_{\rm q})^{\frac{2}{k}(\lambda_{\rm p}/\lambda_{\rm q})^2} = 4.3 \times 10^{-7}{\rm cm} \;. \tag{17}$$

This way, the entrainment current generates a strongly inhomogeneous magnetic structure in the quark condensate: the clusters of quark vortex lines with the fluxes $\Phi_{\rm q}$ and radii $\delta_{\rm q}$, the axes of which are the continuation into the quark phase of the axes of the neutron vortex lines. Since $\delta_{\rm q}$ is by two orders of magnitude smaller than $\delta_{\rm n}$, the mean magnetic induction in the clusters of quark vortex lines increases to a value of the order of 10^{18} G.

When the condition for the applicability of the London approximation $H(0) \ll H_{c2}^{\rm q}$ is fulfilled, then one can apply the modified London equation

$$\boldsymbol{B} + \lambda_{\rm q}^2 {\rm rot\, rot} \boldsymbol{B} = \Phi_{\rm q}\hat{\nu}_{\rm q} \sum \delta(\boldsymbol{r} - \boldsymbol{r}_{\rm q}) \tag{18}$$

for the description of the magnetic structure of the quark condensate. The density of clusters is equal to the density of neutron vortex lines $n_V = 2\,\Omega/\kappa_{\rm n}$, where $\kappa_{\rm n} = \pi\hbar/m_{\rm n}$ is the quantum of neutron circulation.

We note that between the hadronic phase and quark core there is a mixed phase, in which hadrons coexist with a charged lattice of quark droplets (Glendenning 1992). Since the number densities of neutrons and protons in the mixed state are lower than in the hadronic phase, these particles remain superfluid. So neutron vortices and clusters of proton vortex lines continue through the mixed phase. Therefore the magnetic field will pass through it and enter the quark core, see Fig. 2. In the case of small diquark gaps of the order of 1 MeV (Bailin & Love 1984), when the diquark condensate is a superconductor of the first kind, the magnetic field generated in the "npe"-phase penetrates into the quark matter core in the form of ordinary cylindrical regions (Sedrakian et al. 1997). The radii of these regions will be of the order of δ_q since the thermodynamical critical field H_cm is of the same order as the mean magnetic field in the quark cluster. The clusters of quark vortex lines which appear due to the entrainment effect in the "npe"-phase will interact with those which are formed by the initial magnetic field (fossil field). This interaction obviously implies that quark vortex lines will not be expelled from the quark core of the star within a time scale of $\tau = 10^4$ years as suggested in (Bailin & Love 1984).

We note that the evolution of the magnetic field is intimately related to the rotational history of the star. In particular, the magnetic field of the quark core will decay because of the outward motion of neutron vortices when the star spins down. This behavior results from the fact that the magnetic clusters inside the quark core are the continuation of neutron vortices. Therefore the characteristic decay time of the magnetic field for the whole star (and also for quark core) is comparable to the pulsar's slowing down time, which corresponds to the life time of the pulsar.

In conclusion, we find that the occurence of a superconducting quark matter core in pulsars does not contradict the observational data which indicate that magnetic fields of pulsars have life times larger than 10^7 years (Makashima 1992). This holds true for small diquark gaps of the order of 1 MeV (Bailin & Love 1984) as well as for larger ones as obtained recently (Alford et al. 1998; Rapp et al. 1998; Carter & Diakonov 1999; Blaschke & Roberts 1998) using effective models for the nonperturbative quark-quark interaction.

Acknowledgements. K.M.S. and D.M.S. acknowledge the hospitality of the Department of Physics at the University of Rostock where this research has been started. We thank H. Grigorian, K. Rajagopal, G. Röpke, A.D. Sedrakian and D.N. Voskresensky for their discussions and comments.

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