Astron. Astrophys. 350, L57–L61 (1999)

ASTRONOMY AND ASTROPHYSICS

LETTER

Letter to the Editor

Kinematics of LMC stellar populations and self-lensing optical depth

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Received 29 April 1999 / Accepted 25 September 1999

Abstract. Recent observations give some clues that the lenses discovered by the microlensing experiments in the direction of the Magellanic Clouds may be located in these satellite galaxies. We re-examine the possibility that self-lensing alone may account for the optical depth measured towards the Large Magellanic Cloud (LMC). We present a self-consistent multi-component model of the LMC consisting of distinct stellar populations, each associated to a vertical velocity dispersion ranging from 10 to 60 $\mathrm{km\,s^{-1}}$. The present work focuses on showing that such dispersions comply with current 20–30 $\mathrm{km\,s^{-1}}$ limits set by observation on specific LMC populations. We also show that this model reproduces both the 1–2×10^{-7} observed optical depth and the event duration distribution.

Key words: Galaxy: halo – Galaxy: kinematics and dynamics – cosmology: dark matter – cosmology: gravitational lensing – Galaxy: stellar content

Several collaborations (Alcock et al. 1993, Aubourg et al. 1993) are searching for galactic dark matter through the use of gravitational microlensing (Paczyński 1986) towards the Magellanic Clouds. Events have been observed, for which location and mass cannot be determined independently. The current results do not yet yield a coherent explanation: half of the halo of the Milky Way in 0.5 $M_\odot$ objects (Alcock et al. 1997) would require a puzzling star formation history, whereas traditional models of the LMC do not predict a self-lensing optical depth high enough to account for all the observed events (Gould 1995). The only events with additional information all seem to be located in the Clouds themselves (Bennett et al. 1996, Palanque-Delabrouille et al. 1998, Afonso et al. 1999), which makes it worthwhile to re-examine the experimental constraints on the Clouds kinematics and explore more thoroughly models of the LMC. After reviewing the observational constraints on the LMC kinematics (Sect. 1), we show, in Sect. 2, the existence of an age bias: the stars used to derive these constraints are on average both younger and slower than the majority of the LMC objects. We then use a Monte Carlo simulation to show that a maximum velocity dispersion of 60 $\mathrm{km\,s^{-1}}$ reproduces the kinematic observations (Sect. 3) and the microlensing results (Sect. 4).

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1. Present observational constraints

The bulk of the mass of the LMC resides in a nearly face-on disk, with an inclination usually taken to equal the canonical value of $i = 33°$ (Westerlund 1997), although both lower ($27°$) and higher (up to $45°$) values have also been derived from morphological or kinematical studies of the LMC. This disk is observed to rotate with a circular velocity $V_C \sim 80$ $\mathrm{km\,s^{-1}}$ out to at least $8°$ from the LMC center (Schommer et al. 1992). If all the stars belong to the same population, with a vertical (*i.e.* perpendicular to the disk) velocity dispersion σ_W, the microlensing optical depth of such a disk upon its own stars is given by $\tau \sim 2\sigma_W^2 \sec^2 i/c^2$ (Gould 1995). Considering the measured velocity of LMC carbon stars (Cowley & Hartwick 1991), Gould (1995) assumed $\sigma_W = 20\,\mathrm{km\,s^{-1}}$ as a typical velocity dispersion for LMC stars. He thus concluded that $\tau \sim 10^{-8}$, *i.e.* that self-lensing (first suggested by Sahu 1994 and Wu 1994) contributes very little to the observed optical depth towards this line of sight.

Carbon stars however may not be the ultimate probe to infer the velocity dispersion of LMC populations: they actually comprise various ill-defined classes of objects (Menessier 1999), and their prevalence is a complex function of age, metallicity and probably other factors (Gould 1999).

Both observational and theoretical arguments favour the existence of a wide range of velocity dispersions among the various LMC stellar populations. To commence, Meatheringham et al. (1988) have determined the radial velocities of a sample of planetary nebulae (PN) in the LMC. They measured a velocity dispersion of 19.1 $\mathrm{km\,s^{-1}}$, much larger than the value of 5.4 $\mathrm{km\,s^{-1}}$ found for the HI. This was interpreted as being suggestive of orbital heating and diffusion operating in the LMC in the same way as it is observed in the solar neighbourhood. Then, the observations of Hughes et al. (1991) show clear evidence for an increase in the velocity dispersion of long period variables (LPV) as a function of their age. For young LPVs, the velocity dispersion is 12 km s^{-1} whereas for old LPVs, it reaches 35 km s^{-1}. More recently, Zaritsky et al. (1999) found a velocity dispersion of $\sigma = 18.4 \pm 1.4\,\mathrm{km\,s^{-1}}$ for 190 vertical red clump (VRC) stars[1] whereas for the red clump (RC), they measured a value of $\sigma = 32.2 \pm 3.8\,\mathrm{km\,s^{-1}}$ on a sample of 75 objects (throughout this paper, error bars are converted from Zaritsky's 95% confidence levels to standard 1σ). A general trend appears: the velocity dispersion is an increasing function of the age. Just like for our own Milky Way, stars of the LMC disk have been continuously undergoing dynamical scattering by, for instance, molecular clouds or other gravitational inhomogeneities. This results in an increase of the velocity dispersion of a given stellar population with its age, as will be further discussed in Sect. 3. Notice that the main argument in disfavour of a LMC self-lensing explanation is precisely the low value of the measured vertical velocity dispersions. However, the stellar populations so far surveyed predominantly consist of red giants. They are shown in the next section not to be representative of the bulk of the LMC disk stars, and actually biased towards young ages: they are on average ~ 2 Gyr old, to be compared to an LMC age of ~ 12 Gyr.

2. The age bias

The red clump population will illustrate the main thrust of our argument. Clump stars have burning helium cores whose size is approximately independent of the total mass of the object. They also have the same luminosity and hence they spend a fixed amount of time $\tau_{\rm He}$ in the clump, irrespective of their mass m. Such objects are evolved post-MS stars, which does not mean that they are necessarily old. We have assumed a Salpeter Initial Mass Function for the various LMC stellar populations

$$\frac{dN}{dm} \propto m^{-\,(1+\alpha)} \quad , \tag{1}$$

with $\alpha = 1.35$. The stellar formation history has been borrowed from Geha et al. (1998). Their preferred model (e) corresponds to a stellar formation rate $\mathcal{F}(t)$ that has remained constant for 10 Gyr since the formation of the LMC 12 Gyr ago. Then, two Gyr ago, $\mathcal{F}(t)$ has increased by a factor of three. The number of stars that formed at time t and whose mass is comprised between m and $m + dm$ may be expressed as

$$\frac{d^2N}{dm\,dt} = \mathcal{F}(t)\, m^{-\,(1+\alpha)} \quad . \tag{2}$$

We have assumed a mass-luminosity relation $L \propto m^{\beta}$ on the MS so that the stellar lifetime may be expressed as $\tau_{\rm MS}(m) = 12\ {\rm Gyr}/m^{\beta-1}$ (since $\tau \propto m/L$). With these oversimplified but natural assumptions, a star whose initial mass is $\leq 1\,{\rm M}_{\odot}$ is still today on the MS and cannot have reached the clump. Conversely, a heavier star with $m \geq 1\,{\rm M}_{\odot}$ may well be today in a helium core burning stage provided that its formation epoch lies in the range between $t = -\,\tau_{\rm MS}(m)$ (the object has just begun core helium burning) and $t = -\,\tau_{\rm MS}(m) - \tau_{\rm He}(m)$ (the star is about to leave the red clump). The number of RC stars observed today with progenitor mass in the range between m and $m + dm$ is therefore given by

$$dN_{\rm RC} = \mathcal{F}(-\tau_{\rm MS}(m)) \times m^{-\,(1+\alpha)}\, dm \times \tau_{\rm He} \quad . \tag{3}$$

To get more insight into the age bias at stake, we can parameterize the progenitor mass m in terms of the age $\tau \equiv \tau_{\rm MS}(m)$. The previous relation simplifies into

$$\frac{dN_{\rm RC}}{d\tau} = \frac{\mathcal{F}(-\tau)\,\tau_{\rm He}}{(\beta-1)}\,\tau^{(\gamma-1)} \quad , \tag{4}$$

where $\gamma = \alpha/(\beta-1)$. This may be directly compared to the age distribution of the bulk of the LMC stars that goes like $\mathcal{F}(-\tau)$. With a Salpeter mass function and $\beta = 4.5$, we get a value of $\gamma = 0.4$. The excess of young RC stars goes as $1/\tau^{0.6}$ and the bias is obvious. Other IMF are possible and a spectral index as large as $\alpha \sim \beta - 1 \sim 3.5$ would be required to invalidate the effect. HST data analyzed by Holtzman et al. (1997) nevertheless point towards a spectral index α that extends from 0.6 up to 2.1 for stars in the mass range $0.6 \leq m \leq 3\ {\rm M}_{\odot}$. The average value corresponds actually to a Salpeter law.

There has been furthermore a recent burst in the LMC stellar formation rate. In order to model it, we may express the total number of today's RC stars as an integral where the progenitor mass m runs from $m_1 = 1\,{\rm M}_{\odot}$ up to the tip of the IMF whose actual value is irrelevant and has been set equal to infinity here for simplicity. Notice that the specific progenitor mass $m_2 \simeq 1.7\,{\rm M}_{\odot}$ corresponds to stars born 2 Gyr ago, when the stellar formation rate increased by a factor of 3. Stars which formed before that epoch will be referred to as old. Their number is given by

$$N_{\rm RC}^{\rm old} = \int_{m_1}^{m_2} \mathcal{F}\left(-\tau_{\rm MS}\right)\, m^{-\,(1+\alpha)}\, dm\, \tau_{\rm He} \quad . \tag{5}$$

On the other hand, the number $N_{\rm RC}^{\rm young}$ of young clump stars is obtained similarly, with masses in excess of m_2. We readily infer a fraction of young stars

$$N_{\rm RC}^{\rm young}/N_{\rm RC} = \frac{3}{2 + (m_2/m_1)^{\alpha}} \simeq 0.751 \quad . \tag{6}$$

Three quarters of the clump stars observed today in the LMC have thus formed less than 2 Gyr ago, during the recent period of stellar formation mentioned above. Integrating $\tau_{\rm MS}$ over the RC population

$$\langle\tau\rangle = \frac{1}{N_{\rm RC}} \int_{m_1}^{\infty} \tau_{\rm MS}\, dN_{\rm RC} \quad , \tag{7}$$

yields the average age

$$\langle\tau\rangle = (12\ {\rm Gyr}) \times \frac{\alpha}{\alpha+\beta-1} \times \frac{m_1^{1-\alpha-\beta} + 2\,m_2^{1-\alpha-\beta}}{m_1^{-\alpha} + 2\,m_2^{-\alpha}} \ . \tag{8}$$

This gives a numerical value of ~ 1.95 Gyr. We thus conclude that today's clump stars are, on average, much younger than the LMC disk.

[1] see Zaritsky et al. (1999) and Beaulieu and Sackett (1998) for a definition of RC and VRC stars.

3. Distributions of velocity dispersions

This simple analytical result has been checked by means of a Monte Carlo study. We have randomly generated a sample of 10^8 LMC stars. The progenitor mass was drawn in the range $0.1 \leq m \leq 10\ \mathrm{M}_\odot$ according to a Salpeter law. The age of formation was drawn in the range -12 Gyr $\leq t \leq 0$ according to the stellar formation history $\mathcal{F}(t)$ favoured by Geha et al. (1998). The vertical velocity dispersion σ_W was then evolved in time from formation up to now according to Wielen's (1977) relation:

$$\sigma_W^2 \; = \; \sigma_0^2 + C_W \, t. \tag{9}$$

This purely diffusive relation is known to be inadequate to describe velocity dispersions in our Galaxy (Edvardsson et al. 1993). We will however use it in our model, as heating processes in the LMC may be different than those in the galaxy. The LMC is indeed subject to tidal heating by the Milky Way (Weinberg 1999) and has most probably suffered encounters with the SMC. Although this simple relation lacks a theoretical motivation, it will be shown to account for several features of the velocity distributions in the LMC, without being at variance with any observation. The initial velocity dispersion σ_0 was taken to be 10 km s^{-1}, and the diffusion coefficient in velocity space along the vertical direction C_W to be 300 km^2 s^{-2} Gy^{-1} so that our oldest stars have a vertical velocity dispersion reaching up to $\sigma_W^{\mathrm{MAX}} = 60$ km s^{-1}. For each star, the actual vertical velocity was then randomly drawn, assuming a Gaussian distribution with width σ_W.

In order to compare our Monte Carlo results with the Zaritsky et al. (1999) measurements of the radial velocities of LMC clump stars, we selected two groups of stars according to their position in the HR diagram. Following Zaritsky et al., we use their colour index

$$C \; \equiv \; 0.565 \, (B - I) \; + \; 0.825 \, (U - V + 1.15) \; , \tag{10}$$

so that the RC population is defined by $3.1 < C < 3.4$ with a magnitude $19 < V < 19.3$ whereas the VRC stars have the same colour index C and brighter magnitudes $18 < V < 18.75$. In order to infer the colours and magnitudes of the stars that we generated, we used the isochrones computed by Bertelli et al. (1994) for a typical LMC metallicity and helium abundance of $Z = 0.008$ and $Y = 0.25$.

A random sample of 190 stars that passed the VRC selection criteria is presented in Fig. 1 where the vertical velocities are displayed. This histogram may be compared to Fig. 10 of Zaritsky et al. (1999) where no VRC star is found with a velocity in excess of 60 km s^{-1}. With the full statistics, our Monte Carlo generated a population of $\sim$ 2,900 VRC objects whose vertical velocity distribution has a RMS of ~ 18 km s^{-1}. The agreement between the Zaritsky et al. observations and our Monte Carlo results is noteworthy. The average age of our VRC sample is $\sim$ 0.87 Gyr.

We also selected a random sample of 75 RC stars whose velocity distribution is featured in Fig. 2. Even with a diffusion coefficient as large as $C_W = 300$ km^2 s^{-2} Gy^{-1} so as to comply with a large LMC self-lensing optical depth, our full statistics of 18,000 RC objects has a velocity dispersion of ~ 23 km s^{-1}. This is slightly below the value of $\sigma \; = \; 32.2 \pm 3.8$ km s^{-1} quoted by Zaritsky et al. Observations are nevertheless fairly scarce with only 75 RC stars. When Zaristsky et al. fitted a Gaussian to the RC radial velocity distribution featured in the Fig. 11 of their paper, they obtained a 95% C.L. dispersion of $\sigma = 32^{+19}_{-16}$ km s^{-1} with a large uncertainty. Our Monte Carlo velocity dispersion of 23 km s^{-1} is definitely compatible with that result. We infer an average age for the RC population of $\sim$ 1.8 Gyr to be compared to our analytical result of $\sim$ 1.95 Gyr. This agrees well with Beaulieu and Sackett's conclusion that isochrones younger than 2.5 Gyr are necessary to fit the red clump. Notice finally that our age estimates for these various clump populations are in no way related to LMC kinematics. They merely result from the postulated Salpeter IMF, the Geha et al. preferred stellar formation history and the Bertelli et al. isochrones.

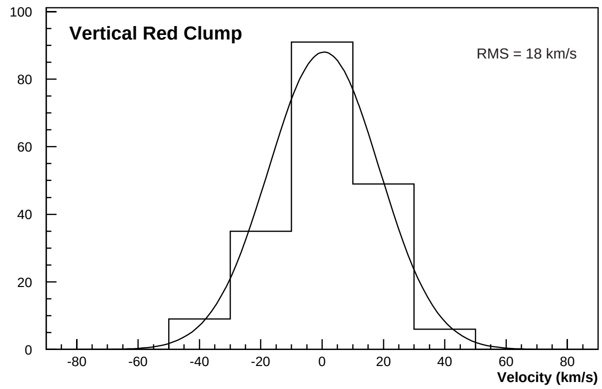

Fig. 1. Velocity distribution for a sample of 190 vertical red clump stars that have been generated by the Monte Carlo discussed in the text. That histogram is similar to Fig. 10 of Zaritsky et al. (1999). A velocity dispersion of 18 km s^{-1} is found for the full sample (solid smooth curve).

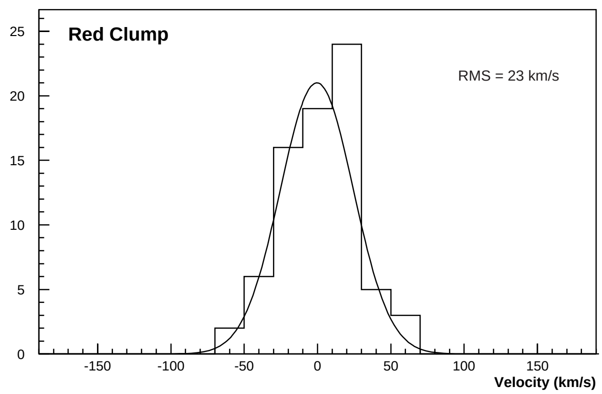

Fig. 2. Like in the previous figure, a distribution of 75 red clump stars is now featured. We inferred a velocity dispersion of 23 km s^{-1} for the full sample (solid smooth curve). Our distribution is similar to that presented in Fig. 11 of Zaritsky et al. (1999). No star exhibits a velocity larger than 70 km s^{-1}.

With this model, 70% in mass of the LMC disk consists of objects whose vertical velocity dispersion is in excess of $25\,\mathrm{km\,s^{-1}}$, although the average vertical velocity dispersion of RC stars, for instance, is only $\sim 23\,\mathrm{km\,s^{-1}}$.

What about the other measurements? The velocity dispersion of PNs has been found equal to $19.1\,\mathrm{km\,s^{-1}}$ (Meatheringham et al. 1988). These authors estimate that the bulk of the PNs have an age near 3.5 Gyr. They also note that younger objects are present down to an age of order 0.5–1.3 Gyr. Meatheringham et al. come finally to the conclusion that the indicative age of the PN population is 2.1 Gyr. This value agrees well once again with our analytical estimate. Our Monte Carlo gives a slightly larger value of 2.4 Gyr for the age of the PNs, with a velocity dispersion of $24.7\,\mathrm{km\,s^{-1}}$. Because the observed sample contains 94 objects, the measured value of $19.1\,\mathrm{km\,s^{-1}}$ suffers presumably from significant uncertainties.

Quite interesting also are the measurements by Hughes et al. (1991) of the velocity dispersions of LPVs as a function of their age. Their sample of 63 "old" LPVs has a velocity dispersion of $\sigma = 35^{+10}_{-4}\,\mathrm{km\,s^{-1}}$. For the bulk of the LMC populations, we obtain an average velocity dispersion of $\sim 37\,\mathrm{km\,s^{-1}}$. The problem at stake is actually the age of those old LPVs. These stars indeed display an age-period relation. However, Hughes et al. derived this relation from kinematics considerations, using precisely Eq. 9, and postulating the same diffusion coefficient as in the Milky Way. They thus inferred an average age of 9.5 Gyr. Finding instead the position of these stars in a colour-magnitude diagram and using LMC isochrones would have led to a clean determination of the age-period relation. A direct determination of the age of LPVs is nevertheless spoilt by a few biases. Some LPVs are carbon stars and the ejected material around them may considerably dim their luminosities. These stars may also pulsate on an harmonic of the fundamental mode. Both effects lead to an under-determination of their luminosity and hence to an overestimate of their age (Menessier 1999). As a matter of fact, Groenewegen and de Jong (1994) conclude that LMC stars whose progenitor mass is less than 1.15 $\mathrm{M}_\odot$ never reach the instability strip on the AGB. This yields an upper limit on the age of LPVs of ~ 7.3 Gyr, in clear contradiction with the average age of 9.5 Gyr inferred by Hughes et al. for old LPVs.

Finally, Schommer et al. (1992) have obtained a velocity dispersion of 21–24 $\mathrm{km\,s^{-1}}$ for 9 old LMC clusters. Their large 1σ error of $\sim 10\,\mathrm{km\,s^{-1}}$ is due to the small size of the sample. It is not clear whether or not these clusters have formed in the disk. If they nevertheless had, they would have undergone a fairly restricted orbital heating with respect to the LMC stars. Those systems and the giant molecular clouds have actually comparable masses and the energy exchange between them does not result in a significant increase of the velocity dispersion of the clusters unlike what happens to the stars.

4. Multi-component model of the LMC

We model the LMC to contain several stellar populations, each associated with a different velocity dispersion $\sigma_{W,i}$ which has evolved according to Eq. 9.

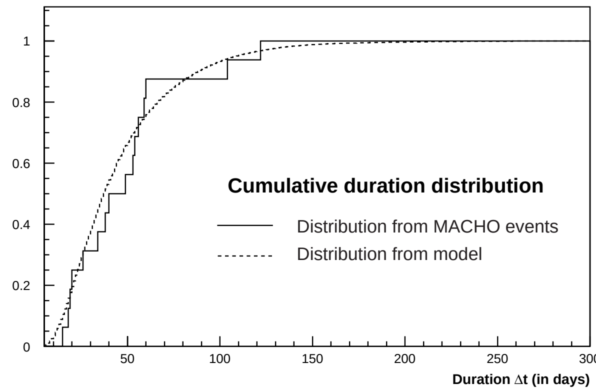

Fig. 3. Predicted distribution of event durations $d\Gamma/d\Delta t$, superimposed with the MACHO experimental distribution. The events are those presented by MACHO at the IVth Microlensing Workshop (Cook 1998), corrected for blending and efficiency using the formulae in Alcock et al. 1997.

We describe each of the ten components of our model by an ellipsoidal density profile

$$\rho_i(R,z) = \frac{\Lambda_i}{R^2 \; + \; z^2/(1-e_i^2)} \quad , \tag{11}$$

up to a cut-off radius $R_{\mathrm{MAX}} = 15\,\mathrm{kpc}$ (Aubourg et al. 1999). The multi-component model based on these profiles is self-consistent in the sense that it satisfies Poisson equation and results in a flat rotation curve with the desired V_C of $80\,\mathrm{km\,s^{-1}}$. We define the set of $\sigma_{W,i}$ so as to sample linearly the range between $\sigma_0 = 10\,\mathrm{km\,s^{-1}}$ and $\sigma_W^{\mathrm{MAX}} = 60\,\mathrm{km\,s^{-1}}$ (see previous section). The parameters Λ_i and the ellipticities e_i are determined so that the model reproduces the set of velocity dispersions $\sigma_{W,i}$ and surface mass densities Σ_i where $d\Sigma_i/d\sigma_i \propto \sigma_i \mathcal{F}(t)$ with $\mathcal{F}(t)$ the stellar formation history of the LMC mentioned in Sect. 2. Assuming a typical M/L of 3, which is a free parameter in our model, we reproduce the observed surface brightness of the LMC.

For a given distribution of objects, one can compute the total self-lensing optical depth τ and the event rate Γ. Both quantities are integrated on all deflectors and sources, considering that only main sequence stars brighter than $V = 20$ and red giants can be potential sources, since they are the only objects bright enough to be visible in microlensing surveys. The computation of Γ requires an estimate of the relative transverse velocity of deflector and source, for which we have assumed an horizontal velocity dispersion equal to the vertical one predicted by the model. Details of this computation can be found in (Aubourg et al. 1999).

For the model described above, one obtains $\tau = 9.3 \times 10^{-8}$ and $\Gamma = 3.5 \times 10^{-7}\,\mathrm{yr^{-1}}$. This can be compared to the EROS and MACHO optical depths, respectively 8.2×10^{-8} (Ansari et al. 1996) and $2.9^{+1.4}_{-0.9} \times 10^{-7}$ (Alcock et al. 1997). A combination of those two results yields an average optical depth of $2.1^{+1.3}_{-0.8} \times 10^{-7}$ (Bennett 1998), but preliminary MACHO results from their five-year analysis (Sutherland 1999) hint to a reduced optical depth as compared to their two-year analysis. The model prediction is thus in good agreement with the results obtained so far from microlensing experiments.

Another relevant prediction of the model is the distribution of event durations, $d\Gamma/d\Delta t$. Fig. 3 illustrates this prediction for our model, along with the distribution of observed MACHO events.

Our model thus reproduces both the total observed optical depth towards the LMC and the observed event duration distribution, while complying with the velocity dispersion measurements. A self-lensing interpretation of *all* the microlensing events observed so far towards the LMC thus appears to be a plausible explanation.

Acknowledgements. We wish to thank M.O. Menessier for useful discussions, and the members of the EROS collaboration for their comments. We thank Andy Gould, our referee, for his useful remarks and suggestions.

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