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The temperature and density structure in the closed field regions of the solar corona

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Abstract. In this paper we study the temperature and density structure in the closed field region of the solar corona using a dipole plus current sheet model to simulate the global solar magnetic field and a heating function of the same type used in models of the fast wind. The heat equation, describing the redistributing effects of heat conduction on the heat input in the presence of radiative losses, is solved simultaneously with hydrostatic pressure balance. At the base we prescribe the temperature and assume that the heat flux is zero there. We also insist that the heat flux is zero at the equator. This ensures that whatever heat has been added is radiated away. From the mathematical viewpoint this additional requirement sets up an eigenvalue problem which implies that the density at the base must be chosen in just the right way to fulfill the condition of zero heat flux at the equator. Thus our model not only provides the temperature and density structure in the closed regions of a global solar magnetic field appropriate to solar minimum but also predicts the latitudinal variation of the base density whose characteristic value is determined by the ratio of the amplitudes of the heating to the cooling. However it should be stressed that this last prediction represents, at best, an approximation to the real stale of affairs which is more complex and involves the connection of the coronal field lines to the magnetic funnels of the chromospheric network.

Key words: Sun: atmosphere – Sun: corona – Sun: magnetic fields

1. Introduction

The temperature and density structure of stationary coronal loop configurations has been studied extensively (Rosner et al. (1978), Serio et al. (1981), Vesecky et al. (1979), Priest (1984), Priest et al. (1998)). The models are based on heat conduction redistributing energy input in the presence of radiative cooling in an atmosphere stratified under gravity. Usually the downward heat conduction at the footpoint is small so that apart from assigning the temperature there its gradient is ta ken to be zero at that point. (But see below for a more detailed discussion). In a symmetrical configuration we impose the condition that the heat flux vanishes at the top of the loop, thus ensuring that whatever heat has been added is radiated away. This additional constraint sets up an "eigenvalue" problem in which the density at the base must be chosen so as to satisfy this condition. It is possible to derive scaling laws relating the maximum temperature at the top of the loop to the pressure at the base, the length of the loop, the pressure scale length and the heating length (Serio et al. (1981)).

The main purpose is to study the temperature and density structure in the closed regions of a *global* solar magnetic field, given a heating function of the type used in models of the fast solar wind (see, e.g. McKenzie et al. (1997)). The motivation here is that arguments can be made (Axford et al. (1999)) for advocating that the same heating process (e.g. dissipation of high frequency Alfven waves generated by microflares in the network) is involved in both open and closed lines. The energy flux required to power the radiation emitted by the quiet corona in closed field regions is about the same as that needed to produce the fast solar wind in open field lines ($\sim 6 \cdot 10^5$ ergs/cm^2 sec). At deeper levels, where the temperature is less then $2{\cdot}10^5$ $^\circ K$ there is no great difference in the morphology between the supergranules and network in closed and open field regions. This suggests that the same heating process is involved in both cases, namely network flaring and the emission of high frequency ($\geq$ 1 Hz) waves. In open field regions the heating drives the fast solar wind whereas in the closed regions it determines, along with cooling and the redistributing effects of heat conduction, the temperature and density structure in the closed field regions of the *global* solar magnetic field. We find that the temperature structure is controlled by a (fairly large) dimensionless number which measures the ratio of the amplitude of the heating function to a characteristic value of the divergence of heat flux at the base. Moreover the density at the base is determined by the ratio of the heating to the cooling at the base and varies with latitude on the solar surface being most dense on the last closed field line.

2. Formulation of coronal heating in closed field regions

For simplicity we assume the solar magnetic field is approximated by a dipole plus a current sheet, the strengths of which

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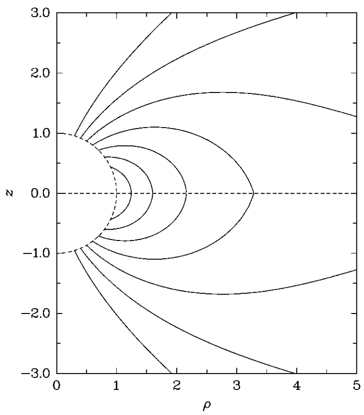

Fig. 1. The magnetic field configuration for a dipole plus current sheet model of the solar magnetic field.

are chosen to match the observed radial component of the field at 1 AU and to ensure that the last open field emanates from latitude 60° on the solar surface (see, e.g., McKenzie et al. (1995), Banaszkiewicz et al. (1998)). In cylindrical coordinates (ρ, z) the magnetic field is given by

$$\frac{B_\rho}{M} = \frac{3\rho z}{r^5}\frac{(4z^2-3\rho^2)}{r^2} + \frac{K}{a_1}\frac{\rho}{[(|z|+a_1)^2+\rho^2]^{3/2}}, \quad (1a)$$

$$\frac{B_z}{M} = \frac{2z^2-\rho^2}{r^5}\frac{(8z^4+3\rho^4-24\rho^2z^2)}{r^9} + \frac{K}{a_1}\frac{|z|+a_1}{[(|z|+a_1)^2+\rho^2]^{3/2}}, \quad (1b)$$

Here we have used M = 7.031, K = 1, a_1 = 3.979, which fulfills the foregoing requrements. The field line topology is shown in Fig. 1, in which distances are measured in solar radii. This configuration is current free except at the equator where there is a current sheet. This of course is merely a convenient approximation which simulates the MHD effect of the plasma in stretching the magnetic field lines near the equator, but nevertheless provides us with a simple model of a global magnetic field. The equation of a field line is

$$\frac{d\rho}{dz} = \frac{B_\rho}{B_z} \quad (1c)$$

so that the element of length ds along a field line is

$$ds = d\rho\sqrt{1+\left(\frac{B_z}{B_\rho}\right)^2} \quad (1d)$$

In the closed field region the plasma is stationary and stratified under gravity on each field line according to hydrostatic pressure balance,

$$\frac{dp}{ds} = -\frac{GM\rho}{r^2}\cos\psi, \quad (2a)$$

where ψ is the angle between the radial direction and field line direction. Noting that $dr/ds = \cos\psi$ this may be written

$$\frac{dp}{dr} = -\frac{GM\rho}{r^2}, r = \sqrt{\rho^2+z^2}, \quad (2b)$$

The energy or heat equation is

$$-B\frac{d}{ds}\left(\frac{K_\parallel}{B}\frac{dT}{ds}\right) = Q - L. \quad (3)$$

Here L represents radiative losses and takes the form (see. e.g. [Priest (1984)])

$$L = \Lambda(T)n^2. \quad (4)$$

Between $T \sim 5\cdot 10^5\,K$ and $3\cdot 10^6\,K$ $\Lambda(T)$ is approximately constant given by

$$\Lambda(T) \approx 10^{-22} \quad ergs \quad sec^{-1}\,cm^3. \quad (5)$$

The conductivity along the field line $K_\parallel$ is given by $K_\parallel = K_0 T^{5/2}$, $g\,\mathrm{cm\,s^{-3}K^{-1}}$, $K_0 \sim 10^{-6}$. We assume the plasma is being heated (e.g. by damping of ion–cyclotron waves) and that Q takes the form

$$Q = Q_0 \exp -(r-r_0)/l_h \quad ergs/cm^5\,sec, \quad (6)$$

where $l_h \approx 0.35r_0$. Because there is only a weak latitudinal variation of the fast solar wind streaming energy, McKenzie et al. (1997) in their model of the fast solar wind chose the amplitude of the heating function Q_0 to vary with latitude θ_0 on the solar surface to ensure the near constancy of the streaming flux. We have adopted a similar viewpoint here and for the present purpose we take $Q_0 = q_0 B_r(\theta_0)/B_r(60°)$ and with $q_0 \approx 5\cdot 10^{-6}$, this yields the amount of heat required to power the fast solar wind namely $6\cdot 10^5$ ergs $\mathrm{cm^{-2}\,sec^{-1}}$. It is convenient to normalize the temperature T to its coronal base value T_0 ($\sim 10^6$ K°), the density to n_0 ($\sim 3\cdot 10^8\,\mathrm{cm^{-3}}$), and distances to the solar radius r_0. The hydrostatic pressure balance may be integrated to yield the density stratification

$$n = \frac{1}{T}\exp -\int_1^r \frac{dr}{l_n}, \quad (7a)$$

where

$$\frac{1}{l_n} = \frac{1}{2}\frac{V_{esc}^2}{V_{th}^2}\frac{1}{Tr^2} \quad (7b)$$

$$V_{th}^2 = \frac{kT_0}{m_p}, \quad \frac{1}{2}V_{esc}^2 = \frac{GM}{r_0} \quad (7c)$$

The heat equation may be cast in the form

$$\frac{2}{7}\left(\frac{d^2T^{7/2}}{ds^2} + \frac{1}{l_b}\frac{dT^{7/2}}{ds}\right) = N((1+\Delta)n^2 - g(\theta_0)e^{-(r-1)/l_h}) \quad (8a)$$

where

$$\frac{1}{l_b} \equiv -\frac{1}{B}\frac{dB}{ds}, \quad (8b)$$

$$N \equiv \frac{r_0^2 q_0}{\kappa_0 T_0^{7/2}}, \quad (8c)$$

$$n_0 = (Q_0/\Lambda(T))^{1/2}\sqrt{1+\Delta}, \quad (8d)$$

$$g(\theta_0) = \frac{B_r(\theta_0)}{B_r(60°)}. \tag{8e}$$

Eqs. (7a) and (8a) form a coupled set of nonlinear equations for the density and the temperature which must be solved subject to certain boundary conditions on each field line. At the base of each field line the conditions could be either that the temperature is fixed but then the heat flux will in general not be zero there and in this sense this base would not be the "end" of the loop, or we could insist that $dT/ds = 0$ implying that no heat flux penetrates further down and that the chosen base is indeed the "end of the loop", with the object as a whole being thought of as a radiator with its extremity being where no heat remains to be radiated away. In this latter case we cannot specify the temperature at this place in advance. Here the fact that we specify both the temperature ($T = 1$) and zero heat flux $dT/ds = 0$ at the base requires some explanation. In the first place the choice of $T = 1$ ($10^6\ °K$) is merely a rough observational constraint. For what we have in mind the choice of $dT/ds = 0$ at the base is an approximation which simplifies a rather more complex problem, namely how the "base" of the field line maps back into the network through magnetic funnels (McKenzie et al. (1997)) and the processes by which a hydrostatic equilibrium is achieved. The funnels themselves are a source of mass and energy so that the mass flux has to be controlled by pressure distribution in the funnel and recombination in the cooler and denser regions.

There is indeed a downward heat flux into these funnels but bearing in mind that the scale height and thermal length scales are large (larger than the depth of the funnel at the foot of which ionization of neutrals takes place), then for the present purpose of deriving the global pattern of density and temperature in the closed regions of the corona it is a reasonably good approximation to take $dT/ds = 0$ at the base. We also insist that $dT/ds = 0$ at the equator, thus ensuring that whatever heat has been added from the base along the field line is radiated away with thermal conduction acting so as to redistribute the energy along each field. It turns out that in order to satisfy this "over–determined" problem the density at the base must vary on each field line in which the parameter Δ determines the latitudial variation of the base density. Although this prediction of the density variation is a deduction from the model it is only an approximation to the real state of affairs which, as we have mentioned above, is much more complex.

In order to indicate how the density parameter Δ is chosen let us briefly consider the case of "short" field lines near the equator, where the temperature will not depart significantly from its value at the base. Put $T = 1 + \Theta$, $\Theta \ll 1$, linearize Eq. 8(a) and seek a power series solution of the form

$$\Theta = \sum_{n=2}^{\infty} a_n x^n \tag{9}$$

where x is a new unit of length along a field line

$$dx = \sqrt{N} ds. \tag{10}$$

It can be shown that the coefficients a_n (up to a_4) are given by

$$\begin{aligned} a_2 &= \frac{\delta}{2} \quad (\equiv 1 + \Delta - g(\theta_0)) \\ a_3 &= -\frac{1}{6}\left(\frac{2(1+\Delta)}{L_n} - \frac{g(\theta_0}{2L_h}\right) - \frac{\delta}{6L_B} \\ a_4 &= \frac{1}{12}\left(\frac{2(1+\Delta)}{L_n}\left(1 + \frac{1}{L_n}\right) - \frac{g(\theta_0)}{2L_H^2}\right) \\ &- \frac{(1+\Delta)\delta}{12} - \frac{a_3}{4L_B}, \end{aligned} \tag{11}$$

etc.

If we insist that in the middle of the field line, $x = x_m$ say, the heat flux is zero, $d\Theta/dx = 0$, we obtain the condition

$$x_m(2a_2 + 3a_3x_m + 4a_4x_m^2 + ...) = 0 \tag{12}$$

For a given x_m this determines the value of Δ (an eigenvalue) which ensures zero heat flux at the equator. The zeroth approximation yields

$$x_m = -\frac{2a_2}{3a_3} \tag{13a}$$

which gives the following for Δ

$$\Delta = \frac{s_m\left(\frac{1}{l_{n_0}} - \frac{1}{2l_h}\right) + (1 - g(\theta_0))\left(1 - \frac{s_m}{2l_b}\right)}{1 - s_m\left(\frac{1}{l_{n_0}} + \frac{1}{2l_b}\right)}, \tag{13b}$$

$$\frac{1}{l_{n_0}} = \frac{1}{2}\frac{V_{esc}^2}{V_{th}^2}. \tag{13c}$$

Since the various length scales have the following ordering $l_{n_0} < 2l_h < l_b$, Eq. (13b) shows that Δ (and hence the density $n_h\sqrt{1+\Delta}$) increases with increasing field line length s_m or increasing latitude at the base. This arises because initially the density scale length ($\sim$ 1/8) is less than the heating length ($\sim$ 1/3) so that at the base the cooling dominates over heating but is quickly overtaken by the latter allowing heat conduction to redistribute the energy in just the right way to ensure zero heat flux at the base and the equator.

3. Temperature and density structure in closed regions of dipole–current sheet magnetic configuration

In the solar context the dimensionless number N appearing in the normalized heat Eq. (8a) is quite large, being between 5 and 50 (for base densities of between 10^8 and $3 \cdot 10^8\ \mathrm{cm}^{-3}$), consequently it is absorbed into a new field line length scale as in (10). In a recent model of the fast solar wind (McKenzie et al. (1997)) it was found that a heating length of about $1/3$ of a solar radius reproduced the properties of the fast wind, and, since our contention is that the same heating mechanism should operate in both open and closed field regions, we have used $l_h = 0.35$. The initial density scale height $l_n \approx 1/8$ and therefore is smaller than l_h, as a result of which the heating begins to dominate over the cooling very close to the Sun. However as we have seen in the foregoing section a delicate balance between

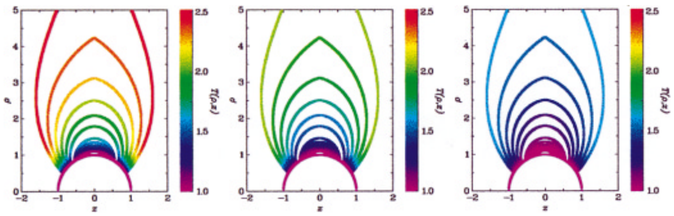

Fig. 2. A colour coded plot of the temperature structure in closed field region for three values of N in which N = 25 for the left, N = 16 for the middle and N = 9 for the right panel.

the heating and cooling must be struck for each closed field line. The longer the field line is the greater the value of Δ (see Eq. 13(b)) that must be chosen in order to ensure there is no heat flux in the middle of the field line. In this way the temperature rises more sharply and attains a broader maximum on longer field lines than on shorter ones so that this effect combined with gravity means that the density scale height increases thereby bringing the cooling ($\propto n^2$) into play in just the right amount to ensure that dT/ds = 0 in the middle of the field line (i. e. at the magnetic equator). Fig. 2 provides a color coded plot of the temperature variation on closed field lines for various values of N. Note that the highest temperatures occur on and near the last closed field line attaining 2.5 million degrees for $N = 25$, 2 million for $N = 16$ and 1.5 million for $N = 9$. Figs. 3a,b illustrate representive profiles of the temperature and density as a function of distance s along the field line. The longer the field line the more sharply the temperature rises to a broad maximum.

The ratio of the base density to $(Q_0/\Lambda)^{1/2}$, namely $(1+\Delta)^{1/2}$, is plotted as a function of latitude θ_0 in Fig. 4, for different values of N. That the density goes to zero at the equator is an artefact of our model heating function (Eq. 8e). The upper curve in Fig. 4 illustrates the base density as a function of latitude for the case in which the heating is independent of latitude.

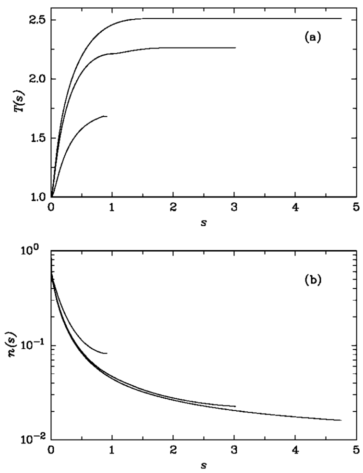

Fig. 3. a The temperature profiles for three characteristic field lines in a dipole plus current sheet model. **b** The corresponding density profiles.

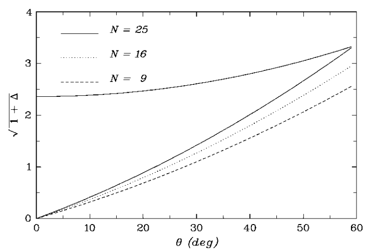

Fig. 4. The variation of the coronal base density (normalized to $(Q_0/\Lambda)^{1/2}$) with latitude for three values of N. The fourth (upper) curve is for the case of heating independent of latitude.

4. Summary and conclusions

In this treatment of the problem of quiet coronal heating we have assumed that, in conditions appropriate to sunspot minimum where active regions are essentially absent and the polar coronal holes are fully developed and long-lasting, the same process heats the corona in magnetically closed regions as drives the high speed solar wind in magnetically open regions. Of course in more active periods well away from sunspot minimum there is additional heating from coronal as distinct from network flares and this can easily be seen in images made in the EUV and soft x-rays. The ‘ground state’ of the corona is in our view one in which only network activity contributes energy which is converted either into coronal heating with its accompanying radiative losses in closed regions and into the high speed solar wind in open regions.

We presume that the heating process for the quiet closed corona is the same as in the high speed solar wind emanating from coronal holes, namely cyclotron absorption of relatively high frequency magnetohydrodynamic waves which are released from small-scale network flares. This is quite different and distinct from the ideas put forward by Gold (1964) and Parker (1988) concerning coronal ‘nanoflares’ which can be regarded as being ‘intra-network” flares rather than flares occurring in the same network element. There are of course similarities in that the energy input comes from the motion of magnetic field footpoints in both cases but for coronal flares this is connected mainly with the slow twisting and shuffling of the footpoints whereas for network flares it is the transport of small closed loops into the network from the sides as a result of chromospheric motions associated with the supergranulation.

It is obvious that field line twisting cannot be the main source of the energy since twists correspond to very low frequency Alfven waves which, in coronal holes would propagate directly away into space. Since the solar wind is accelerated close to the sun and the low frequency waves observed in interplanetary space have only a few percent of the solar wind energy flux and do not seem to have been damped significantly it is difficult to see how such waves could both produce the wind and heat the quiet corona. High frequency waves are need to produce the ground-state (i.e. fast) solar wind and we are therefore forced to consider the same waves for the source of ground-state coronal heating with cyclotron resonant damping being the ultimate dissipation process in both cases.

This study shows that it is possible to obtain a reasonable picture of the temperature structure of the quiet corona in closed field regions with the same heating function used in a fast wind model (McKenzie et al. (1997)). The structure is controlled by the dimensionless number N and maximum temperatures of 2.5 million degrees near last closed field line can be achived. The base density is determined by the ratio of the heating to the cooling with a characteristic value of $\sim 3 \cdot 10^8\,\mathrm{cm}^3$ and increases with latitude.

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